Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.5-Secant/122-4.5.2.3-g-sec-^p-a+b-sec-^m-c+d-sec-^n

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September 27, 2022 Compiled on September 27, 2022 at 6:54pm

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [286]. This is test number [122].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (286)	0.00 (0)
Mathematica	95.45 (273)	4.55 (13)
Maple	92.66 (265)	7.34 (21)
Fricas	82.87 (237)	17.13 (49)
Giac	78.32 (224)	21.68 (62)
Mupad	66.78 (191)	33.22 (95)
Maxima	58.04 (166)	41.96 (120)
Sympy	0.35 (1)	99.65 (285)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

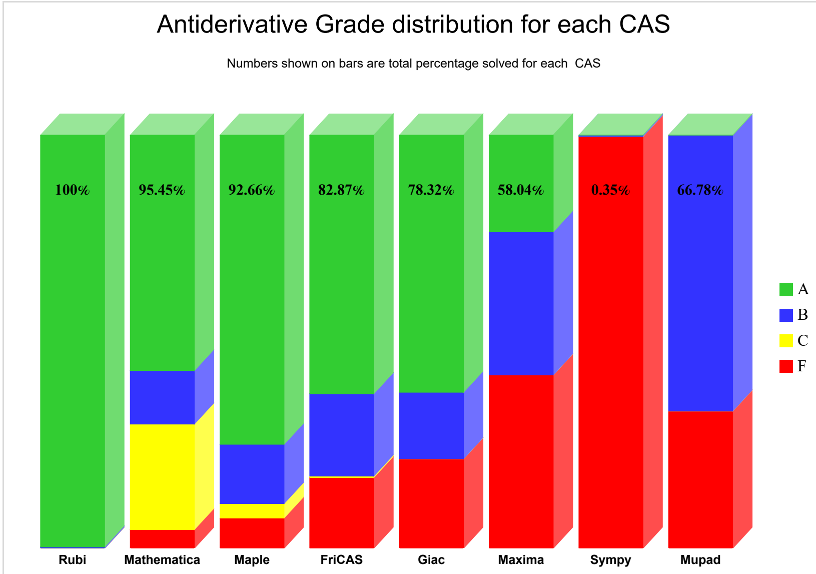
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

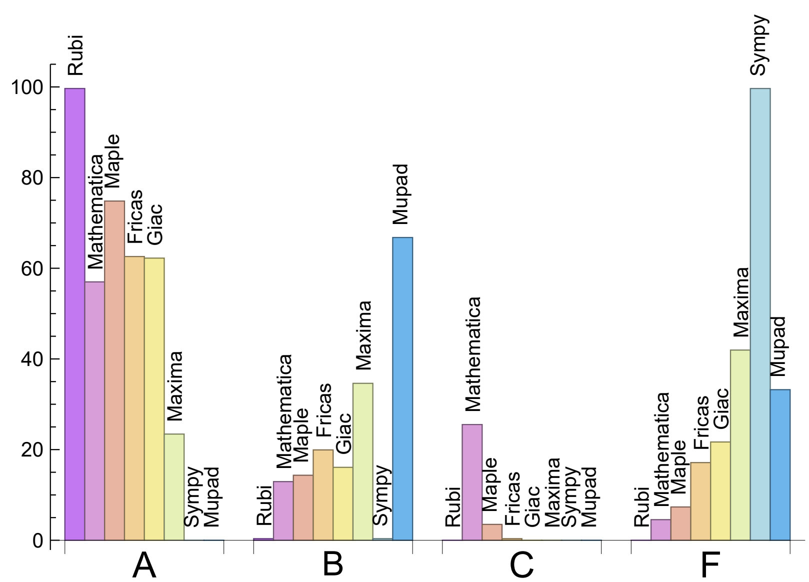
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.65	0.35	0.00	0.00
Maple	74.83	14.34	3.50	7.34
Fricas	62.59	19.93	0.35	17.13
Giac	62.24	16.08	0.00	21.68
Mathematica	56.99	12.94	25.52	4.55
Maxima	23.43	34.62	0.00	41.96
Mupad	N/A	66.78	0.00	33.22
Sympy	0.00	0.35	0.00	99.65

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	13	100.00 %	0.00 %	0.00 %
Maple	21	100.00 %	0.00 %	0.00 %
Fricas	49	69.39 %	30.61 %	0.00 %
Giac	62	77.42 %	0.00 %	22.58 %
Maxima	120	60.83 %	4.17 %	35.00 %
Sympy	285	83.86 %	6.32 %	9.82 %
Mupad	95	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

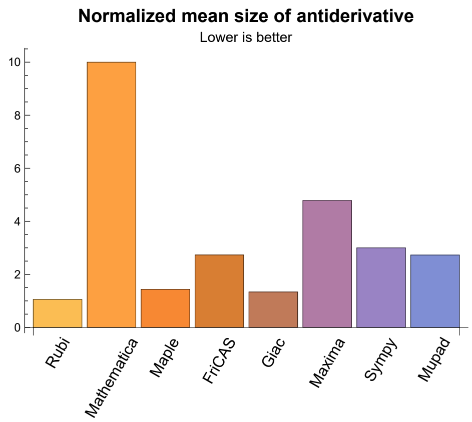
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

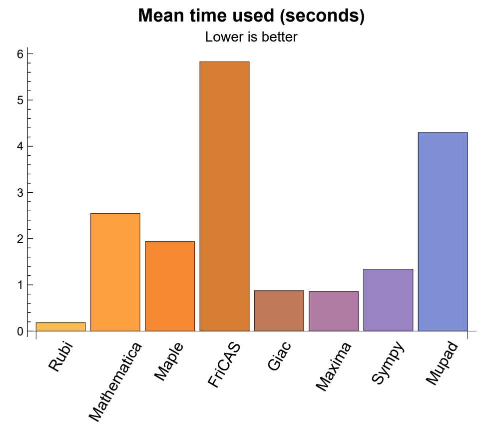
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.18	135.98	1.05	121.00	1.00
Mathematica	2.55	1402.67	10.00	154.00	1.39
Maple	1.94	183.55	1.43	140.00	1.29
Maxima	0.85	488.81	4.78	236.50	2.39
Fricas	5.83	371.68	2.73	189.00	1.84
Sympy	1.34	51.00	3.00	51.00	3.00
Giac	0.87	182.99	1.34	120.50	1.17
Mupad	4.29	365.34	2.73	158.00	1.46

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{174, 176, 205, 206, 207, 216, 224, 232, 233, 239, 240, 265, 269, 275, 277\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

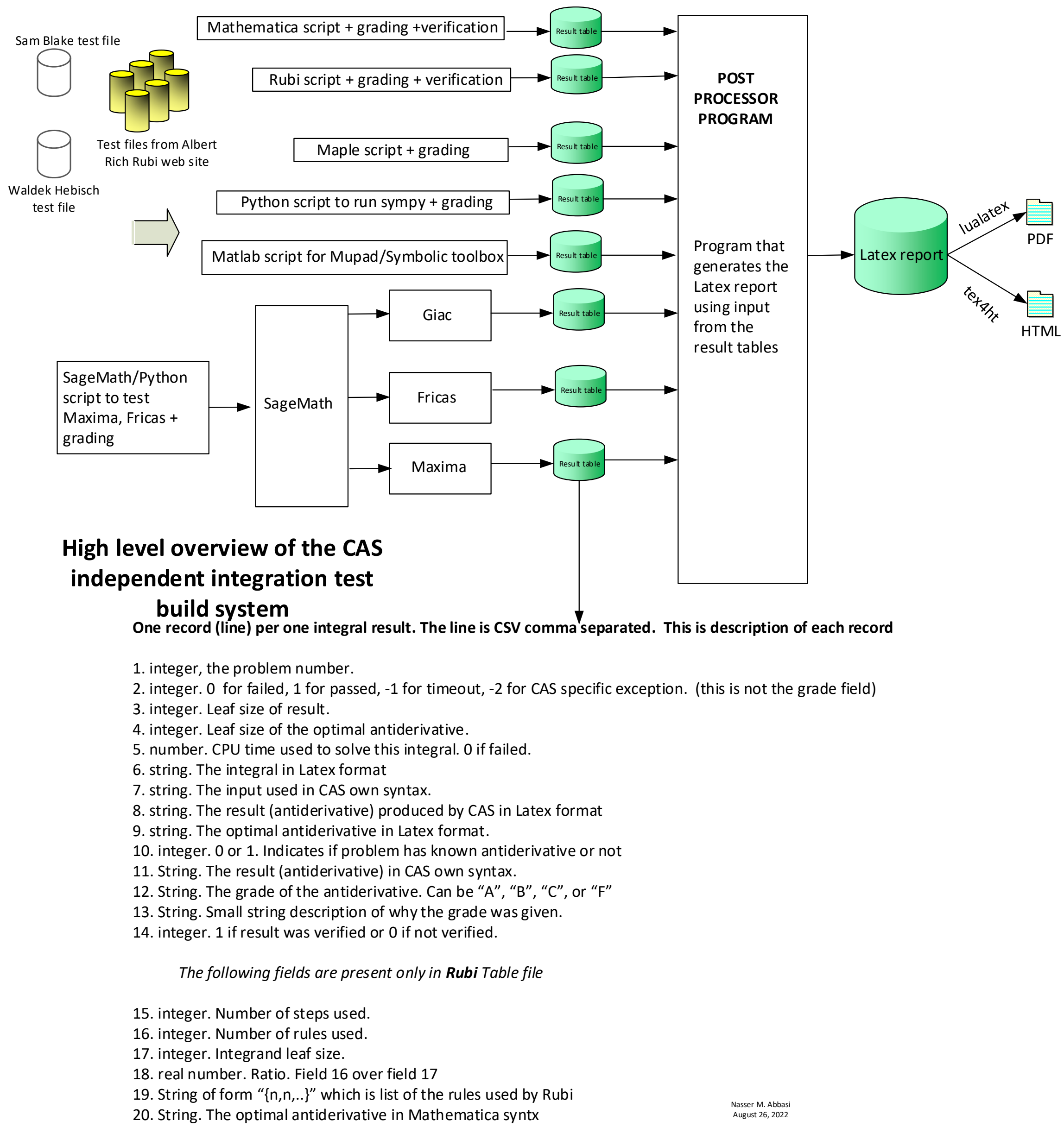
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

B grade: { 197 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 108, 109, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 130, 131, 132, 141, 146, 147, 152, 153, 168, 169, 170, 173, 175, 178, 179, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 200, 201, 202, 203, 209, 217, 218, 221, 226, 227, 229, 230, 234, 235, 236, 237, 238, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 259, 260, 261, 262, 263, 264, 266, 267, 268, 271, 275, 276, 279, 281, 282, 283, 286 }

B grade: { 1, 2, 3, 15, 27, 28, 34, 35, 36, 42, 43, 44, 54, 61, 62, 107, 126, 171, 172, 180, 181, 195, 196, 204, 210, 211, 212, 213, 219, 220, 225, 228, 252, 253, 258, 273, 285 }

C grade: { 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 110, 117, 118, 127, 128, 129, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 148, 149, 150, 158, 162, 163, 164, 165, 174, 176, 183, 197, 198, 199, 205, 206, 207, 208, 214, 215, 216, 222, 223, 224, 231, 232, 233, 239, 240, 265, 269, 270, 272, 277, 280, 284 }

F grade: { 151, 154, 155, 156, 157, 159, 160, 161, 166, 167, 177, 274, 278 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 136, 137, 138, 139, 141, 142, 143, 144, 147, 148, 149, 150, 168, 169, 171, 172, 173, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 269, 273, 275, 276, 279, 282, 285, 286 }

B grade: { 12, 23, 25, 69, 70, 77, 78, 91, 92, 98, 99, 106, 107, 110, 118, 128, 129, 130, 135, 140, 145, 146, 170, 178, 179, 180, 183, 184, 202, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 252, 281 }

C grade: { 267, 270, 271, 272, 274, 277, 278, 280, 283, 284 }

F grade: { 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 174, 175, 176, 177 }

2.1.4 Maxima

A grade: { 2, 3, 4, 7, 8, 9, 14, 26, 38, 39, 40, 41, 47, 48, 49, 50, 51, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 93, 94, 95, 100, 101, 109, 110, 116, 118, 126, 129, 135, 136, 140, 141, 145, 147, 156, 157, 162, 163, 164, 168, 173, 183, 185, 186, 187, 188, 196, 203, 221, 229, 230, 244, 245, 246, 247, 285, 286 }

B grade: { 1, 5, 6, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 89, 96, 102, 103, 107, 108, 111, 112, 113, 114, 115, 117, 119, 120, 121, 122, 123, 124, 125, 127, 128, 130, 131, 132, 133, 134, 137, 138, 139, 142, 143, 146, 148, 150, 158, 169, 170, 171, 172, 178, 180, 181, 182, 193, 194, 195, 202, 204, 210, 211, 212, 213, 217, 218, 219, 220, 225, 226, 227, 228 }

C grade: { }

F grade: { 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 144, 149, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 179, 184, 189, 190, 191, 192, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 214, 215, 216, 222, 223, 224, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 113, 114, 115, 120, 121, 123, 124, 125, 132, 136, 138, 143, 144, 148, 149, 150, 156, 157, 158, 162, 163, 164, 168, 169, 171, 172, 173, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 202, 203, 204, 205, 210, 211, 213, 214, 217, 218, 220, 221, 225, 226, 227, 228, 229, 230, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 262, 286 }

B grade: { 17, 30, 56, 69, 74, 82, 107, 108, 111, 112, 116, 119, 122, 126, 130, 131, 137, 141, 142, 146, 147, 170, 183, 184, 190, 191, 192, 198, 199, 200, 201, 206, 207, 208, 209, 212, 215, 216, 219, 222, 223, 224, 231, 232, 233, 234, 238, 250, 251, 252, 253, 254, 257, 260, 261, 263, 285 }

C grade: { 277 }

F grade: { 110, 117, 118, 127, 128, 129, 133, 134, 135, 139, 140, 145, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 258, 259, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 278, 279, 280, 281, 282, 283, 284 }

2.1.6 Sympy

A grade: { }

B grade: { 170 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

2.1.7 Giac

A grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 111, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 168, 169, 170, 171, 172, 173, 179, 190, 195, 196, 199, 203, 204, 208, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 225, 226, 227, 229, 230, 248, 249, 255, 257, 261, 262, 263, 285, 286 }

B grade: { 3, 14, 107, 108, 112, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 197, 198, 200, 201, 202, 205, 206, 207, 209, 217, 222, 223, 224, 228, 231, 232, 233, 238, 244, 245, 246, 247, 250, 251, 252, 253, 254, 256, 258, 259, 260 }

C grade: { }

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2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 107, 108, 109, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 141, 146, 147, 157, 162, 163, 164, 168, 169, 170, 171, 172, 173, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 285, 286 }

C grade: { }

F grade: { 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 110, 117, 118, 127, 128, 129, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	499	182	233	141	0	145	176
N.S.	1	1.00	4.75	1.73	2.22	1.34	0.00	1.38	1.68
time (sec)	N/A	0.140	1.706	0.270	0.273	3.048	0.000	0.573	6.644

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	887	111	144	126	0	128	146
N.S.	1	1.00	10.31	1.29	1.67	1.47	0.00	1.49	1.70
time (sec)	N/A	0.110	6.491	0.212	0.265	2.843	0.000	0.557	5.197

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	313	98	117	111	0	111	114
N.S.	1	1.00	5.13	1.61	1.92	1.82	0.00	1.82	1.87
time (sec)	N/A	0.070	0.735	0.188	0.265	3.328	0.000	0.577	3.810

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	58	74	73	0	55	77
N.S.	1	1.00	1.00	1.53	1.95	1.92	0.00	1.45	2.03
time (sec)	N/A	0.032	0.028	0.139	0.275	2.900	0.000	0.525	2.238

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	77	50	109	72	0	60	31
N.S.	1	1.00	1.83	1.19	2.60	1.71	0.00	1.43	0.74
time (sec)	N/A	0.037	0.084	0.130	0.271	2.500	0.000	0.569	1.846

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	50	21	105	55	0	20	20
N.S.	1	1.00	1.39	0.58	2.92	1.53	0.00	0.56	0.56
time (sec)	N/A	0.033	0.251	0.138	0.283	2.385	0.000	0.470	2.060

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	87	37	127	84	0	37	35
N.S.	1	1.00	1.14	0.49	1.67	1.11	0.00	0.49	0.46
time (sec)	N/A	0.070	0.345	0.154	0.287	3.273	0.000	0.490	1.710

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	50	193	112	0	51	61
N.S.	1	1.00	0.97	0.43	1.66	0.97	0.00	0.44	0.53
time (sec)	N/A	0.109	0.407	0.167	0.279	4.040	0.000	0.707	1.746

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	139	63	215	138	0	65	106
N.S.	1	1.00	0.88	0.40	1.36	0.87	0.00	0.41	0.67
time (sec)	N/A	0.149	0.366	0.193	0.275	3.370	0.000	0.791	1.843

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	102	299	398	189	0	197	251
N.S.	1	1.00	0.60	1.75	2.33	1.11	0.00	1.15	1.47
time (sec)	N/A	0.207	1.875	0.352	0.283	2.547	0.000	0.824	5.760

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	91	255	347	172	0	178	219
N.S.	1	1.00	0.61	1.70	2.31	1.15	0.00	1.19	1.46
time (sec)	N/A	0.176	1.403	0.307	0.276	2.866	0.000	0.611	5.608

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	82	192	245	155	0	159	187
N.S.	1	1.00	0.87	2.04	2.61	1.65	0.00	1.69	1.99
time (sec)	N/A	0.110	0.766	0.270	0.302	2.373	0.000	0.741	6.503

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	117	162	106	0	87	155
N.S.	1	1.00	0.70	1.60	2.22	1.45	0.00	1.19	2.12
time (sec)	N/A	0.078	0.155	0.193	0.274	3.763	0.000	0.570	5.213

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	96	117	111	0	111	113
N.S.	1	1.00	0.74	1.57	1.92	1.82	0.00	1.82	1.85
time (sec)	N/A	0.073	0.119	0.192	0.263	2.496	0.000	0.552	3.786

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	220	82	243	118	0	100	77
N.S.	1	1.00	2.97	1.11	3.28	1.59	0.00	1.35	1.04
time (sec)	N/A	0.071	1.885	0.155	0.279	2.164	0.000	0.661	1.907

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	109	67	217	138	0	84	63
N.S.	1	1.00	1.22	0.75	2.44	1.55	0.00	0.94	0.71
time (sec)	N/A	0.092	0.088	0.161	0.277	2.611	0.000	0.637	1.779

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	205	89	0	22	22
N.S.	1	1.00	0.66	0.61	5.39	2.34	0.00	0.58	0.58
time (sec)	N/A	0.055	0.125	0.178	0.284	2.496	0.000	0.540	1.643

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	115	39	294	122	0	41	37
N.S.	1	1.00	1.44	0.49	3.68	1.52	0.00	0.51	0.46
time (sec)	N/A	0.109	0.450	0.183	0.310	1.742	0.000	0.582	1.621

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	141	52	293	150	0	57	67
N.S.	1	1.00	1.17	0.43	2.42	1.24	0.00	0.47	0.55
time (sec)	N/A	0.166	0.435	0.214	0.307	3.232	0.000	0.604	1.631

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	167	65	425	180	0	73	108
N.S.	1	1.00	1.02	0.40	2.61	1.10	0.00	0.45	0.66
time (sec)	N/A	0.224	0.660	0.221	0.311	2.582	0.000	0.647	1.691

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	122	345	480	223	0	235	316
N.S.	1	1.00	0.54	1.52	2.11	0.98	0.00	1.04	1.39
time (sec)	N/A	0.255	3.262	0.350	0.282	2.480	0.000	0.634	5.579

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	111	322	442	206	0	216	284
N.S.	1	1.00	0.54	1.56	2.15	1.00	0.00	1.05	1.38
time (sec)	N/A	0.224	2.295	0.389	0.296	2.563	0.000	0.664	5.518

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	302	398	189	0	197	252
N.S.	1	1.00	0.84	2.50	3.29	1.56	0.00	1.63	2.08
time (sec)	N/A	0.128	1.667	0.364	0.295	3.046	0.000	0.605	5.707

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	60	180	264	123	0	103	220
N.S.	1	1.00	0.60	1.80	2.64	1.23	0.00	1.03	2.20
time (sec)	N/A	0.099	0.251	0.243	0.280	3.063	0.000	0.606	5.560

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	81	192	245	155	0	159	188
N.S.	1	1.00	0.86	2.04	2.61	1.65	0.00	1.69	2.00
time (sec)	N/A	0.113	0.831	0.271	0.287	3.071	0.000	0.601	6.265

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	111	144	126	0	128	146
N.S.	1	1.00	0.81	1.29	1.67	1.47	0.00	1.49	1.70
time (sec)	N/A	0.113	0.611	0.210	0.296	3.699	0.000	0.490	4.792

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	287	112	419	136	0	118	105
N.S.	1	1.00	2.87	1.12	4.19	1.36	0.00	1.18	1.05
time (sec)	N/A	0.094	2.644	0.174	0.280	2.629	0.000	0.552	3.172

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	402	97	377	179	0	116	93
N.S.	1	1.00	3.38	0.82	3.17	1.50	0.00	0.97	0.78
time (sec)	N/A	0.128	3.546	0.167	0.279	3.241	0.000	0.545	2.043

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	139	78	335	190	0	102	78
N.S.	1	1.00	1.05	0.59	2.54	1.44	0.00	0.77	0.59
time (sec)	N/A	0.154	0.125	0.206	0.307	2.338	0.000	0.517	1.778

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	388	119	0	22	22
N.S.	1	1.00	0.66	0.61	10.21	3.13	0.00	0.58	0.58
time (sec)	N/A	0.052	0.159	0.196	0.320	2.409	0.000	0.654	1.811

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	141	39	389	150	0	41	37
N.S.	1	1.00	1.76	0.49	4.86	1.88	0.00	0.51	0.46
time (sec)	N/A	0.109	0.407	0.225	0.300	2.202	0.000	0.624	1.621

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	167	52	566	180	0	57	67
N.S.	1	1.00	1.38	0.43	4.68	1.49	0.00	0.47	0.55
time (sec)	N/A	0.164	0.705	0.201	0.303	2.614	0.000	0.821	1.859

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	193	65	565	208	0	73	108
N.S.	1	1.00	1.19	0.40	3.49	1.28	0.00	0.45	0.67
time (sec)	N/A	0.224	0.606	0.234	0.321	4.696	0.000	0.692	1.716

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	1036	140	641	165	0	132	112
N.S.	1	1.00	8.56	1.16	5.30	1.36	0.00	1.09	0.93
time (sec)	N/A	0.120	6.454	0.187	0.295	3.270	0.000	0.556	1.854

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	287	110	418	151	0	116	96
N.S.	1	1.00	2.87	1.10	4.18	1.51	0.00	1.16	0.96
time (sec)	N/A	0.097	2.628	0.175	0.286	2.716	0.000	0.562	1.660

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	220	80	242	129	0	97	77
N.S.	1	1.00	2.97	1.08	3.27	1.74	0.00	1.31	1.04
time (sec)	N/A	0.073	1.672	0.139	0.291	2.820	0.000	0.562	1.636

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	77	48	109	76	0	58	31
N.S.	1	1.00	1.88	1.17	2.66	1.85	0.00	1.41	0.76
time (sec)	N/A	0.039	0.070	0.149	0.271	3.176	0.000	0.642	1.577

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	19	19	19	0	18	18
N.S.	1	1.00	1.00	1.19	1.19	1.19	0.00	1.12	1.12
time (sec)	N/A	0.064	0.030	0.123	0.270	3.248	0.000	0.546	1.556

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	48	83	54	0	56	50
N.S.	1	1.00	1.37	0.81	1.41	0.92	0.00	0.95	0.85
time (sec)	N/A	0.106	0.500	0.142	0.281	1.797	0.000	0.810	1.642

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	107	61	105	80	0	69	63
N.S.	1	1.00	1.37	0.78	1.35	1.03	0.00	0.88	0.81
time (sec)	N/A	0.136	0.845	0.165	0.270	3.277	0.000	0.723	1.722

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	145	74	127	110	0	82	83
N.S.	1	1.00	1.21	0.62	1.06	0.92	0.00	0.68	0.69
time (sec)	N/A	0.172	0.918	0.164	0.295	3.026	0.000	0.578	2.131

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	380	155	829	226	0	156	170
N.S.	1	1.00	2.32	0.95	5.05	1.38	0.00	0.95	1.04
time (sec)	N/A	0.183	1.204	0.228	0.293	3.983	0.000	0.666	1.734

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	349	125	575	212	0	140	136
N.S.	1	1.00	2.33	0.83	3.83	1.41	0.00	0.93	0.91
time (sec)	N/A	0.152	2.004	0.198	0.292	3.481	0.000	0.687	1.681

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	485	95	369	192	0	121	104
N.S.	1	1.00	4.08	0.80	3.10	1.61	0.00	1.02	0.87
time (sec)	N/A	0.134	4.378	0.175	0.281	3.364	0.000	0.610	1.649

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	109	65	212	148	0	89	46
N.S.	1	1.00	1.24	0.74	2.41	1.68	0.00	1.01	0.52
time (sec)	N/A	0.093	0.093	0.171	0.283	2.713	0.000	0.560	1.612

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	23	21	102	57	0	20	20
N.S.	1	1.00	0.64	0.58	2.83	1.58	0.00	0.56	0.56
time (sec)	N/A	0.035	0.090	0.155	0.275	3.412	0.000	0.598	1.567

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	83	48	82	53	0	69	61
N.S.	1	1.00	1.41	0.81	1.39	0.90	0.00	1.17	1.03
time (sec)	N/A	0.103	0.594	0.145	0.279	2.824	0.000	0.534	1.618

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	66	33	53	0	31	28
N.S.	1	1.00	0.87	1.74	0.87	1.39	0.00	0.82	0.74
time (sec)	N/A	0.074	0.052	0.207	0.281	2.394	0.000	0.540	1.565

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	147	76	131	117	0	96	76
N.S.	1	1.00	1.84	0.95	1.64	1.46	0.00	1.20	0.95
time (sec)	N/A	0.110	0.987	0.170	0.297	2.846	0.000	0.498	2.009

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	179	87	152	129	0	109	89
N.S.	1	1.00	1.83	0.89	1.55	1.32	0.00	1.11	0.91
time (sec)	N/A	0.146	0.935	0.193	0.308	2.052	0.000	0.513	2.668

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	211	102	175	175	0	122	102
N.S.	1	1.00	1.50	0.72	1.24	1.24	0.00	0.87	0.72
time (sec)	N/A	0.185	1.253	0.191	0.291	2.114	0.000	0.566	4.237

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	406	168	1015	283	0	176	193
N.S.	1	1.00	1.89	0.78	4.72	1.32	0.00	0.82	0.90
time (sec)	N/A	0.247	2.124	0.212	0.309	2.336	0.000	0.647	1.653

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	380	136	738	269	0	159	159
N.S.	1	1.00	1.97	0.70	3.82	1.39	0.00	0.82	0.82
time (sec)	N/A	0.215	1.307	0.223	0.301	2.227	0.000	0.659	1.642

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	826	108	510	249	0	141	126
N.S.	1	1.00	5.04	0.66	3.11	1.52	0.00	0.86	0.77
time (sec)	N/A	0.188	6.335	0.200	0.289	2.389	0.000	0.605	1.629

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	139	76	330	206	0	109	61
N.S.	1	1.00	1.06	0.58	2.52	1.57	0.00	0.83	0.47
time (sec)	N/A	0.151	0.124	0.195	0.283	3.267	0.000	0.585	1.646

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	201	88	0	22	22
N.S.	1	1.00	0.66	0.61	5.29	2.32	0.00	0.58	0.58
time (sec)	N/A	0.051	0.108	0.167	0.284	2.391	0.000	0.647	1.593

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	87	37	125	85	0	37	35
N.S.	1	1.00	1.14	0.49	1.64	1.12	0.00	0.49	0.46
time (sec)	N/A	0.070	0.347	0.180	0.291	2.570	0.000	0.566	1.576

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	109	61	103	82	0	87	74
N.S.	1	1.00	1.40	0.78	1.32	1.05	0.00	1.12	0.95
time (sec)	N/A	0.140	0.798	0.147	0.276	2.688	0.000	0.528	1.624

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	147	76	130	117	0	103	111
N.S.	1	1.00	1.84	0.95	1.62	1.46	0.00	1.29	1.39
time (sec)	N/A	0.109	0.795	0.166	0.290	4.183	0.000	0.640	1.708

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	95	44	81	0	41	38
N.S.	1	1.00	0.85	1.61	0.75	1.37	0.00	0.69	0.64
time (sec)	N/A	0.083	0.069	0.220	0.316	4.666	0.000	0.577	1.628

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	211	102	173	175	0	128	129
N.S.	1	1.00	2.13	1.03	1.75	1.77	0.00	1.29	1.30
time (sec)	N/A	0.113	1.256	0.191	0.284	3.690	0.000	0.664	2.536

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	257	115	197	204	0	142	109
N.S.	1	1.00	2.14	0.96	1.64	1.70	0.00	1.18	0.91
time (sec)	N/A	0.161	1.503	0.192	0.321	3.671	0.000	0.736	2.882

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	289	128	218	233	0	155	120
N.S.	1	1.00	1.78	0.79	1.35	1.44	0.00	0.96	0.74
time (sec)	N/A	0.194	2.112	0.191	0.278	4.143	0.000	0.683	3.107

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	86	83	0	128	0	107	483
N.S.	1	1.00	0.53	0.51	0.00	0.79	0.00	0.66	2.96
time (sec)	N/A	0.195	0.852	2.042	0.000	3.437	0.000	1.029	9.131

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	76	73	0	113	0	83	384
N.S.	1	1.00	0.62	0.60	0.00	0.93	0.00	0.68	3.15
time (sec)	N/A	0.142	0.470	2.322	0.000	3.766	0.000	1.078	6.004

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	63	0	89	0	56	120
N.S.	1	1.00	0.79	0.78	0.00	1.10	0.00	0.69	1.48
time (sec)	N/A	0.094	0.264	2.413	0.000	2.887	0.000	0.878	5.364

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	51	53	0	71	0	31	87
N.S.	1	1.00	1.31	1.36	0.00	1.82	0.00	0.79	2.23
time (sec)	N/A	0.043	0.181	2.308	0.000	3.310	0.000	0.802	2.749

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	132	85	0	294	0	61	-1
N.S.	1	1.00	1.71	1.10	0.00	3.82	0.00	0.79	-0.01
time (sec)	N/A	0.078	0.636	2.181	0.000	3.536	0.000	1.134	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	246	164	0	370	0	72	-1
N.S.	1	1.00	3.24	2.16	0.00	4.87	0.00	0.95	-0.01
time (sec)	N/A	0.080	1.122	2.254	0.000	3.567	0.000	1.107	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	309	308	0	439	0	104	-1
N.S.	1	1.00	2.73	2.73	0.00	3.88	0.00	0.92	-0.01
time (sec)	N/A	0.112	1.131	1.964	0.000	4.371	0.000	1.232	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	88	85	0	157	0	109	606
N.S.	1	1.00	0.51	0.50	0.00	0.92	0.00	0.64	3.54
time (sec)	N/A	0.310	1.662	2.467	0.000	4.051	0.000	1.129	14.397

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	78	75	0	140	0	85	503
N.S.	1	1.00	0.61	0.59	0.00	1.09	0.00	0.66	3.93
time (sec)	N/A	0.229	1.265	2.193	0.000	4.129	0.000	1.006	7.990

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	0	113	0	58	384
N.S.	1	1.00	0.78	0.76	0.00	1.33	0.00	0.68	4.52
time (sec)	N/A	0.145	0.780	2.319	0.000	3.877	0.000	1.027	6.139

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	0	91	0	33	93
N.S.	1	1.00	1.34	1.34	0.00	2.22	0.00	0.80	2.27
time (sec)	N/A	0.067	0.460	2.197	0.000	2.429	0.000	0.955	5.725

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	123	173	145	0	371	0	82	-1
N.S.	1	1.05	1.48	1.24	0.00	3.17	0.00	0.70	-0.01
time (sec)	N/A	0.159	0.920	2.238	0.000	3.205	0.000	1.170	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	124	184	144	0	400	0	109	-1
N.S.	1	1.10	1.63	1.27	0.00	3.54	0.00	0.96	-0.01
time (sec)	N/A	0.161	1.453	2.401	0.000	2.865	0.000	2.569	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	130	359	230	0	463	0	106	-1
N.S.	1	1.11	3.07	1.97	0.00	3.96	0.00	0.91	-0.01
time (sec)	N/A	0.159	2.486	2.427	0.000	3.238	0.000	1.391	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	398	402	0	557	0	139	-1
N.S.	1	1.00	2.43	2.45	0.00	3.40	0.00	0.85	-0.01
time (sec)	N/A	0.196	5.745	2.481	0.000	3.640	0.000	1.257	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	88	85	0	174	0	109	710
N.S.	1	1.00	0.51	0.50	0.00	1.02	0.00	0.64	4.15
time (sec)	N/A	0.303	2.554	2.397	0.000	2.683	0.000	1.261	14.671

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	78	75	0	157	0	85	607
N.S.	1	1.00	0.61	0.59	0.00	1.23	0.00	0.66	4.74
time (sec)	N/A	0.219	1.563	2.458	0.000	2.342	0.000	1.182	13.711

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	66	65	0	128	0	58	471
N.S.	1	1.00	0.78	0.76	0.00	1.51	0.00	0.68	5.54
time (sec)	N/A	0.141	1.044	2.299	0.000	3.230	0.000	0.983	9.194

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	0	105	0	33	375
N.S.	1	1.00	1.34	1.34	0.00	2.56	0.00	0.80	9.15
time (sec)	N/A	0.065	0.738	2.308	0.000	2.993	0.000	1.004	5.616

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	185	206	0	407	0	111	-1
N.S.	1	1.00	1.13	1.26	0.00	2.48	0.00	0.68	-0.01
time (sec)	N/A	0.228	1.548	2.326	0.000	4.108	0.000	2.097	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	324	157	0	466	0	124	-1
N.S.	1	1.00	1.93	0.93	0.00	2.77	0.00	0.74	-0.01
time (sec)	N/A	0.232	2.940	2.369	0.000	3.673	0.000	1.176	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	263	206	0	475	0	133	-1
N.S.	1	1.00	1.51	1.18	0.00	2.73	0.00	0.76	-0.01
time (sec)	N/A	0.244	4.590	2.520	0.000	3.433	0.000	1.183	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	86	83	175	95	0	108	164
N.S.	1	1.00	0.61	0.58	1.23	0.67	0.00	0.76	1.15
time (sec)	N/A	0.160	0.771	2.323	0.506	2.735	0.000	0.960	6.331

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	74	73	147	83	0	84	125
N.S.	1	1.00	0.69	0.68	1.36	0.77	0.00	0.78	1.16
time (sec)	N/A	0.132	0.430	2.046	0.533	2.672	0.000	0.891	4.229

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	54	63	118	54	0	60	77
N.S.	1	1.00	0.75	0.88	1.64	0.75	0.00	0.83	1.07
time (sec)	N/A	0.105	0.245	2.462	0.518	3.719	0.000	0.813	2.367

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	43	90	49	0	59	40
N.S.	1	1.00	0.74	1.10	2.31	1.26	0.00	1.51	1.03
time (sec)	N/A	0.072	0.142	2.549	0.525	3.388	0.000	0.772	1.770

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	155	107	0	291	0	64	-1
N.S.	1	1.00	1.74	1.20	0.00	3.27	0.00	0.72	-0.01
time (sec)	N/A	0.113	0.501	2.290	0.000	3.776	0.000	0.585	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	183	266	0	357	0	97	-1
N.S.	1	1.00	1.50	2.18	0.00	2.93	0.00	0.80	-0.01
time (sec)	N/A	0.148	1.407	2.231	0.000	4.091	0.000	0.627	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	306	471	0	435	0	128	-1
N.S.	1	1.00	1.96	3.02	0.00	2.79	0.00	0.82	-0.01
time (sec)	N/A	0.180	3.009	2.264	0.000	3.205	0.000	0.680	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	84	85	202	111	0	121	188
N.S.	1	1.00	0.54	0.55	1.30	0.72	0.00	0.78	1.21
time (sec)	N/A	0.223	0.932	2.213	0.506	2.812	0.000	1.039	6.017

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	68	75	175	88	0	99	136
N.S.	1	1.00	0.55	0.61	1.42	0.72	0.00	0.80	1.11
time (sec)	N/A	0.193	0.416	2.210	0.508	2.741	0.000	0.935	5.542

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	60	53	118	78	0	60	134
N.S.	1	1.00	0.67	0.60	1.33	0.88	0.00	0.67	1.51
time (sec)	N/A	0.153	0.259	2.461	0.509	4.022	0.000	0.805	5.233

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	53	117	65	0	62	94
N.S.	1	1.00	1.34	1.29	2.85	1.59	0.00	1.51	2.29
time (sec)	N/A	0.070	0.155	2.493	0.529	3.114	0.000	0.763	5.295

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	259	131	0	359	0	93	-1
N.S.	1	1.00	1.88	0.95	0.00	2.60	0.00	0.67	-0.01
time (sec)	N/A	0.196	2.159	2.295	0.000	3.377	0.000	0.702	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	365	320	0	399	0	129	-1
N.S.	1	1.00	2.16	1.89	0.00	2.36	0.00	0.76	-0.01
time (sec)	N/A	0.243	1.392	2.460	0.000	4.569	0.000	0.677	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	434	551	0	527	0	160	-1
N.S.	1	1.00	2.14	2.71	0.00	2.60	0.00	0.79	-0.00
time (sec)	N/A	0.271	2.412	2.420	0.000	3.958	0.000	0.692	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	78	85	230	117	0	126	492
N.S.	1	1.00	0.46	0.50	1.36	0.69	0.00	0.75	2.91
time (sec)	N/A	0.276	0.623	2.656	0.525	1.976	0.000	1.106	10.238

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	74	65	203	112	0	90	456
N.S.	1	1.00	0.55	0.48	1.50	0.83	0.00	0.67	3.38
time (sec)	N/A	0.240	0.351	2.544	0.503	2.313	0.000	0.982	7.312

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	60	63	175	95	0	58	446
N.S.	1	1.00	0.68	0.72	1.99	1.08	0.00	0.66	5.07
time (sec)	N/A	0.156	0.298	2.422	0.499	3.287	0.000	0.888	7.719

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	55	146	80	0	62	441
N.S.	1	1.00	1.34	1.34	3.56	1.95	0.00	1.51	10.76
time (sec)	N/A	0.070	0.158	2.516	0.525	2.657	0.000	1.272	7.590

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	225	155	0	435	0	119	-1
N.S.	1	1.00	1.24	0.86	0.00	2.40	0.00	0.66	-0.01
time (sec)	N/A	0.283	1.606	2.484	0.000	3.151	0.000	0.668	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	398	370	0	523	0	154	-1
N.S.	1	1.00	1.88	1.75	0.00	2.47	0.00	0.73	-0.00
time (sec)	N/A	0.332	6.438	2.603	0.000	4.311	0.000	0.765	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	468	631	0	499	0	184	-1
N.S.	1	1.00	1.90	2.57	0.00	2.03	0.00	0.75	-0.00
time (sec)	N/A	0.360	6.653	2.874	0.000	2.004	0.000	0.938	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	87	82	688	101	0	104	136
N.S.	1	1.00	2.02	1.91	16.00	2.35	0.00	2.42	3.16
time (sec)	N/A	0.088	0.469	2.522	0.558	2.685	0.000	1.729	3.800

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	73	72	324	85	0	83	78
N.S.	1	1.00	1.70	1.67	7.53	1.98	0.00	1.93	1.81
time (sec)	N/A	0.086	0.309	2.672	0.561	2.759	0.000	1.568	2.637

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	56	62	59	61	0	54	47
N.S.	1	1.00	1.37	1.51	1.44	1.49	0.00	1.32	1.15
time (sec)	N/A	0.079	0.177	2.582	0.507	2.100	0.000	1.397	1.942

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	99	141	98	0	0	0	-1
N.S.	1	1.00	1.94	2.76	1.92	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	0.851	2.757	0.512	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	62	60	556	86	0	58	118
N.S.	1	1.00	1.48	1.43	13.24	2.05	0.00	1.38	2.81
time (sec)	N/A	0.097	0.248	2.801	0.572	3.332	0.000	1.784	2.996

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	69	70	816	115	0	86	203
N.S.	1	1.00	1.60	1.63	18.98	2.67	0.00	2.00	4.72
time (sec)	N/A	0.091	0.387	2.608	0.638	3.253	0.000	1.809	6.610

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	108	103	1804	121	0	132	294
N.S.	1	1.00	1.21	1.16	20.27	1.36	0.00	1.48	3.30
time (sec)	N/A	0.194	1.058	2.542	0.595	2.631	0.000	1.831	6.149

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	97	93	1187	121	0	107	195
N.S.	1	1.00	1.09	1.04	13.34	1.36	0.00	1.20	2.19
time (sec)	N/A	0.187	0.582	2.683	0.568	3.753	0.000	1.745	5.499

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	83	596	89	0	86	108
N.S.	1	1.00	0.88	0.93	6.70	1.00	0.00	0.97	1.21
time (sec)	N/A	0.187	0.425	2.714	0.592	2.339	0.000	1.695	3.145

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	73	73	60	83	0	56	76
N.S.	1	1.00	1.70	1.70	1.40	1.93	0.00	1.30	1.77
time (sec)	N/A	0.092	0.283	2.642	0.500	2.898	0.000	1.593	2.587

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	174	162	296	0	0	0	-1
N.S.	1	1.00	1.83	1.71	3.12	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	1.402	2.928	0.574	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	134	249	130	0	0	0	-1
N.S.	1	1.00	1.35	2.52	1.31	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.656	2.764	0.520	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	63	73	575	103	0	61	165
N.S.	1	1.00	1.50	1.74	13.69	2.45	0.00	1.45	3.93
time (sec)	N/A	0.102	0.470	2.626	0.578	3.025	0.000	1.873	4.830

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	80	83	1676	144	0	89	273
N.S.	1	1.00	0.91	0.94	19.05	1.64	0.00	1.01	3.10
time (sec)	N/A	0.207	0.583	3.005	1.261	2.138	0.000	1.807	7.031

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	90	93	2804	171	0	113	340
N.S.	1	1.00	0.98	1.01	30.48	1.86	0.00	1.23	3.70
time (sec)	N/A	0.196	0.821	2.579	5.952	2.298	0.000	2.152	7.620

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	100	103	4202	199	0	137	407
N.S.	1	1.00	1.09	1.12	45.67	2.16	0.00	1.49	4.42
time (sec)	N/A	0.192	1.205	2.628	33.298	2.315	0.000	1.697	7.680

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	113	105	2620	163	0	134	307
N.S.	1	1.00	0.84	0.78	19.55	1.22	0.00	1.00	2.29
time (sec)	N/A	0.288	1.282	2.917	0.571	2.544	0.000	1.772	6.234

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	92	95	1634	114	0	109	215
N.S.	1	1.00	0.69	0.71	12.19	0.85	0.00	0.81	1.60
time (sec)	N/A	0.285	0.760	2.699	0.561	2.050	0.000	1.761	5.609

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	96	85	1188	121	0	88	195
N.S.	1	1.00	1.08	0.96	13.35	1.36	0.00	0.99	2.19
time (sec)	N/A	0.194	0.592	2.742	0.584	2.753	0.000	1.838	5.442

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	88	75	62	101	0	60	136
N.S.	1	1.00	2.05	1.74	1.44	2.35	0.00	1.40	3.16
time (sec)	N/A	0.089	0.479	2.898	0.498	2.792	0.000	1.746	3.610

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	328	189	791	0	0	0	-1
N.S.	1	1.00	2.33	1.34	5.61	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.275	6.614	2.838	0.595	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	188	289	2185	0	0	0	-1
N.S.	1	1.00	1.30	1.99	15.07	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.282	1.409	2.898	0.741	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	182	366	179	0	0	0	-1
N.S.	1	1.00	1.26	2.52	1.23	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	1.316	2.993	0.492	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	76	75	1949	136	0	65	199
N.S.	1	1.00	1.81	1.79	46.40	3.24	0.00	1.55	4.74
time (sec)	N/A	0.097	0.596	2.909	0.588	2.343	0.000	1.821	6.040

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	92	85	2915	179	0	91	350
N.S.	1	1.00	1.05	0.97	33.12	2.03	0.00	1.03	3.98
time (sec)	N/A	0.204	0.826	2.987	5.786	2.941	0.000	1.908	6.785

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	102	95	4404	209	0	115	419
N.S.	1	1.00	0.77	0.71	33.11	1.57	0.00	0.86	3.15
time (sec)	N/A	0.314	1.214	2.774	32.795	2.869	0.000	1.752	7.169

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	141	165	791	0	0	160	-1
N.S.	1	1.00	1.01	1.19	5.69	0.00	0.00	1.15	-0.01
time (sec)	N/A	0.272	1.481	2.859	0.585	0.000	0.000	1.642	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	173	149	297	0	0	129	-1
N.S.	1	1.00	1.84	1.59	3.16	0.00	0.00	1.37	-0.01
time (sec)	N/A	0.176	1.790	3.125	0.568	0.000	0.000	1.413	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	140	115	68	0	0	59	-1
N.S.	1	1.00	2.80	2.30	1.36	0.00	0.00	1.18	-0.02
time (sec)	N/A	0.082	0.390	3.040	0.511	0.000	0.000	1.245	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	94	85	48	218	0	70	-1
N.S.	1	1.00	2.00	1.81	1.02	4.64	0.00	1.49	-0.02
time (sec)	N/A	0.092	0.814	3.112	0.564	3.169	0.000	1.679	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	131	442	414	0	92	-1
N.S.	1	1.00	0.83	1.38	4.65	4.36	0.00	0.97	-0.01
time (sec)	N/A	0.183	0.883	3.066	0.574	1.984	0.000	1.905	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	91	170	1303	494	0	122	-1
N.S.	1	1.00	0.65	1.21	9.31	3.53	0.00	0.87	-0.01
time (sec)	N/A	0.281	0.818	3.063	0.646	3.499	0.000	1.742	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	183	235	2185	0	0	161	-1
N.S.	1	1.00	1.29	1.65	15.39	0.00	0.00	1.13	-0.01
time (sec)	N/A	0.284	1.031	2.862	0.777	0.000	0.000	1.605	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	132	191	105	0	0	79	-1
N.S.	1	1.00	1.39	2.01	1.11	0.00	0.00	0.83	-0.01
time (sec)	N/A	0.181	1.045	2.911	0.510	0.000	0.000	1.572	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	73	58	85	0	36	50
N.S.	1	1.00	1.00	1.74	1.38	2.02	0.00	0.86	1.19
time (sec)	N/A	0.091	0.181	2.955	0.510	4.239	0.000	1.664	2.548

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	157	123	433	412	0	95	-1
N.S.	1	1.00	1.65	1.29	4.56	4.34	0.00	1.00	-0.01
time (sec)	N/A	0.189	1.390	2.816	0.574	3.193	0.000	1.892	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	69	133	613	434	0	120	-1
N.S.	1	1.00	0.66	1.28	5.89	4.17	0.00	1.15	-0.01
time (sec)	N/A	0.124	0.804	2.967	0.568	2.397	0.000	3.075	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	122	211	0	588	0	153	-1
N.S.	1	1.00	0.84	1.45	0.00	4.03	0.00	1.05	-0.01
time (sec)	N/A	0.235	1.498	2.701	0.000	4.558	0.000	1.930	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	178	281	141	0	0	114	-1
N.S.	1	1.00	1.23	1.94	0.97	0.00	0.00	0.79	-0.01
time (sec)	N/A	0.286	1.395	3.247	0.518	0.000	0.000	1.520	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	68	75	106	103	0	59	119
N.S.	1	1.00	1.62	1.79	2.52	2.45	0.00	1.40	2.83
time (sec)	N/A	0.099	0.291	3.074	0.507	2.058	0.000	1.614	3.512

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	71	75	62	113	0	37	120
N.S.	1	1.00	1.65	1.74	1.44	2.63	0.00	0.86	2.79
time (sec)	N/A	0.093	0.254	3.051	0.497	1.711	0.000	1.544	3.248

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	91	164	1293	494	0	124	-1
N.S.	1	1.00	0.65	1.17	9.24	3.53	0.00	0.89	-0.01
time (sec)	N/A	0.290	1.041	3.190	0.624	1.953	0.000	1.715	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	130	204	0	580	0	151	-1
N.S.	1	1.00	0.89	1.40	0.00	3.97	0.00	1.03	-0.01
time (sec)	N/A	0.228	1.420	3.547	0.000	2.498	0.000	1.958	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	84	173	1791	520	0	182	-1
N.S.	1	1.00	0.52	1.08	11.19	3.25	0.00	1.14	-0.01
time (sec)	N/A	0.141	1.377	3.095	0.878	1.708	0.000	1.917	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	1.302	0.194	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.252	0.229	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.157	0.164	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.582	0.158	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	2.474	0.183	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	A	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	0	0	240	200	0	0	-1
N.S.	1	1.00	0.00	0.00	1.50	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.253	50.897	0.241	0.513	1.980	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	A	A	F(-1)	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	0	0	181	120	0	0	154
N.S.	1	1.00	0.00	0.00	1.81	1.20	0.00	0.00	1.54
time (sec)	N/A	0.146	68.677	0.237	0.502	1.307	0.000	0.000	3.592

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	163	0	122	76	0	0	-1
N.S.	1	1.00	3.54	0.00	2.65	1.65	0.00	0.00	-0.02
time (sec)	N/A	0.071	18.801	0.228	0.506	3.141	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.670	0.225	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.662	0.236	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.899	0.228	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	321	0	164	128	0	0	290
N.S.	1	1.00	1.90	0.00	0.97	0.76	0.00	0.00	1.72
time (sec)	N/A	0.266	9.070	0.358	0.497	1.508	0.000	0.000	10.646

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	250	0	113	100	0	0	145
N.S.	1	1.00	2.40	0.00	1.09	0.96	0.00	0.00	1.39
time (sec)	N/A	0.168	3.079	0.313	0.500	2.545	0.000	0.000	8.002

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	208	0	66	78	0	0	105
N.S.	1	1.00	4.43	0.00	1.40	1.66	0.00	0.00	2.23
time (sec)	N/A	0.078	1.211	0.296	0.497	1.926	0.000	0.000	2.909

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	257	0	0	0	0	0	-1
N.S.	1	1.00	2.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.410	0.240	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	2.288	0.361	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	2.724	0.385	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	68	137	186	141	0	145	175
N.S.	1	1.00	0.65	1.30	1.77	1.34	0.00	1.38	1.67
time (sec)	N/A	0.143	0.312	0.273	0.284	1.397	0.000	0.501	6.320

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	57	126	173	126	0	128	146
N.S.	1	1.00	0.66	1.47	2.01	1.47	0.00	1.49	1.70
time (sec)	N/A	0.116	0.198	0.250	0.278	1.740	0.000	0.504	4.923

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	36	39	38	51	15	15
N.S.	1	1.00	1.00	2.12	2.29	2.24	3.00	0.88	0.88
time (sec)	N/A	0.051	0.018	0.181	0.285	2.306	1.340	0.465	1.704

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	154	78	210	115	0	87	71
N.S.	1	1.00	2.75	1.39	3.75	2.05	0.00	1.55	1.27
time (sec)	N/A	0.090	0.643	0.145	0.298	2.336	0.000	0.481	1.745

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	335	60	156	133	0	81	44
N.S.	1	1.00	4.79	0.86	2.23	1.90	0.00	1.16	0.63
time (sec)	N/A	0.127	0.457	0.197	0.296	1.295	0.000	0.472	1.686

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	43	37	125	84	0	37	35
N.S.	1	1.00	0.50	0.43	1.45	0.98	0.00	0.43	0.41
time (sec)	N/A	0.126	0.170	0.198	0.291	1.550	0.000	0.513	1.700

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	7087	0	0	0	0	0	-1
N.S.	1	1.00	50.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	30.374	0.178	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	72	0	0	0	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.363	0.116	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	3396	0	0	0	0	0	-1
N.S.	1	1.00	18.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	16.813	0.213	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F(-2)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	11.160	0.220	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	143	162	236	1051	369	0	0	-1
N.S.	1	1.38	1.56	2.27	10.11	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.175	2.271	4.122	0.636	1.623	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	116	73	204	0	282	0	132	-1
N.S.	1	1.43	0.90	2.52	0.00	3.48	0.00	1.63	-0.01
time (sec)	N/A	0.150	0.470	3.108	0.000	1.736	0.000	0.989	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	213	724	317	1406	502	0	0	-1
N.S.	1	1.52	5.17	2.26	10.04	3.59	0.00	0.00	-0.01
time (sec)	N/A	0.180	8.048	4.935	0.621	2.352	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	150	236	152	576	356	0	0	-1
N.S.	1	1.29	2.03	1.31	4.97	3.07	0.00	0.00	-0.01
time (sec)	N/A	0.186	2.828	3.978	0.596	1.862	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	242	328	294	1496	614	0	0	-1
N.S.	1	1.35	1.83	1.64	8.36	3.43	0.00	0.00	-0.01
time (sec)	N/A	0.224	2.188	4.504	0.631	2.575	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	94	139	60	274	0	0	-1
N.S.	1	1.00	2.04	3.02	1.30	5.96	0.00	0.00	-0.02
time (sec)	N/A	0.149	0.812	3.198	0.613	2.442	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	98	412	0	374	0	136	-1
N.S.	1	1.00	1.51	6.34	0.00	5.75	0.00	2.09	-0.02
time (sec)	N/A	0.102	0.271	8.211	0.000	2.771	0.000	0.907	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	153	313	410	291	0	566	361
N.S.	1	1.00	0.65	1.33	1.74	1.23	0.00	2.40	1.53
time (sec)	N/A	0.298	1.823	0.399	0.286	2.352	0.000	0.524	5.508

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	103	223	288	220	0	380	255
N.S.	1	1.00	0.60	1.30	1.68	1.29	0.00	2.22	1.49
time (sec)	N/A	0.203	0.715	0.303	0.295	2.001	0.000	0.543	5.337

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	75	143	179	158	0	232	196
N.S.	1	1.00	0.69	1.32	1.66	1.46	0.00	2.15	1.81
time (sec)	N/A	0.115	0.450	0.243	0.285	1.847	0.000	0.513	4.761

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	75	96	103	0	124	111
N.S.	1	1.00	1.34	1.34	1.71	1.84	0.00	2.21	1.98
time (sec)	N/A	0.048	0.041	0.205	0.278	1.617	0.000	0.460	2.572

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	107	90	0	267	0	127	195
N.S.	1	1.00	1.55	1.30	0.00	3.87	0.00	1.84	2.83
time (sec)	N/A	0.098	0.204	0.232	0.000	1.839	0.000	0.509	2.159

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	105	0	371	0	137	85
N.S.	1	1.00	0.95	1.33	0.00	4.70	0.00	1.73	1.08
time (sec)	N/A	0.101	0.241	0.228	0.000	1.659	0.000	0.481	1.914

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	167	178	0	756	0	263	171
N.S.	1	1.00	1.27	1.36	0.00	5.77	0.00	2.01	1.31
time (sec)	N/A	0.191	1.278	0.362	0.000	2.414	0.000	0.577	3.776

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	247	271	0	1304	0	449	321
N.S.	1	1.00	1.31	1.43	0.00	6.90	0.00	2.38	1.70
time (sec)	N/A	0.319	3.342	0.494	0.000	2.513	0.000	0.534	5.283

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	371	460	552	737	398	0	736	484
N.S.	1	1.13	1.41	1.69	2.25	1.22	0.00	2.25	1.48
time (sec)	N/A	0.285	1.977	0.471	0.301	2.134	0.000	0.630	5.354

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	277	326	395	506	304	0	506	394
N.S.	1	1.14	1.35	1.63	2.09	1.26	0.00	2.09	1.63
time (sec)	N/A	0.221	1.368	0.391	0.286	2.159	0.000	0.646	5.485

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	234	479	270	350	218	0	320	237
N.S.	1	1.33	2.72	1.53	1.99	1.24	0.00	1.82	1.35
time (sec)	N/A	0.169	1.003	0.314	0.278	2.306	0.000	0.536	5.464

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	481	145	181	146	0	178	161
N.S.	1	1.00	4.67	1.41	1.76	1.42	0.00	1.73	1.56
time (sec)	N/A	0.083	6.297	0.240	0.288	1.173	0.000	0.514	4.525

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	208	329	150	0	420	0	196	529
N.S.	1	2.19	3.46	1.58	0.00	4.42	0.00	2.06	5.57
time (sec)	N/A	0.168	2.091	0.306	0.000	3.092	0.000	0.517	2.587

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	231	312	157	0	589	0	230	2563
N.S.	1	1.97	2.67	1.34	0.00	5.03	0.00	1.97	21.91
time (sec)	N/A	0.174	1.554	0.385	0.000	1.687	0.000	0.531	4.790

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	184	249	167	0	642	0	211	158
N.S.	1	1.42	1.92	1.28	0.00	4.94	0.00	1.62	1.22
time (sec)	N/A	0.132	1.247	0.388	0.000	2.111	0.000	0.555	3.574

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	268	211	228	0	1260	0	403	286
N.S.	1	1.26	0.99	1.07	0.00	5.92	0.00	1.89	1.34
time (sec)	N/A	0.193	4.860	0.563	0.000	3.945	0.000	0.547	5.092

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	330	322	352	0	1940	0	710	438
N.S.	1	1.20	1.17	1.28	0.00	7.03	0.00	2.57	1.59
time (sec)	N/A	0.385	9.589	0.786	0.000	3.547	0.000	0.693	5.237

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	333	380	573	757	348	0	584	411
N.S.	1	1.16	1.32	1.99	2.63	1.21	0.00	2.03	1.43
time (sec)	N/A	0.283	2.650	0.458	0.295	3.145	0.000	0.601	5.238

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	273	433	385	496	255	0	376	287
N.S.	1	1.06	1.68	1.50	1.93	0.99	0.00	1.46	1.12
time (sec)	N/A	0.198	2.514	0.382	0.294	3.840	0.000	0.620	5.518

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	273	219	284	170	0	212	203
N.S.	1	1.00	2.18	1.75	2.27	1.36	0.00	1.70	1.62
time (sec)	N/A	0.100	1.307	0.313	0.287	2.145	0.000	0.507	5.307

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	257	419	224	0	556	0	285	1902
N.S.	1	1.68	2.74	1.46	0.00	3.63	0.00	1.86	12.43
time (sec)	N/A	0.220	2.430	0.379	0.000	3.448	0.000	0.568	2.892

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	274	455	216	0	891	0	317	3135
N.S.	1	1.70	2.83	1.34	0.00	5.53	0.00	1.97	19.47
time (sec)	N/A	0.241	4.174	0.444	0.000	2.716	0.000	0.505	5.274

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	301	393	227	0	1208	0	376	2500
N.S.	1	1.60	2.09	1.21	0.00	6.43	0.00	2.00	13.30
time (sec)	N/A	0.275	3.480	0.608	0.000	4.272	0.000	0.642	8.500

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	227	398	227	0	1038	0	307	264
N.S.	1	1.28	2.24	1.28	0.00	5.83	0.00	1.72	1.48
time (sec)	N/A	0.163	3.516	0.574	0.000	2.652	0.000	0.600	4.985

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	327	274	303	0	1746	0	601	385
N.S.	1	1.23	1.03	1.14	0.00	6.56	0.00	2.26	1.45
time (sec)	N/A	0.232	9.060	0.897	0.000	2.550	0.000	0.654	5.081

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	236	1243	309	646	309	0	344	211
N.S.	1	1.29	6.79	1.69	3.53	1.69	0.00	1.88	1.15
time (sec)	N/A	0.222	6.494	0.223	0.287	2.513	0.000	0.559	2.451

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	171	275	208	420	227	0	219	139
N.S.	1	1.46	2.35	1.78	3.59	1.94	0.00	1.87	1.19
time (sec)	N/A	0.163	2.640	0.190	0.292	2.275	0.000	0.521	1.938

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	125	237	127	241	165	0	136	85
N.S.	1	1.84	3.49	1.87	3.54	2.43	0.00	2.00	1.25
time (sec)	N/A	0.094	1.749	0.187	0.272	2.031	0.000	0.540	1.824

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	109	61	107	80	0	70	41
N.S.	1	1.00	2.53	1.42	2.49	1.86	0.00	1.63	0.95
time (sec)	N/A	0.054	0.285	0.168	0.284	3.247	0.000	0.489	1.727

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	134	160	74	0	367	0	110	110
N.S.	1	1.61	1.93	0.89	0.00	4.42	0.00	1.33	1.33
time (sec)	N/A	0.111	0.695	0.211	0.000	3.083	0.000	0.472	1.947

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	196	286	146	0	711	0	221	187
N.S.	1	1.35	1.97	1.01	0.00	4.90	0.00	1.52	1.29
time (sec)	N/A	0.171	3.254	0.311	0.000	2.050	0.000	0.465	2.036

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	268	1422	221	0	1357	0	362	379
N.S.	1	1.29	6.87	1.07	0.00	6.56	0.00	1.75	1.83
time (sec)	N/A	0.279	6.836	0.422	0.000	2.465	0.000	0.552	2.975

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	315	446	436	836	472	0	506	268
N.S.	1	1.22	1.73	1.69	3.24	1.83	0.00	1.96	1.04
time (sec)	N/A	0.291	4.178	0.279	0.292	1.619	0.000	0.590	1.973

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	249	310	317	580	376	0	359	193
N.S.	1	1.29	1.61	1.64	3.01	1.95	0.00	1.86	1.00
time (sec)	N/A	0.213	2.883	0.236	0.288	2.316	0.000	0.560	1.912

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	193	294	216	370	282	0	250	136
N.S.	1	1.45	2.21	1.62	2.78	2.12	0.00	1.88	1.02
time (sec)	N/A	0.158	1.641	0.217	0.294	1.482	0.000	0.511	1.880

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	149	181	131	211	165	0	158	89
N.S.	1	1.67	2.03	1.47	2.37	1.85	0.00	1.78	1.00
time (sec)	N/A	0.100	0.781	0.206	0.279	2.867	0.000	0.478	1.771

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	101	62	0	60	45
N.S.	1	1.00	1.17	0.92	1.55	0.95	0.00	0.92	0.69
time (sec)	N/A	0.057	0.223	0.168	0.275	2.226	0.000	0.450	1.714

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	183	209	122	0	618	0	249	168
N.S.	1	1.42	1.62	0.95	0.00	4.79	0.00	1.93	1.30
time (sec)	N/A	0.167	1.719	0.240	0.000	2.362	0.000	0.502	1.915

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	260	376	203	0	1268	0	474	314
N.S.	1	1.23	1.78	0.96	0.00	6.01	0.00	2.25	1.49
time (sec)	N/A	0.251	3.930	0.362	0.000	2.840	0.000	0.527	2.181

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	346	2220	280	0	2062	0	751	505
N.S.	1	1.22	7.82	0.99	0.00	7.26	0.00	2.64	1.78
time (sec)	N/A	0.384	7.391	0.478	0.000	2.369	0.000	0.565	2.282

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	405	1338	579	1026	640	0	672	327
N.S.	1	1.12	3.69	1.60	2.83	1.76	0.00	1.85	0.90
time (sec)	N/A	0.353	6.723	0.266	0.322	1.775	0.000	0.732	1.908

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	329	439	441	747	521	0	504	252
N.S.	1	1.15	1.53	1.54	2.60	1.82	0.00	1.76	0.88
time (sec)	N/A	0.277	2.124	0.280	0.300	1.858	0.000	0.607	1.865

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	265	292	321	515	403	0	374	195
N.S.	1	1.29	1.42	1.57	2.51	1.97	0.00	1.82	0.95
time (sec)	N/A	0.188	2.403	0.240	0.292	1.347	0.000	0.576	1.816

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	193	295	216	333	262	0	259	147
N.S.	1	1.45	2.22	1.62	2.50	1.97	0.00	1.95	1.11
time (sec)	N/A	0.136	1.473	0.225	0.301	2.342	0.000	0.544	1.799

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	180	74	200	119	0	129	79
N.S.	1	1.00	1.57	0.64	1.74	1.03	0.00	1.12	0.69
time (sec)	N/A	0.106	0.491	0.198	0.285	1.436	0.000	0.508	1.833

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	125	99	0	75	66
N.S.	1	1.00	1.32	0.63	1.23	0.97	0.00	0.74	0.65
time (sec)	N/A	0.081	0.349	0.224	0.282	3.146	0.000	0.478	1.738

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	235	345	203	0	1027	0	471	228
N.S.	1	1.30	1.91	1.12	0.00	5.67	0.00	2.60	1.26
time (sec)	N/A	0.234	3.062	0.301	0.000	3.023	0.000	0.529	1.950

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	325	1772	284	0	1725	0	918	464
N.S.	1	1.13	6.15	0.99	0.00	5.99	0.00	3.19	1.61
time (sec)	N/A	0.342	7.103	0.404	0.000	3.516	0.000	0.579	2.120

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	414	1096	365	0	2715	0	1369	655
N.S.	1	1.12	2.98	0.99	0.00	7.38	0.00	3.72	1.78
time (sec)	N/A	0.501	7.882	0.549	0.000	4.294	0.000	0.640	2.359

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	102	302	0	327	0	0	-1
N.S.	1	1.00	1.67	4.95	0.00	5.36	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.247	3.576	0.000	2.531	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	187	503	0	1120	0	0	-1
N.S.	1	1.00	1.34	3.59	0.00	8.00	0.00	0.00	-0.01
time (sec)	N/A	0.295	18.487	3.637	0.000	5.317	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	107	170	0	262	0	0	-1
N.S.	1	1.00	1.37	2.18	0.00	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.218	3.490	0.000	4.101	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	171	403	0	1172	0	0	-1
N.S.	1	1.00	1.21	2.86	0.00	8.31	0.00	0.00	-0.01
time (sec)	N/A	0.335	0.277	3.403	0.000	3.902	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	94	432	0	360	0	139	-1
N.S.	1	1.00	1.54	7.08	0.00	5.90	0.00	2.28	-0.02
time (sec)	N/A	0.090	0.224	8.218	0.000	2.704	0.000	0.868	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	427	568	0	1196	0	0	-1
N.S.	1	1.00	2.87	3.81	0.00	8.03	0.00	0.00	-0.01
time (sec)	N/A	0.368	1.399	13.197	0.000	8.049	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	229015	520	0	1021	0	0	-1
N.S.	1	1.00	1877.17	4.26	0.00	8.37	0.00	0.00	-0.01
time (sec)	N/A	0.192	33.848	9.033	0.000	3.717	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	141	518	0	1099	0	0	-1
N.S.	1	1.00	1.14	4.18	0.00	8.86	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.406	9.424	0.000	3.294	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	198	473	0	1167	0	0	-1
N.S.	1	1.00	1.19	2.83	0.00	6.99	0.00	0.00	-0.01
time (sec)	N/A	0.376	0.412	13.859	0.000	4.416	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	155	727	0	1695	0	0	-1
N.S.	1	1.00	0.67	3.15	0.00	7.34	0.00	0.00	-0.00
time (sec)	N/A	0.514	0.355	13.784	0.000	110.119	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	201	313	410	291	0	850	555
N.S.	1	1.00	0.80	1.25	1.64	1.16	0.00	3.40	2.22
time (sec)	N/A	0.343	4.103	0.399	0.286	3.058	0.000	0.586	5.553

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	143	223	288	220	0	586	395
N.S.	1	1.00	0.79	1.24	1.60	1.22	0.00	3.26	2.19
time (sec)	N/A	0.242	1.157	0.345	0.278	2.394	0.000	0.527	5.494

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	143	179	158	0	294	227
N.S.	1	1.00	0.77	1.24	1.56	1.37	0.00	2.56	1.97
time (sec)	N/A	0.132	0.616	0.259	0.298	3.001	0.000	0.506	5.208

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	75	96	103	0	153	104
N.S.	1	1.00	1.23	1.23	1.57	1.69	0.00	2.51	1.70
time (sec)	N/A	0.055	0.036	0.188	0.279	2.117	0.000	0.465	2.789

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	328	0	127	573
N.S.	1	1.00	1.47	1.21	0.00	4.32	0.00	1.67	7.54
time (sec)	N/A	0.099	0.193	0.272	0.000	5.433	0.000	0.536	2.730

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	97	132	0	403	0	172	106
N.S.	1	1.00	0.98	1.33	0.00	4.07	0.00	1.74	1.07
time (sec)	N/A	0.109	0.373	0.233	0.000	2.625	0.000	0.480	2.122

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	172	236	0	772	0	399	250
N.S.	1	1.00	1.04	1.42	0.00	4.65	0.00	2.40	1.51
time (sec)	N/A	0.222	0.851	0.364	0.000	3.040	0.000	0.551	4.987

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	405	376	0	1264	0	693	439
N.S.	1	1.00	1.71	1.59	0.00	5.33	0.00	2.92	1.85
time (sec)	N/A	0.378	1.039	0.494	0.000	2.975	0.000	0.543	6.395

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	580	480	0	1119	0	606	2500
N.S.	1	1.00	2.35	1.94	0.00	4.53	0.00	2.45	10.12
time (sec)	N/A	0.303	4.456	0.633	0.000	183.450	0.000	0.565	11.315

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	389	289	0	803	0	339	2500
N.S.	1	1.00	2.29	1.70	0.00	4.72	0.00	1.99	14.71
time (sec)	N/A	0.240	1.461	0.484	0.000	38.069	0.000	0.562	9.661

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	135	165	0	540	0	195	2500
N.S.	1	1.00	1.31	1.60	0.00	5.24	0.00	1.89	24.27
time (sec)	N/A	0.204	0.832	0.366	0.000	8.621	0.000	0.507	7.316

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	321	0	127	571
N.S.	1	1.00	1.47	1.21	0.00	4.22	0.00	1.67	7.51
time (sec)	N/A	0.089	0.182	0.294	0.000	2.907	0.000	0.518	2.810

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	108	0	1072	0	522	2665
N.S.	1	1.00	0.98	0.89	0.00	8.86	0.00	4.31	22.02
time (sec)	N/A	0.191	0.234	0.467	0.000	5.018	0.000	0.580	4.358

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	229	208	0	2863	0	331	2500
N.S.	1	1.00	1.22	1.11	0.00	15.31	0.00	1.77	13.37
time (sec)	N/A	0.422	0.709	1.177	0.000	98.584	0.000	0.544	15.423

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	1137	689	0	0	0	857	2500
N.S.	1	1.00	3.00	1.82	0.00	0.00	0.00	2.26	6.60
time (sec)	N/A	0.477	6.527	1.465	0.000	0.000	0.000	0.624	16.949

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	511	465	0	0	0	551	2500
N.S.	1	1.00	1.72	1.57	0.00	0.00	0.00	1.86	8.42
time (sec)	N/A	0.382	4.008	1.089	0.000	0.000	0.000	0.554	14.374

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	362	314	0	1358	0	539	2500
N.S.	1	1.00	1.59	1.38	0.00	5.96	0.00	2.36	10.96
time (sec)	N/A	0.337	1.764	0.778	0.000	106.479	0.000	0.581	11.297

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	180	215	0	820	0	267	2500
N.S.	1	1.00	0.91	1.09	0.00	4.14	0.00	1.35	12.63
time (sec)	N/A	0.261	0.710	0.559	0.000	14.501	0.000	0.514	9.749

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	97	132	0	408	0	173	106
N.S.	1	1.00	0.97	1.32	0.00	4.08	0.00	1.73	1.06
time (sec)	N/A	0.099	0.396	0.242	0.000	2.765	0.000	0.495	2.199

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	176	210	0	2900	0	330	2500
N.S.	1	1.00	0.95	1.13	0.00	15.59	0.00	1.77	13.44
time (sec)	N/A	0.416	1.026	1.204	0.000	166.846	0.000	0.524	15.563

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	183	355	0	0	0	0	-1
N.S.	1	1.00	0.86	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	3.779	4.942	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	44216	351	0	0	0	0	-1
N.S.	1	1.00	225.59	1.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	35.522	4.161	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	233	219	0	0	0	0	-1
N.S.	1	1.00	1.21	1.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	3.520	3.926	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	176	177	0	0	0	0	-1
N.S.	1	1.00	1.60	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.796	9.728	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	176	170	0	0	0	0	-1
N.S.	1	1.00	1.41	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.448	11.069	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	39039	291	0	0	0	0	-1
N.S.	1	1.00	98.58	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	35.332	4.191	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	223	481	0	0	0	0	-1
N.S.	1	1.00	1.31	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.574	13.602	6.339	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	254	0	0	0	0	-1
N.S.	1	1.00	1.00	3.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.284	0.250	6.539	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	222	461	0	0	0	0	-1
N.S.	1	1.00	1.32	2.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.703	13.515	6.229	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	264	153	0	0	0	0	-1
N.S.	1	1.00	2.78	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	5.361	5.011	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	0	308	0	0	0	0	-1
N.S.	1	1.00	0.00	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.617	19.733	6.759	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	375	225	0	0	0	0	-1
N.S.	1	1.00	1.79	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	13.976	4.812	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	156	224	0	0	0	0	-1
N.S.	1	1.00	0.73	1.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.246	4.842	5.118	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	1019	238	0	535	0	0	-1
N.S.	1	1.00	4.45	1.04	0.00	2.34	0.00	0.00	-0.00
time (sec)	N/A	0.303	8.308	6.823	0.000	1.017	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	0	357	0	0	0	0	-1
N.S.	1	1.00	0.00	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	14.730	6.546	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	183	355	0	0	0	0	-1
N.S.	1	1.00	0.86	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.405	5.237	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	223	481	0	0	0	0	-1
N.S.	1	1.00	1.31	2.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.570	13.484	6.856	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	187	236	0	0	0	0	-1
N.S.	1	1.00	1.83	2.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	6.739	5.589	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	165	236	0	0	0	0	-1
N.S.	1	1.00	0.79	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.252	3.080	5.815	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	254	0	0	0	0	-1
N.S.	1	1.00	1.00	3.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.295	0.239	6.736	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	246	346	0	0	0	0	-1
N.S.	1	1.00	1.48	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.599	13.676	6.277	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	151	49	74	131	0	47	47
N.S.	1	1.00	2.25	0.73	1.10	1.96	0.00	0.70	0.70
time (sec)	N/A	0.218	0.974	0.251	0.280	2.448	0.000	1.259	1.934

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	175	62	96	158	0	60	60
N.S.	1	1.00	1.97	0.70	1.08	1.78	0.00	0.67	0.67
time (sec)	N/A	0.234	1.141	0.270	0.287	2.636	0.000	1.336	2.429

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [178] had the largest ratio of [40]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	7	1.00	30	0.233
2	A	9	6	1.00	30	0.200
3	A	6	5	1.00	30	0.167
4	A	3	3	1.00	28	0.107
5	A	2	2	1.00	30	0.067
6	A	1	1	1.00	30	0.033
7	A	2	2	1.00	30	0.067
8	A	3	2	1.00	30	0.067
9	A	4	2	1.00	30	0.067
10	A	14	7	1.00	32	0.219
11	A	11	6	1.00	32	0.188
12	A	7	5	1.00	32	0.156
13	A	4	3	1.00	32	0.094
14	A	6	5	1.00	30	0.167
15	A	5	5	1.00	32	0.156
16	A	3	2	1.00	32	0.062
17	A	1	1	1.00	32	0.031
18	A	2	2	1.00	32	0.062
19	A	3	2	1.00	32	0.062
20	A	4	2	1.00	32	0.062
21	A	16	7	1.00	32	0.219
22	A	13	6	1.00	32	0.188
23	A	8	5	1.00	32	0.156
24	A	5	3	1.00	32	0.094
25	A	7	5	1.00	32	0.156

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	9	6	1.00	30	0.200
27	A	6	6	1.00	32	0.188
28	A	6	5	1.00	32	0.156
29	A	4	2	1.00	32	0.062
30	A	1	1	1.00	32	0.031
31	A	2	2	1.00	32	0.062
32	A	3	2	1.00	32	0.062
33	A	4	2	1.00	32	0.062
34	A	10	6	1.00	32	0.188
35	A	6	6	1.00	32	0.188
36	A	5	5	1.00	32	0.156
37	A	2	2	1.00	30	0.067
38	A	3	3	1.00	32	0.094
39	A	6	4	1.00	32	0.125
40	A	10	6	1.00	32	0.188
41	A	13	7	1.00	32	0.219
42	A	11	6	1.00	32	0.188
43	A	7	6	1.00	32	0.188
44	A	6	5	1.00	32	0.156
45	A	3	2	1.00	32	0.062
46	A	1	1	1.00	30	0.033
47	A	6	4	1.00	32	0.125
48	A	3	2	1.00	32	0.062
49	A	7	5	1.00	32	0.156
50	A	10	6	1.00	32	0.188
51	A	13	7	1.00	32	0.219
52	A	12	6	1.00	32	0.188
53	A	8	6	1.00	32	0.188
54	A	7	5	1.00	32	0.156
55	A	4	2	1.00	32	0.062
56	A	1	1	1.00	32	0.031
57	A	2	2	1.00	30	0.067
58	A	10	6	1.00	32	0.188
59	A	7	5	1.00	32	0.156
60	A	4	3	1.00	32	0.094

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	5	1.00	32	0.156
62	A	10	6	1.00	32	0.188
63	A	13	7	1.00	32	0.219
64	A	4	2	1.00	32	0.062
65	A	3	2	1.00	32	0.062
66	A	2	2	1.00	32	0.062
67	A	1	1	1.00	32	0.031
68	A	3	3	1.00	32	0.094
69	A	3	3	1.00	32	0.094
70	A	4	4	1.00	32	0.125
71	A	4	2	1.00	34	0.059
72	A	3	2	1.00	34	0.059
73	A	2	2	1.00	34	0.059
74	A	1	1	1.00	34	0.029
75	A	4	3	1.05	34	0.088
76	A	4	4	1.10	34	0.118
77	A	4	3	1.11	34	0.088
78	A	5	4	1.00	34	0.118
79	A	4	2	1.00	34	0.059
80	A	3	2	1.00	34	0.059
81	A	2	2	1.00	34	0.059
82	A	1	1	1.00	34	0.029
83	A	5	3	1.00	34	0.088
84	A	5	4	1.00	34	0.118
85	A	5	4	1.00	34	0.118
86	A	4	3	1.00	34	0.088
87	A	3	3	1.00	34	0.088
88	A	2	2	1.00	34	0.059
89	A	1	1	1.00	34	0.029
90	A	3	3	1.00	34	0.088
91	A	4	4	1.00	34	0.118
92	A	5	4	1.00	34	0.118
93	A	4	3	1.00	34	0.088
94	A	3	2	1.00	34	0.059
95	A	2	2	1.00	34	0.059

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	1	1	1.00	34	0.029
97	A	4	3	1.00	34	0.088
98	A	5	4	1.00	34	0.118
99	A	6	4	1.00	34	0.118
100	A	4	2	1.00	34	0.059
101	A	3	2	1.00	34	0.059
102	A	2	2	1.00	34	0.059
103	A	1	1	1.00	34	0.029
104	A	5	3	1.00	34	0.088
105	A	6	4	1.00	34	0.118
106	A	7	4	1.00	34	0.118
107	A	1	1	1.00	36	0.028
108	A	1	1	1.00	36	0.028
109	A	1	1	1.00	36	0.028
110	A	1	1	1.00	36	0.028
111	A	1	1	1.00	36	0.028
112	A	1	1	1.00	36	0.028
113	A	2	2	1.00	36	0.056
114	A	2	2	1.00	36	0.056
115	A	2	2	1.00	36	0.056
116	A	1	1	1.00	36	0.028
117	A	2	2	1.00	36	0.056
118	A	2	2	1.00	36	0.056
119	A	1	1	1.00	36	0.028
120	A	2	2	1.00	36	0.056
121	A	2	2	1.00	36	0.056
122	A	2	2	1.00	36	0.056
123	A	3	2	1.00	36	0.056
124	A	3	2	1.00	36	0.056
125	A	2	2	1.00	36	0.056
126	A	1	1	1.00	36	0.028
127	A	3	2	1.00	36	0.056
128	A	3	3	1.00	36	0.083
129	A	3	2	1.00	36	0.056
130	A	1	1	1.00	36	0.028

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	36	0.056
132	A	3	2	1.00	36	0.056
133	A	3	2	1.00	36	0.056
134	A	2	2	1.00	36	0.056
135	A	1	1	1.00	36	0.028
136	A	2	2	1.00	36	0.056
137	A	3	3	1.00	36	0.083
138	A	4	3	1.00	36	0.083
139	A	3	3	1.00	36	0.083
140	A	2	2	1.00	36	0.056
141	A	1	1	1.00	36	0.028
142	A	3	3	1.00	36	0.083
143	A	3	3	1.00	36	0.083
144	A	4	4	1.00	36	0.111
145	A	3	2	1.00	36	0.056
146	A	1	1	1.00	36	0.028
147	A	1	1	1.00	36	0.028
148	A	4	3	1.00	36	0.083
149	A	4	4	1.00	36	0.111
150	A	4	3	1.00	36	0.083
151	A	3	3	1.00	32	0.094
152	A	3	3	1.00	32	0.094
153	A	3	3	1.00	30	0.100
154	A	3	3	1.00	32	0.094
155	A	3	3	1.00	32	0.094
156	A	3	2	1.00	34	0.059
157	A	2	2	1.00	34	0.059
158	A	1	1	1.00	34	0.029
159	A	2	2	1.00	34	0.059
160	A	2	2	1.00	34	0.059
161	A	2	2	1.00	34	0.059
162	A	3	2	1.00	36	0.056
163	A	2	2	1.00	36	0.056
164	A	1	1	1.00	36	0.028
165	A	3	3	1.00	34	0.088

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	3	1.00	36	0.083
167	A	3	3	1.00	36	0.083
168	A	10	7	1.00	32	0.219
169	A	7	6	1.00	32	0.188
170	A	3	3	1.00	30	0.100
171	A	5	5	1.00	32	0.156
172	A	4	4	1.00	32	0.125
173	A	3	3	1.00	32	0.094
174	A	5	3	1.00	34	0.088
175	A	2	2	1.00	32	0.062
176	A	6	4	1.00	34	0.118
177	A	7	4	1.00	34	0.118
178	A	5	5	1.38	40	0.125
179	A	4	4	1.43	36	0.111
180	A	8	8	1.52	38	0.210
181	A	4	4	1.29	40	0.100
182	A	8	8	1.35	40	0.200
183	A	3	3	1.00	38	0.079
184	A	2	2	1.00	34	0.059
185	A	8	6	1.00	29	0.207
186	A	7	6	1.00	29	0.207
187	A	6	6	1.00	29	0.207
188	A	5	5	1.00	27	0.185
189	A	5	5	1.00	29	0.172
190	A	5	5	1.00	29	0.172
191	A	6	5	1.00	29	0.172
192	A	7	5	1.00	29	0.172
193	A	9	8	1.13	31	0.258
194	A	8	7	1.14	31	0.226
195	A	8	7	1.33	31	0.226
196	A	6	6	1.00	29	0.207
197	B	8	8	2.19	31	0.258
198	A	8	8	1.97	31	0.258
199	A	5	4	1.42	31	0.129
200	A	6	5	1.26	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	8	6	1.20	31	0.194
202	A	9	7	1.16	31	0.226
203	A	9	7	1.06	31	0.226
204	A	10	6	1.00	29	0.207
205	A	9	9	1.68	31	0.290
206	A	9	9	1.70	31	0.290
207	A	9	9	1.60	31	0.290
208	A	6	4	1.28	31	0.129
209	A	7	5	1.23	31	0.161
210	A	7	7	1.29	31	0.226
211	A	6	6	1.46	31	0.194
212	A	6	6	1.84	31	0.194
213	A	3	3	1.00	29	0.103
214	A	4	4	1.61	31	0.129
215	A	6	6	1.35	31	0.194
216	A	7	7	1.29	31	0.226
217	A	8	8	1.22	31	0.258
218	A	7	7	1.29	31	0.226
219	A	6	6	1.45	31	0.194
220	A	6	6	1.67	31	0.194
221	A	2	2	1.00	29	0.069
222	A	6	6	1.42	31	0.194
223	A	7	6	1.23	31	0.194
224	A	8	7	1.22	31	0.226
225	A	9	8	1.12	31	0.258
226	A	8	7	1.15	31	0.226
227	A	7	7	1.29	31	0.226
228	A	6	6	1.45	31	0.194
229	A	4	4	1.00	31	0.129
230	A	3	3	1.00	29	0.103
231	A	7	6	1.30	31	0.194
232	A	8	6	1.13	31	0.194
233	A	9	7	1.12	31	0.226
234	A	2	2	1.00	35	0.057
235	A	5	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
236	A	2	2	1.00	35	0.057
237	A	5	5	1.00	37	0.135
238	A	2	2	1.00	33	0.061
239	A	5	4	1.00	39	0.103
240	A	5	5	1.00	33	0.152
241	A	5	5	1.00	35	0.143
242	A	5	4	1.00	39	0.103
243	A	8	6	1.00	39	0.154
244	A	8	6	1.00	29	0.207
245	A	7	6	1.00	29	0.207
246	A	6	6	1.00	29	0.207
247	A	5	5	1.00	27	0.185
248	A	5	5	1.00	29	0.172
249	A	5	5	1.00	29	0.172
250	A	6	5	1.00	29	0.172
251	A	7	5	1.00	29	0.172
252	A	12	8	1.00	31	0.258
253	A	10	8	1.00	31	0.258
254	A	8	7	1.00	31	0.226
255	A	5	5	1.00	29	0.172
256	A	6	4	1.00	31	0.129
257	A	7	5	1.00	31	0.161
258	A	16	10	1.00	31	0.323
259	A	14	10	1.00	31	0.323
260	A	12	9	1.00	31	0.290
261	A	10	7	1.00	31	0.226
262	A	5	5	1.00	29	0.172
263	A	7	5	1.00	31	0.161
264	A	3	3	1.00	33	0.091
265	A	1	1	1.00	35	0.029
266	A	1	1	1.00	35	0.029
267	A	1	1	1.00	35	0.029
268	A	1	1	1.00	35	0.029
269	A	3	3	1.00	37	0.081
270	A	7	5	1.00	39	0.128

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	3	3	1.00	39	0.077
272	A	8	6	1.00	39	0.154
273	A	1	1	1.00	33	0.030
274	A	11	11	1.00	39	0.282
275	A	3	3	1.00	33	0.091
276	A	3	3	1.00	35	0.086
277	A	7	7	1.00	39	0.180
278	A	11	11	1.00	39	0.282
279	A	3	3	1.00	33	0.091
280	A	7	5	1.00	39	0.128
281	A	1	1	1.00	33	0.030
282	A	3	3	1.00	35	0.086
283	A	3	3	1.00	39	0.077
284	A	7	5	1.00	39	0.128
285	A	4	2	1.00	28	0.071
286	A	4	2	1.00	28	0.071

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=105

$$\frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^4\sec(e+fx)\tan(e+fx)}{8f}-\frac{3ac^4\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{4ac^4\tan^3(e+fx)}{3f}$$

```
[Out] 7/8*a*c^4*arctanh(sin(f*x+e))/f-1/8*a*c^4*sec(f*x+e)*tan(f*x+e)/f-3/4*a*c^4
*sec(f*x+e)^3*tan(f*x+e)/f+4/3*a*c^4*tan(f*x+e)^3/f+1/5*a*c^4*tan(f*x+e)^5/
f
```

Rubi [A]
time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {4043, 2691, 3855, 2687, 30, 3853, 14}

$$\frac{ac^4\tan^5(e+fx)}{5f}+\frac{4ac^4\tan^3(e+fx)}{3f}+\frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3ac^4\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{ac^4\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (7*a*c^4*ArcTanh[Sin[e + f*x]])/(8*f) - (a*c^4*Sec[e + f*x]*Tan[e + f*x])/(
8*f) - (3*a*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (4*a*c^4*Tan[e + f*x]^
3)/(3*f) + (a*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx &= -\left((ac)\int \left(c^3\sec(e+fx)\tan^2(e+fx)-3c^3\sec^{2}\right.\right. \\ &= -\left(\left(ac^4\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)+\left(ac^4\right. \\ &= -\frac{ac^4\sec(e+fx)\tan(e+fx)}{2f}-\frac{3ac^4\sec^3(e+fx}{4f} \\ &= \frac{ac^4\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^4\sec(e+fx)\tan(e}{8f} \\ &= \frac{7ac^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^4\sec(e+fx)\tan(e}{8f}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 499 vs. 2(105) = 210.

time = 1.71, size = 499, normalized size = 4.75

$$-\frac{ac^4\sec(e)\sec^5(e+fx)\left(525\cos(2e+3fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+525\cos(4e+3fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+105\cos(4e+5fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+105\cos(6e+5fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+1050\cos(fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+1050\cos(2e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-525\cos(2e+3fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-525\cos(4e+3fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-105\cos(4e+5fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-105\cos(6e+5fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+800\sin(fx)-1920\sin(2e+fx)+780\sin(e+2fx)+780\sin(3e+2fx)+640\sin(2e+3fx)-720\sin(4e+3fx)+30\sin(3e+4fx)+30\sin(5e+4fx)+272\sin(4e+5fx)\right)}{3840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/3840*(a*c^4*Sec[e]*Sec[e + f*x]^5*(525*Cos[2*e + 3*f*x]*Log[Cos[(e + f*x
)/2] - Sin[(e + f*x)/2]] + 525*Cos[4*e + 3*f*x]*Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] + 105*Cos[4*e + 5*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]] + 105*Cos[6*e + 5*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 1050*C
os[f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]]) + 1050*Cos[2*e + f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 525*Cos[2*e + 3*f*x]*
Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 525*Cos[4*e + 3*f*x]*Log[Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]] - 105*Cos[4*e + 5*f*x]*Log[Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]] - 105*Cos[6*e + 5*f*x]*Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]] + 800*Sin[f*x] - 1920*Sin[2*e + f*x] + 780*Sin[e + 2*f*x] + 780*Sin
[3*e + 2*f*x] + 640*Sin[2*e + 3*f*x] - 720*Sin[4*e + 3*f*x] + 30*Sin[3*e +
4*f*x] + 30*Sin[5*e + 4*f*x] + 272*Sin[4*e + 5*f*x]))/f
```

Maple [A]

time = 0.27, size = 182, normalized size = 1.73

method	result
norman	$\frac{\frac{7a\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{49a\,c^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f}+\frac{224a\,c^4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15f}-\frac{79a\,c^4\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f}-\frac{7a\,c^4\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{7a\,c^4}{\ldots}$ …
risch	$\frac{ia\,c^4\left(15\,\mathrm{e}^{9i(fx+e)}-360\,\mathrm{e}^{8i(fx+e)}+390\,\mathrm{e}^{7i(fx+e)}-960\,\mathrm{e}^{6i(fx+e)}-400\,\mathrm{e}^{4i(fx+e)}-390\,\mathrm{e}^{3i(fx+e)}-320\,\mathrm{e}^{2i(fx+e)}-15\,\mathrm{e}^{i(fx\ldots}\right.}{60f\left(\mathrm{e}^{2i(fx+e)}+1\right)^5}$ …
derivativedivides	$\frac{-a\,c^4\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-3a\,c^4\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec\ldots}{}\right.}{\ldots}$ …
default	$\frac{-a\,c^4\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-3a\,c^4\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec\ldots}{}\right.}{\ldots}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^4*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-3*a*c^4*(
-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)
))-2*a*c^4*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a*c^4*(1/2*sec(f*x+e)*tan(f
*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-3*a*c^4*tan(f*x+e)+a*c^4*ln(sec(f*x+e)
+tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(102) = 204$.
time = 0.27, size = 233, normalized size = 2.22

$$\frac{16\left(3\tan\left(fx+e\right)^{5}+10\tan\left(fx+e\right)^{3}+15\tan\left(fx+e\right)\right)ac^{4}+160\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)ac^{4}+45\,ac^{4}\left(\frac{2\left(3\sin(fx+e)^{3}-5\sin(fx+e)\right)}{\sin(fx+e)^{4}-2\sin(fx+e)^{2}+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-120\,ac^{4}\left(\frac{2\sin(fx+e)}{\sin(fx+e)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+240\,ac^{4}\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-720\,ac^{4}\tan\left(fx+e\right)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a*c^4 +
160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^4 + 45*a*c^4*(2*(3*sin(f*x + e)^3
 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
+ e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a*c^4*(2*sin(f*x + e)/(sin(f*x +
 e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*log
(sec(f*x + e) + tan(f*x + e)) - 720*a*c^4*tan(f*x + e))/f
```

Fricas [A]
time = 3.05, size = 141, normalized size = 1.34

$$\frac{105\,ac^{4}\cos\left(fx+e\right)^{5}\log\left(\sin\left(fx+e\right)+1\right)-105\,ac^{4}\cos\left(fx+e\right)^{5}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(136\,ac^{4}\cos\left(fx+e\right)^{4}+15\,ac^{4}\cos\left(fx+e\right)^{3}-112\,ac^{4}\cos\left(fx+e\right)^{2}+90\,ac^{4}\cos\left(fx+e\right)-24\,ac^{4}\right)\sin\left(fx+e\right)}{240\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(105*a*c^4*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 105*a*c^4*cos(f*x +
 e)^5*log(-sin(f*x + e) + 1) - 2*(136*a*c^4*cos(f*x + e)^4 + 15*a*c^4*cos(f
*x + e)^3 - 112*a*c^4*cos(f*x + e)^2 + 90*a*c^4*cos(f*x + e) - 24*a*c^4)*si
n(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ac^{4}\left(\int\sec\left(e+fx\right)dx+\int\left(-3\sec^{2}\left(e+fx\right)\right)dx+\int 2\sec^{3}\left(e+fx\right)dx+\int 2\sec^{4}\left(e+fx\right)dx+\int\left(-3\sec^{5}\left(e+fx\right)\right)dx+\int\sec^{6}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a*c**4*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + Integ
ral(2*sec(e + f*x)**3, x) + Integral(2*sec(e + f*x)**4, x) + Integral(-3*se
c(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A]
time = 0.57, size = 145, normalized size = 1.38

$$\frac{105\,ac^{4}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,ac^{4}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,ac^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+790\,ac^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}-896\,ac^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+490\,ac^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-105\,ac^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{5}}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(105*a*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a*c^4*log(abs(tan
(1/2*f*x + 1/2*e) - 1)) - 2*(105*a*c^4*tan(1/2*f*x + 1/2*e)^9 + 790*a*c^4*t
an(1/2*f*x + 1/2*e)^7 - 896*a*c^4*tan(1/2*f*x + 1/2*e)^5 + 490*a*c^4*tan(1/
2*f*x + 1/2*e)^3 - 105*a*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2
- 1)^5)/f
```

Mupad [B]
time = 6.64, size = 176, normalized size = 1.68

$$\frac{7\,a\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{7\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}+\frac{79\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{6}-\frac{224\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}+\frac{49\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{6}-\frac{7\,a\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (7*a*c^4*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((49*a*c^4*tan(e/2 + (f*x)/2)^3
)/6 - (7*a*c^4*tan(e/2 + (f*x)/2))/4 - (224*a*c^4*tan(e/2 + (f*x)/2)^5)/15
+ (79*a*c^4*tan(e/2 + (f*x)/2)^7)/6 + (7*a*c^4*tan(e/2 + (f*x)/2)^9)/4)/(f*
(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6
 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.2 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=86

$$\frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3ac^3\sec(e+fx)\tan(e+fx)}{8f}-\frac{ac^3\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{2ac^3\tan^3(e+fx)}{3f}$$

```
[Out] 5/8*a*c^3*arctanh(sin(f*x+e))/f-3/8*a*c^3*sec(f*x+e)*tan(f*x+e)/f-1/4*a*c^3
*sec(f*x+e)^3*tan(f*x+e)/f+2/3*a*c^3*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4043, 2691, 3855, 2687, 30, 3853}

$$\frac{2ac^3\tan^3(e+fx)}{3f}+\frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{ac^3\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3ac^3\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (5*a*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a*c^3*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a*c^3*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (2*a*c^3*Tan[e + f*x]^
3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx = -\left((ac)\int \left(c^2\sec(e+fx)\tan^2(e+fx)-2c^2\text{se}\right.\right.$$
$$= -\left(\left(ac^3\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right) - \left(ac\right.$$
$$= -\frac{ac^3\sec(e+fx)\tan(e+fx)}{2f} - \frac{ac^3\sec^3(e+fx}{4}$$
$$= \frac{ac^3\tanh^{-1}(\sin(e+fx))}{2f} - \frac{3ac^3\sec(e+fx)\tan}{8f}$$
$$= \frac{5ac^3\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3ac^3\sec(e+fx)\text{ta}}{8f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 887 vs. 2(86) = 172.
time = 6.49, size = 887, normalized size = 10.31

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] a*((5*Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*Log[Cos[e/2 + (f*x)/2] - Sin[e/2
+ (f*x)/2]]*(c - c*Sec[e + f*x])^3)/(64*f) - (5*Cos[e + f*x]^3*Csc[e/2 + (f
*x)/2]^6*Log[Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^
3)/(64*f) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3)/(1
28*f*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^4) - (Cos[e + f*x]^3*Csc[e/2
 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(24*f*(Cos[e/2] - Sin[e/
2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Csc[e/2
+ (f*x)/2]^6*(c - c*Sec[e + f*x])^3*(Cos[e/2] - 17*Sin[e/2]))/(384*f*(Cos[e
/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^2) + (Cos[e + f*x
]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(12*f*(Cos[e/
2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) - (Cos[e + f*x]^3
*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^3)/(128*f*(Cos[e/2 + (f*x)/2] +
Sin[e/2 + (f*x)/2])^4) - (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e
+ f*x])^3*Sin[(f*x)/2])/(24*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + S
in[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e +
 f*x])^3*(-Cos[e/2] - 17*Sin[e/2]))/(384*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 +
 (f*x)/2] + Sin[e/2 + (f*x)/2])^2) + (Cos[e + f*x]^3*Csc[e/2 + (f*x)/2]^6*(
c - c*Sec[e + f*x])^3*Sin[(f*x)/2])/(12*f*(Cos[e/2] + Sin[e/2])*(Cos[e/2 +
(f*x)/2] + Sin[e/2 + (f*x)/2])))
```

Maple [A]
time = 0.21, size = 111, normalized size = 1.29

method	result
derivativedivides	$\frac{-a\,c^{3}\left(-\left(-\frac{\left(\sec^{3}(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-2a\,c^{3}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(fx+e)\right)}{3}\right)\tan(fx+e}{f}$
default	$\frac{-a\,c^{3}\left(-\left(-\frac{\left(\sec^{3}(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-2a\,c^{3}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(fx+e)\right)}{3}\right)\tan(fx+e}{f}$
norman	$\frac{-\frac{5a\,c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{55a\,c^{3}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}-\frac{73a\,c^{3}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}-\frac{5a\,c^{3}\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{5a\,c^{3}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f}+$
risch	$\frac{ia\,c^{3}\left(9\,e^{7i(fx+e)}-48\,e^{6i(fx+e)}+33\,e^{5i(fx+e)}-48\,e^{4i(fx+e)}-33\,e^{3i(fx+e)}-16\,e^{2i(fx+e)}-9\,e^{i(fx+e)}-16\right)}{12f\left(e^{2i(fx+e)}+1\right)^{4}}-\frac{5a\,c^{3}\ln(e^{i(f}}{8f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+
e)+tan(f*x+e)))-2*a*c^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a*c^3*tan(f*x+
e)+a*c^3*ln(sec(f*x+e)+tan(f*x+e)))
```

Maxima [A]
time = 0.27, size = 144, normalized size = 1.67

$$\frac{32\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)ac^{3}+3\,ac^{3}\left(\frac{2\left(3\,\sin(fx+e)^{3}-5\,\sin(fx+e)\right)}{\sin(fx+e)^{4}-2\,\sin(fx+e)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx+e\right)-1\right)\right)+48\,ac^{3}\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-96\,ac^{3}\tan\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^3 + 3*a*c^3*(2*(3*sin(f*x +
e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(
f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 48*a*c^3*log(sec(f*x + e) + tan(
f*x + e)) - 96*a*c^3*tan(f*x + e))/f
```

Fricas [A]
time = 2.84, size = 126, normalized size = 1.47

$$\frac{15\,ac^3\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-15\,ac^3\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)-2\left(16\,ac^3\cos\left(fx+e\right)^3+9\,ac^3\cos\left(fx+e\right)^2-16\,ac^3\cos\left(fx+e\right)+6\,ac^3\right)\sin\left(fx+e\right)}{48\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(15*a*c^3*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a*c^3*cos(f*x + e)
^4*log(-sin(f*x + e) + 1) - 2*(16*a*c^3*cos(f*x + e)^3 + 9*a*c^3*cos(f*x +
e)^2 - 16*a*c^3*cos(f*x + e) + 6*a*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ac^3\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int 2\sec^2\left(e+fx\right)dx+\int\left(-2\sec^4\left(e+fx\right)\right)dx+\int\sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a*c**3*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + Inte
gral(-2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Giac [A]
time = 0.56, size = 128, normalized size = 1.49

$$\frac{15\,ac^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,ac^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+73\,ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-55\,ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(15*a*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a*c^3*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a*c^3*tan(1/2*f*x + 1/2*e)^7 + 73*a*c^3*tan(1/
```

```
2*f*x + 1/2*e)^5 - 55*a*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a*c^3*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B]
time = 5.20, size = 146, normalized size = 1.70

$$\frac{5\,a\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{5\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}+\frac{73\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}-\frac{55\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{5\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (5*a*c^3*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((5*a*c^3*tan(e/2 + (f*x)/2))/4
 - (55*a*c^3*tan(e/2 + (f*x)/2)^3)/12 + (73*a*c^3*tan(e/2 + (f*x)/2)^5)/12
+ (5*a*c^3*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2
+ (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.3 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=61

$$\frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{ac^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*a*c^2*arctanh(sin(f*x+e))/f-1/2*a*c^2*sec(f*x+e)*tan(f*x+e)/f+1/3*a*c^2
*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4043, 2691, 3855, 2687, 30}

$$\frac{ac^2\tan^3(e+fx)}{3f}+\frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac^2\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a*c^2*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c^2*Sec[e + f*x]*Tan[e + f*x])/(2*
f) + (a*c^2*Tan[e + f*x]^3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = -\left((ac)\int \left(c\sec(e+fx)\tan^2(e+fx) - c\sec^2(e\right.\right.$$

$$= -\left(\left(ac^2\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right) + \left(ac^2\right.$$

$$= -\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}\left(ac^2\right)\int \sec(e$$

$$= \frac{ac^2\tanh^{-1}(\sin(e+fx))}{2f} - \frac{ac^2\sec(e+fx)\tan(e}{2f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(61) = 122.
time = 0.73, size = 313, normalized size = 5.13

$$-\frac{ac^2\sec(e)\sec^3(e+fx)\left(3\cos(2e+3fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+3\cos(4e+3fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+9\cos(fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+9\cos(2e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-3\cos(2e+3fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-3\cos(4e+3fx)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-12\sin(2e+fx)+6\sin(e+2fx)+6\sin(3e+2fx)+4\sin(2e+3fx)\right)}{48f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/48*(a*c^2*Sec[e]*Sec[e + f*x]^3*(3*Cos[2*e + 3*f*x]*Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] + 3*Cos[4*e + 3*f*x]*Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] + 9*Cos[f*x]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]) + 9*Cos[2*e + f*x]*(Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 3*Cos[2*e +
 3*f*x]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 3*Cos[4*e + 3*f*x]*Log[C
os[(e + f*x)/2] + Sin[(e + f*x)/2]] - 12*Sin[2*e + f*x] + 6*Sin[e + 2*f*x]
+ 6*Sin[3*e + 2*f*x] + 4*Sin[2*e + 3*f*x]))/f
```

Maple [A]
time = 0.19, size = 98, normalized size = 1.61

method	result
derivativedivides	$\frac{-a\,c^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-a\,c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,c^2\tan(fx+e)+a\,c^2\ln(}{f}$
default	$\frac{-a\,c^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-a\,c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,c^2\tan(fx+e)+a\,c^2\ln(}{f}$
risch	$\frac{ia\,c^2\left(3\,{\rm e}^{5i(fx+e)}-6\,{\rm e}^{4i(fx+e)}-3\,{\rm e}^{i(fx+e)}-2\right)}{3f\left({\rm e}^{2i(fx+e)}+1\right)^3}+\frac{a\,c^2\ln\left({\rm e}^{i(fx+e)}+i\right)}{2f}-\frac{a\,c^2\ln\left({\rm e}^{i(fx+e)}-i\right)}{2f}$
norman	$\frac{\frac{a\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{8a\,c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{a\,c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{a\,c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a\,c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-a*c^2*(1/2*sec(f*x+e)*tan(f*
x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a*c^2*tan(f*x+e)+a*c^2*ln(sec(f*x+e)+ta
n(f*x+e)))
```

Maxima [A]
time = 0.26, size = 117, normalized size = 1.92

$$\frac{4\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ac^2+3\,ac^2\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+12\,ac^2\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-12\,ac^2\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2 + 3*a*c^2*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 12*a
*c^2*log(sec(f*x + e) + tan(f*x + e)) - 12*a*c^2*tan(f*x + e))/f
```

Fricas [A]
time = 3.33, size = 111, normalized size = 1.82

$$\frac{3\,ac^2\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,ac^2\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)-2\left(2\,ac^2\cos\left(fx+e\right)^2+3\,ac^2\cos\left(fx+e\right)-2\,ac^2\right)\sin\left(fx+e\right)}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*a*c^2*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a*c^2*cos(f*x + e)^3
*log(-sin(f*x + e) + 1) - 2*(2*a*c^2*cos(f*x + e)^2 + 3*a*c^2*cos(f*x + e)
- 2*a*c^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$ac^2\left(\int \sec(e+fx)\,dx + \int \left(-\sec^2(e+fx)\right)\,dx + \int \left(-\sec^3(e+fx)\right)\,dx + \int \sec^4(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a*c**2*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integra
l(-sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(55) = 110.
time = 0.58, size = 111, normalized size = 1.82

$$\frac{3\,ac^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) - 3\,ac^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) - \frac{2\left(3\,ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+8\,ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*a*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a*c^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1)) - 2*(3*a*c^2*tan(1/2*f*x + 1/2*e)^5 + 8*a*c^2*tan(1/2*f*x
 + 1/2*e)^3 - 3*a*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)
/f
```

Mupad [B]
time = 3.81, size = 114, normalized size = 1.87

$$\frac{a\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f} - \frac{a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\frac{8\,a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}-a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a*c^2*atanh(tan(e/2 + (f*x)/2)))/f - ((8*a*c^2*tan(e/2 + (f*x)/2)^3)/3 - a
*c^2*tan(e/2 + (f*x)/2) + a*c^2*tan(e/2 + (f*x)/2)^5)/(f*(3*tan(e/2 + (f*x)
/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

3.4 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=38

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*a*c*arctanh(sin(f*x+e))/f-1/2*a*c*sec(f*x+e)*tan(f*x+e)/f
```

Rubi [A]

time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.107$, Rules used = {4043, 2691, 3855}

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{2f}-\frac{ac\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\left((ac)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)$$
$$= -\frac{ac\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}(ac)\int \sec(e+f$$
$$= \frac{ac\tanh^{-1}(\sin(e+fx))}{2f} - \frac{ac\sec(e+fx)\tan(e+}{2f}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.00

$$-ac\left(-\frac{\tanh^{-1}(\sin(e+fx))}{2f} + \frac{\sec(e+fx)\tan(e+fx)}{2f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
[Out] -(a*c*(-1/2*ArcTanh[Sin[e + f*x]]/f + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))
```

Maple [A]
time = 0.14, size = 58, normalized size = 1.53

method	result	size
derivativedivides	$\frac{-ac\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+ac\ln(\sec(fx+e)+\tan(fx+e))}{f}$	58
default	$\frac{-ac\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+ac\ln(\sec(fx+e)+\tan(fx+e))}{f}$	58
risch	$\frac{iac\left(e^{3i(fx+e)}-e^{i(fx+e)}\right)}{f\left(e^{2i(fx+e)}+1\right)^2}+\frac{ac\ln\left(e^{i(fx+e)}+i\right)}{2f}-\frac{ac\ln\left(e^{i(fx+e)}-i\right)}{2f}$	84
norman	$\frac{-\frac{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{ac\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{ac\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{ac\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*(-a*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a*c*ln(
sec(f*x+e)+tan(f*x+e)))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 1.95

$$\frac{ac\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1} - \log(\sin(fx+e)+1) + \log(\sin(fx+e)-1)\right) + 4\,ac\log(\sec(fx+e)+\tan(fx+e))}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] 1/4*(a*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) + 4*a*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

Fricas [A]
time = 2.90, size = 73, normalized size = 1.92

$$\frac{ac\cos\left(fx+e\right)^{2}\log\left(\sin\left(fx+e\right)+1\right)-ac\cos\left(fx+e\right)^{2}\log\left(-\sin\left(fx+e\right)+1\right)-2\,ac\sin\left(fx+e\right)}{4\,f\cos\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*(a*c*cos(f*x + e)^2*log(sin(f*x + e) + 1) - a*c*cos(f*x + e)^2*log(-sin
(f*x + e) + 1) - 2*a*c*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-ac\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int\sec^{3}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] -a*c*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**3, x))
```

Giac [A]
time = 0.53, size = 55, normalized size = 1.45

$$\frac{ac\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-ac\log\left(\left|\sin\left(fx+e\right)-1\right|\right)+\frac{2\,ac\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*(a*c*log(abs(sin(f*x + e) + 1)) - a*c*log(abs(sin(f*x + e) - 1)) + 2*a*
c*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Mupad [B]
time = 2.24, size = 77, normalized size = 2.03

$$\frac{a\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}+a\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (a*c*atanh(tan(e/2 + (f*x)/2)))/f - (a*c*tan(e/2 + (f*x)/2)^3 + a*c*tan(e/2
 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

3.5 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=42

$$-\frac{a\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -a*arctanh(sin(f*x+e))/c/f-2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4042, 3855}

$$-\frac{a\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[e + f*x]])/(c*f)) - (2*a*Tan[e + f*x])/(f*(c - c*Sec[e + f
*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx &= -\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}-\frac{a\int \sec(e+fx)\,dx}{c}\\ &= -\frac{a\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 1.83

$$-\frac{a\left(-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)}{f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*((-2*Cot[(e + f*x)/2])/f - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f
+ Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f))/c)
```

Maple [A]
time = 0.13, size = 50, normalized size = 1.19

method	result	size
derivativedivides	$\frac{2a\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$	50
default	$\frac{2a\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$	50
risch	$\frac{4ia}{fc\left(e^{i(fx+e)}-1\right)}+\frac{a\ln\left(e^{i(fx+e)}-i\right)}{cf}-\frac{a\ln\left(e^{i(fx+e)}+i\right)}{cf}$	68
norman	$\frac{-\frac{2a}{cf}+\frac{2a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{a\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a/c*(-1/2*ln(tan(1/2*f*x+1/2*e)+1)+1/2*ln(tan(1/2*f*x+1/2*e)-1)+1/tan(1
/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(46) = 92$.
time = 0.27, size = 109, normalized size = 2.60

$$-\frac{a\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}-\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)-\frac{a(\cos(fx+e)+1)}{c\sin(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -(a*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a*(cos(f*x + e)
+ 1)/(c*sin(f*x + e)))/f
```

Fricas [A]
time = 2.50, size = 72, normalized size = 1.71

$$-\frac{a\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-a\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-4\,a\cos\left(fx+e\right)-4\,a}{2\,cf\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] -1/2*(a*log(sin(f*x + e) + 1)*sin(f*x + e) - a*log(-sin(f*x + e) + 1)*sin(f
*x + e) - 4*a*cos(f*x + e) - 4*a)/(c*f*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**2
/(sec(e + f*x) - 1), x))/c
```

Giac [A]
time = 0.57, size = 60, normalized size = 1.43

$$-\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}-\frac{2\,a}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - a*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/c - 2*a/(c*tan(1/2*f*x + 1/2*e)))/f
```

Mupad [B]
time = 1.85, size = 31, normalized size = 0.74

$$-\frac{2\,a\,\left(\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] -(2*a*(atanh(tan(e/2 + (f*x)/2)) - cot(e/2 + (f*x)/2)))/(c*f)
```

3.6 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{(a+a\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] -1/3*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^2
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}$$

Mathematica [A]
time = 0.25, size = 50, normalized size = 1.39

$$\frac{a\csc\left(\frac{e}{2}\right)\csc^3\left(\frac{1}{2}(e+fx)\right)\left(-3\sin\left(e+\frac{fx}{2}\right)+\sin\left(e+\frac{3fx}{2}\right)\right)}{12c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a*Csc[e/2]*Csc[(e + f*x)/2]^3*(-3*Sin[e + (f*x)/2] + Sin[e + (3*f*x)/2]))/
(12*c^2*f)
```

Maple [A]
time = 0.14, size = 21, normalized size = 0.58

method	result	size
derivativedivides	$-\frac{a}{3 f \,c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	21
default	$-\frac{a}{3 f \,c^{2} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	21
risch	$\frac{2 i a \left(3 \,{\mathrm e}^{2 i \left(f x +e \right)}+1\right)}{3 f \,c^{2} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{3}}$	37
norman	$\frac{\frac{a}{3 c f}-\frac{a \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 c f}}{c \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right) \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*a/c^2/tan(1/2*f*x+1/2*e)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(38) = 76$.
time = 0.28, size = 105, normalized size = 2.92

$$-\frac{\frac{a \left(\frac{3 \sin\left(f x+e\right)^{2}}{\left(\cos\left(f x+e\right)+1\right)^{2}}+1\right) \left(\cos\left(f x+e\right)+1\right)^{3}}{c^{2} \sin\left(f x+e\right)^{3}}-\frac{a \left(\frac{3 \sin\left(f x+e\right)^{2}}{\left(\cos\left(f x+e\right)+1\right)^{2}}-1\right) \left(\cos\left(f x+e\right)+1\right)^{3}}{c^{2} \sin\left(f x+e\right)^{3}}}{6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c
^2*sin(f*x + e)^3) - a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x
 + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Fricas [A]
time = 2.39, size = 55, normalized size = 1.53

$$\frac{a \cos\left(f x + e\right)^{2} + 2 \, a \cos\left(f x + e\right) + a}{3 \, {\left(c^{2} f \cos\left(f x + e\right) - c^{2} f\right)} \sin\left(f x + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

[Out] 1/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx+\int \frac{\sec^2(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A]
time = 0.47, size = 20, normalized size = 0.56

$$-\frac{a}{3\,c^2 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/3*a/(c^2*f*tan(1/2*f*x + 1/2*e)^3)
```

Mupad [B]
time = 2.06, size = 20, normalized size = 0.56

$$-\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,c^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] -(a*cot(e/2 + (f*x)/2)^3)/(3*c^2*f)
```

3.7 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=76

$$-\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3}-\frac{(a+a\sec(e+fx))\tan(e+fx)}{15cf(c-c\sec(e+fx))^2}$$

```
[Out] -1/5*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^3-1/15*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^2
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4036, 4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -1/5*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^3) - ((a +
 a*Sec[e + f*x])*Tan[e + f*x])/(15*c*f*(c - c*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx}{5c}$$
$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{15cf(c-c\sec(e+fx))^2}$$

Mathematica [A]
time = 0.34, size = 87, normalized size = 1.14

$$-\frac{a\csc\left(\frac{e}{2}\right)\csc^5\left(\frac{1}{2}(e+fx)\right)\left(25\sin\left(\frac{fx}{2}\right)+15\sin\left(e+\frac{fx}{2}\right)-5\sin\left(e+\frac{3fx}{2}\right)-15\sin\left(2e+\frac{3fx}{2}\right)+4\sin\left(2e+\frac{5fx}{2}\right)\right)}{240c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
[Out] -1/240*(a*Csc[e/2]*Csc[(e + f*x)/2]^5*(25*Sin[(f*x)/2] + 15*Sin[e + (f*x)/2
] - 5*Sin[e + (3*f*x)/2] - 15*Sin[2*e + (3*f*x)/2] + 4*Sin[2*e + (5*f*x)/2]
))/(c^3*f)
```

Maple [A]
time = 0.15, size = 37, normalized size = 0.49

method	result	size
derivativedivides	$\frac{a\left(\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}\right)}{2f\,c^3}$	37
default	$\frac{a\left(\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}\right)}{2f\,c^3}$	37
risch	$\frac{2ia\left(15\,e^{4i(fx+e)}-15\,e^{3i(fx+e)}+25\,e^{2i(fx+e)}-5\,e^{i(fx+e)}+4\right)}{15f\,c^3\left(e^{i(fx+e)}-1\right)^5}$	70
norman	$\frac{-\frac{a}{10cf}+\frac{4a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{a\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/2/f*a/c^3*(1/5/tan(1/2*f*x+1/2*e)^5-1/3/tan(1/2*f*x+1/2*e)^3)
```

Maxima [A]
time = 0.29, size = 127, normalized size = 1.67

$$-\frac{\frac{a\left(\frac{10\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}+\frac{3\,a\left(\frac{5\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/60*(a*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 3*a*(5*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)
^5))/f
```

Fricas [A]
time = 3.27, size = 84, normalized size = 1.11

$$\frac{4\,a\cos\left(fx+e\right)^{3}+7\,a\cos\left(fx+e\right)^{2}+2\,a\cos\left(fx+e\right)-a}{15\,\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*a*cos(f*x + e)^3 + 7*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/((c^3
*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a\left(\int\frac{\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)-3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)-1}\,dx\right)}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e +
f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A]
time = 0.49, size = 37, normalized size = 0.49

$$-\frac{5\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-3\,a}{30\,c^{3}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] -1/30*(5*a*tan(1/2*f*x + 1/2*e)^2 - 3*a)/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

Mupad [B]
time = 1.71, size = 35, normalized size = 0.46

$$\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-5\right)}{30\,c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
[Out] (a*cot(e/2 + (f*x)/2)^3*(3*cot(e/2 + (f*x)/2)^2 - 5))/(30*c^3*f)
```

3.8 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4} dx$

Optimal. Leaf size=116

$$-\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4}-\frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}-\frac{2(a+a\sec(e+fx))\tan(e+fx)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}$$

```
[Out] -1/7*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^4-2/35*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^3-2/105*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c^2
-c^2*sec(f*x+e))^2
```

Rubi [A]
time = 0.11, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4036, 4035}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/7*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4) - (2*(a
 + a*Sec[e + f*x])*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3) - (2*(a +
a*Sec[e + f*x])*Tan[e + f*x])/(105*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx}{7c}$$
$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx}{35cf(c-c\sec(e+fx))^3}$$
$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx}{35cf(c-c\sec(e+fx))^3}$$

Mathematica [A]
time = 0.41, size = 113, normalized size = 0.97

$$-\frac{a\csc\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\left(350\sin\left(\frac{fx}{2}\right)+455\sin\left(e+\frac{fx}{2}\right)-273\sin\left(e+\frac{3fx}{2}\right)-210\sin\left(2e+\frac{3fx}{2}\right)+56\sin\left(2e+\frac{5fx}{2}\right)+105\sin\left(3e+\frac{5fx}{2}\right)-23\sin\left(3e+\frac{7fx}{2}\right)\right)}{6720c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/6720*(a*Csc[e/2]*Csc[(e + f*x)/2]^7*(350*Sin[(f*x)/2] + 455*Sin[e + (f*x
)/2] - 273*Sin[e + (3*f*x)/2] - 210*Sin[2*e + (3*f*x)/2] + 56*Sin[2*e + (5*
f*x)/2] + 105*Sin[3*e + (5*f*x)/2] - 23*Sin[3*e + (7*f*x)/2]))/(c^4*f)
```

Maple [A]
time = 0.17, size = 50, normalized size = 0.43

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}\right)}{4f\,c^4}$	50
default	$\frac{a\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}\right)}{4f\,c^4}$	50
risch	$\frac{2ia\left(105\,e^{6i(fx+e)}-210\,e^{5i(fx+e)}+455\,e^{4i(fx+e)}-350\,e^{3i(fx+e)}+273\,e^{2i(fx+e)}-56\,e^{i(fx+e)}+23\right)}{105f\,c^4\left(e^{i(fx+e)}-1\right)^7}$	92
norman	$\frac{\frac{a}{28cf}-\frac{19a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{140cf}+\frac{11a\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{60cf}-\frac{a\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f*a/c^4*(-1/7/tan(1/2*f*x+1/2*e)^7-1/3/tan(1/2*f*x+1/2*e)^3+2/5/tan(1/2
*f*x+1/2*e)^5)
```

Maxima [A]
time = 0.28, size = 193, normalized size = 1.66

$$\frac{\frac{a\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}+\frac{3a\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}}{840f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e
) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Fricas [A]
time = 4.04, size = 112, normalized size = 0.97

$$\frac{23\,a\cos\left(fx+e\right)^4+36\,a\cos\left(fx+e\right)^3+5\,a\cos\left(fx+e\right)^2-6\,a\cos\left(fx+e\right)+2\,a}{105\,\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*(23*a*cos(f*x + e)^4 + 36*a*cos(f*x + e)^3 + 5*a*cos(f*x + e)^2 - 6*a
*cos(f*x + e) + 2*a)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^
4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a\left(\int\frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f
*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**
4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [A]
time = 0.71, size = 51, normalized size = 0.44

$$-\frac{35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,a}{420\,c^4f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/420*(35*a*tan(1/2*f*x + 1/2*e)^4 - 42*a*tan(1/2*f*x + 1/2*e)^2 + 15*a)/(
c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B]
time = 1.75, size = 61, normalized size = 0.53

$$\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{10\,c^4\,f}-\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12\,c^4\,f}-\frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{28\,c^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

```
[Out] (a*cot(e/2 + (f*x)/2)^5)/(10*c^4*f) - (a*cot(e/2 + (f*x)/2)^3)/(12*c^4*f) -
 (a*cot(e/2 + (f*x)/2)^7)/(28*c^4*f)
```

3.9 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=158

$$-\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5}-\frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4}-\frac{2(a+a\sec(e+fx))\tan(e+fx)}{105c^2f(c-c\sec(e+fx))^3}-2($$

```
[Out] -1/9*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-1/21*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4-2/105*(a+a*sec(f*x+e))*tan(f*x+e)/c^2/f/
(c-c*sec(f*x+e))^3-2/315*(a+a*sec(f*x+e))*tan(f*x+e)/c/f/(c^2-c^2*sec(f*x+e
))^2
```

Rubi [A]
time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4036, 4035}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{315cf\left(c^2-c^2\sec(e+fx)\right)^2}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105c^2f(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{21cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/9*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) - ((a +
 a*Sec[e + f*x])*Tan[e + f*x])/(21*c*f*(c - c*Sec[e + f*x])^4) - (2*(a + a*
Sec[e + f*x])*Tan[e + f*x])/(105*c^2*f*(c - c*Sec[e + f*x])^3) - (2*(a + a*
Sec[e + f*x])*Tan[e + f*x])/(315*c*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx}{3c}$$
$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx}{21cf(c-c\sec(e+fx))^4}$$
$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx}{21cf(c-c\sec(e+fx))^4}$$
$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx}{21cf(c-c\sec(e+fx))^4}$$

Mathematica [A]
time = 0.37, size = 139, normalized size = 0.88

$$-\frac{a\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(3843\sin\left(\frac{fx}{2}\right)+3465\sin\left(e+\frac{fx}{2}\right)-2247\sin\left(e+\frac{3fx}{2}\right)-2625\sin\left(2e+\frac{3fx}{2}\right)+1143\sin\left(2e+\frac{5fx}{2}\right)+945\sin\left(3e+\frac{5fx}{2}\right)-207\sin\left(3e+\frac{7fx}{2}\right)-315\sin\left(4e+\frac{7fx}{2}\right)+58\sin\left(4e+\frac{9fx}{2}\right)\right)}{80640c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/80640*(a*Csc[e/2]*Csc[(e + f*x)/2]^9*(3843*Sin[(f*x)/2] + 3465*Sin[e + (
f*x)/2] - 2247*Sin[e + (3*f*x)/2] - 2625*Sin[2*e + (3*f*x)/2] + 1143*Sin[2*
e + (5*f*x)/2] + 945*Sin[3*e + (5*f*x)/2] - 207*Sin[3*e + (7*f*x)/2] - 315*
Sin[4*e + (7*f*x)/2] + 58*Sin[4*e + (9*f*x)/2]))/(c^5*f)
```

Maple [A]
time = 0.19, size = 63, normalized size = 0.40

method	result
derivativedivides	$\frac{a\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{8f\,c^5}$
default	$\frac{a\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{8f\,c^5}$
risch	$\frac{2ia(315\,e^{8i(fx+e)}-945\,e^{7i(fx+e)}+2625\,e^{6i(fx+e)}-3465\,e^{5i(fx+e)}+3843\,e^{4i(fx+e)}-2247\,e^{3i(fx+e)}+1143\,e^{2i(fx+e)}-207\,e}{315f\,c^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{-\frac{a}{72cf}+\frac{17a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{252cf}-\frac{9a\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{70cf}+\frac{7a\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{60cf}-\frac{a\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f*a/c^5*(1/9/tan(1/2*f*x+1/2*e)^9-1/3/tan(1/2*f*x+1/2*e)^3+3/5/tan(1/2*
f*x+1/2*e)^5-3/7/tan(1/2*f*x+1/2*e)^7)
```

Maxima [A]
time = 0.27, size = 215, normalized size = 1.36

$$-\frac{\frac{a\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{5a\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}}{5040f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="maxi
ma")
```

```
[Out] -1/5040*(a*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)
 + 5*a*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) +
1)^9/(c^5*sin(f*x + e)^9))/f
```

Fricas [A]
time = 3.37, size = 138, normalized size = 0.87

$$\frac{58\,a\cos\left(fx+e\right)^5+83\,a\cos\left(fx+e\right)^4+4\,a\cos\left(fx+e\right)^3-11\,a\cos\left(fx+e\right)^2+8\,a\cos\left(fx+e\right)-2\,a}{315\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="fric
as")
```

```
[Out] 1/315*(58*a*cos(f*x + e)^5 + 83*a*cos(f*x + e)^4 + 4*a*cos(f*x + e)^3 - 11*
a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - 2*a)/((c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*si
n(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a\left(\int\frac{\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx+\int\frac{\sec^2(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e +
 f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e +
```

```
f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(
e + f*x)**2 + 5*sec(e + f*x) - 1), x))/c**5
```

Giac [A]
time = 0.79, size = 65, normalized size = 0.41

$$-\frac{105\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-189\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+135\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-35\,a}{2520\,c^{5}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="giac
")
```

```
[Out] -1/2520*(105*a*tan(1/2*f*x + 1/2*e)^6 - 189*a*tan(1/2*f*x + 1/2*e)^4 + 135*
a*tan(1/2*f*x + 1/2*e)^2 - 35*a)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B]
time = 1.84, size = 106, normalized size = 0.67

$$\frac{a\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(35\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-135\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+189\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-105\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\right)}{2520\,c^5\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (a*cos(e/2 + (f*x)/2)^3*(35*cos(e/2 + (f*x)/2)^6 - 105*sin(e/2 + (f*x)/2)^6
 + 189*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 135*cos(e/2 + (f*x)/2)^4
*sin(e/2 + (f*x)/2)^2))/(2520*c^5*f*sin(e/2 + (f*x)/2)^9)
```

3.10 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

Optimal. Leaf size=171

$$\frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}-\frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{16f}-\frac{3a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^5\sec(e+f}{}$$

```
[Out] 9/16*a^2*c^5*arctanh(sin(f*x+e))/f-3/16*a^2*c^5*sec(f*x+e)*tan(f*x+e)/f-3/8
*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+1/4*a^2*c^5*sec(f*x+e)*tan(f*x+e)^3/f+1/
2*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)^3/f-4/5*a^2*c^5*tan(f*x+e)^5/f-1/7*a^2*c^
5*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.21, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {4043, 2691, 3855, 2687, 30, 3853, 14}

$$-\frac{a^2c^5\tan^7(e+fx)}{7f}-\frac{4a^2c^5\tan^5(e+fx)}{5f}+\frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^5\tan^3(e+fx)\sec^3(e+fx)}{2f}-\frac{3a^2c^5\tan(e+fx)\sec^3(e+fx)}{8f}+\frac{a^2c^5\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^5\tan(e+fx)\sec(e+fx)}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (9*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (3*a^2*c^5*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (3*a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Sec
[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(
2*f) - (4*a^2*c^5*Tan[e + f*x]^5)/(5*f) - (a^2*c^5*Tan[e + f*x]^7)/(7*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = \left(a^2c^2\right)\int \left(c^3\sec(e+fx)\tan^4(e+fx) - 3c^3\sec^2\right.$$
$$= \left(a^2c^5\right)\int \sec(e+fx)\tan^4(e+fx)\,dx - \left(a^2c^5\right)\int$$
$$= \frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^5\sec^3(e+f}{2}$$
$$= -\frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{8f} - \frac{3a^2c^5\sec^3(e}{}$$
$$= \frac{3a^2c^5\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3a^2c^5\sec(e+fx)\,t}{16f}$$
$$= \frac{9a^2c^5\tanh^{-1}(\sin(e+fx))}{16f} - \frac{3a^2c^5\sec(e+fx)\,t}{16f}$$

Mathematica [A]
time = 1.87, size = 102, normalized size = 0.60

$$\frac{a^2c^5(10080\tanh^{-1}(\sin(e+fx))-\sec^7(e+fx)(2520\sin(e+fx)-455\sin(2(e+fx))-616\sin(3(e+fx))+2380\sin(4(e+fx))-392\sin(5(e+fx))+245\sin(6(e+fx))+184\sin(7(e+fx))))}{17920f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*(10080*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(2520*Sin[e + f*x] -
 455*Sin[2*(e + f*x)] - 616*Sin[3*(e + f*x)] + 2380*Sin[4*(e + f*x)] - 392*
Sin[5*(e + f*x)] + 245*Sin[6*(e + f*x)] + 184*Sin[7*(e + f*x)])))/(17920*f)
```

Maple [A]
time = 0.35, size = 299, normalized size = 1.75

method	result
risch	$\frac{ic^5a^2\left(245\,e^{13i(fx+e)}-1680\,e^{12i(fx+e)}+2380\,e^{11i(fx+e)}-4480\,e^{10i(fx+e)}-455\,e^{9i(fx+e)}-3920\,e^{8i(fx+e)}-8960\,e^{6i(fx+ \right.}{280f\left(e^{2i(fx+e)}+1\right)^7}$
norman	$\frac{\frac{9c^5a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}-\frac{15c^5a^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}+\frac{849c^5a^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{40f}-\frac{1152c^5a^2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{35f}+\frac{1199c^5a^2\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{40f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}$
derivativedivides	$c^5a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+3c^5a^2\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-5\right.\right.$
default	$c^5a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)+3c^5a^2\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-5\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(c^5*a^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*
tan(f*x+e)+3*c^5*a^2*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e)
)*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+c^5*a^2*(-8/15-1/5*sec(f*x+e)^
4-4/15*sec(f*x+e)^2)*tan(f*x+e)-5*c^5*a^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+
e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-5*c^5*a^2*(-2/3-1/3*sec(f*x+e
)^2)*tan(f*x+e)+c^5*a^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*
x+e)))-3*c^5*a^2*tan(f*x+e)+c^5*a^2*ln(sec(f*x+e)+tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(168) = 336.
time = 0.28, size = 398, normalized size = 2.33

$$-\frac{96\left(5\tan(fx+e)^7+21\tan(fx+e)^5+35\tan(fx+e)^3+35\tan(fx+e)\right)a^2c^5+224\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^2c^5-5600\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2c^5+105\,a^2c^5\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log(\sin(fx+e)+1)+15\log(\sin(fx+e)-1)\right)-1050\,a^2c^5\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log(\sin(fx+e)-1)\right)+840\,a^2c^5\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-3360\,a^2c^5\log(\sec(fx+e)+\tan(fx+e))+10080\,a^2c^5\tan(fx+e)}{3360\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*
tan(f*x + e))*a^2*c^5 + 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(
f*x + e))*a^2*c^5 - 5600*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 105*a^
2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x
 + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) +
1) + 15*log(sin(f*x + e) - 1)) - 1050*a^2*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(
f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1)
 + 3*log(sin(f*x + e) - 1)) + 840*a^2*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 -
 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3360*a^2*c^5*log(sec
(f*x + e) + tan(f*x + e)) + 10080*a^2*c^5*tan(f*x + e))/f
```

Fricas [A]
time = 2.55, size = 189, normalized size = 1.11

$$\frac{315\,a^2c^5\cos\left(fx+e\right)^7\log\left(\sin\left(fx+e\right)+1\right)-315\,a^2c^5\cos\left(fx+e\right)^7\log\left(-\sin\left(fx+e\right)+1\right)-2\left(368\,a^2c^5\cos\left(fx+e\right)^6+245\,a^2c^5\cos\left(fx+e\right)^5-656\,a^2c^5\cos\left(fx+e\right)^4+350\,a^2c^5\cos\left(fx+e\right)^3+208\,a^2c^5\cos\left(fx+e\right)^2-280\,a^2c^5\cos\left(fx+e\right)+80\,a^2c^5\right)\sin\left(fx+e\right)}{1120\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/1120*(315*a^2*c^5*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 315*a^2*c^5*cos(
f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(368*a^2*c^5*cos(f*x + e)^6 + 245*a^2
*c^5*cos(f*x + e)^5 - 656*a^2*c^5*cos(f*x + e)^4 + 350*a^2*c^5*cos(f*x + e)
^3 + 208*a^2*c^5*cos(f*x + e)^2 - 280*a^2*c^5*cos(f*x + e) + 80*a^2*c^5)*si
n(f*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2c^5\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int 3\sec^2\left(e+fx\right)dx+\int\left(-\sec^3\left(e+fx\right)\right)dx+\int\left(-5\sec^4\left(e+fx\right)\right)dx+\int 5\sec^5\left(e+fx\right)dx+\int\sec^6\left(e+fx\right)dx+\int\left(-3\sec^7\left(e+fx\right)\right)dx+\int\sec^8\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*c**5*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**2, x) + I
ntegral(-sec(e + f*x)**3, x) + Integral(-5*sec(e + f*x)**4, x) + Integral(5
*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x) + Integral(-3*sec(e + f
*x)**7, x) + Integral(sec(e + f*x)**8, x))
```

Giac [A]
time = 0.82, size = 197, normalized size = 1.15

$$\frac{315\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-315\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(315\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-2100\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}-8393\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+9216\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-5943\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+2100\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-315\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^7}}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/560*(315*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 315*a^2*c^5*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(315*a^2*c^5*tan(1/2*f*x + 1/2*e)^13 - 2100
*a^2*c^5*tan(1/2*f*x + 1/2*e)^11 - 8393*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 + 92
16*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 - 5943*a^2*c^5*tan(1/2*f*x + 1/2*e)^5 + 2
100*a^2*c^5*tan(1/2*f*x + 1/2*e)^3 - 315*a^2*c^5*tan(1/2*f*x + 1/2*e))/(tan
(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B]
time = 5.76, size = 251, normalized size = 1.47

$$\frac{-\frac{9\,a^2\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}+\frac{15\,a^2\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{2}+\frac{1199\,a^2\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{40}-\frac{1152\,a^2\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{35}+\frac{849\,a^2\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{40}-\frac{15\,a^2\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{2}+\frac{9\,a^2\,c^5\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{14}-7\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}+21\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-35\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{8}+35\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{6}-21\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{4}+7\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{2}-1\right)}+\frac{9\,a^2\,c^5\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5)/cos(e + f*x),x)
```

```
[Out] ((849*a^2*c^5*tan(e/2 + (f*x)/2)^5)/40 - (15*a^2*c^5*tan(e/2 + (f*x)/2)^3)/
2 - (1152*a^2*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1199*a^2*c^5*tan(e/2 + (f*x)/
2)^9)/40 + (15*a^2*c^5*tan(e/2 + (f*x)/2)^11)/2 - (9*a^2*c^5*tan(e/2 + (f*x
)/2)^13)/8 + (9*a^2*c^5*tan(e/2 + (f*x)/2))/8)/(f*(7*tan(e/2 + (f*x)/2)^2 -
 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^
8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)
^14 - 1)) + (9*a^2*c^5*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

3.11 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=150

$$\frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{5a^2c^4\sec(e+fx)\tan(e+fx)}{16f}-\frac{a^2c^4\sec^3(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^4\sec(e+fx)}{4f}$$

```
[Out] 7/16*a^2*c^4*arctanh(sin(f*x+e))/f-5/16*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f-1/8
*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)/f+1/4*a^2*c^4*sec(f*x+e)*tan(f*x+e)^3/f+1/
6*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)^3/f-2/5*a^2*c^4*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.18, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4043, 2691, 3855, 2687, 30, 3853}

$$-\frac{2a^2c^4\tan^5(e+fx)}{5f}+\frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^2c^4\tan^3(e+fx)\sec^3(e+fx)}{6f}-\frac{a^2c^4\tan(e+fx)\sec^3(e+fx)}{8f}+\frac{a^2c^4\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{5a^2c^4\tan(e+fx)\sec(e+fx)}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (7*a^2*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^2*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^4*Sec[e
 + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6*
f) - (2*a^2*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx &= \left(a^2c^2\right)\int \left(c^2\sec(e+fx)\tan^4(e+fx)-2c^2\sec\right.\\ &= \left(a^2c^4\right)\int \sec(e+fx)\tan^4(e+fx)\,dx+\left(a^2c^4\right)\\ &= \frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f}+\frac{a^2c^4\sec^3(e+}{}\\ &= -\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^2c^4\sec^3(e+}{}\\ &= \frac{3a^2c^4\tanh^{-1}(\sin(e+fx))}{8f}-\frac{5a^2c^4\sec(e+fx)}{16f}\\ &= \frac{7a^2c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{5a^2c^4\sec(e+fx)}{16f}\end{aligned}$$

Mathematica [A]
time = 1.40, size = 91, normalized size = 0.61

$$\frac{a^2c^4\left(1680\tanh^{-1}(\sin(e+fx))+\sec^6(e+fx)(330\sin(e+fx)-240\sin(2(e+fx))-445\sin(3(e+fx))+192\sin(4(e+fx))-135\sin(5(e+fx))-48\sin(6(e+fx)))\right)}{3840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*c^4*(1680*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^6*(330*Sin[e + f*x] - 2
40*Sin[2*(e + f*x)] - 445*Sin[3*(e + f*x)] + 192*Sin[4*(e + f*x)] - 135*Sin
[5*(e + f*x)] - 48*Sin[6*(e + f*x)])))/(3840*f)
```

Maple [A]
time = 0.31, size = 255, normalized size = 1.70

method	result
norman	$\frac{-\frac{7a^2c^4\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f}-\frac{7c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{119c^4a^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}-\frac{231c^4a^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{20f}+\frac{281c^4a^2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{20f}+}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}$
risch	$\frac{ia^2c^4\left(135\,e^{11i(fx+e)}-480\,e^{10i(fx+e)}+445\,e^{9i(fx+e)}-480\,e^{8i(fx+e)}-330\,e^{7i(fx+e)}-960\,e^{6i(fx+e)}+330\,e^{5i(fx+e)}-960\,e^{4}\right.}{120f\left(e^{2i(fx+e)}+1\right)^6}$
derivativedivides	$a^2c^4\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+2c^4a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx\right.}{5}\right.$
default	$a^2c^4\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+2c^4a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(fx\right.}{5}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(a^2*c^4*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*
x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+2*c^4*a^2*(-8/15-1/5*sec(f*x+e)^4-4/15
*sec(f*x+e)^2)*tan(f*x+e)-c^4*a^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(
f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-4*c^4*a^2*(-2/3-1/3*sec(f*x+e)^2)*tan
(f*x+e)-c^4*a^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-2
*c^4*a^2*tan(f*x+e)+c^4*a^2*ln(sec(f*x+e)+tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(148) = 296.
time = 0.28, size = 347, normalized size = 2.31

$$-\frac{64\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^2c^4-640\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c^4+5\,a^2c^4\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)-30\,a^2c^4\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-120\,a^2c^4\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-480\,a^2c^4\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)+960\,a^2c^4\tan\left(fx+e\right)}{480\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] -1/480*(64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4
 - 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 5*a^2*c^4*(2*(15*sin(f*x
```

```
 + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x
+ e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x
+ e) - 1)) - 30*a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e
)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e)
- 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e)
 + 1) + log(sin(f*x + e) - 1)) - 480*a^2*c^4*log(sec(f*x + e) + tan(f*x + e
)) + 960*a^2*c^4*tan(f*x + e))/f
```

Fricas [A]
time = 2.87, size = 172, normalized size = 1.15

$$\frac{105 \, a^{2} c^{4} \cos\left(f x + e\right)^{6} \log\left(\sin\left(f x + e\right) + 1\right) - 105 \, a^{2} c^{4} \cos\left(f x + e\right)^{6} \log\left(-\sin\left(f x + e\right) + 1\right) - 2 \, {\left(96 \, a^{2} c^{4} \cos\left(f x + e\right)^{5} + 135 \, a^{2} c^{4} \cos\left(f x + e\right)^{4} - 192 \, a^{2} c^{4} \cos\left(f x + e\right)^{3} + 10 \, a^{2} c^{4} \cos\left(f x + e\right)^{2} + 96 \, a^{2} c^{4} \cos\left(f x + e\right) - 40 \, a^{2} c^{4}\right)} \sin\left(f x + e\right)}{480 \, f \cos\left(f x + e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*a^2*c^4*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 105*a^2*c^4*cos(f
*x + e)^6*log(-sin(f*x + e) + 1) - 2*(96*a^2*c^4*cos(f*x + e)^5 + 135*a^2*c
^4*cos(f*x + e)^4 - 192*a^2*c^4*cos(f*x + e)^3 + 10*a^2*c^4*cos(f*x + e)^2
+ 96*a^2*c^4*cos(f*x + e) - 40*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2} c^{4} \left(\int \sec{\left(e + f x \right)}\, dx + \int \left(- 2 \sec^{2}{\left(e + f x \right)}\right)\, dx + \int \left(- \sec^{3}{\left(e + f x \right)}\right)\, dx + \int 4 \sec^{4}{\left(e + f x \right)}\, dx + \int \left(- \sec^{5}{\left(e + f x \right)}\right)\, dx + \int \left(- 2 \sec^{6}{\left(e + f x \right)}\right)\, dx + \int \sec^{7}{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*c**4*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + In
tegral(-sec(e + f*x)**3, x) + Integral(4*sec(e + f*x)**4, x) + Integral(-se
c(e + f*x)**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(sec(e + f*x)
**7, x))
```

Giac [A]
time = 0.61, size = 178, normalized size = 1.19

$$\frac{105 \, a^{2} c^{4} \log\left({\left| \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 1 \right|}\right) - 105 \, a^{2} c^{4} \log\left({\left| \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) - 1 \right|}\right) - \frac{2 \, {\left(105 \, a^{2} c^{4} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{11} - 595 \, a^{2} c^{4} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{9} - 1686 \, a^{2} c^{4} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{7} + 1386 \, a^{2} c^{4} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{5} - 595 \, a^{2} c^{4} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} + 105 \, a^{2} c^{4} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right)}}{{\left(\tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{2} - 1\right)}^{6}}}{240 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a^2*c^4*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(105*a^2*c^4*tan(1/2*f*x + 1/2*e)^11 - 595*
```

```
a^2*c^4*tan(1/2*f*x + 1/2*e)^9 - 1686*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 1386
*a^2*c^4*tan(1/2*f*x + 1/2*e)^5 - 595*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 105*
a^2*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B]
time = 5.61, size = 219, normalized size = 1.46

$$\frac{-\frac{7\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}+\frac{119\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{24}+\frac{281\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{20}-\frac{231\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20}+\frac{119\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{24}-\frac{7\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-20\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{7\,a^2\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] ((119*a^2*c^4*tan(e/2 + (f*x)/2)^3)/24 - (231*a^2*c^4*tan(e/2 + (f*x)/2)^5)
/20 + (281*a^2*c^4*tan(e/2 + (f*x)/2)^7)/20 + (119*a^2*c^4*tan(e/2 + (f*x)/
2)^9)/24 - (7*a^2*c^4*tan(e/2 + (f*x)/2)^11)/8 - (7*a^2*c^4*tan(e/2 + (f*x)
/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 +
 (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 +
 (f*x)/2)^12 + 1)) + (7*a^2*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

3.12 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=94

$$\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^2c^3\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{a^2c^3\tan^5(e+f$$

```
[Out] 3/8*a^2*c^3*arctanh(sin(f*x+e))/f-3/8*a^2*c^3*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^2*c^3*sec(f*x+e)*tan(f*x+e)^3/f-1/5*a^2*c^3*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4043, 2691, 3855, 2687, 30}

$$-\frac{a^2c^3\tan^5(e+fx)}{5f}+\frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^3\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^3\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^3*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^3*Tan[e +
 f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \left(a^2c^2\right)\int \left(c\sec(e+fx)\tan^4(e+fx)-c\sec^2(e\right.$$
$$= \left(a^2c^3\right)\int \sec(e+fx)\tan^4(e+fx)\,dx-\left(a^2c^3\right)\int$$
$$= \frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{1}{4}\left(3a^2c^3\right)\int \sec$$
$$= -\frac{3a^2c^3\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^3\sec(e+}{}$$
$$= \frac{3a^2c^3\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^2c^3\sec(e+fx)\mathrm{t}}{8f}$$

Mathematica [A]
time = 0.77, size = 82, normalized size = 0.87

$$\frac{a^2c^3(120\tanh^{-1}(\sin(e+fx))-\sec^5(e+fx)(40\sin(e+fx)+10\sin(2(e+fx))-20\sin(3(e+fx))+25\sin(4(e+fx))+4\sin(5(e+fx))))}{320f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*(120*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^5*(40*Sin[e + f*x] + 10*
Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] + 25*Sin[4*(e + f*x)] + 4*Sin[5*(e +
 f*x)])))/(320*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(86) = 172.
time = 0.27, size = 192, normalized size = 2.04

method	result
risch	$\frac{i c^3 a^2 \left(25 \mathrm{e}^{9 i (f x+e)}-40 \mathrm{e}^{8 i (f x+e)}+10 \mathrm{e}^{7 i (f x+e)}-80 \mathrm{e}^{4 i (f x+e)}-10 \mathrm{e}^{3 i (f x+e)}-25 \mathrm{e}^{i (f x+e)}-8\right)}{20 f \left(\mathrm{e}^{2 i (f x+e)}+1\right)^5}+\frac{3 c^3 a^2 \ln \left(\mathrm{e}^{i (f x+e)}+i\right)}{8 f}$
norman	$\frac{\frac{3 c^3 a^2 \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{4 f}-\frac{7 c^3 a^2 \left(\tan^3\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2 f}+\frac{32 c^3 a^2 \left(\tan^5\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{5 f}+\frac{7 c^3 a^2 \left(\tan^7\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2 f}-\frac{3 c^3 a^2 \left(\tan^9\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{4 f}}{\left(\tan^2\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{3 c^3 a}{}$
derivativedivides	$c^3 a^2 \left(-\frac{8}{15}-\frac{\left(\sec^4(f x+e)\right)}{5}-\frac{4 \left(\sec^2(f x+e)\right)}{15}\right) \tan (f x+e)+c^3 a^2 \left(-\left(-\frac{\left(\sec^3(f x+e)\right)}{4}-\frac{3 \sec (f x+e)}{8}\right) \tan (f x+e)+\frac{3 \ln (\sec (f}{}\right.\right.$
default	$c^3 a^2 \left(-\frac{8}{15}-\frac{\left(\sec^4(f x+e)\right)}{5}-\frac{4 \left(\sec^2(f x+e)\right)}{15}\right) \tan (f x+e)+c^3 a^2 \left(-\left(-\frac{\left(\sec^3(f x+e)\right)}{4}-\frac{3 \sec (f x+e)}{8}\right) \tan (f x+e)+\frac{3 \ln (\sec (f}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(c^3*a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+c^3*a^2*
(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e
)))-2*c^3*a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*c^3*a^2*(1/2*sec(f*x+e)*
tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-c^3*a^2*tan(f*x+e)+c^3*a^2*ln(sec
(f*x+e)+tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.
time = 0.30, size = 245, normalized size = 2.61

$$-\frac{16 \left(3 \tan \left(f x+e\right)^5+10 \tan \left(f x+e\right)^3+15 \tan \left(f x+e\right)\right) a^2 c^3-160 \left(\tan \left(f x+e\right)^3+3 \tan \left(f x+e\right)\right) a^2 c^3+15 a^2 c^3 \left(\frac{2 \left(3 \sin(f x+e)^3-5 \sin(f x+e)\right)}{\sin(f x+e)^4-2 \sin(f x+e)^2+1}-3 \log \left(\sin \left(f x+e\right)+1\right)+3 \log \left(\sin \left(f x+e\right)-1\right)\right)-120 a^2 c^3 \left(\frac{2 \sin(f x+e)}{\sin(f x+e)^2-1}-\log \left(\sin \left(f x+e\right)+1\right)+\log \left(\sin \left(f x+e\right)-1\right)\right)-240 a^2 c^3 \log \left(\sec \left(f x+e\right)+\tan \left(f x+e\right)\right)+240 a^2 c^3 \tan \left(f x+e\right)}{240 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^3
 - 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 15*a^2*c^3*(2*(3*sin(f*x
 + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(s
in(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^3*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 240*
a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 240*a^2*c^3*tan(f*x + e))/f
```

Fricas [A]
time = 2.37, size = 155, normalized size = 1.65

$$\frac{15 a^2 c^3 \cos \left(f x+e\right)^5 \log \left(\sin \left(f x+e\right)+1\right)-15 a^2 c^3 \cos \left(f x+e\right)^5 \log \left(-\sin \left(f x+e\right)+1\right)-2 \left(8 a^2 c^3 \cos \left(f x+e\right)^4+25 a^2 c^3 \cos \left(f x+e\right)^3-16 a^2 c^3 \cos \left(f x+e\right)^2-10 a^2 c^3 \cos \left(f x+e\right)+8 a^2 c^3\right) \sin \left(f x+e\right)}{80 f \cos \left(f x+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*a^2*c^3*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^2*c^3*cos(f*x
+ e)^5*log(-sin(f*x + e) + 1) - 2*(8*a^2*c^3*cos(f*x + e)^4 + 25*a^2*c^3*co
s(f*x + e)^3 - 16*a^2*c^3*cos(f*x + e)^2 - 10*a^2*c^3*cos(f*x + e) + 8*a^2*
c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2c^3\left(\int(-\sec(e+fx))\,dx+\int\sec^2(e+fx)\,dx+\int 2\sec^3(e+fx)\,dx+\int\left(-2\sec^4(e+fx)\right)\,dx+\int\left(-\sec^5(e+fx)\right)\,dx+\int\sec^6(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*c**3*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Int
egral(2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(-s
ec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A]
time = 0.74, size = 159, normalized size = 1.69

$$\frac{15\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-70\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-128\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+70\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/40*(15*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^2*c^3*log(abs(ta
n(1/2*f*x + 1/2*e) - 1)) - 2*(15*a^2*c^3*tan(1/2*f*x + 1/2*e)^9 - 70*a^2*c^
3*tan(1/2*f*x + 1/2*e)^7 - 128*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 70*a^2*c^3*
tan(1/2*f*x + 1/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/
2*e)^2 - 1)^5)/f
```

Mupad [B]
time = 6.50, size = 187, normalized size = 1.99

$$\frac{-\frac{3\,a^2\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}+\frac{7\,a^2\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2}+\frac{32\,a^2\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{7\,a^2\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2}+\frac{3\,a^2\,c^3\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}+\frac{3\,a^2\,c^3\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] ((32*a^2*c^3*tan(e/2 + (f*x)/2)^5)/5 - (7*a^2*c^3*tan(e/2 + (f*x)/2)^3)/2 +
 (7*a^2*c^3*tan(e/2 + (f*x)/2)^7)/2 - (3*a^2*c^3*tan(e/2 + (f*x)/2)^9)/4 +
(3*a^2*c^3*tan(e/2 + (f*x)/2))/4)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 +
 (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 +
(f*x)/2)^10 - 1)) + (3*a^2*c^3*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

3.13 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=73

$$\frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^2c^2\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f}$$

```
[Out] 3/8*a^2*c^2*arctanh(sin(f*x+e))/f-3/8*a^2*c^2*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^2*c^2*sec(f*x+e)*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4043, 2691, 3855}

$$\frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^2\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (3*a^2*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \sec(e+fx)\tan^4(e+fx)\,dx$$
$$= \frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{1}{4}\left(3a^2c^2\right)\int \sec$$
$$= -\frac{3a^2c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^2\sec(e+}{}$$
$$= \frac{3a^2c^2\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3a^2c^2\sec(e+fx)\,t}{8f}$$

Mathematica [A]
time = 0.15, size = 51, normalized size = 0.70

$$\frac{a^2c^2\left(6\tanh^{-1}(\sin(e+fx)) - (1+5\cos(2(e+fx)))\sec^3(e+fx)\tan(e+fx)\right)}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
[Out] (a^2*c^2*(6*ArcTanh[Sin[e + f*x]] - (1 + 5*Cos[2*(e + f*x)])*Sec[e + f*x]^3
*Tan[e + f*x]))/(16*f)
```

Maple [A]
time = 0.19, size = 117, normalized size = 1.60

method	result
derivativedivides	$\frac{a^2c^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-2a^2c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+}{}\right.}{f}$
default	$\frac{a^2c^2\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-2a^2c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+}{}\right.}{f}$
risch	$\frac{ia^2c^2\left(5\,e^{7i(fx+e)}-3\,e^{5i(fx+e)}+3\,e^{3i(fx+e)}-5\,e^{i(fx+e)}\right)}{4f\left(e^{2i(fx+e)}+1\right)^4}+\frac{3a^2c^2\ln\left(e^{i(fx+e)}+i\right)}{8f}-\frac{3a^2c^2\ln\left(e^{i(fx+e)}-i\right)}{8f}$
norman	$\frac{-\frac{3a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{11a^2c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{11a^2c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}-\frac{3a^2c^2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{3a^2c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(a^2*c^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x
+e)+tan(f*x+e)))-2*a^2*c^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan
(f*x+e)))+a^2*c^2*ln(sec(f*x+e)+tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(72) = 144$.
time = 0.27, size = 162, normalized size = 2.22

$$-\frac{a^2c^2\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-8a^2c^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-16a^2c^2\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/16*(a^2*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*s
in(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 8
*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) - 16*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)))/f
```

Fricas [A]
time = 3.76, size = 106, normalized size = 1.45

$$\frac{3\,a^2c^2\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c^2\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)-2\left(5\,a^2c^2\cos\left(fx+e\right)^2-2\,a^2c^2\right)\sin\left(fx+e\right)}{16\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/16*(3*a^2*c^2*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c^2*cos(f*x +
e)^4*log(-sin(f*x + e) + 1) - 2*(5*a^2*c^2*cos(f*x + e)^2 - 2*a^2*c^2)*sin(
f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2c^2\left(\int\sec\left(e+fx\right)dx+\int\left(-2\sec^3\left(e+fx\right)\right)\,dx+\int\sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*c**2*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**3, x) + In
tegral(sec(e + f*x)**5, x))
```

Giac [A]
time = 0.57, size = 87, normalized size = 1.19

$$\frac{3\,a^2c^2\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\,a^2c^2\log\left(\left|\sin\left(fx+e\right)-1\right|\right)+\frac{2\left(5\,a^2c^2\sin(fx+e)^3-3\,a^2c^2\sin(fx+e)\right)}{\left(\sin(fx+e)^2-1\right)^2}}{16\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/16*(3*a^2*c^2*log(abs(sin(f*x + e) + 1)) - 3*a^2*c^2*log(abs(sin(f*x + e)
 - 1)) + 2*(5*a^2*c^2*sin(f*x + e)^3 - 3*a^2*c^2*sin(f*x + e))/(sin(f*x + e
)^2 - 1)^2)/f
```

Mupad [B]
time = 5.21, size = 155, normalized size = 2.12

$$\frac{-\frac{3\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}+\frac{11\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{4}+\frac{11\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{4}-\frac{3\,a^2\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{3\,a^2\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] ((11*a^2*c^2*tan(e/2 + (f*x)/2)^3)/4 + (11*a^2*c^2*tan(e/2 + (f*x)/2)^5)/4
- (3*a^2*c^2*tan(e/2 + (f*x)/2)^7)/4 - (3*a^2*c^2*tan(e/2 + (f*x)/2))/4)/(f
*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6
+ tan(e/2 + (f*x)/2)^8 + 1)) + (3*a^2*c^2*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

3.14 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=61

$$\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f}-\frac{a^2c\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*a^2*c*arctanh(sin(f*x+e))/f-1/2*a^2*c*sec(f*x+e)*tan(f*x+e)/f-1/3*a^2*c
*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4043, 2691, 3855, 2687, 30}

$$-\frac{a^2c\tan^3(e+fx)}{3f}+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f}-\frac{a^2c\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(2*
f) - (a^2*c*Tan[e + f*x]^3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a\sec(e+fx)\tan^2(e+fx)+a\sec^2(e\right.\right.$$
$$= -\left(\left(a^2c\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right) - \left(a^2c\right.$$
$$= -\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}\left(a^2c\right)\int \sec(e$$
$$= \frac{a^2c\tanh^{-1}(\sin(e+fx))}{2f} - \frac{a^2c\sec(e+fx)\tan(e}{2f}$$

Mathematica [A]
time = 0.12, size = 45, normalized size = 0.74

$$\frac{a^2c\left(3\tanh^{-1}(\sin(e+fx)) - 3\sec(e+fx)\tan(e+fx) - 2\tan^3(e+fx)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
[Out] (a^2*c*(3*ArcTanh[Sin[e + f*x]] - 3*Sec[e + f*x]*Tan[e + f*x] - 2*Tan[e + f
*x]^3))/(6*f)
```

Maple [A]
time = 0.19, size = 96, normalized size = 1.57

method	result
derivativedivides	$\frac{a^2c\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^2c\tan(fx+e)+a^2c\ln(\sec(f}{f}$

default	$\frac{a^2c\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^2c\tan(fx+e)+a^2c\ln(\sec(}{f}$
risch	$\frac{ia^2c\left(3\,e^{5i(fx+e)}+6\,e^{4i(fx+e)}-3\,e^{i(fx+e)}+2\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}+\frac{a^2c\ln\left(e^{i(fx+e)}+i\right)}{2f}-\frac{a^2c\ln\left(e^{i(fx+e)}-i\right)}{2f}$
norman	$\frac{\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{8a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{a^2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-a^2*c*(1/2*sec(f*x+e)*tan(f*x
+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^2*c*tan(f*x+e)+a^2*c*ln(sec(f*x+e)+tan
(f*x+e)))
```

Maxima [A]

time = 0.26, size = 117, normalized size = 1.92

$$-\frac{4\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c-3\,a^2c\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-12\,a^2c\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-12\,a^2c\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*
a^2*c*log(sec(f*x + e) + tan(f*x + e)) - 12*a^2*c*tan(f*x + e))/f
```

Fricas [A]

time = 2.50, size = 111, normalized size = 1.82

$$\frac{3\,a^2c\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2\,a^2c\cos\left(fx+e\right)^2-3\,a^2c\cos\left(fx+e\right)-2\,a^2c\right)\sin\left(fx+e\right)}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*a^2*c*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^3
*log(-sin(f*x + e) + 1) + 2*(2*a^2*c*cos(f*x + e)^2 - 3*a^2*c*cos(f*x + e)
- 2*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$-a^2c\left(\int\left(-\sec\left(e+fx\right)\right)\,dx+\int\left(-\sec^2\left(e+fx\right)\right)\,dx+\int\sec^3\left(e+fx\right)dx+\int\sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integ
ral(sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(55) = 110.
time = 0.55, size = 111, normalized size = 1.82

$$\frac{3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-8\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^2*c*log(abs(tan(1/2*f
*x + 1/2*e) - 1)) - 2*(3*a^2*c*tan(1/2*f*x + 1/2*e)^5 - 8*a^2*c*tan(1/2*f*x
 + 1/2*e)^3 - 3*a^2*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)
/f
```

Mupad [B]
time = 3.79, size = 113, normalized size = 1.85

$$\frac{-c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\frac{8\,c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}+\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (a^2*c*tan(e/2 + (f*x)/2) + (8*a^2*c*tan(e/2 + (f*x)/2)^3)/3 - a^2*c*tan(e/
2 + (f*x)/2)^5)/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e
/2 + (f*x)/2)^6 - 1)) + (a^2*c*atanh(tan(e/2 + (f*x)/2)))/f
```

3.15 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{3a^2\tan(e+fx)}{cf}-\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -3*a^2*arctanh(sin(f*x+e))/c/f-3*a^2*tan(f*x+e)/c/f-2*(a^2+a^2*sec(f*x+e))*
tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}=0.156$, Rules used = {4042, 3872, 3855, 3852, 8}

$$-\frac{3a^2\tan(e+fx)}{cf}-\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf}-\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (-3*a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (3*a^2*Tan[e + f*x])/(c*f) - (2*(a^2
 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx &= -\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(3a)\int \sec(e+fx)(a+a\sec}{c} \\ &= -\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(3a^2)\int \sec(e+fx)\,dx}{c} - \frac{(3}{} \\ &= -\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf} - \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))} + \\ &= -\frac{3a^2\tanh^{-1}(\sin(e+fx))}{cf} - \frac{3a^2\tan(e+fx)}{cf} - \frac{2(a^2+a^2\sec(e+f}{f(c-c\sec(}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(74) = 148.
time = 1.89, size = 220, normalized size = 2.97

$$\frac{2a^2\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)\left(4\csc\left(\frac{e}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{fx}{2}\right)+\left(-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{\sin(fx)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (2*a^2*Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]*(4*Csc[e/2]*Sec[(e +
f*x)/2]*Sin[(f*x)/2] + (-3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 3*Log
[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[f*x]/((Cos[e/2] - Sin[e/2])*(Co
s[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])))*Tan[(e + f*x)/2]))/(f*(c - c*Sec[e + f*x]))
```

Maple [A]
time = 0.16, size = 82, normalized size = 1.11

method	result	size
derivativedivides	$\frac{4a^2\left(\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fc}$	82

default	$\frac{4a^2\left(\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fc}$	82
risch	$\frac{2ia^2\left(4\,e^{2i(fx+e)}-e^{i(fx+e)}+5\right)}{fc\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)}+\frac{3a^2\ln\left(e^{i(fx+e)}-i\right)}{cf}-\frac{3a^2\ln\left(e^{i(fx+e)}+i\right)}{cf}$	112
norman	$\frac{\frac{4a^2}{cf}-\frac{10a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{6a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{3a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{3a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a^2/c*(1/tan(1/2*f*x+1/2*e)+1/4/(tan(1/2*f*x+1/2*e)-1)+3/4*ln(tan(1/2*f
*x+1/2*e)-1)+1/4/(tan(1/2*f*x+1/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(80) = 160$.
time = 0.28, size = 243, normalized size = 3.28

$$-\frac{a^2\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+2a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}-\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)-\frac{a^2(\cos(fx+e)+1)}{c\sin(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(a^2*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*x
 + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) + 2*a^2
*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a^2*(cos(f*x + e) +
 1)/(c*sin(f*x + e)))/f
```

Fricas [A]
time = 2.16, size = 118, normalized size = 1.59

$$-\frac{3\,a^2\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-3\,a^2\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-10\,a^2\cos\left(fx+e\right)^2-8\,a^2\cos\left(fx+e\right)+2\,a^2}{2\,cf\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/2*(3*a^2*cos(f*x + e)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*a^2*cos(f*x
 + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) - 10*a^2*cos(f*x + e)^2 - 8*a^2*c
os(f*x + e) + 2*a^2)/(c*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^{2}\left(\int \frac{\sec (e+fx)}{\sec (e+fx)-1}\,dx+\int \frac{2\sec ^{2}(e+fx)}{\sec (e+fx)-1}\,dx+\int \frac{\sec ^{3}(e+fx)}{\sec (e+fx)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(2*sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) - 1),
 x))/c
```

Giac [A]
time = 0.66, size = 100, normalized size = 1.35

$$-\frac{\frac{3\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}-\frac{3\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-2\,a^{2}\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(3*a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - 3*a^2*log(abs(tan(1/2*f*x +
1/2*e) - 1))/c - 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^2)/((tan(1/2*f*x + 1
/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f
```

Mupad [B]
time = 1.91, size = 77, normalized size = 1.04

$$\frac{6\,a^{2}\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-4\,a^{2}}{c\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-1\right)}-\frac{6\,a^{2}\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] (6*a^2*tan(e/2 + (f*x)/2)^2 - 4*a^2)/(c*f*tan(e/2 + (f*x)/2)*(tan(e/2 + (f*
x)/2)^2 - 1)) - (6*a^2*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

3.16 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f}-\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}+\frac{2a^2\tan(e+fx)}{f\,(c^2-c^2\sec(e+fx))}$$

```
[Out] a^2*arctanh(sin(f*x+e))/c^2/f-2/3*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/f/(c-c*se
c(f*x+e))^2+2*a^2*tan(f*x+e)/f/(c^2-c^2*sec(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4042, 3855}

$$\frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{2a^2\tan(e+fx)}{f\,(c^2-c^2\sec(e+fx))}-\frac{2\tan(e+fx)\,(a^2\sec(e+fx)+a^2)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[e + f*x]])/(c^2*f) - (2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(3*f*(c - c*Sec[e + f*x])^2) + (2*a^2*Tan[e + f*x])/(f*(c^2 - c^2*Sec[
e + f*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = -\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx}{c}$$

$$= -\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{2a^2\tan(e+fx)}{f\left(c^2-c^2\sec(e+fx)\right)} + \frac{a}{}$$

$$= \frac{a^2\tanh^{-1}(\sin(e+fx))}{c^2f} - \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{}{f(}$$

Mathematica [A]
time = 0.09, size = 109, normalized size = 1.22

$$\frac{a^2\left(-\frac{4\cot\left(\frac{1}{2}(e+fx)\right)}{3f} - \frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{3f} - \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f} + \frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*((-4*Cot[(e + f*x)/2])/(3*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)
/(3*f) - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f))/c^2
```

Maple [A]
time = 0.16, size = 67, normalized size = 0.75

method	result
derivativedivides	$\frac{2a^2\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2} - \frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3} - \frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)} + \frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,c^2}$
default	$\frac{2a^2\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2} - \frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3} - \frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)} + \frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,c^2}$
risch	$\frac{8ia^2\left(3\,e^{i(fx+e)}-1\right)}{3f\,c^2\left(e^{i(fx+e)}-1\right)^3} + \frac{a^2\ln\left(e^{i(fx+e)}+i\right)}{c^2f} - \frac{a^2\ln\left(e^{i(fx+e)}-i\right)}{c^2f}$
norman	$\frac{-\frac{2a^2}{3cf} - \frac{2a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf} + \frac{10a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf} - \frac{2a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2 c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3} + \frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{c^2f} - \frac{a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{c^2f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 2/f*a^2/c^2*(-1/2*ln(tan(1/2*f*x+1/2*e)-1)-1/3/tan(1/2*f*x+1/2*e)^3-1/tan(1
/2*f*x+1/2*e)+1/2*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(95) = 190$.
time = 0.28, size = 217, normalized size = 2.44

$$\frac{a^2\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2} - \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2} - \frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right) - \frac{2a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3} + \frac{a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(a^2*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 2*a^2*(3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + a^2*(3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x +
e)^3))/f
```

Fricas [A]
time = 2.61, size = 138, normalized size = 1.55

$$-\frac{8a^2\cos(fx+e)^2 - 8a^2\cos(fx+e) - 3(a^2\cos(fx+e)-a^2)\log(\sin(fx+e)+1)\sin(fx+e) + 3(a^2\cos(fx+e)-a^2)\log(-\sin(fx+e)+1)\sin(fx+e) - 16a^2}{6(c^2f\cos(fx+e)-c^2f)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(8*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) - 3*(a^2*cos(f*x + e) - a^2
)*log(sin(f*x + e) + 1)*sin(f*x + e) + 3*(a^2*cos(f*x + e) - a^2)*log(-sin(
f*x + e) + 1)*sin(f*x + e) - 16*a^2)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\frac{\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int\frac{2\sec^2(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int\frac{\sec^3(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + In
tegral(2*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A]
time = 0.64, size = 84, normalized size = 0.94

$$\frac{\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2}-\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2}-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^2\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 3*a^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/c^2 - 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 + a^2)/(c^2*tan(1/2
*f*x + 1/2*e)^3))/f
```

Mupad [B]
time = 1.78, size = 63, normalized size = 0.71

$$\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^2\,f}-\frac{2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\frac{2\,a^2}{3}}{c^2\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (2*a^2*atanh(tan(e/2 + (f*x)/2)))/(c^2*f) - (2*a^2*tan(e/2 + (f*x)/2)^2 + (
2*a^2)/3)/(c^2*f*tan(e/2 + (f*x)/2)^3)
```

3.17 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=38

$$-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}$$

```
[Out] -1/5*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^3
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{4035\}$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -1/5*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}$$

Mathematica [A]
time = 0.12, size = 25, normalized size = 0.66

$$\frac{a^2\cot^5\left(\frac{1}{2}(e+fx)\right)}{5c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*Cot[(e + f*x)/2]^5)/(5*c^3*f)
```

Maple [A]
time = 0.18, size = 23, normalized size = 0.61

method	result	size
derivativedivides	$\frac{a^{2}}{5 f\,c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	23
default	$\frac{a^{2}}{5 f\,c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	23
risch	$\frac{2 i a^{2} \left(5\,{\mathrm e}^{4 i (f x+e)}+10\,{\mathrm e}^{2 i (f x+e)}+1\right)}{5 f\,c^{3} \left({\mathrm e}^{i (f x+e)}-1\right)^{5}}$	50
norman	$\frac{\frac{a^{2}}{5 c f}-\frac{2 a^{2} \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{5 c f}+\frac{a^{2} \left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{5 c f}}{\left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{2} c^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/5/f*a^2/c^3/tan(1/2*f*x+1/2*e)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(40) = 80$.
time = 0.28, size = 205, normalized size = 5.39

$$\frac{\frac{a^{2} \left(\frac{10 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{15 \sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+3\right) (\cos(fx+e)+1)^{5}}{c^{3} \sin(fx+e)^{5}}-\frac{a^{2} \left(\frac{10 \sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{15 \sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-3\right) (\cos(fx+e)+1)^{5}}{c^{3} \sin(fx+e)^{5}}-\frac{6\,a^{2} \left(\frac{5 \sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-1\right) (\cos(fx+e)+1)^{5}}{c^{3} \sin(fx+e)^{5}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(a^2*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - a^2*(10*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - 6*a^2*(5*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(40) = 80$.
time = 2.50, size = 89, normalized size = 2.34

$$\frac{a^{2} \cos\left(f x+e\right)^{3}+3\,a^{2} \cos\left(f x+e\right)^{2}+3\,a^{2} \cos\left(f x+e\right)+a^{2}}{5\,\left(c^{3} f \cos\left(f x+e\right)^{2}-2\,c^{3} f \cos\left(f x+e\right)+c^{3} f\right) \sin\left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)/
((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{2\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int \frac{\sec^3(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) - 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)*
*3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A]
time = 0.54, size = 22, normalized size = 0.58

$$\frac{a^2}{5\,c^3 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/5*a^2/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

Mupad [B]
time = 1.64, size = 22, normalized size = 0.58

$$\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (a^2*cot(e/2 + (f*x)/2)^5)/(5*c^3*f)
```

3.18 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=80

$$-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4}-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}$$

```
[Out] -1/7*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^4-1/35*(a+a*sec(f*x+e
))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^3
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4036, 4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/7*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4) - ((a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx}{7c}$$
$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{(a+a\sec(e+fx))^2\tan(e+}{35cf(c-c\sec(e+fx))^3}$$

Mathematica [A]
time = 0.45, size = 115, normalized size = 1.44

$$-\frac{a^2\csc\left(\frac{e}{2}\right)\csc^7\left(\frac{1}{2}(e+fx)\right)\left(70\sin\left(\frac{fx}{2}\right)+140\sin\left(e+\frac{fx}{2}\right)-91\sin\left(e+\frac{3fx}{2}\right)-35\sin\left(2e+\frac{3fx}{2}\right)+7\sin\left(2e+\frac{5fx}{2}\right)+35\sin\left(3e+\frac{5fx}{2}\right)-6\sin\left(3e+\frac{7fx}{2}\right)\right)}{2240c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/2240*(a^2*Csc[e/2]*Csc[(e + f*x)/2]^7*(70*Sin[(f*x)/2] + 140*Sin[e + (f*
x)/2] - 91*Sin[e + (3*f*x)/2] - 35*Sin[2*e + (3*f*x)/2] + 7*Sin[2*e + (5*f*
x)/2] + 35*Sin[3*e + (5*f*x)/2] - 6*Sin[3*e + (7*f*x)/2]))/(c^4*f)
```

Maple [A]
time = 0.18, size = 39, normalized size = 0.49

method	result	size
derivativedivides	$\frac{a^2\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}\right)}{2f\,c^4}$	39
default	$\frac{a^2\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}\right)}{2f\,c^4}$	39
risch	$\frac{2ia^2\left(35\,e^{6i(fx+e)}-35\,e^{5i(fx+e)}+140\,e^{4i(fx+e)}-70\,e^{3i(fx+e)}+91\,e^{2i(fx+e)}-7\,e^{i(fx+e)}+6\right)}{35f\,c^4\left(e^{i(fx+e)}-1\right)^7}$	94
norman	$\frac{-\frac{a^2}{14cf}+\frac{17a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{70cf}-\frac{19a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{70cf}+\frac{a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f*a^2/c^4*(-1/7/tan(1/2*f*x+1/2*e)^7+1/5/tan(1/2*f*x+1/2*e)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(84) = 168.
time = 0.31, size = 294, normalized size = 3.68

$$\frac{\frac{2a^2\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}+\frac{3a^2\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}-\frac{a^2\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}}{840\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(2*a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x
 + e) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a^2*(21*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^2*(21*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^4*sin
(f*x + e)^7))/f
```

Fricas [A]
time = 1.74, size = 122, normalized size = 1.52

$$\frac{6\,a^2\cos\left(fx+e\right)^4+17\,a^2\cos\left(fx+e\right)^3+15\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)-a^2}{35\,\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/35*(6*a^2*cos(f*x + e)^4 + 17*a^2*cos(f*x + e)^3 + 15*a^2*cos(f*x + e)^2
+ 3*a^2*cos(f*x + e) - a^2)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2
 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\left(\int\frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^3\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
+ f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f
*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [A]
time = 0.58, size = 41, normalized size = 0.51

$$\frac{7\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-5\,a^2}{70\,c^4f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/70*(7*a^2*tan(1/2*f*x + 1/2*e)^2 - 5*a^2)/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B]
time = 1.62, size = 37, normalized size = 0.46

$$-\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(5\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-7\right)}{70\,c^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

```
[Out] -(a^2*cot(e/2 + (f*x)/2)^5*(5*cot(e/2 + (f*x)/2)^2 - 7))/(70*c^4*f)
```

3.19 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=121

$$-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5}-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{315c^2f(c-c\sec(e+fx))^3}$$

```
[Out] -1/9*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-2/63*(a+a*sec(f*x+e
))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4-2/315*(a+a*sec(f*x+e))^2*tan(f*x+e)/
c^2/f/(c-c*sec(f*x+e))^3
```

Rubi [A]
time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4036, 4035}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{315c^2f(c-c\sec(e+fx))^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/9*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) - (2*
(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4) - (2*(
a + a*Sec[e + f*x])^2*Tan[e + f*x])/(315*c^2*f*(c - c*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx}{9c}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^2\tan(e}{63cf(c-c\sec(e+fx))}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^2\tan(e}{63cf(c-c\sec(e+fx))}$$

Mathematica [A]
time = 0.43, size = 141, normalized size = 1.17

$$-\frac{a^2\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(3402\sin\left(\frac{fx}{2}\right)+2520\sin\left(e+\frac{fx}{2}\right)-1638\sin\left(e+\frac{3fx}{2}\right)-2310\sin\left(2e+\frac{3fx}{2}\right)+1062\sin\left(2e+\frac{5fx}{2}\right)+630\sin\left(3e+\frac{5fx}{2}\right)-108\sin\left(3e+\frac{7fx}{2}\right)-315\sin\left(4e+\frac{7fx}{2}\right)+47\sin\left(4e+\frac{9fx}{2}\right)\right)}{80640c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/80640*(a^2*Csc[e/2]*Csc[(e + f*x)/2]^9*(3402*Sin[(f*x)/2] + 2520*Sin[e +
 (f*x)/2] - 1638*Sin[e + (3*f*x)/2] - 2310*Sin[2*e + (3*f*x)/2] + 1062*Sin[
2*e + (5*f*x)/2] + 630*Sin[3*e + (5*f*x)/2] - 108*Sin[3*e + (7*f*x)/2] - 31
5*Sin[4*e + (7*f*x)/2] + 47*Sin[4*e + (9*f*x)/2]))/(c^5*f)
```

Maple [A]
time = 0.21, size = 52, normalized size = 0.43

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{4f\,c^5}$
default	$\frac{a^2\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{4f\,c^5}$
risch	$\frac{2ia^2\left(315\,e^{8i(fx+e)}-630\,e^{7i(fx+e)}+2310\,e^{6i(fx+e)}-2520\,e^{5i(fx+e)}+3402\,e^{4i(fx+e)}-1638\,e^{3i(fx+e)}+1062\,e^{2i(fx+e)}-1\right.}{315f\,c^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{a^2}{36cf}-\frac{8a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{63cf}+\frac{139a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{630cf}-\frac{6a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{35cf}+\frac{a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{20cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f*a^2/c^5*(1/9/tan(1/2*f*x+1/2*e)^9+1/5/tan(1/2*f*x+1/2*e)^5-2/7/tan(1/
2*f*x+1/2*e)^7)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(127) = 254$.
time = 0.31, size = 293, normalized size = 2.42

$$-\frac{\frac{a^2\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{10\,a^2\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{7\,a^2\left(\frac{18\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{45\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-5\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}}{5040\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/5040*(a^2*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^
9) + 10*a^2*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
e) + 1)^9/(c^5*sin(f*x + e)^9) + 7*a^2*(18*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5
*sin(f*x + e)^9))/f
```

Fricas [A]
time = 3.23, size = 150, normalized size = 1.24

$$\frac{47\,a^2\cos\left(fx+e\right)^5+127\,a^2\cos\left(fx+e\right)^4+101\,a^2\cos\left(fx+e\right)^3+11\,a^2\cos\left(fx+e\right)^2-8\,a^2\cos\left(fx+e\right)+2\,a^2}{315\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/315*(47*a^2*cos(f*x + e)^5 + 127*a^2*cos(f*x + e)^4 + 101*a^2*cos(f*x + e
)^3 + 11*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) + 2*a^2)/((c^5*f*cos(f*x +
 e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x +
 e) + c^5*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\left(\int\frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{2\sec^2\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^3\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(
e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(2*sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
```

```
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(
e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 +
 5*sec(e + f*x) - 1), x))/c**5
```

Giac [A]
time = 0.60, size = 57, normalized size = 0.47

$$\frac{63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-90\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^2}{1260\,c^5f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/1260*(63*a^2*tan(1/2*f*x + 1/2*e)^4 - 90*a^2*tan(1/2*f*x + 1/2*e)^2 + 35*
a^2)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B]
time = 1.63, size = 67, normalized size = 0.55

$$\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20\,c^5\,f}-\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{14\,c^5\,f}+\frac{a^2\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{36\,c^5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (a^2*cot(e/2 + (f*x)/2)^5)/(20*c^5*f) - (a^2*cot(e/2 + (f*x)/2)^7)/(14*c^5*
f) + (a^2*cot(e/2 + (f*x)/2)^9)/(36*c^5*f)
```

3.20 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=163

$$-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6}-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5}-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{231c^2f(c-c\sec(e+fx))^4}-2$$

```
[Out] -1/11*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^6-1/33*(a+a*sec(f*x+
e))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^5-2/231*(a+a*sec(f*x+e))^2*tan(f*x+e)
/c^2/f/(c-c*sec(f*x+e))^4-2/1155*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c^2-c^2*s
ec(f*x+e))^3
```

Rubi [A]
time = 0.22, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4036, 4035}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\left(c^2-c^2\sec(e+fx)\right)^3}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{231c^2f(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{33cf(c-c\sec(e+fx))^5}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/11*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^6) - ((
a + a*Sec[e + f*x])^2*Tan[e + f*x])/(33*c*f*(c - c*Sec[e + f*x])^5) - (2*(a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(231*c^2*f*(c - c*Sec[e + f*x])^4) - (2*
(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*(c^2 - c^2*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} + \frac{3\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx}{11c}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+}{33cf(c-c\sec(e+fx))^5}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+}{33cf(c-c\sec(e+fx))^5}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+}{33cf(c-c\sec(e+fx))^5}$$

Mathematica [A]

time = 0.66, size = 167, normalized size = 1.02

$$-\frac{a^2\csc\left(\frac{e}{2}\right)\csc^{11}\left(\frac{1}{2}(e+fx)\right)\left(32802\sin\left(\frac{fx}{2}\right)+37422\sin\left(e+\frac{fx}{2}\right)-27060\sin\left(e+\frac{3fx}{2}\right)-23100\sin\left(2e+\frac{3fx}{2}\right)+11220\sin\left(2e+\frac{5fx}{2}\right)+13860\sin\left(3e+\frac{5fx}{2}\right)-4895\sin\left(3e+\frac{7fx}{2}\right)-3465\sin\left(4e+\frac{7fx}{2}\right)+517\sin\left(4e+\frac{9fx}{2}\right)+1155\sin\left(5e+\frac{9fx}{2}\right)-152\sin\left(5e+\frac{11fx}{2}\right)\right)}{1182720c^6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/1182720*(a^2*Csc[e/2]*Csc[(e + f*x)/2]^11*(32802*Sin[(f*x)/2] + 37422*Si
n[e + (f*x)/2] - 27060*Sin[e + (3*f*x)/2] - 23100*Sin[2*e + (3*f*x)/2] + 11
220*Sin[2*e + (5*f*x)/2] + 13860*Sin[3*e + (5*f*x)/2] - 4895*Sin[3*e + (7*f
*x)/2] - 3465*Sin[4*e + (7*f*x)/2] + 517*Sin[4*e + (9*f*x)/2] + 1155*Sin[5*
e + (9*f*x)/2] - 152*Sin[5*e + (11*f*x)/2]))/(c^6*f)
```

Maple [A]

time = 0.22, size = 65, normalized size = 0.40

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}\right)}{8fc^6}$
default	$\frac{a^2\left(\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}\right)}{8fc^6}$
risch	$\frac{2ia^2\left(1155\,e^{10i(fx+e)}-3465\,e^{9i(fx+e)}+13860\,e^{8i(fx+e)}-23100\,e^{7i(fx+e)}+37422\,e^{6i(fx+e)}-32802\,e^{5i(fx+e)}+27060\,e^{4i(}\right.}{1155fc^6\left(e^{i(fx+e)}-1\right)^{11}}$
norman	$\frac{-\frac{a^2}{88cf}+\frac{17a^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{264cf}-\frac{137a^2\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{924cf}+\frac{73a^2\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{420cf}-\frac{29a^2\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{280cf}+\frac{a^2\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{40cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/f*a^2/c^6*(1/5/tan(1/2*f*x+1/2*e)^5+1/3/tan(1/2*f*x+1/2*e)^9-3/7/tan(1/
2*f*x+1/2*e)^7-1/11/tan(1/2*f*x+1/2*e)^11)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(171) = 342$.
time = 0.31, size = 425, normalized size = 2.61

$$\frac{\frac{a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{990\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{1386\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{3465\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-315\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}+\frac{6a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{330\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{462\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{1155\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-105\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}+\frac{5a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{990\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{1386\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{693\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-63\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}}{110880f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/110880*(a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 6*a^2*(385*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x + e) + 1)
^11/(c^6*sin(f*x + e)^11) + 5*a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

Fricas [A]
time = 2.58, size = 180, normalized size = 1.10

$$\frac{152\,a^2\cos(fx+e)^6+395\,a^2\cos(fx+e)^5+289\,a^2\cos(fx+e)^4+15\,a^2\cos(fx+e)^3-19\,a^2\cos(fx+e)^2+10\,a^2\cos(fx+e)-2\,a^2}{1155\,\left(c^6f\cos(fx+e)^5-5\,c^6f\cos(fx+e)^4+10\,c^6f\cos(fx+e)^3-10\,c^6f\cos(fx+e)^2+5\,c^6f\cos(fx+e)-c^6f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/1155*(152*a^2*cos(f*x + e)^6 + 395*a^2*cos(f*x + e)^5 + 289*a^2*cos(f*x +
 e)^4 + 15*a^2*cos(f*x + e)^3 - 19*a^2*cos(f*x + e)^2 + 10*a^2*cos(f*x + e)
 - 2*a^2)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*
x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{2}\left(\int \frac{\sec\left(e+fx\right)}{\sec^{6}\left(e+fx\right)-6\sec^{5}\left(e+fx\right)+15\sec^{4}\left(e+fx\right)-20\sec^{3}\left(e+fx\right)+15\sec^{2}\left(e+fx\right)-6\sec\left(e+fx\right)+1}\,dx+\int \frac{2\sec^{2}\left(e+fx\right)}{\sec^{6}\left(e+fx\right)-6\sec^{5}\left(e+fx\right)+15\sec^{4}\left(e+fx\right)-20\sec^{3}\left(e+fx\right)+15\sec^{2}\left(e+fx\right)-6\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^{3}\left(e+fx\right)}{\sec^{6}\left(e+fx\right)-6\sec^{5}\left(e+fx\right)+15\sec^{4}\left(e+fx\right)-20\sec^{3}\left(e+fx\right)+15\sec^{2}\left(e+fx\right)-6\sec\left(e+fx\right)+1}\,dx\right)}{c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e
 + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),
 x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*
sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x)
+ 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 +
15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*
x) + 1), x))/c**6
```

Giac [A]
time = 0.65, size = 73, normalized size = 0.45

$$\frac{231\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-495\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+385\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-105\,a^{2}}{9240\,c^{6}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/9240*(231*a^2*tan(1/2*f*x + 1/2*e)^6 - 495*a^2*tan(1/2*f*x + 1/2*e)^4 + 3
85*a^2*tan(1/2*f*x + 1/2*e)^2 - 105*a^2)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B]
time = 1.69, size = 108, normalized size = 0.66

$$-\frac{a^{2}\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}\left(105\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}-385\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+495\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}-231\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}\right)}{9240\,c^{6}\,f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^6),x)
```

```
[Out] -(a^2*cos(e/2 + (f*x)/2)^5*(105*cos(e/2 + (f*x)/2)^6 - 231*sin(e/2 + (f*x)/
2)^6 + 495*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 385*cos(e/2 + (f*x)/
2)^4*sin(e/2 + (f*x)/2)^2))/(9240*c^6*f*sin(e/2 + (f*x)/2)^11)
```

3.21 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$

Optimal. Leaf size=227

$$\frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{25a^3c^6\sec(e+fx)\tan(e+fx)}{128f}-\frac{15a^3c^6\sec^3(e+fx)\tan(e+fx)}{64f}+\frac{5a^3c^6\sec(e$$

```
[Out] 55/128*a^3*c^6*arctanh(sin(f*x+e))/f-25/128*a^3*c^6*sec(f*x+e)*tan(f*x+e)/f
-15/64*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)/f+5/24*a^3*c^6*sec(f*x+e)*tan(f*x+e)
^3/f+5/16*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)^3/f-1/6*a^3*c^6*sec(f*x+e)*tan(f*
x+e)^5/f-3/8*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)^5/f+4/7*a^3*c^6*tan(f*x+e)^7/f
+1/9*a^3*c^6*tan(f*x+e)^9/f
```

Rubi [A]
time = 0.26, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {4043, 2691, 3855, 2687, 30, 3853, 14}

$$\frac{a^3c^6\tan^9(e+fx)}{9f}+\frac{4a^3c^6\tan^7(e+fx)}{7f}+\frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{3a^3c^6\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^6\tan^3(e+fx)\sec^3(e+fx)}{16f}-\frac{15a^3c^6\tan(e+fx)\sec^3(e+fx)}{64f}-\frac{a^3c^6\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^6\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{25a^3c^6\tan(e+fx)\sec(e+fx)}{128f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

```
[Out] (55*a^3*c^6*ArcTanh[Sin[e + f*x]])/(128*f) - (25*a^3*c^6*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (15*a^3*c^6*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3
*c^6*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^6*Sec[e + f*x]^3*Tan[e
+ f*x]^3)/(16*f) - (a^3*c^6*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (3*a^3*c^6
*Sec[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (4*a^3*c^6*Tan[e + f*x]^7)/(7*f) +
(a^3*c^6*Tan[e + f*x]^9)/(9*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx = -\left(\left(a^3c^3\right)\int \left(c^3\sec(e+fx)\tan^6(e+fx)-3c^3\right.\right.$$

$$= -\left(\left(a^3c^6\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)+(a$$

$$= -\frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f}-\frac{3a^3c^6\sec^3(e+}{}$$

$$= \frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f}+\frac{5a^3c^6\sec^3(e+}{}$$

$$= -\frac{5a^3c^6\sec(e+fx)\tan(e+fx)}{16f}-\frac{15a^3c^6\sec^3(e}{}$$

$$= \frac{5a^3c^6\tanh^{-1}(\sin(e+fx))}{16f}-\frac{25a^3c^6\sec(e+fx)}{128f}$$

$$= \frac{55a^3c^6\tanh^{-1}(\sin(e+fx))}{128f}-\frac{25a^3c^6\sec(e+fx}{128f}$$

Mathematica [A]

time = 3.26, size = 122, normalized size = 0.54

$$\frac{a^3c^6(443520\tanh^{-1}(\sin(e+fx))-\sec^9(e+fx)(-88704\sin(e+fx)+88074\sin(2(e+fx))+37632\sin(3(e+fx))-2142\sin(4(e+fx))+2304\sin(5(e+fx))+39858\sin(6(e+fx))-7488\sin(7(e+fx))+4599\sin(8(e+fx))+1856\sin(9(e+fx))))}{1032192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

```
[Out] (a^3*c^6*(443520*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^9*(-88704*Sin[e + f*x
] + 88074*Sin[2*(e + f*x)] + 37632*Sin[3*(e + f*x)] - 2142*Sin[4*(e + f*x)]
 + 2304*Sin[5*(e + f*x)] + 39858*Sin[6*(e + f*x)] - 7488*Sin[7*(e + f*x)] +
 4599*Sin[8*(e + f*x)] + 1856*Sin[9*(e + f*x)])))/(1032192*f)
```

Maple [A]

time = 0.35, size = 345, normalized size = 1.52

method	result
risch	$\frac{ia^3c^6\left(4599\,e^{17i(fx+e)}-24192\,e^{16i(fx+e)}+39858\,e^{15i(fx+e)}-64512\,e^{14i(fx+e)}-2142\,e^{13i(fx+e)}-118272\,e^{12i(fx+e)}+8807\right.}{}$
derivativedivides	$\frac{-a^3c^6\left(-\frac{128}{315}-\frac{\left(\sec^8(fx+e)\right)}{9}-\frac{8\left(\sec^6(fx+e)\right)}{63}-\frac{16\left(\sec^4(fx+e)\right)}{105}-\frac{64\left(\sec^2(fx+e)\right)}{315}\right)\tan(fx+e)-3a^3c^6\tan(fx+e)+a^3c^6\ln(}{}$
default	$\frac{-a^3c^6\left(-\frac{128}{315}-\frac{\left(\sec^8(fx+e)\right)}{9}-\frac{8\left(\sec^6(fx+e)\right)}{63}-\frac{16\left(\sec^4(fx+e)\right)}{105}-\frac{64\left(\sec^2(fx+e)\right)}{315}\right)\tan(fx+e)-3a^3c^6\tan(fx+e)+a^3c^6\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-a^3*c^6*(-128/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e
)^4-64/315*sec(f*x+e)^2)*tan(f*x+e)-3*a^3*c^6*tan(f*x+e)+a^3*c^6*ln(sec(f*x
+e)+tan(f*x+e))+8*a^3*c^6*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f
*x+e))*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+6*a^3*c^6*(-8/15-1/5*sec(
f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-6*a^3*c^6*(-(-1/4*sec(f*x+e)^3-3/8*s
ec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-8*a^3*c^6*(-2/3-1/3*se
c(f*x+e)^2)*tan(f*x+e)-3*a^3*c^6*(-(-1/8*sec(f*x+e)^7-7/48*sec(f*x+e)^5-35/
192*sec(f*x+e)^3-35/128*sec(f*x+e))*tan(f*x+e)+35/128*ln(sec(f*x+e)+tan(f*x
+e))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(224) = 448$.
time = 0.28, size = 480, normalized size = 2.11

$$\frac{256\left(35\tan\left(fx+e\right)^{9}+180\tan\left(fx+e\right)^{7}+378\tan\left(fx+e\right)^{5}+420\tan\left(fx+e\right)^{3}+315\tan\left(fx+e\right)\right)a^{3}c^{6}-32256\left(3\tan\left(fx+e\right)^{5}+10\tan\left(fx+e\right)^{3}+15\tan\left(fx+e\right)\right)a^{3}c^{6}+215040\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)a^{3}c^{6}+315a^{3}c^{6}\left(\frac{2\left(105\sin(fx+e)^{7}-385\sin(fx+e)^{5}+511\sin(fx+e)^{3}-279\sin(fx+e)\right)}{\sin(fx+e)^{8}-4\sin(fx+e)^{6}+6\sin(fx+e)^{4}-4\sin(fx+e)^{2}+1}-105\log\left(\sin\left(fx+e\right)+1\right)+105\log\left(\sin\left(fx+e\right)-1\right)\right)-6720a^{3}c^{6}\left(\frac{2\left(15\sin(fx+e)^{5}-40\sin(fx+e)^{3}+33\sin(fx+e)\right)}{\sin(fx+e)^{6}-3\sin(fx+e)^{4}+3\sin(fx+e)^{2}-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)+30240a^{3}c^{6}\left(\frac{2\left(3\sin(fx+e)^{3}-5\sin(fx+e)\right)}{\sin(fx+e)^{4}-2\sin(fx+e)^{2}+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)+80640a^{3}c^{6}\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-241920a^{3}c^{6}\tan\left(fx+e\right)}{80640f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/80640*(256*(35*tan(f*x + e)^9 + 180*tan(f*x + e)^7 + 378*tan(f*x + e)^5 +
 420*tan(f*x + e)^3 + 315*tan(f*x + e))*a^3*c^6 - 32256*(3*tan(f*x + e)^5 +
 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^6 + 215040*(tan(f*x + e)^3 + 3*
tan(f*x + e))*a^3*c^6 + 315*a^3*c^6*(2*(105*sin(f*x + e)^7 - 385*sin(f*x +
e)^5 + 511*sin(f*x + e)^3 - 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x +
 e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1
) + 105*log(sin(f*x + e) - 1)) - 6720*a^3*c^6*(2*(15*sin(f*x + e)^5 - 40*si
n(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(
f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 30
240*a^3*c^6*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(
f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 8064
0*a^3*c^6*log(sec(f*x + e) + tan(f*x + e)) - 241920*a^3*c^6*tan(f*x + e))/f
```

Fricas [A]
time = 2.48, size = 223, normalized size = 0.98

$$\frac{3465a^{3}c^{6}\cos\left(fx+e\right)^{9}\log\left(\sin\left(fx+e\right)+1\right)-3465a^{3}c^{6}\cos\left(fx+e\right)^{9}\log\left(-\sin\left(fx+e\right)+1\right)-2\left(3712a^{3}c^{6}\cos\left(fx+e\right)^{8}+4599a^{3}c^{6}\cos\left(fx+e\right)^{7}-10240a^{3}c^{6}\cos\left(fx+e\right)^{6}+3066a^{3}c^{6}\cos\left(fx+e\right)^{5}+8448a^{3}c^{6}\cos\left(fx+e\right)^{4}-7224a^{3}c^{6}\cos\left(fx+e\right)^{3}-1024a^{3}c^{6}\cos\left(fx+e\right)^{2}+3024a^{3}c^{6}\cos\left(fx+e\right)-896a^{3}c^{6}\right)\sin\left(fx+e\right)}{16128f\cos\left(fx+e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/16128*(3465*a^3*c^6*cos(f*x + e)^9*log(sin(f*x + e) + 1) - 3465*a^3*c^6*c
os(f*x + e)^9*log(-sin(f*x + e) + 1) - 2*(3712*a^3*c^6*cos(f*x + e)^8 + 459
9*a^3*c^6*cos(f*x + e)^7 - 10240*a^3*c^6*cos(f*x + e)^6 + 3066*a^3*c^6*cos(
f*x + e)^5 + 8448*a^3*c^6*cos(f*x + e)^4 - 7224*a^3*c^6*cos(f*x + e)^3 - 10
24*a^3*c^6*cos(f*x + e)^2 + 3024*a^3*c^6*cos(f*x + e) - 896*a^3*c^6)*sin(f*
x + e))/(f*cos(f*x + e)^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}c^{6}\left(\int \sec{\left(e+fx\right)}\,dx+\int \left(-3\sec^{2}{\left(e+fx\right)}\right)dx+\int 8\sec^{4}{\left(e+fx\right)}\,dx+\int \left(-6\sec^{5}{\left(e+fx\right)}\right)dx+\int \left(-6\sec^{6}{\left(e+fx\right)}\right)dx+\int 8\sec^{7}{\left(e+fx\right)}\,dx+\int \left(-3\sec^{9}{\left(e+fx\right)}\right)dx+\int \sec^{10}{\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**3*c**6*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + In
tegral(8*sec(e + f*x)**4, x) + Integral(-6*sec(e + f*x)**5, x) + Integral(-
6*sec(e + f*x)**6, x) + Integral(8*sec(e + f*x)**7, x) + Integral(-3*sec(e
+ f*x)**9, x) + Integral(sec(e + f*x)**10, x))
```

Giac [A]
time = 0.63, size = 235, normalized size = 1.04

$$\frac{3465\,a^{3}c^{6}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3465\,a^{3}c^{6}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\,\left(3465\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{17}-30030\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{15}+115038\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+334602\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}-360448\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+255222\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}-115038\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+30030\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-3465\,a^{3}c^{6}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{9}}}{8064\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/8064*(3465*a^3*c^6*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3465*a^3*c^6*log(
abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(3465*a^3*c^6*tan(1/2*f*x + 1/2*e)^17 -
30030*a^3*c^6*tan(1/2*f*x + 1/2*e)^15 + 115038*a^3*c^6*tan(1/2*f*x + 1/2*e)
^13 + 334602*a^3*c^6*tan(1/2*f*x + 1/2*e)^11 - 360448*a^3*c^6*tan(1/2*f*x +
 1/2*e)^9 + 255222*a^3*c^6*tan(1/2*f*x + 1/2*e)^7 - 115038*a^3*c^6*tan(1/2*
f*x + 1/2*e)^5 + 30030*a^3*c^6*tan(1/2*f*x + 1/2*e)^3 - 3465*a^3*c^6*tan(1/
2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^9)/f
```

Mupad [B]
time = 5.58, size = 316, normalized size = 1.39

$$\frac{55\,a^3\,c^6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{64\,f}-\frac{\frac{55\,a^3\,c^6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{17}}{64}-\frac{715\,a^3\,c^6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{15}}{96}+\frac{913\,a^3\,c^6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{13}}{32}+\frac{18589\,a^3\,c^6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}}{224}-\frac{5632\,a^3\,c^6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{9}}{63}+\frac{14179\,a^3\,c^6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{7}}{224}-\frac{913\,a^3\,c^6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{5}}{32}+\frac{715\,a^3\,c^6\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{3}}{96}-\frac{55\,a^3\,c^6\,\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{64}}{f\,\left({\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{18}-9\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{16}+36\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{14}-84\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}+126\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-126\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{8}+84\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{6}-36\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{4}+9\,{\mathrm{tan}\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6)/cos(e + f*x),x)

[Out] (55*a^3*c^6*atanh(tan(e/2 + (f*x)/2)))/(64*f) - ((715*a^3*c^6*tan(e/2 + (f*
x)/2)^3)/96 - (913*a^3*c^6*tan(e/2 + (f*x)/2)^5)/32 + (14179*a^3*c^6*tan(e/
2 + (f*x)/2)^7)/224 - (5632*a^3*c^6*tan(e/2 + (f*x)/2)^9)/63 + (18589*a^3*c
^6*tan(e/2 + (f*x)/2)^11)/224 + (913*a^3*c^6*tan(e/2 + (f*x)/2)^13)/32 - (7
15*a^3*c^6*tan(e/2 + (f*x)/2)^15)/96 + (55*a^3*c^6*tan(e/2 + (f*x)/2)^17)/6
4 - (55*a^3*c^6*tan(e/2 + (f*x)/2))/64)/(f*(9*tan(e/2 + (f*x)/2)^2 - 36*tan
(e/2 + (f*x)/2)^4 + 84*tan(e/2 + (f*x)/2)^6 - 126*tan(e/2 + (f*x)/2)^8 + 12
6*tan(e/2 + (f*x)/2)^10 - 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (f*x)/2)^
14 - 9*tan(e/2 + (f*x)/2)^16 + tan(e/2 + (f*x)/2)^18 - 1))
```

3.22 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$

Optimal. Leaf size=206

$$\frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f}-\frac{35a^3c^5\sec(e+fx)\tan(e+fx)}{128f}-\frac{5a^3c^5\sec^3(e+fx)\tan(e+fx)}{64f}+\frac{5a^3c^5\sec(e+$$

```
[Out] 45/128*a^3*c^5*arctanh(sin(f*x+e))/f-35/128*a^3*c^5*sec(f*x+e)*tan(f*x+e)/f
-5/64*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)/f+5/24*a^3*c^5*sec(f*x+e)*tan(f*x+e)^
3/f+5/48*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)^3/f-1/6*a^3*c^5*sec(f*x+e)*tan(f*x
+e)^5/f-1/8*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)^5/f+2/7*a^3*c^5*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.22, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4043, 2691, 3855, 2687, 30, 3853}

$$\frac{2a^3c^5\tan^7(e+fx)}{7f}+\frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f}-\frac{a^3c^5\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^5\tan^3(e+fx)\sec^3(e+fx)}{48f}-\frac{5a^3c^5\tan(e+fx)\sec^3(e+fx)}{64f}-\frac{a^3c^5\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^5\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{35a^3c^5\tan(e+fx)\sec(e+fx)}{128f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (45*a^3*c^5*ArcTanh[Sin[e + f*x]])/(128*f) - (35*a^3*c^5*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (5*a^3*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3*
c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^5*Sec[e + f*x]^3*Tan[e +
 f*x]^3)/(48*f) - (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (a^3*c^5*Se
c[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (2*a^3*c^5*Tan[e + f*x]^7)/(7*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = -\left(\left(a^3c^3\right)\int \left(c^2\sec(e+fx)\tan^6(e+fx)-2c\right.\right.$$

$$= -\left(\left(a^3c^5\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right) - ($$

$$= -\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{a^3c^5\sec^3(e}{}$$

$$= \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f} + \frac{5a^3c^5\sec^3(e}{}$$

$$= -\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{16f} - \frac{5a^3c^5\sec^3(e}{}$$

$$= \frac{5a^3c^5\tanh^{-1}(\sin(e+fx))}{16f} - \frac{35a^3c^5\sec(e+fx}{128}$$

$$= \frac{45a^3c^5\tanh^{-1}(\sin(e+fx))}{128f} - \frac{35a^3c^5\sec(e+f}{128}$$

Mathematica [A]

time = 2.29, size = 111, normalized size = 0.54

$$-\frac{a^3c^5\left(-20160\tanh^{-1}(\sin(e+fx))+\sec^8(e+fx)(5705\sin(e+fx)-1792\sin(2(e+fx))+21\sin(3(e+fx))+1792\sin(4(e+fx))+2065\sin(5(e+fx))-768\sin(6(e+fx))+581\sin(7(e+fx))+128\sin(8(e+fx)))\right)}{57344f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/57344*(a^3*c^5*(-20160*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^8*(5705*Sin[
e + f*x] - 1792*Sin[2*(e + f*x)] + 21*Sin[3*(e + f*x)] + 1792*Sin[4*(e + f*
x)] + 2065*Sin[5*(e + f*x)] - 768*Sin[6*(e + f*x)] + 581*Sin[7*(e + f*x)] +
 128*Sin[8*(e + f*x)])))/f
```

Maple [A]
time = 0.39, size = 322, normalized size = 1.56 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-c^5*a^3*(-(-1/8*sec(f*x+e)^7-7/48*sec(f*x+e)^5-35/192*sec(f*x+e)^3-35
/128*sec(f*x+e))*tan(f*x+e)+35/128*ln(sec(f*x+e)+tan(f*x+e)))-2*c^5*a^3*(-1
6/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f*x+e)+2*c^5
*a^3*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)+5/1
6*ln(sec(f*x+e)+tan(f*x+e)))+6*c^5*a^3*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x
+e)^2)*tan(f*x+e)-6*c^5*a^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*c^5*a^3*(1
/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-2*c^5*a^3*tan(f*x+e
)+c^5*a^3*ln(sec(f*x+e)+tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(204) = 408$.
time = 0.30, size = 442, normalized size = 2.15

$$\frac{1536\left(5\tan(fx+e)^7+21\tan(fx+e)^5+35\tan(fx+e)^3+35\tan(fx+e)\right)a^3c^5-10752\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c^5+53760\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c^5+35a^3c^5\left(\frac{2\left(105\sin(fx+e)^7-385\sin(fx+e)^5+511\sin(fx+e)^3-279\sin(fx+e)\right)}{\sin(fx+e)^8-4\sin(fx+e)^6+6\sin(fx+e)^4-4\sin(fx+e)^2+1}-105\log(\sin(fx+e)+1)+105\log(\sin(fx+e)-1)\right)-560a^3c^5\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log(\sin(fx+e)+1)+15\log(\sin(fx+e)-1)\right)+13440a^3c^5\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+26880a^3c^5\log(\sec(fx+e)+\tan(fx+e))-53760a^3c^5\tan(fx+e)}{26880f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/26880*(1536*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 3
5*tan(f*x + e))*a^3*c^5 - 10752*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*
tan(f*x + e))*a^3*c^5 + 53760*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^5 + 3
5*a^3*c^5*(2*(105*sin(f*x + e)^7 - 385*sin(f*x + e)^5 + 511*sin(f*x + e)^3
- 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 -
 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1) + 105*log(sin(f*x + e) -
 1)) - 560*a^3*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x +
 e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(si
```

```
n(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 13440*a^3*c^5*(2*sin(f*x + e)
/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 26
880*a^3*c^5*log(sec(f*x + e) + tan(f*x + e)) - 53760*a^3*c^5*tan(f*x + e))/
f
```

Fricas [A]
time = 2.56, size = 206, normalized size = 1.00

$$\frac{315\,a^3c^5\cos\left(fx+e\right)^8\log\left(\sin\left(fx+e\right)+1\right)-315\,a^3c^5\cos\left(fx+e\right)^8\log\left(-\sin\left(fx+e\right)+1\right)-2\left(256\,a^3c^5\cos\left(fx+e\right)^7+581\,a^3c^5\cos\left(fx+e\right)^6-768\,a^3c^5\cos\left(fx+e\right)^5-210\,a^3c^5\cos\left(fx+e\right)^4+768\,a^3c^5\cos\left(fx+e\right)^3-168\,a^3c^5\cos\left(fx+e\right)^2-256\,a^3c^5\cos\left(fx+e\right)+112\,a^3c^5\right)\sin\left(fx+e\right)}{1792\,f\cos\left(fx+e\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/1792*(315*a^3*c^5*cos(f*x + e)^8*log(sin(f*x + e) + 1) - 315*a^3*c^5*cos(
f*x + e)^8*log(-sin(f*x + e) + 1) - 2*(256*a^3*c^5*cos(f*x + e)^7 + 581*a^3
*c^5*cos(f*x + e)^6 - 768*a^3*c^5*cos(f*x + e)^5 - 210*a^3*c^5*cos(f*x + e)
^4 + 768*a^3*c^5*cos(f*x + e)^3 - 168*a^3*c^5*cos(f*x + e)^2 - 256*a^3*c^5*
cos(f*x + e) + 112*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^3c^5\left(\int\left(-\sec\left(e+fx\right)\right)dx+\int 2\sec^2\left(e+fx\right)dx+\int 2\sec^3\left(e+fx\right)dx+\int\left(-6\sec^4\left(e+fx\right)\right)dx+\int 6\sec^6\left(e+fx\right)dx+\int\left(-2\sec^7\left(e+fx\right)\right)dx+\int\left(-2\sec^8\left(e+fx\right)\right)dx+\int\sec^9\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*c**5*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + I
ntegral(2*sec(e + f*x)**3, x) + Integral(-6*sec(e + f*x)**4, x) + Integral(
6*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x) + Integral(-2*sec(e
 + f*x)**8, x) + Integral(sec(e + f*x)**9, x))
```

Giac [A]
time = 0.66, size = 216, normalized size = 1.05

$$\frac{315\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-315\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(315\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{15}-2415\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}+8043\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+17609\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}-15159\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{7}+8043\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-2415\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+315\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^8}}{896\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/896*(315*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 315*a^3*c^5*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(315*a^3*c^5*tan(1/2*f*x + 1/2*e)^15 - 2415
*a^3*c^5*tan(1/2*f*x + 1/2*e)^13 + 8043*a^3*c^5*tan(1/2*f*x + 1/2*e)^11 + 1
7609*a^3*c^5*tan(1/2*f*x + 1/2*e)^9 - 15159*a^3*c^5*tan(1/2*f*x + 1/2*e)^7
```

```
+ 8043*a^3*c^5*tan(1/2*f*x + 1/2*e)^5 - 2415*a^3*c^5*tan(1/2*f*x + 1/2*e)^3
 + 315*a^3*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^8)/f
```

Mupad [B]
time = 5.52, size = 284, normalized size = 1.38

$$\frac{45\,a^3\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{64\,f}-\frac{\frac{45\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{15}}{64}-\frac{345\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{64}+\frac{1149\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{64}+\frac{17609\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{448}-\frac{15159\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{448}+\frac{1149\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{64}-\frac{345\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{64}+\frac{45\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{64}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}-8\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}+28\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-56\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+70\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-56\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+28\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-8\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5)/cos(e + f*x),x)
```

```
[Out] (45*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(64*f) - ((1149*a^3*c^5*tan(e/2 + (f
*x)/2)^5)/64 - (345*a^3*c^5*tan(e/2 + (f*x)/2)^3)/64 - (15159*a^3*c^5*tan(e
/2 + (f*x)/2)^7)/448 + (17609*a^3*c^5*tan(e/2 + (f*x)/2)^9)/448 + (1149*a^3
*c^5*tan(e/2 + (f*x)/2)^11)/64 - (345*a^3*c^5*tan(e/2 + (f*x)/2)^13)/64 + (
45*a^3*c^5*tan(e/2 + (f*x)/2)^15)/64 + (45*a^3*c^5*tan(e/2 + (f*x)/2))/64)/
(f*(28*tan(e/2 + (f*x)/2)^4 - 8*tan(e/2 + (f*x)/2)^2 - 56*tan(e/2 + (f*x)/2
)^6 + 70*tan(e/2 + (f*x)/2)^8 - 56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*
x)/2)^12 - 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)^16 + 1))
```

3.23 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

Optimal. Leaf size=121

$$\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{5a^3c^4\sec(e+fx)\tan(e+fx)}{16f}+\frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f}-\frac{a^3c^4\sec(e+f}{}$$

```
[Out] 5/16*a^3*c^4*arctanh(sin(f*x+e))/f-5/16*a^3*c^4*sec(f*x+e)*tan(f*x+e)/f+5/2
4*a^3*c^4*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^3*c^4*sec(f*x+e)*tan(f*x+e)^5/f+1
/7*a^3*c^4*tan(f*x+e)^7/f
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4043, 2691, 3855, 2687, 30}

$$\frac{a^3c^4\tan^7(e+fx)}{7f}+\frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^4\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^4\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c^4\tan(e+fx)\sec(e+fx)}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^4*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f) + (a^3*c^4*Tan[e + f*x]^7)/(7*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = -\left(\left(a^3c^3\right)\int \left(c\sec(e+fx)\tan^6(e+fx)-c\sec\right.\right.$$
$$= -\left(\left(a^3c^4\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)+\left(a\right.$$
$$= -\frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f}+\frac{1}{6}\left(5a^3c^4\right)\int \mathrm{s}$$
$$= \frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f}-\frac{a^3c^4\sec(e+f}{6}$$
$$= -\frac{5a^3c^4\sec(e+fx)\tan(e+fx)}{16f}+\frac{5a^3c^4\sec(e+}{}$$
$$= \frac{5a^3c^4\tanh^{-1}(\sin(e+fx))}{16f}-\frac{5a^3c^4\sec(e+fx)\,\mathrm{t}}{16f}$$

Mathematica [A]
time = 1.67, size = 102, normalized size = 0.84

$$\frac{a^3c^4(3360\tanh^{-1}(\sin(e+fx))-\sec^7(e+fx)(-840\sin(e+fx)+595\sin(2(e+fx))+504\sin(3(e+fx))+196\sin(4(e+fx))-168\sin(5(e+fx))+231\sin(6(e+fx))+24\sin(7(e+fx))))}{10752f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*(3360*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(-840*Sin[e + f*x] +
595*Sin[2*(e + f*x)] + 504*Sin[3*(e + f*x)] + 196*Sin[4*(e + f*x)] - 168*Si
n[5*(e + f*x)] + 231*Sin[6*(e + f*x)] + 24*Sin[7*(e + f*x)])))/(10752*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(111) = 222$.
time = 0.36, size = 302, normalized size = 2.50

method	result
risch	$\frac{ic^4a^3\left(231\,\mathrm{e}^{13i(fx+e)}-336\,\mathrm{e}^{12i(fx+e)}+196\,\mathrm{e}^{11i(fx+e)}+595\,\mathrm{e}^{9i(fx+e)}-1680\,\mathrm{e}^{8i(fx+e)}-595\,\mathrm{e}^{5i(fx+e)}-1008\,\mathrm{e}^{4i(fx+e)}-1}{168f\left(\mathrm{e}^{2i(fx+e)}+1\right)^7}$
norman	$\frac{\frac{5c^4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}-\frac{25c^4a^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f}+\frac{283c^4a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}-\frac{128c^4a^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{7f}-\frac{283c^4a^3\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f}+}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}$
derivativedivides	$-c^4a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)-c^4a^3\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\right.\right.$
default	$-c^4a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(fx+e)\right)}{7}-\frac{6\left(\sec^4(fx+e)\right)}{35}-\frac{8\left(\sec^2(fx+e)\right)}{35}\right)\tan(fx+e)-c^4a^3\left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6}-\frac{5\left(\sec^3(fx+e)\right)}{24}-\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-c^4*a^3*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)
*tan(f*x+e)-c^4*a^3*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))
*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+3*c^4*a^3*(-8/15-1/5*sec(f*x+e)
^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+3*c^4*a^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x
+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-3*c^4*a^3*(-2/3-1/3*sec(f*x+
e)^2)*tan(f*x+e)-3*c^4*a^3*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan
(f*x+e)))-c^4*a^3*tan(f*x+e)+c^4*a^3*ln(sec(f*x+e)+tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(119) = 238$.
time = 0.30, size = 398, normalized size = 3.29

$$\frac{96\left(5\tan(fx+e)^7+21\tan(fx+e)^5+35\tan(fx+e)^3+35\tan(fx+e)\right)a^3c^4-672\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c^4+3360\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c^4+35a^3c^4\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log(\sin(fx+e)+1)+15\log(\sin(fx+e)-1)\right)-630a^3c^4\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log(\sin(fx+e)-1)\right)+2520a^3c^4\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+3360a^3c^4\log(\sec(fx+e)+\tan(fx+e))-3360a^3c^4\tan(fx+e)}{3360f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*t
an(f*x + e))*a^3*c^4 - 672*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f
*x + e))*a^3*c^4 + 3360*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 + 35*a^3*
c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x +
 e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1)
 + 15*log(sin(f*x + e) - 1)) - 630*a^3*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x
 + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) +
3*log(sin(f*x + e) - 1)) + 2520*a^3*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1
) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^4*log(sec(f
*x + e) + tan(f*x + e)) - 3360*a^3*c^4*tan(f*x + e))/f
```

Fricas [A]
time = 3.05, size = 189, normalized size = 1.56

$$\frac{105\,a^3c^4\cos\left(fx+e\right)^7\log\left(\sin\left(fx+e\right)+1\right)-105\,a^3c^4\cos\left(fx+e\right)^7\log\left(-\sin\left(fx+e\right)+1\right)-2\left(48\,a^3c^4\cos\left(fx+e\right)^6+231\,a^3c^4\cos\left(fx+e\right)^5-144\,a^3c^4\cos\left(fx+e\right)^4-182\,a^3c^4\cos\left(fx+e\right)^3+144\,a^3c^4\cos\left(fx+e\right)^2+56\,a^3c^4\cos\left(fx+e\right)-48\,a^3c^4\right)\sin\left(fx+e\right)}{672\,f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/672*(105*a^3*c^4*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 105*a^3*c^4*cos(f
*x + e)^7*log(-sin(f*x + e) + 1) - 2*(48*a^3*c^4*cos(f*x + e)^6 + 231*a^3*c
^4*cos(f*x + e)^5 - 144*a^3*c^4*cos(f*x + e)^4 - 182*a^3*c^4*cos(f*x + e)^3
 + 144*a^3*c^4*cos(f*x + e)^2 + 56*a^3*c^4*cos(f*x + e) - 48*a^3*c^4)*sin(f
*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3c^4\left(\int\sec\left(e+fx\right)dx+\int\left(-\sec^2\left(e+fx\right)\right)dx+\int\left(-3\sec^3\left(e+fx\right)\right)dx+\int3\sec^4\left(e+fx\right)dx+\int3\sec^5\left(e+fx\right)dx+\int\left(-3\sec^6\left(e+fx\right)\right)dx+\int\left(-\sec^7\left(e+fx\right)\right)dx+\int\sec^8\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*c**4*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Inte
gral(-3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4, x) + Integral(3*s
ec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x) + Integral(-sec(e + f*
x)**7, x) + Integral(sec(e + f*x)**8, x))
```

Giac [A]
time = 0.61, size = 197, normalized size = 1.63

$$\frac{105\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-700\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+1981\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+3072\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-1981\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+700\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-105\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^7}}{336\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/336*(105*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a^3*c^4*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(105*a^3*c^4*tan(1/2*f*x + 1/2*e)^13 - 700*
a^3*c^4*tan(1/2*f*x + 1/2*e)^11 + 1981*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 + 307
2*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 1981*a^3*c^4*tan(1/2*f*x + 1/2*e)^5 + 70
0*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 - 105*a^3*c^4*tan(1/2*f*x + 1/2*e))/(tan(1
/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B]
time = 5.71, size = 252, normalized size = 2.08

$$\frac{5\,a^3\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}-\frac{\frac{5\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}-\frac{25\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{6}+\frac{283\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{128\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{7}-\frac{283\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{24}+\frac{25\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{6}-\frac{5\,a^3\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-21\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}+7\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (5*a^3*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f) - ((25*a^3*c^4*tan(e/2 + (f*x)/
2)^3)/6 - (283*a^3*c^4*tan(e/2 + (f*x)/2)^5)/24 + (128*a^3*c^4*tan(e/2 + (f
*x)/2)^7)/7 + (283*a^3*c^4*tan(e/2 + (f*x)/2)^9)/24 - (25*a^3*c^4*tan(e/2 +
 (f*x)/2)^11)/6 + (5*a^3*c^4*tan(e/2 + (f*x)/2)^13)/8 - (5*a^3*c^4*tan(e/2
+ (f*x)/2))/8)/(f*(7*tan(e/2 + (f*x)/2)^2 - 21*tan(e/2 + (f*x)/2)^4 + 35*ta
n(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7
*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 - 1))
```

3.24 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

Optimal. Leaf size=100

$$\frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f}-\frac{5a^3c^3\sec(e+fx)\tan(e+fx)}{16f}+\frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f}-\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f}$$

```
[Out] 5/16*a^3*c^3*arctanh(sin(f*x+e))/f-5/16*a^3*c^3*sec(f*x+e)*tan(f*x+e)/f+5/2
4*a^3*c^3*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^3*c^3*sec(f*x+e)*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4043, 2691, 3855}

$$\frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^3c^3\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^3\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c^3\tan(e+fx)\sec(e+fx)}{16f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (5*a^3*c^3*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^3*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^3*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -\left(\left(a^3c^3\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)$$
$$= -\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f} + \frac{1}{6}\left(5a^3c^3\right)\int$$
$$= \frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f} - \frac{a^3c^3\sec(e+$$
$$= -\frac{5a^3c^3\sec(e+fx)\tan(e+fx)}{16f} + \frac{5a^3c^3\sec(e$$
$$= \frac{5a^3c^3\tanh^{-1}(\sin(e+fx))}{16f} - \frac{5a^3c^3\sec(e+fx)}{16f}$$

Mathematica [A]
time = 0.25, size = 60, normalized size = 0.60

$$-\frac{a^3c^3\left(-120\tanh^{-1}(\sin(e+fx)) + (59+28\cos(2(e+fx))+33\cos(4(e+fx)))\sec^5(e+fx)\tan(e+fx)\right)}{384f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
[Out] -1/384*(a^3*c^3*(-120*ArcTanh[Sin[e + f*x]] + (59 + 28*Cos[2*(e + f*x)] + 3
3*Cos[4*(e + f*x)])*Sec[e + f*x]^5*Tan[e + f*x]))/f
```

Maple [A]
time = 0.24, size = 180, normalized size = 1.80

method	result
risch	$\frac{i a^3 c^3 \left(33\,e^{11 i (fx+e)} - 5\,e^{9 i (fx+e)} + 90\,e^{7 i (fx+e)} - 90\,e^{5 i (fx+e)} + 5\,e^{3 i (fx+e)} - 33\,e^{i (fx+e)}\right)}{24 f \left(e^{2 i (fx+e)}+1\right)^6} - \frac{5 c^3 a^3 \ln\left(e^{i (fx+e)} - i\right)}{16 f} + \frac{5}{}$
derivativedivides	$\frac{-c^3 a^3 \left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6} - \frac{5\left(\sec^3(fx+e)\right)}{24} - \frac{5\sec(fx+e)}{16}\right)\tan(fx+e) + \frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right) + 3c^3a^3\left(-\left(-\frac{\left(\sec^3\right.}{}\right.\right.}{}$
default	$\frac{-c^3 a^3 \left(-\left(-\frac{\left(\sec^5(fx+e)\right)}{6} - \frac{5\left(\sec^3(fx+e)\right)}{24} - \frac{5\sec(fx+e)}{16}\right)\tan(fx+e) + \frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right) + 3c^3a^3\left(-\left(-\frac{\left(\sec^3\right.}{}\right.\right.}{}$
norman	$\frac{-\frac{5c^3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f} + \frac{85c^3a^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f} - \frac{33c^3a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f} - \frac{33c^3a^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f} + \frac{85c^3a^3\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{24f} - \frac{5c}{}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-c^3*a^3*(-(-1/6*sec(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f
*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+3*c^3*a^3*(-(-1/4*sec(f*x+e)^3-3/8*se
c(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-3*c^3*a^3*(1/2*sec(f*x+
e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+c^3*a^3*ln(sec(f*x+e)+tan(f*x+
e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(99) = 198$.
time = 0.28, size = 264, normalized size = 2.64

$$\frac{a^3c^3\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)-18a^3c^3\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)+72a^3c^3\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+96a^3c^3\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)}{96f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/96*(a^3*c^3*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/
(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
 + e) + 1) + 15*log(sin(f*x + e) - 1)) - 18*a^3*c^3*(2*(3*sin(f*x + e)^3 -
5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e
) + 1) + 3*log(sin(f*x + e) - 1)) + 72*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e
)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 96*a^3*c^3*log(
sec(f*x + e) + tan(f*x + e)))/f
```

Fricas [A]
time = 3.06, size = 123, normalized size = 1.23

$$\frac{15\,a^3c^3\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c^3\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-2\left(33\,a^3c^3\cos\left(fx+e\right)^4-26\,a^3c^3\cos\left(fx+e\right)^2+8\,a^3c^3\right)\sin\left(fx+e\right)}{96\,f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/96*(15*a^3*c^3*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*a^3*c^3*cos(f*x
+ e)^6*log(-sin(f*x + e) + 1) - 2*(33*a^3*c^3*cos(f*x + e)^4 - 26*a^3*c^3*c
os(f*x + e)^2 + 8*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^3c^3\left(\int\left(-\sec\left(e+fx\right)\right)\,dx+\int 3\sec^3\left(e+fx\right)dx+\int\left(-3\sec^5\left(e+fx\right)\right)\,dx+\int\sec^7\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*c**3*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**3, x) + I
ntegral(-3*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**7, x))
```

Giac [A]
time = 0.61, size = 103, normalized size = 1.03

$$\frac{15\,a^3c^3\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-15\,a^3c^3\log\left(\left|\sin\left(fx+e\right)-1\right|\right)+\frac{2\left(33\,a^3c^3\sin(fx+e)^5-40\,a^3c^3\sin(fx+e)^3+15\,a^3c^3\sin(fx+e)\right)}{\left(\sin(fx+e)^2-1\right)^3}}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/96*(15*a^3*c^3*log(abs(sin(f*x + e) + 1)) - 15*a^3*c^3*log(abs(sin(f*x +
e) - 1)) + 2*(33*a^3*c^3*sin(f*x + e)^5 - 40*a^3*c^3*sin(f*x + e)^3 + 15*a^
3*c^3*sin(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

Mupad [B]
time = 5.56, size = 220, normalized size = 2.20

$$\frac{5\,a^3\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}-\frac{\frac{5\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}-\frac{85\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{33\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{4}+\frac{33\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{4}-\frac{85\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24}+\frac{5\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{8}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (5*a^3*c^3*atanh(tan(e/2 + (f*x)/2)))/(8*f) - ((33*a^3*c^3*tan(e/2 + (f*x)/
2)^5)/4 - (85*a^3*c^3*tan(e/2 + (f*x)/2)^3)/24 + (33*a^3*c^3*tan(e/2 + (f*x
)/2)^7)/4 - (85*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (5*a^3*c^3*tan(e/2 + (f*
x)/2)^11)/8 + (5*a^3*c^3*tan(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4
 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)
^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1))
```

3.25 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=94

$$\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f}+\frac{a^3c^2\tan^5(e+fx}{5f}$$

```
[Out] 3/8*a^3*c^2*arctanh(sin(f*x+e))/f-3/8*a^3*c^2*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^3*c^2*sec(f*x+e)*tan(f*x+e)^3/f+1/5*a^3*c^2*tan(f*x+e)^5/f
```

Rubi [A]
time = 0.11, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4043, 2691, 3855, 2687, 30}

$$\frac{a^3c^2\tan^5(e+fx)}{5f}+\frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3c^2\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^3c^2\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^3*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^3*c^2*Tan[e +
 f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \left(a^2c^2\right)\int \left(a\sec(e+fx)\tan^4(e+fx)+a\sec^2(\right.$$
$$= \left(a^3c^2\right)\int \sec(e+fx)\tan^4(e+fx)\,dx + \left(a^3c^2\right)$$
$$= \frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{1}{4}\left(3a^3c^2\right)\int \mathrm{se}$$
$$= -\frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^3c^2\sec(e+}{}$$
$$= \frac{3a^3c^2\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3a^3c^2\sec(e+fx)}{8f}$$

Mathematica [A]
time = 0.83, size = 81, normalized size = 0.86

$$\frac{a^3c^2\left(120\tanh^{-1}(\sin(e+fx))+\sec^5(e+fx)(40\sin(e+fx)-10\sin(2(e+fx))-20\sin(3(e+fx))-25\sin(4(e+fx))+4\sin(5(e+fx)))\right)}{320f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*(120*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^5*(40*Sin[e + f*x] - 10*
Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] - 25*Sin[4*(e + f*x)] + 4*Sin[5*(e +
 f*x)])))/(320*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(86) = 172$.
time = 0.27, size = 192, normalized size = 2.04

method	result
risch	$\frac{i c^2 a^3 \left(25\,\mathrm{e}^{9 i (f x+e)}+40\,\mathrm{e}^{8 i (f x+e)}+10\,\mathrm{e}^{7 i (f x+e)}+80\,\mathrm{e}^{4 i (f x+e)}-10\,\mathrm{e}^{3 i (f x+e)}-25\,\mathrm{e}^{i (f x+e)}+8\right)}{20 f \left(\mathrm{e}^{2 i (f x+e)}+1\right)^5}-\frac{3 c^2 a^3 \ln\left(\mathrm{e}^{i (f x+e)}-i\right)}{8 f}+$
norman	$\frac{\frac{3 c^2 a^3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{4 f}-\frac{7 c^2 a^3 \left(\tan^3\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2 f}-\frac{32 c^2 a^3 \left(\tan^5\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{5 f}+\frac{7 c^2 a^3 \left(\tan^7\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{2 f}-\frac{3 c^2 a^3 \left(\tan^9\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{4 f}}{\left(\tan^2\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{3 c^2 a^3}{}$
derivativedivides	$\frac{-c^2 a^3 \left(-\frac{8}{15}-\frac{\left(\sec^4(f x+e)\right)}{5}-\frac{4 \left(\sec^2(f x+e)\right)}{15}\right) \tan(f x+e)+c^2 a^3 \left(-\left(-\frac{\left(\sec^3(f x+e)\right)}{4}-\frac{3 \sec(f x+e)}{8}\right) \tan(f x+e)+\frac{3 \ln(\sec(f}{}\right.}{}$
default	$\frac{-c^2 a^3 \left(-\frac{8}{15}-\frac{\left(\sec^4(f x+e)\right)}{5}-\frac{4 \left(\sec^2(f x+e)\right)}{15}\right) \tan(f x+e)+c^2 a^3 \left(-\left(-\frac{\left(\sec^3(f x+e)\right)}{4}-\frac{3 \sec(f x+e)}{8}\right) \tan(f x+e)+\frac{3 \ln(\sec(f}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-c^2*a^3*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+c^2*a^3
*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+
e)))+2*c^2*a^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*c^2*a^3*(1/2*sec(f*x+e)
*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+c^2*a^3*tan(f*x+e)+c^2*a^3*ln(se
c(f*x+e)+tan(f*x+e)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.
time = 0.29, size = 245, normalized size = 2.61

$$\frac{16 \left(3 \tan(fx+e)^5+10 \tan(fx+e)^3+15 \tan(fx+e)\right)a^3c^2-160 \left(\tan(fx+e)^3+3 \tan(fx+e)\right)a^3c^2-15 a^3c^2\left(\frac{2 \left(3 \sin(fx+e)^3-5 \sin(fx+e)\right)}{\sin(fx+e)^4-2 \sin(fx+e)^2+1}-3 \log(\sin(fx+e)+1)+3 \log(\sin(fx+e)-1)\right)+120 a^3c^2\left(\frac{2 \sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+240 a^3c^2 \log(\sec(fx+e)+\tan(fx+e))+240 a^3c^2 \tan(fx+e)}{240 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^2
- 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 - 15*a^3*c^2*(2*(3*sin(f*x
+ e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(si
n(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 120*a^3*c^2*(2*sin(f*x + e)/(s
in(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a
^3*c^2*log(sec(f*x + e) + tan(f*x + e)) + 240*a^3*c^2*tan(f*x + e))/f
```

Fricas [A]
time = 3.07, size = 155, normalized size = 1.65

$$\frac{15 a^3c^2 \cos(fx+e)^5 \log(\sin(fx+e)+1)-15 a^3c^2 \cos(fx+e)^5 \log(-\sin(fx+e)+1)+2 \left(8 a^3c^2 \cos(fx+e)^4-25 a^3c^2 \cos(fx+e)^3-16 a^3c^2 \cos(fx+e)^2+10 a^3c^2 \cos(fx+e)+8 a^3c^2\right)\sin(fx+e)}{80 f \cos(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/80*(15*a^3*c^2*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c^2*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^3*c^2*cos(f*x + e)^4 - 25*a^3*c^2*cos(f*x + e)^3 - 16*a^3*c^2*cos(f*x + e)^2 + 10*a^3*c^2*cos(f*x + e) + 8*a^3*c^2)*sin(f*x + e))/(f*cos(f*x + e)^5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3c^2\left(\int \sec(e+fx)\,dx + \int \sec^2(e+fx)\,dx + \int \left(-2\sec^3(e+fx)\right)\,dx + \int \left(-2\sec^4(e+fx)\right)\,dx + \int \sec^5(e+fx)\,dx + \int \sec^6(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)

[Out] a**3*c**2*(Integral(sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))

Giac [A]
time = 0.60, size = 159, normalized size = 1.69

$$\frac{15\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right) - 15\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right) - \frac{2\left(15\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9 - 70\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7 + 128\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 70\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 15\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="giac")

[Out] 1/40*(15*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c^2*tan(1/2*f*x + 1/2*e)^9 - 70*a^3*c^2*tan(1/2*f*x + 1/2*e)^7 + 128*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 70*a^3*c^2*tan(1/2*f*x + 1/2*e)^3 - 15*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f

Mupad [B]
time = 6.26, size = 188, normalized size = 2.00

$$\frac{3\,a^3\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f} - \frac{\frac{3\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4} - \frac{7\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2} + \frac{32\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5} + \frac{7\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2} - \frac{3\,a^3\,c^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10} - 5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8 + 10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6 - 10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 + 5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)

[Out] (3*a^3*c^2*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((7*a^3*c^2*tan(e/2 + (f*x)/2)^3)/2 + (32*a^3*c^2*tan(e/2 + (f*x)/2)^5)/5 - (7*a^3*c^2*tan(e/2 + (f*x)/2)^7)/2 + (3*a^3*c^2*tan(e/2 + (f*x)/2)^9)/4 - (3*a^3*c^2*tan(e/2 + (f*x)/2))/4)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))

3.26 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=86

$$\frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{3a^3c\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{4f}-\frac{2a^3c\tan^3(e+fx)}{3f}$$

```
[Out] 5/8*a^3*c*arctanh(sin(f*x+e))/f-3/8*a^3*c*sec(f*x+e)*tan(f*x+e)/f-1/4*a^3*c
*sec(f*x+e)^3*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4043, 2691, 3855, 2687, 30, 3853}

$$-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{3a^3c\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (5*a^3*c*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a^2\sec(e+fx)\tan^2(e+fx)+2a^2\text{se}\right.\right.$$

$$= -\left((a^3c)\int \sec(e+fx)\tan^2(e+fx)\,dx\right) - (a^3$$

$$= -\frac{a^3c\sec(e+fx)\tan(e+fx)}{2f} - \frac{a^3c\sec^3(e+fx}{4}$$

$$= \frac{a^3c\tanh^{-1}(\sin(e+fx))}{2f} - \frac{3a^3c\sec(e+fx)\tan}{8f}$$

$$= \frac{5a^3c\tanh^{-1}(\sin(e+fx))}{8f} - \frac{3a^3c\sec(e+fx)\text{ta}}{8f}$$

Mathematica [A]
time = 0.61, size = 70, normalized size = 0.81

$$\frac{a^3c(60\tanh^{-1}(\sin(e+fx))-\sec^4(e+fx)(33\sin(e+fx)+16\sin(2(e+fx))+9\sin(3(e+fx))-8\sin(4(e+fx))))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*(60*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^4*(33*Sin[e + f*x] + 16*Sin
[2*(e + f*x)] + 9*Sin[3*(e + f*x)] - 8*Sin[4*(e + f*x)])))/(96*f)
```

Maple [A]
time = 0.21, size = 111, normalized size = 1.29

method	result
derivativedivides	$\frac{-a^{3} c \left(-\left(-\frac{\left(\sec^{3}\left(f x+e\right)\right)}{4}-\frac{3 \sec\left(f x+e\right)}{8}\right) \tan\left(f x+e\right)+\frac{3 \ln\left(\sec\left(f x+e\right)+\tan\left(f x+e\right)\right)}{8}\right)+2 a^{3} c \left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(f x+e\right)\right)}{3}\right) \tan\left(f x+e\right)}{f}$
default	$\frac{-a^{3} c \left(-\left(-\frac{\left(\sec^{3}\left(f x+e\right)\right)}{4}-\frac{3 \sec\left(f x+e\right)}{8}\right) \tan\left(f x+e\right)+\frac{3 \ln\left(\sec\left(f x+e\right)+\tan\left(f x+e\right)\right)}{8}\right)+2 a^{3} c \left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(f x+e\right)\right)}{3}\right) \tan\left(f x+e\right)}{f}$
norman	$\frac{-\frac{5 a^{3} c \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{4 f}-\frac{73 a^{3} c \left(\tan^{3}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{12 f}+\frac{55 a^{3} c \left(\tan^{5}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{12 f}-\frac{5 a^{3} c \left(\tan^{7}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{4 f}}{\left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{5 a^{3} c \ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{8 f}+$
risch	$\frac{i a^{3} c \left(9 \,{\mathrm e}^{7 i \left(f x+e\right)}+48 \,{\mathrm e}^{6 i \left(f x+e\right)}+33 \,{\mathrm e}^{5 i \left(f x+e\right)}+48 \,{\mathrm e}^{4 i \left(f x+e\right)}-33 \,{\mathrm e}^{3 i \left(f x+e\right)}+16 \,{\mathrm e}^{2 i \left(f x+e\right)}-9 \,{\mathrm e}^{i \left(f x+e\right)}+16\right)}{12 f \left({\mathrm e}^{2 i \left(f x+e\right)}+1\right)^{4}}-\frac{5 a^{3} c \ln\left({\mathrm e}^{i \left(f x\right.}\right.}{8 f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^3*c*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+
e)+tan(f*x+e)))+2*a^3*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a^3*c*tan(f*x+
e)+a^3*c*ln(sec(f*x+e)+tan(f*x+e)))
```

Maxima [A]
time = 0.30, size = 144, normalized size = 1.67

$$-\frac{32 \, {\left(\tan\left(f x + e\right)^{3} + 3 \, \tan\left(f x + e\right)\right)} a^{3} c - 3 \, a^{3} c {\left(\frac{2 \, {\left(3 \, \sin\left(f x + e\right)^{3} - 5 \, \sin\left(f x + e\right)\right)}}{\sin\left(f x + e\right)^{4} - 2 \, \sin\left(f x + e\right)^{2} + 1} - 3 \, \log\left(\sin\left(f x + e\right) + 1\right) + 3 \, \log\left(\sin\left(f x + e\right) - 1\right)\right)} - 48 \, a^{3} c \log\left(\sec\left(f x + e\right) + \tan\left(f x + e\right)\right) - 96 \, a^{3} c \tan\left(f x + e\right)}{48 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 3*a^3*c*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin
(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 48*a^3*c*log(sec(f*x + e) + tan
(f*x + e)) - 96*a^3*c*tan(f*x + e))/f
```

Fricas [A]
time = 3.70, size = 126, normalized size = 1.47

$$\frac{15 \, a^{3} c \cos\left(f x + e\right)^{4} \log\left(\sin\left(f x + e\right) + 1\right) - 15 \, a^{3} c \cos\left(f x + e\right)^{4} \log\left(-\sin\left(f x + e\right) + 1\right) + 2 \, {\left(16 \, a^{3} c \cos\left(f x + e\right)^{3} - 9 \, a^{3} c \cos\left(f x + e\right)^{2} - 16 \, a^{3} c \cos\left(f x + e\right) - 6 \, a^{3} c\right)} \sin\left(f x + e\right)}{48 \, f \cos\left(f x + e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*a^3*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e)
^4*log(-sin(f*x + e) + 1) + 2*(16*a^3*c*cos(f*x + e)^3 - 9*a^3*c*cos(f*x +
e)^2 - 16*a^3*c*cos(f*x + e) - 6*a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^3c\left(\int(-\sec(e+fx))\,dx+\int\left(-2\sec^2(e+fx)\right)\,dx+\int 2\sec^4(e+fx)\,dx+\int\sec^5(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + Int
egral(2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Giac [A]
time = 0.49, size = 128, normalized size = 1.49

$$\frac{15\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-55\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+73\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/24*(15*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c*tan(1/2*f*x + 1/2*e)^7 - 55*a^3*c*tan(1/
2*f*x + 1/2*e)^5 + 73*a^3*c*tan(1/2*f*x + 1/2*e)^3 + 15*a^3*c*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B]
time = 4.79, size = 146, normalized size = 1.70

$$\frac{5\,a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{5\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{55\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{73\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{5\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (5*a^3*c*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((5*a^3*c*tan(e/2 + (f*x)/2))/4
 + (73*a^3*c*tan(e/2 + (f*x)/2)^3)/12 - (55*a^3*c*tan(e/2 + (f*x)/2)^5)/12
+ (5*a^3*c*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2
+ (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.27 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=100

$$-\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf}-\frac{10a^3\tan(e+fx)}{cf}-\frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf}-\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -15/2*a^3*arctanh(sin(f*x+e))/c/f-10*a^3*tan(f*x+e)/c/f-5/2*a^3*sec(f*x+e)*
tan(f*x+e)/c/f-2*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A]

time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4042, 3873, 3852, 8, 4131, 3855}

$$-\frac{10a^3\tan(e+fx)}{cf}-\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf}-\frac{5a^3\tan(e+fx)\sec(e+fx)}{2cf}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (-15*a^3*ArcTanh[Sin[e + f*x]])/(2*c*f) - (10*a^3*Tan[e + f*x])/(c*f) - (5*
a^3*Sec[e + f*x]*Tan[e + f*x])/(2*c*f) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(c - c*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx &= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))}-\frac{(5a)\int \sec(e+fx)(a+a\,\mathrm{s}}{c} \\
&= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))}-\frac{(5a)\int \sec(e+fx)\left(a^2+a\right.}{c} \\
&= -\frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf}-\frac{2a(a+a\sec(e+fx))^2\tan(e+f}{f(c-c\sec(e+fx))} \\
&= -\frac{15a^3\tanh^{-1}(\sin(e+fx))}{2cf}-\frac{10a^3\tan(e+fx)}{cf}-\frac{5a^3\sec(e+fx}{2c}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 287 vs. 2(100) = 200.

time = 2.64, size = 287, normalized size = 2.87

$$\frac{a^3\cos^2(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\tan\left(\frac{1}{2}(e+fx)\right)\left(32\csc\left(\frac{e}{2}\right)\sec\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{fx}{2}\right)+\left(-30\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+30\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{1}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{16\sin(fx)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)\tan\left(\frac{1}{2}(e+fx)\right)\right)}{16f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^3*Tan[(e + f*x)/2
]*(32*Csc[e/2]*Sec[(e + f*x)/2]*Sin[(f*x)/2] + (-30*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] + 30*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^(
-2) + (16*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e +
```

```
f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))*Tan[(e
+ f*x)/2]))/(16*f*(c - c*Sec[e + f*x]))
```

Maple [A]
time = 0.17, size = 112, normalized size = 1.12

method	result
derivativedivides	$\frac{8a^3\left(\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{15\ln(\text{t}}{}\right.}{fc}$
default	$\frac{8a^3\left(\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{15\ln(\text{t}}{}\right.}{fc}$
risch	$\frac{ia^3\left(17\,e^{4i(fx+e)}-9\,e^{3i(fx+e)}+39\,e^{2i(fx+e)}-7\,e^{i(fx+e)}+24\right)}{fc\left(e^{i(fx+e)}-1\right)\left(e^{2i(fx+e)}+1\right)^2}-\frac{15a^3\ln\left(e^{i(fx+e)}+i\right)}{2cf}+\frac{15a^3\ln\left(e^{i(fx+e)}-i\right)}{2cf}$
norman	$\frac{-\frac{8a^3}{cf}+\frac{33a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{40a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{15a^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{15a^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2cf}-\frac{15a^3\ln\left(\tan\left(\frac{f}{}\right.\right.}{2cf}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*a^3/c*(1/16/(tan(1/2*f*x+1/2*e)+1)^2+7/16/(tan(1/2*f*x+1/2*e)+1)-15/16*
ln(tan(1/2*f*x+1/2*e)+1)-1/16/(tan(1/2*f*x+1/2*e)-1)^2+7/16/(tan(1/2*f*x+1/
2*e)-1)+15/16*ln(tan(1/2*f*x+1/2*e)-1)+1/tan(1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(104) = 208.
time = 0.28, size = 419, normalized size = 4.19

$$-\frac{a^3\left(\frac{2\left(\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+6a^3\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+6a^3\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}-\frac{\cos(fx+e)+1}{c\sin(fx+e)}\right)-\frac{2a^3(\cos(fx+e)+1)}{c\sin(fx+e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(a^3*(2*(5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 1)/(c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*c*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/c - 3*log(sin(f*x + e)/(cos(f*x + e) + 1
) - 1)/c) + 6*a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x +
 e)/(cos(f*x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)/c) + 6*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - 2*a^3*(
cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Fricas [A]
time = 2.63, size = 136, normalized size = 1.36

$$-\frac{15\,a^{3}\cos\left(fx+e\right)^{2}\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-15\,a^{3}\cos\left(fx+e\right)^{2}\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-48\,a^{3}\cos\left(fx+e\right)^{3}-34\,a^{3}\cos\left(fx+e\right)^{2}+16\,a^{3}\cos\left(fx+e\right)+2\,a^{3}}{4\,cf\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/4*(15*a^3*cos(f*x + e)^2*log(sin(f*x + e) + 1)*sin(f*x + e) - 15*a^3*cos
(f*x + e)^2*log(-sin(f*x + e) + 1)*sin(f*x + e) - 48*a^3*cos(f*x + e)^3 - 3
4*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 2*a^3)/(c*f*cos(f*x + e)^2*sin
(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^{3}\left(\int\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^{3}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^{4}\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x) - 1
), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) - 1), x))/c
```

Giac [A]
time = 0.55, size = 118, normalized size = 1.18

$$-\frac{\frac{15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}-\frac{15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}-\frac{16\,a^{3}}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}-\frac{2\,\left(7\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-9\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}c}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/2*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - 15*a^3*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/c - 16*a^3/(c*tan(1/2*f*x + 1/2*e)) - 2*(7*a^3*tan(1/2*f
*x + 1/2*e)^3 - 9*a^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2
*c))/f
```

Mupad [B]
time = 3.17, size = 105, normalized size = 1.05

$$\frac{15\,a^{3}\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-25\,a^{3}\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+8\,a^{3}}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}-2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{15\,a^{3}\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] (15*a^3*tan(e/2 + (f*x)/2)^4 - 25*a^3*tan(e/2 + (f*x)/2)^2 + 8*a^3)/(f*(c*t
an(e/2 + (f*x)/2) - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)/2)^5)) - (
15*a^3*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

3.28 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{5a^3\tan(e+fx)}{c^2f}-\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2}+\frac{10(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\left(c^2-c^2\sec(e+fx)\right)}$$

```
[Out] 5*a^3*arctanh(sin(f*x+e))/c^2/f+5*a^3*tan(f*x+e)/c^2/f-2/3*a*(a+a*sec(f*x+e
))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^2+10/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f
/(c^2-c^2*sec(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4042, 3872, 3855, 3852, 8}

$$\frac{5a^3\tan(e+fx)}{c^2f}+\frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f}+\frac{10\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\left(c^2-c^2\sec(e+fx)\right)}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) + (5*a^3*Tan[e + f*x])/(c^2*f) - (2*a
*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2) + (10*(a
^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c^2 - c^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} - \frac{(5a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}}{3c}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10(a^3+a^3\sec(e+fx))\tan}{3f\left(c^2-c^2\sec(e+f\right.}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10(a^3+a^3\sec(e+fx))\tan}{3f\left(c^2-c^2\sec(e+f\right.}$$

$$= \frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{1}{}$$

$$= \frac{5a^3\tanh^{-1}(\sin(e+fx))}{c^2f} + \frac{5a^3\tan(e+fx)}{c^2f} - \frac{2a(a+a\sec(e+fx)}{3f(c-c\sec(e}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 402 vs. 2(119) = 238.
time = 3.55, size = 402, normalized size = 3.38

$$\frac{a^3(1+\cos(e+fx))^3\sec^2\left(\tfrac{1}{2}(e+fx)\right)\tan\left(\tfrac{1}{2}(e+fx)\right)\left(\tfrac{1}{16}(-74+42\cos(e)-76\cos(fx)+120\cos(e+fx)-46\cos(2(e+fx))-76\cos(2e+fx)+23\cos(e+2fx)+23\cos(3e+2fx))\csc^3\left(\tfrac{e}{2}\right)\sec^5\left(\tfrac{1}{2}(e+fx)\right)\sin\left(\tfrac{fx}{2}\right)-48\csc^3\left(\tfrac{e}{2}\right)\csc^4(e+fx)\sin(e)\sin\left(\tfrac{fx}{2}\right)\sin^7\left(\tfrac{1}{2}(e+fx)\right)+\cos(e)\cos(e+fx)\csc^2\left(\tfrac{e}{2}\right)\sec^4\left(\tfrac{1}{2}(e+fx)\right)\left(4\cot\left(\tfrac{e}{2}\right)+15\left(\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right)\right)\sin^2\left(\tfrac{1}{2}(e+fx)\right)\right)\tan\left(\tfrac{1}{2}(e+fx)\right)-4(-1+5\cos(e+fx))\cot^2\left(\tfrac{e}{2}\right)\csc\left(\tfrac{e}{2}\right)\sec^3\left(\tfrac{1}{2}(e+fx)\right)\sin\left(\tfrac{fx}{2}\right)\tan^2\left(\tfrac{1}{2}(e+fx)\right)\right)}{6c^2f(-1+\cos(e+fx))^2\left(-1+\cot\left(\tfrac{e}{2}\right)\right)\left(1+\cot\left(\tfrac{e}{2}\right)\right)\left(-1+\tan\left(\tfrac{1}{2}(e+fx)\right)\right)\left(1+\tan\left(\tfrac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(((-74 + 42*C
os[e] - 76*Cos[f*x] + 120*Cos[e + f*x] - 46*Cos[2*(e + f*x)] - 76*Cos[2*e +
 f*x] + 23*Cos[e + 2*f*x] + 23*Cos[3*e + 2*f*x])*Csc[e/2]^3*Sec[(e + f*x)/2
]^5*Sin[(f*x)/2])/16 - 48*Csc[e/2]^3*Csc[e + f*x]^4*Sin[e]*Sin[(f*x)/2]*Sin
[(e + f*x)/2]^7 + Cos[e]*Cos[e + f*x]*Csc[e/2]^2*Sec[(e + f*x)/2]^4*(4*Cot[
e/2] + 15*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]])*Sin[(e + f*x)/2]^2)*Tan[(e + f*x)/2] - 4*(-1 + 5*Cos[e
 + f*x])*Cot[e/2]^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sin[(f*x)/2]*Tan[(e + f*x)/
```

```
2]^2))/(6*c^2*f*(-1 + Cos[e + f*x])^2*(-1 + Cot[e/2])*(1 + Cot[e/2])*(-1 +
Tan[(e + f*x)/2])*(1 + Tan[(e + f*x)/2]))
```

Maple [A]
time = 0.17, size = 97, normalized size = 0.82

method	result
derivativedivides	$\frac{4a^{3}\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right.}{f\,c^{2}}$
default	$\frac{4a^{3}\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right.}{f\,c^{2}}$
risch	$-\frac{2ia^{3}\left(12\,e^{4i(fx+e)}-51\,e^{3i(fx+e)}+41\,e^{2i(fx+e)}-57\,e^{i(fx+e)}+23\right)}{3f\,c^{2}\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{3}}-\frac{5a^{3}\ln\left(e^{i(fx+e)}-i\right)}{c^{2}f}+\frac{5a^{3}\ln\left(e^{i(fx+e)}+i\right)}{c^{2}f}$
norman	$\frac{\frac{4a^{3}}{3cf}+\frac{4a^{3}\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}-\frac{22a^{3}\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}+\frac{80a^{3}\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}-\frac{10a^{3}\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{5a^{3}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{c^{2}f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 4/f*a^3/c^2*(-1/3/tan(1/2*f*x+1/2*e)^3-2/tan(1/2*f*x+1/2*e)-1/4/(tan(1/2*f*
x+1/2*e)-1)-5/4*ln(tan(1/2*f*x+1/2*e)-1)-1/4/(tan(1/2*f*x+1/2*e)+1)+5/4*ln(
tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(125) = 250$.
time = 0.28, size = 377, normalized size = 3.17

$$-\frac{a^{3}\left(\frac{\frac{14\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{27\,\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+1}{\frac{c^{2}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}-\frac{c^{2}\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}-\frac{12\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{2}}+\frac{12\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{2}}\right)-3\,a^{3}\left(\frac{6\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{2}}-\frac{6\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{2}}-\frac{\left(\frac{9\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)(\cos(fx+e)+1)^{3}}{c^{2}\sin(fx+e)^{3}}\right)+\frac{3\,a^{3}\left(\frac{3\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+1\right)(\cos(fx+e)+1)^{3}}{c^{2}\sin(fx+e)^{3}}-\frac{a^{3}\left(\frac{3\,\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-1\right)(\cos(fx+e)+1)^{3}}{c^{2}\sin(fx+e)^{3}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(a^3*((14*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 27*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 1)/(c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - c^2*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/c^2 + 12*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^2) - 3*a^3*(6*log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1
)^3/(c^2*sin(f*x + e)^3)) + 3*a^3*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) - a^3*(3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Fricas [A]
time = 3.24, size = 179, normalized size = 1.50

$$-\frac{46\,a^{3}\cos\left(fx+e\right)^{3}-22\,a^{3}\cos\left(fx+e\right)^{2}-62\,a^{3}\cos\left(fx+e\right)+6\,a^{3}-15\,\left(a^{3}\cos\left(fx+e\right)^{2}-a^{3}\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+15\,\left(a^{3}\cos\left(fx+e\right)^{2}-a^{3}\cos\left(fx+e\right)\right)\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)}{6\,\left(c^{2}f\cos\left(fx+e\right)^{2}-c^{2}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(46*a^3*cos(f*x + e)^3 - 22*a^3*cos(f*x + e)^2 - 62*a^3*cos(f*x + e) +
 6*a^3 - 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(sin(f*x + e) + 1)*s
in(f*x + e) + 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(-sin(f*x + e)
+ 1)*sin(f*x + e))/((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{3}\left(\int\frac{\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{3\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{3\sec^{3}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx+\int\frac{\sec^{4}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)-2\sec\left(e+fx\right)+1}\,dx\right)}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + In
tegral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(3*sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral
(sec(e + f*x)**4/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Giac [A]
time = 0.54, size = 116, normalized size = 0.97

$$\frac{\frac{15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^{2}}-\frac{15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^{2}}-\frac{6\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)c^{2}}-\frac{4\,\left(6\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a^{3}\right)}{c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 15*a^3*log(abs(tan(1/2
*f*x + 1/2*e) - 1))/c^2 - 6*a^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)
^2 - 1)*c^2) - 4*(6*a^3*tan(1/2*f*x + 1/2*e)^2 + a^3)/(c^2*tan(1/2*f*x + 1/
2*e)^3))/f
```

Mupad [B]
time = 2.04, size = 93, normalized size = 0.78

$$\frac{10\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^2\,f}+\frac{-10\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{20\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+\frac{4\,a^3}{3}}{c^2\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (10*a^3*atanh(tan(e/2 + (f*x)/2)))/(c^2*f) + ((20*a^3*tan(e/2 + (f*x)/2)^2)
/3 - 10*a^3*tan(e/2 + (f*x)/2)^4 + (4*a^3)/3)/(c^2*f*tan(e/2 + (f*x)/2)^3*(
tan(e/2 + (f*x)/2)^2 - 1))
```

3.29 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=132

$$-\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3f}-\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}+\frac{2(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf(c-c\sec(e+fx))^2}-\frac{2a^3}{f\,(c^3-}$$

```
[Out] -a^3*arctanh(sin(f*x+e))/c^3/f-2/5*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*s
ec(f*x+e))^3+2/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^2-2*a
^3*tan(f*x+e)/f/(c^3-c^3*sec(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4042, 3855}

$$-\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3f}-\frac{2a^3\tan(e+fx)}{f\,(c^3-c^3\sec(e+fx))}+\frac{2\tan(e+fx)\,(a^3\sec(e+fx)+a^3)}{3cf(c-c\sec(e+fx))^2}-\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Sin[e + f*x]])/(c^3*f)) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(5*f*(c - c*Sec[e + f*x])^3) + (2*(a^3 + a^3*Sec[e + f*x])*Tan[e +
f*x])/(3*c*f*(c - c*Sec[e + f*x])^2) - (2*a^3*Tan[e + f*x])/(f*(c^3 - c^3*S
ec[e + f*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}}{c}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2(a^3+a^3\sec(e+fx))\tan}{3cf(c-c\sec(e+f}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2(a^3+a^3\sec(e+fx))\tan}{3cf(c-c\sec(e+f}$$

$$= -\frac{a^3\tanh^{-1}(\sin(e+fx))}{c^3f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} +$$

Mathematica [A]

time = 0.13, size = 139, normalized size = 1.05

$$-\frac{a^3\left(-\frac{26\cot\left(\frac{1}{2}(e+fx)\right)}{15f}+\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{15f}-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)}{5f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*((-26*Cot[(e + f*x)/2])/(15*f) + (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2
]^2)/(15*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4)/(5*f) - Log[Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f)
)/c^3)
```

Maple [A]

time = 0.21, size = 78, normalized size = 0.59

method	result
derivativedivides	$\frac{2a^3\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,c^3}$
default	$\frac{2a^3\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,c^3}$
risch	$\frac{4ia^3\left(15\,e^{4i(fx+e)}-30\,e^{3i(fx+e)}+100\,e^{2i(fx+e)}-50\,e^{i(fx+e)}+13\right)}{15f\,c^3\left(e^{i(fx+e)}-1\right)^5}+\frac{a^3\ln\left(e^{i(fx+e)}-i\right)}{c^3f}-\frac{a^3\ln\left(e^{i(fx+e)}+i\right)}{c^3f}$
norman	$\frac{-\frac{2a^3}{5cf}+\frac{8a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15cf}-\frac{6a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5cf}+\frac{22a^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5cf}-\frac{16a^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3cf}+\frac{2a^3\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 2/f*a^3/c^3*(1/2*ln(tan(1/2*f*x+1/2*e)-1)+1/5/tan(1/2*f*x+1/2*e)^5+1/3/tan(
1/2*f*x+1/2*e)^3+1/tan(1/2*f*x+1/2*e)-1/2*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(140) = 280$.
time = 0.31, size = 335, normalized size = 2.54

$$-\frac{a^3\left(\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right)-\frac{3a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}+\frac{a^3\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}+\frac{9a^3\left(\frac{5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(a^3*(60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/c^3 - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*
sin(f*x + e)^5)) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^
5) + a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 9*a^3*(5*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x +
e)^5))/f
```

Fricas [A]
time = 2.34, size = 190, normalized size = 1.44

$$\frac{52\,a^3\cos(fx+e)^3-44\,a^3\cos(fx+e)^2-4\,a^3\cos(fx+e)+92\,a^3-15\left(a^3\cos(fx+e)^2-2\,a^3\cos(fx+e)+a^3\right)\log(\sin(fx+e)+1)\sin(fx+e)+15\left(a^3\cos(fx+e)^2-2\,a^3\cos(fx+e)+a^3\right)\log(-\sin(fx+e)+1)\sin(fx+e)}{30\left(c^3f\cos(fx+e)^2-2\,c^3f\cos(fx+e)+c^3f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(52*a^3*cos(f*x + e)^3 - 44*a^3*cos(f*x + e)^2 - 4*a^3*cos(f*x + e) +
92*a^3 - 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*log(sin(f*x + e
) + 1)*sin(f*x + e) + 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*lo
g(-sin(f*x + e) + 1)*sin(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x
 + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3\left(\int\frac{\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int\frac{3\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int\frac{3\sec^3(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx+\int\frac{\sec^4(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x
)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)*
*4/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Giac [A]
time = 0.52, size = 102, normalized size = 0.77

$$-\frac{\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^3}-\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^3}-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+5\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,a^3\right)}{c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/15*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^3 - 15*a^3*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/c^3 - 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 + 5*a^3*tan(1/
2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B]
time = 1.78, size = 78, normalized size = 0.59

$$\frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+\frac{2\,a^3}{5}}{c^3\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}-\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] ((2*a^3*tan(e/2 + (f*x)/2)^2)/3 + 2*a^3*tan(e/2 + (f*x)/2)^4 + (2*a^3)/5)/(
c^3*f*tan(e/2 + (f*x)/2)^5) - (2*a^3*atanh(tan(e/2 + (f*x)/2)))/(c^3*f)
```

3.30 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=38

$$-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{7f(c-c\sec(e+fx))^4}$$

```
[Out] -1/7*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^4
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{4035\}$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{7f(c-c\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/7*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{7f(c-c\sec(e+fx))^4}$$

Mathematica [A]
time = 0.16, size = 25, normalized size = 0.66

$$-\frac{a^3\cot^7\left(\frac{1}{2}(e+fx)\right)}{7c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

[Out] -1/7*(a^3*Cot[(e + f*x)/2]^7)/(c^4*f)

Maple [A]
time = 0.20, size = 23, normalized size = 0.61

method	result	size
derivativedivides	$-\frac{a^{3}}{7 f \,c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$	23
default	$-\frac{a^{3}}{7 f \,c^{4} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$	23
risch	$\frac{2 i a^{3} \left(7 \,{\mathrm e}^{6 i \left(f x +e \right)}+35 \,{\mathrm e}^{4 i \left(f x +e \right)}+21 \,{\mathrm e}^{2 i \left(f x +e \right)}+1\right)}{7 f \,c^{4} \left({\mathrm e}^{i \left(f x +e \right)}-1\right)^{7}}$	61
norman	$\frac{\frac{a^{3}}{7 c f}-\frac{3 a^{3} \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{7 c f}+\frac{3 a^{3} \left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{7 c f}-\frac{a^{3} \left(\tan^{6}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{7 c f}}{\left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{3} c^{3} \tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$	109

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)

[Out] -1/7/f*a^3/c^4/tan(1/2*f*x+1/2*e)^7

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(40) = 80$.
time = 0.32, size = 388, normalized size = 10.21

$$-\frac{\frac{a^{3}\left(\frac{21 \sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{35 \sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{35 \sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+5\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{c^{4}\sin\left(fx+e\right)^{7}}-\frac{a^{3}\left(\frac{21 \sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{35 \sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{105 \sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-15\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{c^{4}\sin\left(fx+e\right)^{7}}-\frac{a^{3}\left(\frac{21 \sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{35 \sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}+\frac{35 \sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-5\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{c^{4}\sin\left(fx+e\right)^{7}}+\frac{a^{3}\left(\frac{21 \sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{35 \sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{105 \sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}+15\right)\left(\cos\left(fx+e\right)+1\right)^{7}}{c^{4}\sin\left(fx+e\right)^{7}}}{280 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="maxima")

[Out] -1/280*(a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) + a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(40) = 80$.

time = 2.41, size = 119, normalized size = 3.13

$$\frac{a^3 \cos(fx+e)^4 + 4a^3 \cos(fx+e)^3 + 6a^3 \cos(fx+e)^2 + 4a^3 \cos(fx+e) + a^3}{7\left(c^4 f \cos(fx+e)^3 - 3c^4 f \cos(fx+e)^2 + 3c^4 f \cos(fx+e) - c^4 f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + a^3)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*
c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{3\sec^3(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{\sec^4(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
+ f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
 f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)
**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Giac [A]
time = 0.65, size = 22, normalized size = 0.58

$$-\frac{a^3}{7\,c^4 f \tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/7*a^3/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B]
time = 1.81, size = 22, normalized size = 0.58

$$-\frac{a^3 \cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{7\,c^4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

```
[Out] -(a^3*cot(e/2 + (f*x)/2)^7)/(7*c^4*f)
```

3.31 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=80

$$-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5}-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}$$

```
[Out] -1/9*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-1/63*(a+a*sec(f*x+e
))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4036, 4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/9*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) - ((a
 + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx}{9c}$$
$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))^3\tan(e+}{63cf(c-c\sec(e+fx))^4}$$

Mathematica [A]
time = 0.41, size = 141, normalized size = 1.76

$$-\frac{a^3\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(693\sin\left(\frac{fx}{2}\right)+315\sin\left(e+\frac{fx}{2}\right)-189\sin\left(e+\frac{3fx}{2}\right)-483\sin\left(2e+\frac{3fx}{2}\right)+225\sin\left(2e+\frac{5fx}{2}\right)+63\sin\left(3e+\frac{5fx}{2}\right)-9\sin\left(3e+\frac{7fx}{2}\right)-63\sin\left(4e+\frac{7fx}{2}\right)+8\sin\left(4e+\frac{9fx}{2}\right)\right)}{16128c^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/16128*(a^3*Csc[e/2]*Csc[(e + f*x)/2]^9*(693*Sin[(f*x)/2] + 315*Sin[e + (
f*x)/2] - 189*Sin[e + (3*f*x)/2] - 483*Sin[2*e + (3*f*x)/2] + 225*Sin[2*e +
 (5*f*x)/2] + 63*Sin[3*e + (5*f*x)/2] - 9*Sin[3*e + (7*f*x)/2] - 63*Sin[4*e
 + (7*f*x)/2] + 8*Sin[4*e + (9*f*x)/2]))/(c^5*f)
```

Maple [A]
time = 0.22, size = 39, normalized size = 0.49

method	result
derivativedivides	$\frac{a^3\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}\right)}{2f\,c^5}$
default	$\frac{a^3\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}\right)}{2f\,c^5}$
risch	$\frac{2ia^3\left(63\,e^{8i(fx+e)}-63\,e^{7i(fx+e)}+483\,e^{6i(fx+e)}-315\,e^{5i(fx+e)}+693\,e^{4i(fx+e)}-189\,e^{3i(fx+e)}+225\,e^{2i(fx+e)}-9\,e^{i(fx+e)}+\right.}{63f\,c^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{-\frac{a^3}{18cf}+\frac{5a^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{21cf}-\frac{8a^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{21cf}+\frac{17a^3\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{63cf}-\frac{a^3\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{14cf}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f*a^3/c^5*(-1/7/tan(1/2*f*x+1/2*e)^7+1/9/tan(1/2*f*x+1/2*e)^9)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(84) = 168$.
time = 0.30, size = 389, normalized size = 4.86

$$-\frac{\frac{a^3\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{15a^3\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}-\frac{5a^3\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{63\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+7\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{21a^3\left(\frac{18\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{45\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-5\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}}{5040f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/5040*(a^3*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^
9) + 15*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 63*sin(f*x + e)^8/(cos(f*x +
 e) + 1)^8 + 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) + 21*a^3*(18*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 -
5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

Fricas [A]
time = 2.20, size = 150, normalized size = 1.88

$$\frac{8\,a^3\cos\left(fx+e\right)^5+31\,a^3\cos\left(fx+e\right)^4+44\,a^3\cos\left(fx+e\right)^3+26\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)-a^3}{63\,\left(c^5f\cos\left(fx+e\right)^4-4\,c^5f\cos\left(fx+e\right)^3+6\,c^5f\cos\left(fx+e\right)^2-4\,c^5f\cos\left(fx+e\right)+c^5f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/63*(8*a^3*cos(f*x + e)^5 + 31*a^3*cos(f*x + e)^4 + 44*a^3*cos(f*x + e)^3
+ 26*a^3*cos(f*x + e)^2 + 4*a^3*cos(f*x + e) - a^3)/((c^5*f*cos(f*x + e)^4
- 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) +
c^5*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3\left(\int\frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{3\sec^3\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx+\int\frac{\sec^4\left(e+fx\right)}{\sec^5\left(e+fx\right)-5\sec^4\left(e+fx\right)+10\sec^3\left(e+fx\right)-10\sec^2\left(e+fx\right)+5\sec\left(e+fx\right)-1}\,dx\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(
e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(se
c(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2
 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**5 - 5*
sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x)
- 1), x))/c**5
```

Giac [A]
time = 0.62, size = 41, normalized size = 0.51

$$-\frac{9\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-7\,a^{3}}{126\,c^{5}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] -1/126*(9*a^3*tan(1/2*f*x + 1/2*e)^2 - 7*a^3)/(c^5*f*tan(1/2*f*x + 1/2*e)^9
)
```

Mupad [B]
time = 1.62, size = 37, normalized size = 0.46

$$\frac{a^{3}\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}\left(7\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-9\right)}{126\,c^{5}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (a^3*cot(e/2 + (f*x)/2)^7*(7*cot(e/2 + (f*x)/2)^2 - 9))/(126*c^5*f)
```

3.32 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=121

$$-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6}-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5}-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{693c^2f(c-c\sec(e+fx))^4}$$

```
[Out] -1/11*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^6-2/99*(a+a*sec(f*x+
e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^5-2/693*(a+a*sec(f*x+e))^3*tan(f*x+e)
/c^2/f/(c-c*sec(f*x+e))^4
```

Rubi [A]
time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4036, 4035}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{693c^2f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{99cf(c-c\sec(e+fx))^5}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/11*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^6) - (2
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(99*c*f*(c - c*Sec[e + f*x])^5) - (2*
(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*c^2*f*(c - c*Sec[e + f*x])^4)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx}{11c}$$
$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+}{99cf(c-c\sec(e+fx))^5}$$
$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+}{99cf(c-c\sec(e+fx))^5}$$

Mathematica [A]
time = 0.70, size = 167, normalized size = 1.38

$$-\frac{a^3\csc\left(\frac{e}{2}\right)\csc^{11}\left(\frac{1}{2}(e+fx)\right)\left(15246\sin\left(\frac{fx}{2}\right)+21252\sin\left(e+\frac{fx}{2}\right)-15444\sin\left(e+\frac{3fx}{2}\right)-10626\sin\left(2e+\frac{3fx}{2}\right)+4950\sin\left(2e+\frac{5fx}{2}\right)+8085\sin\left(3e+\frac{5fx}{2}\right)-2959\sin\left(3e+\frac{7fx}{2}\right)-1386\sin\left(4e+\frac{7fx}{2}\right)+176\sin\left(4e+\frac{9fx}{2}\right)+693\sin\left(5e+\frac{9fx}{2}\right)-79\sin\left(5e+\frac{11fx}{2}\right)\right)}{709632c^6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/709632*(a^3*Csc[e/2]*Csc[(e + f*x)/2]^11*(15246*Sin[(f*x)/2] + 21252*Sin
[e + (f*x)/2] - 15444*Sin[e + (3*f*x)/2] - 10626*Sin[2*e + (3*f*x)/2] + 495
0*Sin[2*e + (5*f*x)/2] + 8085*Sin[3*e + (5*f*x)/2] - 2959*Sin[3*e + (7*f*x)
/2] - 1386*Sin[4*e + (7*f*x)/2] + 176*Sin[4*e + (9*f*x)/2] + 693*Sin[5*e +
(9*f*x)/2] - 79*Sin[5*e + (11*f*x)/2]))/(c^6*f)
```

Maple [A]
time = 0.20, size = 52, normalized size = 0.43

method	result
derivativedivides	$\frac{a^3\left(-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{2}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}\right)}{4f\,c^6}$
default	$\frac{a^3\left(-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{2}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}\right)}{4f\,c^6}$
risch	$\frac{2ia^3\left(693\,e^{10i(fx+e)}-1386\,e^{9i(fx+e)}+8085\,e^{8i(fx+e)}-10626\,e^{7i(fx+e)}+21252\,e^{6i(fx+e)}-15246\,e^{5i(fx+e)}+15444\,e^{4i(fx+}\right.}{693f\,c^6\left(e^{i(fx+e)}-1\right)^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f*a^3/c^6*(-1/11/tan(1/2*f*x+1/2*e)^11-1/7/tan(1/2*f*x+1/2*e)^7+2/9/tan
(1/2*f*x+1/2*e)^9)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(127) = 254$.
time = 0.30, size = 566, normalized size = 4.68

$$\frac{\frac{3a^3\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{990\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{1386\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{3465\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-315\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}+\frac{9a^3\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{330\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{462\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{1155\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-105\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}+\frac{5a^3\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{990\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{1386\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{693\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-63\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}-\frac{a^3\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{990\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{1386\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{3465\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+315\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}}{110880\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/110880*(3*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1
)^10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 9*a^3*(385*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4
62*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x + e) +
1)^11/(c^6*sin(f*x + e)^11) + 5*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*x + e)^1
0/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) -
 a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 990*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 315)
*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

Fricas [A]
time = 2.61, size = 180, normalized size = 1.49

$$\frac{79\,a^3\cos(fx+e)^6+298\,a^3\cos(fx+e)^5+404\,a^3\cos(fx+e)^4+216\,a^3\cos(fx+e)^3+19\,a^3\cos(fx+e)^2-10\,a^3\cos(fx+e)+2\,a^3}{693\left(c^6f\cos(fx+e)^5-5\,c^6f\cos(fx+e)^4+10\,c^6f\cos(fx+e)^3-10\,c^6f\cos(fx+e)^2+5\,c^6f\cos(fx+e)-c^6f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/693*(79*a^3*cos(f*x + e)^6 + 298*a^3*cos(f*x + e)^5 + 404*a^3*cos(f*x + e
)^4 + 216*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 10*a^3*cos(f*x + e)
+ 2*a^3)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x
 + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x +
 e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3\left(\int\frac{\sec(e+fx)}{\sec^6(e+fx)-6\sec^5(e+fx)+15\sec^4(e+fx)-20\sec^3(e+fx)+15\sec^2(e+fx)-6\sec(e+fx)+1}\,dx+\int\frac{3\sec^2(e+fx)}{\sec^6(e+fx)-6\sec^5(e+fx)+15\sec^4(e+fx)-20\sec^3(e+fx)+15\sec^2(e+fx)-6\sec(e+fx)+1}\,dx+\int\frac{3\sec^3(e+fx)}{\sec^6(e+fx)-6\sec^5(e+fx)+15\sec^4(e+fx)-20\sec^3(e+fx)+15\sec^2(e+fx)-6\sec(e+fx)+1}\,dx+\int\frac{\sec^4(e+fx)}{\sec^6(e+fx)-6\sec^5(e+fx)+15\sec^4(e+fx)-20\sec^3(e+fx)+15\sec^2(e+fx)-6\sec(e+fx)+1}\,dx\right)}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e
 + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),
 x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*
sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x)
+ 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x)**5
+ 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e +
f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**6 - 6*sec(e + f*x)*
*5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e
 + f*x) + 1), x))/c**6
```

Giac [A]
time = 0.82, size = 57, normalized size = 0.47

$$-\frac{99\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-154\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+63\,a^3}{2772\,c^6f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] -1/2772*(99*a^3*tan(1/2*f*x + 1/2*e)^4 - 154*a^3*tan(1/2*f*x + 1/2*e)^2 + 6
3*a^3)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B]
time = 1.86, size = 67, normalized size = 0.55

$$\frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{18\,c^6\,f}-\frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{28\,c^6\,f}-\frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{44\,c^6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^6),x)
```

```
[Out] (a^3*cot(e/2 + (f*x)/2)^9)/(18*c^6*f) - (a^3*cot(e/2 + (f*x)/2)^7)/(28*c^6*
f) - (a^3*cot(e/2 + (f*x)/2)^11)/(44*c^6*f)
```

3.33 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7} dx$

Optimal. Leaf size=162

$$-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7}-\frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6}-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{429c^2f(c-c\sec(e+fx))^5}$$

```
[Out] -1/13*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^7-3/143*(a+a*sec(f*x
+e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^6-2/429*(a+a*sec(f*x+e))^3*tan(f*x+e
)/c^2/f/(c-c*sec(f*x+e))^5-2/3003*(a+a*sec(f*x+e))^3*tan(f*x+e)/c^3/f/(c-c*
sec(f*x+e))^4
```

Rubi [A]
time = 0.22, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4036, 4035}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{3003c^3f(c-c\sec(e+fx))^4}-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{429c^2f(c-c\sec(e+fx))^5}-\frac{3\tan(e+fx)(a\sec(e+fx)+a)^3}{143cf(c-c\sec(e+fx))^6}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] -1/13*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^7) - (3
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(143*c*f*(c - c*Sec[e + f*x])^6) - (2
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(429*c^2*f*(c - c*Sec[e + f*x])^5) -
(2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*c^3*f*(c - c*Sec[e + f*x])^4)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx &= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7}+\frac{3\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx}{13c} \\
&= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7}-\frac{3(a+a\sec(e+fx))^3\tan(e+\ldots}{143cf(c-c\sec(e+fx))^6} \\
&= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7}-\frac{3(a+a\sec(e+fx))^3\tan(e+\ldots}{143cf(c-c\sec(e+fx))^6} \\
&= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7}-\frac{3(a+a\sec(e+fx))^3\tan(e+\ldots}{143cf(c-c\sec(e+fx))^6}
\end{aligned}
$$

Mathematica [A]
time = 0.61, size = 193, normalized size = 1.19

$$
-\frac{a^3\csc\left(\frac{e}{2}\right)\csc^{13}\left(\frac{1}{2}(e+fx)\right)\left(285714\sin\left(\frac{fx}{2}\right)+246246\sin\left(e+\frac{fx}{2}\right)-182754\sin\left(e+\frac{3fx}{2}\right)-216216\sin\left(2e+\frac{3fx}{2}\right)+122551\sin\left(2e+\frac{5fx}{2}\right)+99099\sin\left(3e+\frac{5fx}{2}\right)-37609\sin\left(3e+\frac{7fx}{2}\right)-51051\sin\left(4e+\frac{7fx}{2}\right)+15171\sin\left(4e+\frac{9fx}{2}\right)+9009\sin\left(5e+\frac{9fx}{2}\right)-1027\sin\left(5e+\frac{11fx}{2}\right)-3003\sin\left(6e+\frac{11fx}{2}\right)+310\sin\left(6e+\frac{13fx}{2}\right)\right)}{12300288c^7f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] -1/12300288*(a^3*Csc[e/2]*Csc[(e + f*x)/2]^13*(285714*Sin[(f*x)/2] + 246246
*Sin[e + (f*x)/2] - 182754*Sin[e + (3*f*x)/2] - 216216*Sin[2*e + (3*f*x)/2]
 + 122551*Sin[2*e + (5*f*x)/2] + 99099*Sin[3*e + (5*f*x)/2] - 37609*Sin[3*e
 + (7*f*x)/2] - 51051*Sin[4*e + (7*f*x)/2] + 15171*Sin[4*e + (9*f*x)/2] + 9
009*Sin[5*e + (9*f*x)/2] - 1027*Sin[5*e + (11*f*x)/2] - 3003*Sin[6*e + (11*
f*x)/2] + 310*Sin[6*e + (13*f*x)/2]))/(c^7*f)
```

Maple [A]
time = 0.23, size = 65, normalized size = 0.40

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{13\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}-\frac{3}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}\right)}{8f\,c^7}$
default	$\frac{a^3\left(\frac{1}{13\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}-\frac{3}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}\right)}{8f\,c^7}$
risch	$\frac{2ia^3\left(3003\,\mathrm{e}^{12i(fx+e)}-9009\,\mathrm{e}^{11i(fx+e)}+51051\,\mathrm{e}^{10i(fx+e)}-99099\,\mathrm{e}^{9i(fx+e)}+216216\,\mathrm{e}^{8i(fx+e)}-246246\,\mathrm{e}^{7i(fx+e)}+285714\ldots\right.}{3003f\,c^7\left(\mathrm{e}^{i(fx+e)}-1\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/f*a^3/c^7*(1/13/tan(1/2*f*x+1/2*e)^13-3/11/tan(1/2*f*x+1/2*e)^11+1/3/ta
n(1/2*f*x+1/2*e)^9-1/7/tan(1/2*f*x+1/2*e)^7)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(170) = 340$.
time = 0.32, size = 565, normalized size = 3.49

$$-\frac{\frac{a^3\left(\frac{8190\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5005\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{25740\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{9009\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{30030\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-\frac{45045\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}-3465\right)(\cos(fx+e)+1)^{13}}{c^7\sin(fx+e)^{13}}+\frac{5a^3\left(\frac{1638\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5005\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{8580\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{9009\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{6006\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-\frac{3003\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}-231\right)(\cos(fx+e)+1)^{13}}{c^7\sin(fx+e)^{13}}+\frac{35a^3\left(\frac{468\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{715\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{1287\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-\frac{1716\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{1287\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}-99\right)(\cos(fx+e)+1)^{13}}{c^7\sin(fx+e)^{13}}+\frac{77a^3\left(\frac{65\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{117\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{195\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}-15\right)(\cos(fx+e)+1)^{13}}{c^7\sin(fx+e)^{13}}}{960960f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="ma
xima")
```

```
[Out] -1/960960*(a^3*(8190*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5005*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 25740*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9009
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30030*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 - 45045*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 3465)*(cos(f*x + e)
 + 1)^13/(c^7*sin(f*x + e)^13) + 5*a^3*(1638*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 5005*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8580*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 9009*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 6006*sin(f*x
+ e)^10/(cos(f*x + e) + 1)^10 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
- 231)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13) + 35*a^3*(468*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 715*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1287*
sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 1716*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + 1287*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 99)*(cos(f*x + e) + 1)
^13/(c^7*sin(f*x + e)^13) + 77*a^3*(65*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 117*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^12/(cos(f*x +
e) + 1)^12 - 15)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13))/f
```

Fricas [**A**]
time = 4.70, size = 208, normalized size = 1.28

$$\frac{310\,a^3\cos(fx+e)^7+1143\,a^3\cos(fx+e)^6+1492\,a^3\cos(fx+e)^5+736\,a^3\cos(fx+e)^4+34\,a^3\cos(fx+e)^3-29\,a^3\cos(fx+e)^2+12\,a^3\cos(fx+e)-2\,a^3}{3003\left(c^7f\cos(fx+e)^6-6\,c^7f\cos(fx+e)^5+15\,c^7f\cos(fx+e)^4-20\,c^7f\cos(fx+e)^3+15\,c^7f\cos(fx+e)^2-6\,c^7f\cos(fx+e)+c^7f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="fr
icas")
```

```
[Out] 1/3003*(310*a^3*cos(f*x + e)^7 + 1143*a^3*cos(f*x + e)^6 + 1492*a^3*cos(f*x
 + e)^5 + 736*a^3*cos(f*x + e)^4 + 34*a^3*cos(f*x + e)^3 - 29*a^3*cos(f*x +
 e)^2 + 12*a^3*cos(f*x + e) - 2*a^3)/((c^7*f*cos(f*x + e)^6 - 6*c^7*f*cos(f
*x + e)^5 + 15*c^7*f*cos(f*x + e)^4 - 20*c^7*f*cos(f*x + e)^3 + 15*c^7*f*co
s(f*x + e)^2 - 6*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^{3}\left(\int \frac{\sec \left(e+fx\right)}{\sec ^{7}\left(e+fx\right)-7\sec ^{6}\left(e+fx\right)+21\sec ^{5}\left(e+fx\right)-35\sec ^{4}\left(e+fx\right)+35\sec ^{3}\left(e+fx\right)-21\sec ^{2}\left(e+fx\right)+7\sec \left(e+fx\right)-1}\,dx+\int \frac{3\sec ^{2}\left(e+fx\right)}{\sec ^{7}\left(e+fx\right)-7\sec ^{6}\left(e+fx\right)+21\sec ^{5}\left(e+fx\right)-35\sec ^{4}\left(e+fx\right)+35\sec ^{3}\left(e+fx\right)-21\sec ^{2}\left(e+fx\right)+7\sec \left(e+fx\right)-1}\,dx+\int \frac{3\sec ^{3}\left(e+fx\right)}{\sec ^{7}\left(e+fx\right)-7\sec ^{6}\left(e+fx\right)+21\sec ^{5}\left(e+fx\right)-35\sec ^{4}\left(e+fx\right)+35\sec ^{3}\left(e+fx\right)-21\sec ^{2}\left(e+fx\right)+7\sec \left(e+fx\right)-1}\,dx+\int \frac{\sec ^{4}\left(e+fx\right)}{\sec ^{7}\left(e+fx\right)-7\sec ^{6}\left(e+fx\right)+21\sec ^{5}\left(e+fx\right)-35\sec ^{4}\left(e+fx\right)+35\sec ^{3}\left(e+fx\right)-21\sec ^{2}\left(e+fx\right)+7\sec \left(e+fx\right)-1}\,dx\right)}{c^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**7,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec(
e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2
+ 7*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**7 - 7
*sec(e + f*x)**6 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x
)**3 - 21*sec(e + f*x)**2 + 7*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*
x)**3/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec(e + f*x)**5 - 35*sec(e
+ f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2 + 7*sec(e + f*x) - 1),
x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec
(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2
 + 7*sec(e + f*x) - 1), x))/c**7
```

Giac [A]
time = 0.69, size = 73, normalized size = 0.45

$$-\frac{429\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-1001\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+819\,a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-231\,a^{3}}{24024\,c^{7}f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="gi
ac")
```

```
[Out] -1/24024*(429*a^3*tan(1/2*f*x + 1/2*e)^6 - 1001*a^3*tan(1/2*f*x + 1/2*e)^4
+ 819*a^3*tan(1/2*f*x + 1/2*e)^2 - 231*a^3)/(c^7*f*tan(1/2*f*x + 1/2*e)^13)
```

Mupad [B]
time = 1.72, size = 108, normalized size = 0.67

$$\frac{a^{3}\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}\left(231\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-819\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1001\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-429\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}\right)}{24024\,c^{7}\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^7),x)
```

```
[Out] (a^3*cos(e/2 + (f*x)/2)^7*(231*cos(e/2 + (f*x)/2)^6 - 429*sin(e/2 + (f*x)/2
)^6 + 1001*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 819*cos(e/2 + (f*x)/
2)^4*sin(e/2 + (f*x)/2)^2))/(24024*c^7*f*sin(e/2 + (f*x)/2)^13)
```

3.34 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=121

$$-\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af}+\frac{28c^4\tan(e+fx)}{af}-\frac{21c^4\sec(e+fx)\tan(e+fx)}{2af}+\frac{2c(c-c\sec(e+fx))^3\tan(e}{f(a+a\sec(e+fx))}$$

```
[Out] -35/2*c^4*arctanh(sin(f*x+e))/a/f+28*c^4*tan(f*x+e)/a/f-21/2*c^4*sec(f*x+e)
*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))+7/3*c^
4*tan(f*x+e)^3/a/f
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4042, 3876, 3855, 3852, 8, 3853}

$$\frac{7c^4\tan^3(e+fx)}{3af}+\frac{28c^4\tan(e+fx)}{af}-\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af}-\frac{21c^4\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-35*c^4*ArcTanh[Sin[e + f*x]])/(2*a*f) + (28*c^4*Tan[e + f*x])/(a*f) - (21
*c^4*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^3*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x])) + (7*c^4*Tan[e + f*x]^3)/(3*a*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx &= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c)\int \sec(e+fx)(c-c\sec(e\\
&= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c)\int (c^3\sec(e+fx)-3c^3\text{se}\\
&= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c^4)\int \sec(e+fx)\,dx}{a} + \frac{(7c^4)\\
&= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{af} - \frac{21c^4\sec(e+fx)\tan(e+fx)}{2af} + \frac{2c(c-\\
&= -\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2af} + \frac{28c^4\tan(e+fx)}{af} - \frac{21c^4\sec(e+fx)\,\text{t}}{2af}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1036 vs. $2(121) = 242$.
time = 6.45, size = 1036, normalized size = 8.56

$$
\frac{35\cos^3(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(c-c\sec(e+fx))^4}{16f(a+a\sec(e+fx))} - \frac{35\cos^3(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(c-c\sec(e+fx))^4}{16f(a+a\sec(e+fx))} + \frac{2\cos^3(e+fx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^7\left(\frac{e}{2}+\frac{fx}{2}\right)\sec\left(\frac{e}{2}\right)(c-c\sec(e+fx))^4\sin\left(\frac{fx}{2}\right)}{f(a+a\sec(e+fx))} + \frac{\cos^3(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)(c-c\sec(e+fx))^4\sin\left(\frac{fx}{2}\right)}{48f(a+a\sec(e+fx))\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^3} + \frac{\cos^3(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)(c-c\sec(e+fx))^4\left(-7\cos\left(\frac{e}{2}\right)+8\sin\left(\frac{e}{2}\right)\right)}{48f(a+a\sec(e+fx))\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2} + \frac{35\cos^3(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)(c-c\sec(e+fx))^4\sin\left(\frac{fx}{2}\right)}{24f(a+a\sec(e+fx))\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)} + \frac{\cos^3(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)(c-c\sec(e+fx))^4\sin\left(\frac{fx}{2}\right)}{48f(a+a\sec(e+fx))\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^3} + \frac{\cos^3(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)(c-c\sec(e+fx))^4\left(7\cos\left(\frac{e}{2}\right)+8\sin\left(\frac{e}{2}\right)\right)}{48f(a+a\sec(e+fx))\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^2} + \frac{35\cos^3(e+fx)\cot^2\left(\frac{e}{2}+\frac{fx}{2}\right)\csc^6\left(\frac{e}{2}+\frac{fx}{2}\right)(c-c\sec(e+fx))^4\sin\left(\frac{fx}{2}\right)}{24f(a+a\sec(e+fx))\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*Log[Cos[e/2 +
(f*x)/2] - Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(16*f*(a + a*Sec[e +
```

```
f*x])) - (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*Log[
Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(16*f*(a +
 a*Sec[e + f*x])) + (2*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]*Csc[e/2 + (f*x)/2]
^7*Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(f*(a + a*Sec[e + f*x])) +
 (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f
*x])^4*Sin[(f*x)/2])/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] - Sin[e/2])*(Cos[
e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2
]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^4*(-7*Cos[e/2] + 8*Sin[e/2]))
/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin
[e/2 + (f*x)/2])^2) + (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*
x)/2]^6*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(24*f*(a + a*Sec[e + f*x])*(Co
s[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2])) + (Cos[e + f*
x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c - c*Sec[e + f*x])^4*Sin[(
f*x)/2])/(48*f*(a + a*Sec[e + f*x])*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/
2] + Sin[e/2 + (f*x)/2])^3) + (Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2
+ (f*x)/2]^6*(c - c*Sec[e + f*x])^4*(7*Cos[e/2] + 8*Sin[e/2]))/(48*f*(a + a
*Sec[e + f*x])*(Cos[e/2] + Sin[e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/
2])^2) + (35*Cos[e + f*x]^3*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2]^6*(c -
c*Sec[e + f*x])^4*Sin[(f*x)/2])/(24*f*(a + a*Sec[e + f*x])*(Cos[e/2] + Sin[
e/2])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]))
```

Maple [A]
time = 0.19, size = 140, normalized size = 1.16

method	result
derivativedivides	$\frac{16c^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{3}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{29}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}-\frac{}{48\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{fa}$
default	$\frac{16c^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{3}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{29}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}-\frac{}{48\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{fa}$
risch	$\frac{ic^4\left(111\,e^{6i(fx+e)}+81\,e^{5i(fx+e)}+354\,e^{4i(fx+e)}+144\,e^{3i(fx+e)}+417\,e^{2i(fx+e)}+55\,e^{i(fx+e)}+166\right)}{3af\left(e^{2i(fx+e)}+1\right)^3\left(e^{i(fx+e)}+1\right)}-\frac{35c^4\ln\left(e^{i(fx+e)}+\right.}{2af}$
norman	$\frac{\frac{35c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{385c^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{511c^4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{93c^4\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{16c^4\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}+\frac{35c^4\ln\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 16/f/a*c^4*(tan(1/2*f*x+1/2*e)-1/48/(tan(1/2*f*x+1/2*e)-1)^3-3/16/(tan(1/2*
f*x+1/2*e)-1)^2-29/32/(tan(1/2*f*x+1/2*e)-1)+35/32*ln(tan(1/2*f*x+1/2*e)-1)
-1/48/(tan(1/2*f*x+1/2*e)+1)^3+3/16/(tan(1/2*f*x+1/2*e)+1)^2-29/32/(tan(1/2
*f*x+1/2*e)+1)-35/32*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(124) = 248$.

time = 0.29, size = 641, normalized size = 5.30

$$\frac{c^4\left(\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{16\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a-\frac{3a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{a\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}-\frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{6\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+12c^4\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-36c^4\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-24c^4\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+\frac{6c^4\sin(fx+e)}{a(\cos(fx+e)+1)}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(c^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(a*(co
s(f*x + e) + 1))) + 12*c^4*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)
/(a*(cos(f*x + e) + 1))) - 36*c^4*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(c
os(f*x + e) + 1))) - 24*c^4*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) +
 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 3.27, size = 165, normalized size = 1.36

$$-\frac{105\left(c^4\cos(fx+e)^4+c^4\cos(fx+e)^3\right)\log(\sin(fx+e)+1)-105\left(c^4\cos(fx+e)^4+c^4\cos(fx+e)^3\right)\log(-\sin(fx+e)+1)-2\left(166c^4\cos(fx+e)^3+55c^4\cos(fx+e)^2-13c^4\cos(fx+e)+2c^4\right)\sin(fx+e)}{12\left(af\cos(fx+e)^4+af\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/12*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(sin(f*x + e) + 1)
- 105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*
(166*c^4*cos(f*x + e)^3 + 55*c^4*cos(f*x + e)^2 - 13*c^4*cos(f*x + e) + 2*c
^4)*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{\sec(e+fx)}{\sec(e+fx)+1}\,dx+\int\left(-\frac{4\sec^2(e+fx)}{\sec(e+fx)+1}\right)dx+\int\frac{6\sec^3(e+fx)}{\sec(e+fx)+1}\,dx+\int\left(-\frac{4\sec^4(e+fx)}{\sec(e+fx)+1}\right)dx+\int\frac{\sec^5(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*
x)**2/(sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x) + 1
), x) + Integral(-4*sec(e + f*x)**4/(sec(e + f*x) + 1), x) + Integral(sec(e
 + f*x)**5/(sec(e + f*x) + 1), x))/a
```

Giac [A]
time = 0.56, size = 132, normalized size = 1.09

$$-\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{96\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{2\left(87\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-136\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+57\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/6*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 105*c^4*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a - 96*c^4*tan(1/2*f*x + 1/2*e)/a + 2*(87*c^4*tan(1/2*
f*x + 1/2*e)^5 - 136*c^4*tan(1/2*f*x + 1/2*e)^3 + 57*c^4*tan(1/2*f*x + 1/2*
e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

Mupad [B]
time = 1.85, size = 112, normalized size = 0.93

$$\frac{16\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}-\frac{29\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{136\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+19\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}-\frac{35\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (16*c^4*tan(e/2 + (f*x)/2))/(a*f) - (29*c^4*tan(e/2 + (f*x)/2)^5 - (136*c^4
*tan(e/2 + (f*x)/2)^3)/3 + 19*c^4*tan(e/2 + (f*x)/2))/(a*f*(tan(e/2 + (f*x)
/2)^2 - 1)^3) - (35*c^4*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

3.35 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=100

$$-\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af}+\frac{10c^3\tan(e+fx)}{af}-\frac{5c^3\sec(e+fx)\tan(e+fx)}{2af}+\frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] -15/2*c^3*arctanh(sin(f*x+e))/a/f+10*c^3*tan(f*x+e)/a/f-5/2*c^3*sec(f*x+e)*
tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4042, 3873, 3852, 8, 4131, 3855}

$$\frac{10c^3\tan(e+fx)}{af}-\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af}-\frac{5c^3\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-15*c^3*ArcTanh[Sin[e + f*x]])/(2*a*f) + (10*c^3*Tan[e + f*x])/(a*f) - (5*
c^3*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx &= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(5c)\int\sec(e+fx)(c-c\sec(\ldots}{a}\\ &= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(5c)\int\sec(e+fx)\left(c^2+c^2\mathrm{se}\ldots\right.}{a}\\ &= -\frac{5c^3\sec(e+fx)\tan(e+fx)}{2af}+\frac{2c(c-c\sec(e+fx))^2\tan(e+fx\ldots}{f(a+a\sec(e+fx))}\\ &= -\frac{15c^3\tanh^{-1}(\sin(e+fx))}{2af}+\frac{10c^3\tan(e+fx)}{af}-\frac{5c^3\sec(e+fx)\ldots}{2af}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 287 vs. 2(100) = 200.
time = 2.63, size = 287, normalized size = 2.87

$$\frac{\cos^2(e+fx)\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^3\left(-32\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+\cot\left(\frac{1}{2}(e+fx)\right)\left(-30\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+30\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{1}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{16\sin(fx)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)\right)}{16af(1+\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*(c - c*Sec[e + f*x])^3*
(-32*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2] + Cot[(e + f*x)/2]*(-30*Log[Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]] + 30*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]] + (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^(-2) - (16*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[
```

```
e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])))))/(16*a*f*(1 + Sec[e + f*x]))
```

Maple [A]
time = 0.18, size = 110, normalized size = 1.10

method	result
derivativedivides	$\frac{8c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{9}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{9}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{fa}$
default	$\frac{8c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{9}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{9}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{fa}$
risch	$\frac{ic^3\left(17\,e^{4i(fx+e)}+9\,e^{3i(fx+e)}+39\,e^{2i(fx+e)}+7\,e^{i(fx+e)}+24\right)}{fa\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\right)^2}+\frac{15c^3\ln\left(e^{i(fx+e)}-i\right)}{2af}-\frac{15c^3\ln\left(e^{i(fx+e)}+i\right)}{2af}$
norman	$\frac{\frac{40c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{33c^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{8c^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{15c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}+\frac{15c^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2af}-\frac{15c^3\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*c^3/a*(tan(1/2*f*x+1/2*e)+1/16/(tan(1/2*f*x+1/2*e)+1)^2-9/16/(tan(1/2*f
*x+1/2*e)+1)-15/16*ln(tan(1/2*f*x+1/2*e)+1)-1/16/(tan(1/2*f*x+1/2*e)-1)^2-9
/16/(tan(1/2*f*x+1/2*e)-1)+15/16*ln(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(104) = 208.
time = 0.29, size = 418, normalized size = 4.18

$$\frac{c^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-6c^3\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-6c^3\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+\frac{2c^3\sin(fx+e)}{a(\cos(fx+e)+1)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(c^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x + e)
+ 1))) - 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1)))
- 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^3*sin(f*
x + e)/(a*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 2.72, size = 151, normalized size = 1.51

$$-\frac{15\left(c^{3}\cos\left(fx+e\right)^{3}+c^{3}\cos\left(fx+e\right)^{2}\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^{3}\cos\left(fx+e\right)^{3}+c^{3}\cos\left(fx+e\right)^{2}\right)\log\left(-\sin\left(fx+e\right)+1\right)-2\left(24\,c^{3}\cos\left(fx+e\right)^{2}+7\,c^{3}\cos\left(fx+e\right)-c^{3}\right)\sin\left(fx+e\right)}{4\left(af\cos\left(fx+e\right)^{3}+af\cos\left(fx+e\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/4*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(sin(f*x + e) + 1) -
15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(24
*c^3*cos(f*x + e)^2 + 7*c^3*cos(f*x + e) - c^3)*sin(f*x + e))/(a*f*cos(f*x
+ e)^3 + a*f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{3\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{3\sec^{3}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^{4}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(3*sec(e + f
*x)**2/(sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f*x) +
 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) + 1), x))/a
```

Giac [A]
time = 0.56, size = 116, normalized size = 1.16

$$-\frac{\frac{15\,c^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{15\,c^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{16\,c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{2\left(9\,c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-7\,c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/2*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 15*c^3*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/a - 16*c^3*tan(1/2*f*x + 1/2*e)/a + 2*(9*c^3*tan(1/2*f*x
 + 1/2*e)^3 - 7*c^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a
))/f
```

Mupad [B]
time = 1.66, size = 96, normalized size = 0.96

$$\frac{8\,c^{3}\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}-\frac{9\,c^{3}\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}-7\,c^{3}\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}-1\right)^{2}}-\frac{15\,c^{3}\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (8*c^3*tan(e/2 + (f*x)/2))/(a*f) - (9*c^3*tan(e/2 + (f*x)/2)^3 - 7*c^3*tan(
e/2 + (f*x)/2))/(a*f*(tan(e/2 + (f*x)/2)^2 - 1)^2) - (15*c^3*atanh(tan(e/2
+ (f*x)/2)))/(a*f)
```

3.36 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{3c^2\tan(e+fx)}{af}+\frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] -3*c^2*arctanh(sin(f*x+e))/a/f+3*c^2*tan(f*x+e)/a/f+2*(c^2-c^2*sec(f*x+e))*
tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4042, 3872, 3855, 3852, 8}

$$\frac{3c^2\tan(e+fx)}{af}-\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{2\tan(e+fx)\,(c^2-c^2\sec(e+fx))}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-3*c^2*ArcTanh[Sin[e + f*x]])/(a*f) + (3*c^2*Tan[e + f*x])/(a*f) + (2*(c^2
 - c^2*Sec[e + f*x])*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx &= \frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(3c)\int \sec(e+fx)(c-c\sec(e\\ &= \frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(3c^2)\int \sec(e+fx)\,dx}{a}+\frac{(3c^2)\\ &= -\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(\\ &= -\frac{3c^2\tanh^{-1}(\sin(e+fx))}{af}+\frac{3c^2\tan(e+fx)}{af}+\frac{2(c^2-c^2\sec(e+fx}{f(a+a\sec(e}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(74) = 148.
time = 1.67, size = 220, normalized size = 2.97

$$\frac{2c^2\cos\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)\left(4\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+\cot\left(\frac{1}{2}(e+fx)\right)\left(3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{\sin(fx)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)\right)}{af(1+\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c^2*Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]*(4*Csc[(e + f*x)/2]*S
ec[e/2]*Sin[(f*x)/2] + Cot[(e + f*x)/2]*(3*Log[Cos[(e + f*x)/2] - Sin[(e +
f*x)/2]] - 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + Sin[f*x]/((Cos[e/2]
 - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])))))/(a*f*(1 + Sec[e + f*x]))
```

Maple [A]
time = 0.14, size = 80, normalized size = 1.08

method	result	s
derivativedivides	$\frac{4c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fa}$	8

default	$\frac{4c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fa}$
risch	$\frac{2ic^2\left(4\,\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}+5\right)}{fa\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)}+\frac{3c^2\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{af}-\frac{3c^2\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{af}$
norman	$\frac{\frac{6c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{10c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{4c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{3c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}-\frac{3c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*c^2/a*(tan(1/2*f*x+1/2*e)-1/4/(tan(1/2*f*x+1/2*e)-1)+3/4*ln(tan(1/2*f*x
+1/2*e)-1)-1/4/(tan(1/2*f*x+1/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(80) = 160$.
time = 0.29, size = 242, normalized size = 3.27

$$-\frac{c^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+2\,c^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-\frac{c^2\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(c^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^2*(
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)/(a*(
cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 2.82, size = 129, normalized size = 1.74

$$-\frac{3\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\log(\sin(fx+e)+1)-3\left(c^2\cos(fx+e)^2+c^2\cos(fx+e)\right)\log(-\sin(fx+e)+1)-2\left(5\,c^2\cos(fx+e)+c^2\right)\sin(fx+e)}{2\left(af\cos(fx+e)^2+af\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/2*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*(
c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(5*c^2*co
s(f*x + e) + c^2)*sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int \frac{\sec(e+fx)}{\sec(e+fx)+1}\,dx+\int\left(-\frac{2\sec^2(e+fx)}{\sec(e+fx)+1}\right)dx+\int\frac{\sec^3(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-2*sec(e + f*
x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) + 1),
 x))/a
```

Giac [A]
time = 0.56, size = 97, normalized size = 1.31

$$-\frac{\frac{3c^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{3c^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}-\frac{4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}+\frac{2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 3*c^2*log(abs(tan(1/2*f*x +
1/2*e) - 1))/a - 4*c^2*tan(1/2*f*x + 1/2*e)/a + 2*c^2*tan(1/2*f*x + 1/2*e)/
((tan(1/2*f*x + 1/2*e)^2 - 1)*a))/f
```

Mupad [B]
time = 1.64, size = 77, normalized size = 1.04

$$\frac{4c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(a-a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\right)}-\frac{6c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (4*c^2*tan(e/2 + (f*x)/2))/(a*f) + (2*c^2*tan(e/2 + (f*x)/2))/(f*(a - a*tan
(e/2 + (f*x)/2)^2)) - (6*c^2*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

3.37 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{c\tanh^{-1}(\sin(e+fx))}{af}+\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] -c*arctanh(sin(f*x+e))/a/f+2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4042, 3855}

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\tanh^{-1}(\sin(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a*f)) + (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f
*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx &= \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{c\int \sec(e+fx)\,dx}{a}\\ &= -\frac{c\tanh^{-1}(\sin(e+fx))}{af}+\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 77, normalized size = 1.88

$$-\frac{c\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{f}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f - (2*Tan[(e + f*x)/2])/f))/a)
```

Maple [A]
time = 0.15, size = 48, normalized size = 1.17

method	result	size
derivativedivides	$\frac{2c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{fa}$	48
default	$\frac{2c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{fa}$	48
risch	$\frac{4ic}{fa\left(e^{i(fx+e)}+1\right)}+\frac{c\ln\left(e^{i(fx+e)}-i\right)}{af}-\frac{c\ln\left(e^{i(fx+e)}+i\right)}{af}$	68
norman	$\frac{-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{2c\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}-\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c/a*(tan(1/2*f*x+1/2*e)+1/2*ln(tan(1/2*f*x+1/2*e)-1)-1/2*ln(tan(1/2*f*x
+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(44) = 88.
time = 0.27, size = 109, normalized size = 2.66

$$-\frac{c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -(c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c*sin(f*x + e)/(
a*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 3.18, size = 76, normalized size = 1.85

$$-\frac{(c\cos(fx+e)+c)\log(\sin(fx+e)+1)-(c\cos(fx+e)+c)\log(-\sin(fx+e)+1)-4\,c\sin(fx+e)}{2\,(af\cos(fx+e)+af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] -1/2*((c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - (c*cos(f*x + e) + c)*log
(-sin(f*x + e) + 1) - 4*c*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec(e+fx)}{\sec(e+fx)+1}\right)dx+\int\frac{\sec^2(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**
2/(sec(e + f*x) + 1), x))/a
```

Giac [A]
time = 0.64, size = 58, normalized size = 1.41

$$-\frac{\frac{c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{2\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] -(c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - c*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/a - 2*c*tan(1/2*f*x + 1/2*e)/a)/f
```

Mupad [B]
time = 1.58, size = 31, normalized size = 0.76

$$-\frac{2\,c\left(\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(2*c*(atanh(tan(e/2 + (f*x)/2)) - tan(e/2 + (f*x)/2)))/(a*f)
```

3.38 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=16

$$\frac{\csc(e+fx)}{acf}$$

[Out] csc(f*x+e)/a/c/f

Rubi [A]
time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4043, 2686, 8}

$$\frac{\csc(e+fx)}{acf}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]

[Out] Csc[e + f*x]/(a*c*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = -\frac{\int \cot(e+fx)\csc(e+fx)\,dx}{ac}$$
$$= \frac{\text{Subst}(\int 1\,dx, x, \csc(e+fx))}{acf}$$
$$= \frac{\csc(e+fx)}{acf}$$

Mathematica [A]
time = 0.03, size = 16, normalized size = 1.00

$$\frac{\csc(e+fx)}{acf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]
[Out] Csc[e + f*x]/(a*c*f)
```

Maple [A]
time = 0.12, size = 19, normalized size = 1.19

method	result	size
default	$\frac{1}{caf\sin(fx+e)}$	19
norman	$\frac{\frac{1}{2acf}+\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{2acf}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	47
risch	$\frac{2ie^{i(fx+e)}}{fca\left(e^{i(fx+e)}-1\right)\left(e^{i(fx+e)}+1\right)}$	48
derivativedivides	`error in RationalFunction: argument is not a rational function\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/c/a/f/sin(f*x+e)
```

Maxima [A]
time = 0.27, size = 19, normalized size = 1.19

$$\frac{1}{acf\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] 1/(a*c*f*sin(f*x + e))
```

Fricas [A]
time = 3.25, size = 19, normalized size = 1.19

$$\frac{1}{acf \sin (fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/(a*c*f*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec (e+fx)}{\sec^2 (e+fx)-1}\, dx}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**2 - 1), x)/(a*c)
```

Giac [A]
time = 0.55, size = 18, normalized size = 1.12

$$\frac{1}{acf \sin (fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/(a*c*f*sin(f*x + e))
```

Mupad [B]
time = 1.56, size = 18, normalized size = 1.12

$$\frac{1}{a\, c\, f\, \sin (e + f\, x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))),x)
```

```
[Out] 1/(a*c*f*sin(e + f*x))
```

3.39 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=59

$$-\frac{\cot^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}$$

```
[Out] -1/3*cot(f*x+e)^3/a/c^2/f+csc(f*x+e)/a/c^2/f-1/3*csc(f*x+e)^3/a/c^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4043, 2686, 2687, 30}

$$-\frac{\cot^3(e+fx)}{3ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -1/3*Cot[e + f*x]^3/(a*c^2*f) + Csc[e + f*x]/(a*c^2*f) - Csc[e + f*x]^3/(3*
a*c^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
```

```
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\int \left(a\cot^3(e+fx)\csc(e+fx)+a\cot^2(e+fx)\csc^2(e+fx)\right)}{a^2c^2}$$

$$= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{ac^2} + \frac{\int \cot^2(e+fx)\csc^2(e+fx}{ac^2}$$

$$= \frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(e+fx)\right)}{ac^2f} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x\right.}{ac^2f}$$

$$= -\frac{\cot^3(e+fx)}{3ac^2f} + \frac{\csc(e+fx)}{ac^2f} - \frac{\csc^3(e+fx)}{3ac^2f}$$

Mathematica [A]
time = 0.50, size = 81, normalized size = 1.37

$$\frac{\csc(e)\csc^2\left(\frac{1}{2}(e+fx)\right)\csc(e+fx)(6\sin(e)+2\sin(fx)-10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)+2\sin(e+2fx))}{24ac^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]*(6*Sin[e] + 2*Sin[f*x] - 10*Sin[e +
 f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] + 2*Sin[e + 2*f*x]))/(24*a*c^
2*f)
```

Maple [A]
time = 0.14, size = 48, normalized size = 0.81

method	result	size
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}}{4fac^2}$	48
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}}{4fac^2}$	48
norman	$\frac{-\frac{1}{12acf}+\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{2acf}+\frac{\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)}{4acf}}{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	72
risch	$\frac{2i\left(3\,e^{3i(fx+e)}-3\,e^{2i(fx+e)}+e^{i(fx+e)}+1\right)}{3fac^2\left(e^{i(fx+e)}-1\right)^3\left(e^{i(fx+e)}+1\right)}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a/c^2*(tan(1/2*f*x+1/2*e)+2/tan(1/2*f*x+1/2*e)-1/3/tan(1/2*f*x+1/2*e)
^3)
```

Maxima [A]
time = 0.28, size = 83, normalized size = 1.41

$$\frac{\frac{\left(\frac{6\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{ac^2\sin(fx+e)^3}+\frac{3\sin(fx+e)}{ac^2(\cos(fx+e)+1)}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*((6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a*c
^2*sin(f*x + e)^3) + 3*sin(f*x + e)/(a*c^2*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 1.80, size = 54, normalized size = 0.92

$$\frac{\cos(fx+e)^2+2\cos(fx+e)-2}{3\left(ac^2f\cos(fx+e)-ac^2f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(cos(f*x + e)^2 + 2*cos(f*x + e) - 2)/((a*c^2*f*cos(f*x + e) - a*c^2*f)
*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec(e+fx)}{\sec^3(e+fx)-\sec^2(e+fx)-\sec(e+fx)+1}\,dx}{ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**3 - sec(e + f*x)**2 - sec(e + f*x) + 1
), x)/(a*c**2)
```

Giac [A]
time = 0.81, size = 56, normalized size = 0.95

$$\frac{\frac{3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^2}+\frac{6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1}{ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/12*(3*tan(1/2*f*x + 1/2*e)/(a*c^2) + (6*tan(1/2*f*x + 1/2*e)^2 - 1)/(a*c^
2*tan(1/2*f*x + 1/2*e)^3))/f
```

Mupad [B]
time = 1.64, size = 50, normalized size = 0.85

$$\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}{12\,a\,c^2\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (6*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^4 - 1)/(12*a*c^2*f*tan(e/2 +
 (f*x)/2)^3)
```

3.40 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3} dx$

Optimal. Leaf size=78

$$\frac{2\cot^5(e+fx)}{5ac^3f}+\frac{\csc(e+fx)}{ac^3f}-\frac{\csc^3(e+fx)}{ac^3f}+\frac{2\csc^5(e+fx)}{5ac^3f}$$

```
[Out] 2/5*cot(f*x+e)^5/a/c^3/f+csc(f*x+e)/a/c^3/f-csc(f*x+e)^3/a/c^3/f+2/5*csc(f*
x+e)^5/a/c^3/f
```

Rubi [A]
time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4043, 2686, 200, 2687, 30, 14}

$$\frac{2\cot^5(e+fx)}{5ac^3f}+\frac{2\csc^5(e+fx)}{5ac^3f}-\frac{\csc^3(e+fx)}{ac^3f}+\frac{\csc(e+fx)}{ac^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*Cot[e + f*x]^5)/(5*a*c^3*f) + Csc[e + f*x]/(a*c^3*f) - Csc[e + f*x]^3/(a
*c^3*f) + (2*Csc[e + f*x]^5)/(5*a*c^3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = -\frac{\int \left(a^2\cot^5(e+fx)\csc(e+fx)+2a^2\cot^4(e+fx)\csc^2(e+\right.}{a^3c^3}$$

$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{ac^3} - \frac{\int \cot^3(e+fx)\csc^3(e+f}{ac^3}$$

$$= \frac{\text{Subst}\left(\int x^2(-1+x^2)\,dx, x, \csc(e+fx)\right)}{ac^3f} + \frac{\text{Subst}\Big(\int (-1+}{}$$

$$= \frac{2\cot^5(e+fx)}{5ac^3f} + \frac{\text{Subst}\left(\int (1-2x^2+x^4)\,dx, x, \csc(e+fx)\right)}{ac^3f}$$

$$= \frac{2\cot^5(e+fx)}{5ac^3f} + \frac{\csc(e+fx)}{ac^3f} - \frac{\csc^3(e+fx)}{ac^3f} + \frac{2\csc^5(e+f}{5ac^3f}$$

Mathematica [A]
time = 0.85, size = 107, normalized size = 1.37

$$-\frac{\csc(e)\csc^4\left(\frac{1}{2}(e+fx)\right)\csc(e+fx)(-40\sin(e)+65\sin(e+fx)-52\sin(2(e+fx))+13\sin(3(e+fx))+40\sin(2e+fx)-12\sin(e+2fx)-20\sin(3e+2fx)+8\sin(2e+3fx))}{320ac^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -1/320*(Csc[e]*Csc[(e + f*x)/2]^4*Csc[e + f*x]*(-40*Sin[e] + 65*Sin[e + f*x
] - 52*Sin[2*(e + f*x)] + 13*Sin[3*(e + f*x)] + 40*Sin[2*e + f*x] - 12*Sin[
e + 2*f*x] - 20*Sin[3*e + 2*f*x] + 8*Sin[2*e + 3*f*x]))/(a*c^3*f)
```

Maple [A]
time = 0.16, size = 61, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{8fa\,c^{3}}$	61
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{8fa\,c^{3}}$	61
risch	$\frac{2i\left(5\,\mathrm{e}^{5i(fx+e)}-10\,\mathrm{e}^{4i(fx+e)}+10\,\mathrm{e}^{3i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}+2\right)}{5fa\,c^{3}\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)^{5}}$	85
norman	$\frac{\frac{1}{40acf}-\frac{\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}{8acf}+\frac{3\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}+\frac{\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)}{8acf}}{c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/a/c^3*(tan(1/2*f*x+1/2*e)-1/tan(1/2*f*x+1/2*e)^3+3/tan(1/2*f*x+1/2*e)
+1/5/tan(1/2*f*x+1/2*e)^5)
```

Maxima [A]
time = 0.27, size = 105, normalized size = 1.35

$$-\frac{\frac{\left(\frac{5\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{15\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-1\right)\left(\cos\left(fx+e\right)+1\right)^{5}}{ac^{3}\sin\left(fx+e\right)^{5}}-\frac{5\sin\left(fx+e\right)}{ac^{3}\left(\cos\left(fx+e\right)+1\right)}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/40*((5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a*c^3*sin(f*x + e)^5) - 5*sin(f*x +
e)/(a*c^3*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 3.28, size = 80, normalized size = 1.03

$$\frac{2\,\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-4\,\cos\left(fx+e\right)+2}{5\left(ac^{3}f\cos\left(fx+e\right)^{2}-2\,ac^{3}f\cos\left(fx+e\right)+ac^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/5*(2*cos(f*x + e)^3 + cos(f*x + e)^2 - 4*cos(f*x + e) + 2)/((a*c^3*f*cos(
f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec(e+fx)}{\sec^4(e+fx)-2\sec^3(e+fx)+2\sec(e+fx)-1}\,dx}{ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x
) - 1), x)/(a*c**3)
```

Giac [A]
time = 0.72, size = 69, normalized size = 0.88

$$\frac{\frac{5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^3}+\frac{15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1}{ac^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}}{40f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/40*(5*tan(1/2*f*x + 1/2*e)/(a*c^3) + (15*tan(1/2*f*x + 1/2*e)^4 - 5*tan(1
/2*f*x + 1/2*e)^2 + 1)/(a*c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B]
time = 1.72, size = 63, normalized size = 0.81

$$\frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1}{40\,a\,c^3\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (15*tan(e/2 + (f*x)/2)^4 - 5*tan(e/2 + (f*x)/2)^2 + 5*tan(e/2 + (f*x)/2)^6
+ 1)/(40*a*c^3*f*tan(e/2 + (f*x)/2)^5)
```

3.41 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=120

$$-\frac{\cot^5(e+fx)}{5ac^4f}-\frac{4\cot^7(e+fx)}{7ac^4f}+\frac{\csc(e+fx)}{ac^4f}-\frac{2\csc^3(e+fx)}{ac^4f}+\frac{9\csc^5(e+fx)}{5ac^4f}-\frac{4\csc^7(e+fx)}{7ac^4f}$$

```
[Out] -1/5*cot(f*x+e)^5/a/c^4/f-4/7*cot(f*x+e)^7/a/c^4/f+csc(f*x+e)/a/c^4/f-2*csc
(f*x+e)^3/a/c^4/f+9/5*csc(f*x+e)^5/a/c^4/f-4/7*csc(f*x+e)^7/a/c^4/f
```

Rubi [A]
time = 0.17, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {4043, 2686, 200, 2687, 30, 276, 14}

$$-\frac{4\cot^7(e+fx)}{7ac^4f}-\frac{\cot^5(e+fx)}{5ac^4f}-\frac{4\csc^7(e+fx)}{7ac^4f}+\frac{9\csc^5(e+fx)}{5ac^4f}-\frac{2\csc^3(e+fx)}{ac^4f}+\frac{\csc(e+fx)}{ac^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -1/5*Cot[e + f*x]^5/(a*c^4*f) - (4*Cot[e + f*x]^7)/(7*a*c^4*f) + Csc[e + f*
x]/(a*c^4*f) - (2*Csc[e + f*x]^3)/(a*c^4*f) + (9*Csc[e + f*x]^5)/(5*a*c^4*f
) - (4*Csc[e + f*x]^7)/(7*a*c^4*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{\int (a^3\cot^7(e+fx)\csc(e+fx)+3a^3\cot^6(e+fx)\csc^2(e+f}{}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{ac^4} + \frac{\int \cot^4(e+fx)\csc^4(e+fx}{ac^4}$$

$$= -\frac{\operatorname{Subst}\left(\int (-1+x^2)^3\,dx, x, \csc(e+fx)\right)}{ac^4 f} + \frac{\operatorname{Subst}(\int x^4(1+}{}$$

$$= -\frac{3\cot^7(e+fx)}{7ac^4 f} - \frac{\operatorname{Subst}(\int (-1+3x^2-3x^4+x^6)\,dx, x, \mathrm{cs}}{ac^4 f}$$

$$= -\frac{\cot^5(e+fx)}{5ac^4 f} - \frac{4\cot^7(e+fx)}{7ac^4 f} + \frac{\csc(e+fx)}{ac^4 f} - \frac{2\csc^3(e+}{ac^4 f}$$

Mathematica [A]

time = 0.92, size = 145, normalized size = 1.21

$$\frac{\csc(e)\csc^6\left(\frac{1}{2}(e+fx)\right)\csc(e+fx)(840\sin(e)-56\sin(fx)-1946\sin(e+fx)+1946\sin(2(e+fx))-834\sin(3(e+fx))+139\sin(4(e+fx))-1400\sin(2e+fx)+616\sin(e+2fx)+840\sin(3e+2fx)-344\sin(2e+3fx)-280\sin(4e+3fx)+104\sin(3e+4fx))}{17920ac^4 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^6*Csc[e + f*x]*(840*Sin[e] - 56*Sin[f*x] - 1946*Si
n[e + f*x] + 1946*Sin[2*(e + f*x)] - 834*Sin[3*(e + f*x)] + 139*Sin[4*(e +
f*x)] - 1400*Sin[2*e + f*x] + 616*Sin[e + 2*f*x] + 840*Sin[3*e + 2*f*x] - 3
44*Sin[2*e + 3*f*x] - 280*Sin[4*e + 3*f*x] + 104*Sin[3*e + 4*f*x]))/(17920*
a*c^4*f)
```

Maple [A]
time = 0.16, size = 74, normalized size = 0.62

method	result	size
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16 f a\,c^{4}}$	74
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16 f a\,c^{4}}$	74
norman	$\frac{-\frac{1}{112acf}+\frac{\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}{20acf}-\frac{\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)}{8acf}+\frac{\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)}{4acf}+\frac{\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)}{16acf}}{c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$	116
risch	$\frac{2i\left(35\,\mathrm{e}^{7i(fx+e)}-105\,\mathrm{e}^{6i(fx+e)}+175\,\mathrm{e}^{5i(fx+e)}-105\,\mathrm{e}^{4i(fx+e)}-7\,\mathrm{e}^{3i(fx+e)}+77\,\mathrm{e}^{2i(fx+e)}-43\,\mathrm{e}^{i(fx+e)}+13\right)}{35 f a\,c^{4}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}\left(\mathrm{e}^{i(fx+e)}+1\right)}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/a/c^4*(tan(1/2*f*x+1/2*e)+4/5/tan(1/2*f*x+1/2*e)^5-1/7/tan(1/2*f*x+1
/2*e)^7-2/tan(1/2*f*x+1/2*e)^3+4/tan(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.29, size = 127, normalized size = 1.06

$$\frac{\frac{\left(\frac{28\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{70\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{140\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-5\right)(\cos(fx+e)+1)^{7}}{ac^{4}\sin(fx+e)^{7}}+\frac{35\sin(fx+e)}{ac^{4}(\cos(fx+e)+1)}}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/560*((28*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 140*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) +
 1)^7/(a*c^4*sin(f*x + e)^7) + 35*sin(f*x + e)/(a*c^4*(cos(f*x + e) + 1)))/
f
```

Fricas [A]
time = 3.03, size = 110, normalized size = 0.92

$$\frac{13 \cos(fx+e)^4 - 4 \cos(fx+e)^3 - 20 \cos(fx+e)^2 + 24 \cos(fx+e) - 8}{35 \left(ac^4 f \cos(fx+e)^3 - 3\,ac^4 f \cos(fx+e)^2 + 3\,ac^4 f \cos(fx+e) - ac^4 f\right) \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/35*(13*cos(f*x + e)^4 - 4*cos(f*x + e)^3 - 20*cos(f*x + e)^2 + 24*cos(f*x
 + e) - 8)/((a*c^4*f*cos(f*x + e)^3 - 3*a*c^4*f*cos(f*x + e)^2 + 3*a*c^4*f*
cos(f*x + e) - a*c^4*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{\sec^5(e+fx) - 3\sec^4(e+fx) + 2\sec^3(e+fx) + 2\sec^2(e+fx) - 3\sec(e+fx) + 1}\,dx}{ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**5 - 3*sec(e + f*x)**4 + 2*sec(e + f*x)
**3 + 2*sec(e + f*x)**2 - 3*sec(e + f*x) + 1), x)/(a*c**4)
```

Giac [A]
time = 0.58, size = 82, normalized size = 0.68

$$\frac{\frac{35 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac^4} + \frac{140 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 70 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 28 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 5}{ac^4 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}}{560\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/560*(35*tan(1/2*f*x + 1/2*e)/(a*c^4) + (140*tan(1/2*f*x + 1/2*e)^6 - 70*t
an(1/2*f*x + 1/2*e)^4 + 28*tan(1/2*f*x + 1/2*e)^2 - 5)/(a*c^4*tan(1/2*f*x +
 1/2*e)^7))/f
```

Mupad [B]
time = 2.13, size = 83, normalized size = 0.69

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{16\,a\,c^4\,f} + \frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{4} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{8} + \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{20} - \frac{1}{112}}{a\,c^4\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^4),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(16*a*c^4*f) + (tan(e/2 + (f*x)/2)^2/20 - tan(e/2 + (f*x
)/2)^4/8 + tan(e/2 + (f*x)/2)^6/4 - 1/112)/(a*c^4*f*tan(e/2 + (f*x)/2)^7)
```

3.42 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{84c^5\tan(e+fx)}{a^2f}+\frac{63c^5\sec(e+fx)\tan(e+fx)}{2a^2f}-\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))}$$

```
[Out] 105/2*c^5*arctanh(sin(f*x+e))/a^2/f-84*c^5*tan(f*x+e)/a^2/f+63/2*c^5*sec(f*
x+e)*tan(f*x+e)/a^2/f-6*c^2*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a^2+a^2*sec(f*
x+e))+2/3*c*(c-c*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-7*c^5*tan(f*
x+e)^3/a^2/f
```

Rubi [A]
time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4042, 3876, 3855, 3852, 8, 3853}

$$-\frac{7c^5\tan^3(e+fx)}{a^2f}-\frac{84c^5\tan(e+fx)}{a^2f}+\frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{63c^5\tan(e+fx)\sec(e+fx)}{2a^2f}-\frac{6c^2\tan(e+fx)(c-c\sec(e+fx))^3}{f\,(a^2\sec(e+fx)+a^2)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (105*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (84*c^5*Tan[e + f*x])/(a^2*f) +
 (63*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^2*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^4
*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (7*c^5*Tan[e + f*x]^3)/(a^2*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx &= \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(3c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,d}{a} \\
&= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan}{3f(a+a\sec(e+fx} \\
&= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan}{3f(a+a\sec(e+fx} \\
&= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan}{3f(a+a\sec(e+fx} \\
&= \frac{21c^5\tanh^{-1}(\sin(e+fx))}{a^2f} + \frac{63c^5\sec(e+fx)\tan(e+fx)}{2a^2f} - \frac{6c^2(c}{} \\
&= \frac{105c^5\tanh^{-1}(\sin(e+fx))}{2a^2f} - \frac{84c^5\tan(e+fx)}{a^2f} + \frac{63c^5\sec(e+fx)}{2a^2f}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 380 vs. 2(164) = 328.
time = 1.20, size = 380, normalized size = 2.32

$$
\frac{\cot\left(\frac{1}{2}(e+fx)\right)\csc^6\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^5\left(20160\cos^3(e+fx)\cot^3\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sec(e)\left(-1323\sin\left(\frac{fx}{2}\right)+3247\sin\left(\frac{3fx}{2}\right)-2901\sin\left(e-\frac{fx}{2}\right)+1197\sin\left(e+\frac{fx}{2}\right)-3027\sin\left(2e+\frac{fx}{2}\right)-273\sin\left(e+\frac{3fx}{2}\right)+1827\sin\left(2e+\frac{3fx}{2}\right)-1693\sin\left(3e+\frac{3fx}{2}\right)+1995\sin\left(e+\frac{5fx}{2}\right)-117\sin\left(2e+\frac{5fx}{2}\right)+1143\sin\left(3e+\frac{5fx}{2}\right)-969\sin\left(4e+\frac{5fx}{2}\right)+1173\sin\left(2e+\frac{7fx}{2}\right)+117\sin\left(3e+\frac{7fx}{2}\right)+747\sin\left(4e+\frac{7fx}{2}\right)-309\sin\left(5e+\frac{7fx}{2}\right)+494\sin\left(3e+\frac{9fx}{2}\right)+142\sin\left(4e+\frac{9fx}{2}\right)+352\sin\left(5e+\frac{9fx}{2}\right)\right)\right)}{3072a^2f(1+\sec(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^6*(c - c*Sec[e + f*x])^5*(20160*Cos[e +
f*x]^3*Cot[(e + f*x)/2]^3*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[C
os[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Csc[(e + f*x)/2]^3*Sec[e/2]*Sec[e]*(
-1323*Sin[(f*x)/2] + 3247*Sin[(3*f*x)/2] - 2901*Sin[e - (f*x)/2] + 1197*Sin
[e + (f*x)/2] - 3027*Sin[2*e + (f*x)/2] - 273*Sin[e + (3*f*x)/2] + 1827*Sin
[2*e + (3*f*x)/2] - 1693*Sin[3*e + (3*f*x)/2] + 1995*Sin[e + (5*f*x)/2] - 1
17*Sin[2*e + (5*f*x)/2] + 1143*Sin[3*e + (5*f*x)/2] - 969*Sin[4*e + (5*f*x)
/2] + 1173*Sin[2*e + (7*f*x)/2] + 117*Sin[3*e + (7*f*x)/2] + 747*Sin[4*e +
(7*f*x)/2] - 309*Sin[5*e + (7*f*x)/2] + 494*Sin[3*e + (9*f*x)/2] + 142*Sin[
4*e + (9*f*x)/2] + 352*Sin[5*e + (9*f*x)/2])))/(3072*a^2*f*(1 + Sec[e + f*x
])^2)
```

Maple [A]
time = 0.23, size = 155, normalized size = 0.95

method	result
derivativedivides	$\frac{16c^5\left(-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{55}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{105\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{32}\right.}{f\,a^2}$
default	$\frac{16c^5\left(-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{55}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{105\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{32}\right.}{f\,a^2}$
risch	$-\frac{ic^5\left(309\,e^{8i(fx+e)}+969\,e^{7i(fx+e)}+1693\,e^{6i(fx+e)}+3027\,e^{5i(fx+e)}+2901\,e^{4i(fx+e)}+3247\,e^{3i(fx+e)}+1995\,e^{2i(fx+e)}+11\right.}{3a^2f\left(e^{2i(fx+e)}+1\right)^3\left(e^{i(fx+e)}+1\right)^3}$
norman	$\frac{-\frac{16c^5\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{105c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{490c^5\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{896c^5\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{790c^5\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{965c^5\left(\tan\right.}{3}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/f*c^5/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-4*tan(1/2*f*x+1/2*e)+1/48/(tan(1/2*
f*x+1/2*e)+1)^3-1/4/(tan(1/2*f*x+1/2*e)+1)^2+55/32/(tan(1/2*f*x+1/2*e)+1)+1
05/32*ln(tan(1/2*f*x+1/2*e)+1)+1/48/(tan(1/2*f*x+1/2*e)-1)^3+1/4/(tan(1/2*f
*x+1/2*e)-1)^2+55/32/(tan(1/2*f*x+1/2*e)-1)-105/32*ln(tan(1/2*f*x+1/2*e)-1)
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(171) = 342$.
time = 0.29, size = 829, normalized size = 5.05

$$-\frac{c^5\left(\frac{4\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^2-\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{a^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}+\frac{\frac{27\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+5c^5\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+10c^5\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+10c^5\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{5c^5\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c^5\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x + e)
+ 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(cos(
f*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) +
 5*c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1)
+ 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10*c^5*((15*s
in(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 -
 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(cos(f*x + e
) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) +
 5*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 - c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 3.98, size = 226, normalized size = 1.38

$$\frac{315 \left(c^5 \cos(fx+e)^5 + 2 c^5 \cos(fx+e)^4 + c^5 \cos(fx+e)^3\right) \log(\sin(fx+e)+1) - 315 \left(c^5 \cos(fx+e)^5 + 2 c^5 \cos(fx+e)^4 + c^5 \cos(fx+e)^3\right) \log(-\sin(fx+e)+1) - 2 \left(494 c^5 \cos(fx+e)^4 + 679 c^5 \cos(fx+e)^3 + 102 c^5 \cos(fx+e)^2 - 17 c^5 \cos(fx+e) + 2 c^5\right) \sin(fx+e)}{12 \left(a^2 f \cos(fx+e)^5 + 2 a^2 f \cos(fx+e)^4 + a^2 f \cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^5*cos(f*x + e)^3)*
log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^
5*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*(494*c^5*cos(f*x + e)^4 + 679*
c^5*cos(f*x + e)^3 + 102*c^5*cos(f*x + e)^2 - 17*c^5*cos(f*x + e) + 2*c^5)*
sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f*cos(f*
x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^5 \left(\int \left(-\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right) dx + \int \frac{5\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\, dx + \int \left(-\frac{10\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right) dx + \int \frac{10\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\, dx + \int \left(-\frac{5\sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right) dx + \int \frac{\sec^6(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\, dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(5*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(10*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integr
al(-5*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral
(sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.67, size = 156, normalized size = 0.95

$$\frac{\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}+\frac{2\left(165\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-280\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+123\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^2}-\frac{32\left(a^4c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+12\,a^4c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(315*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 315*c^5*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^2 + 2*(165*c^5*tan(1/2*f*x + 1/2*e)^5 - 280*c^5*tan
(1/2*f*x + 1/2*e)^3 + 123*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^
2 - 1)^3*a^2) - 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 12*a^4*c^5*tan(1/2*f*x
 + 1/2*e))/a^6)/f
```

Mupad [B]
time = 1.73, size = 170, normalized size = 1.04

$$\frac{55\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{280\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+41\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}-\frac{64\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}-\frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}+\frac{105\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (55*c^5*tan(e/2 + (f*x)/2)^5 - (280*c^5*tan(e/2 + (f*x)/2)^3)/3 + 41*c^5*ta
n(e/2 + (f*x)/2))/(f*(3*a^2*tan(e/2 + (f*x)/2)^2 - 3*a^2*tan(e/2 + (f*x)/2)
^4 + a^2*tan(e/2 + (f*x)/2)^6 - a^2)) - (64*c^5*tan(e/2 + (f*x)/2))/(a^2*f)
 - (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) + (105*c^5*atanh(tan(e/2 + (f*x)
/2)))/(a^2*f)
```

3.43 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{70c^4\tan(e+fx)}{3a^2f}+\frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f}+\frac{2c(c-c\sec(e+fx))^3\tan(e+}{3f(a+a\sec(e+fx))^2}$$

```
[Out] 35/2*c^4*arctanh(sin(f*x+e))/a^2/f-70/3*c^4*tan(f*x+e)/a^2/f+35/6*c^4*sec(f
*x+e)*tan(f*x+e)/a^2/f+2/3*c*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e
))^2-14/3*(c^2-c^2*sec(f*x+e))^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A]
time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4042, 3873, 3852, 8, 4131, 3855}

$$-\frac{70c^4\tan(e+fx)}{3a^2f}+\frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{35c^4\tan(e+fx)\sec(e+fx)}{6a^2f}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (35*c^4*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (70*c^4*Tan[e + f*x])/(3*a^2*f)
+ (35*c^4*Sec[e + f*x]*Tan[e + f*x])/(6*a^2*f) + (2*c*(c - c*Sec[e + f*x])^
3*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (14*(c^2 - c^2*Sec[e + f*x])
^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx &= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(7c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx}{3a} \\
&= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e}{3f\left(a^2+a^2\sec(e+fx)\right)} \\
&= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e}{3f\left(a^2+a^2\sec(e+fx)\right)} \\
&= \frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&= \frac{35c^4\tanh^{-1}(\sin(e+fx))}{2a^2f} - \frac{70c^4\tan(e+fx)}{3a^2f} + \frac{35c^4\sec(e+fx)\tan}{6a^2f}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 349 vs. 2(150) = 300.
time = 2.00, size = 349, normalized size = 2.33

$$
\frac{c^4\cos\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sin^3\left(\frac{1}{2}(e+fx)\right)\left(-256\cot^2\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-32\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+3\cot^3\left(\frac{1}{2}(e+fx)\right)\left(-70\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+70\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{1}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{24\sin(fx)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)-32\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)\right)}{3a^2f(1+\sec(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

[Out] (c^4*Cos[(e + f*x)/2]*Sec[e + f*x]^2*Sin[(e + f*x)/2]^3*(-256*Cot[(e + f*x)/2]^2*Csc[(e + f*x)/2]*Sec[e/2]*Sin[(f*x)/2] - 32*Csc[(e + f*x)/2]^3*Sec[e/2]*Sin[(f*x)/2] + 3*Cot[(e + f*x)/2]^3*(-70*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 70*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^(-2) - (24*Sin[f*x])/((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))) - 32*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*Tan[e/2]))/(3*a^2*f*(1 + Sec[e + f*x])^2)

Maple [A]
time = 0.20, size = 125, normalized size = 0.83

method	result
derivativedivides	$\frac{8c^4\left(-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{13}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\cdots\right.\right.}\right.}{f a^2}$
default	$\frac{8c^4\left(-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{13}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\cdots\right.\right.}\right.}{f a^2}$
risch	$-\frac{ic^4\left(99\,\mathrm{e}^{6i(fx+e)}+333\,\mathrm{e}^{5i(fx+e)}+434\,\mathrm{e}^{4i(fx+e)}+714\,\mathrm{e}^{3i(fx+e)}+487\,\mathrm{e}^{2i(fx+e)}+393\,\mathrm{e}^{i(fx+e)}+164\right)}{3f a^2\left(\mathrm{e}^{2i(fx+e)}+1\right)^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3}+\frac{35c^4\ln\left(\mathrm{e}^{i(fx+\cdots}\right.}{2a^2 f}$
norman	$\frac{-\frac{35c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{385c^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{511c^4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{93c^4\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{40c^4\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{8c^4\left(\tan^{11}\left(\cdots\right.\right.}{3a\cdots}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4 a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 8/f*c^4/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-3*tan(1/2*f*x+1/2*e)-1/16/(tan(1/2*f*x+1/2*e)+1)^2+13/16/(tan(1/2*f*x+1/2*e)+1)+35/16*ln(tan(1/2*f*x+1/2*e)+1)+1/16/(tan(1/2*f*x+1/2*e)-1)^2+13/16/(tan(1/2*f*x+1/2*e)-1)-35/16*ln(tan(1/2*f*x+1/2*e)-1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(152) = 304.
time = 0.29, size = 575, normalized size = 3.83

$$-\frac{c^4\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+4c^4\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+6c^4\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{4c^4\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c^4\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="maxima")

[Out] -1/6*(c^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*

```
x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 4*c^4*((15
*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(c
os(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^4*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2)
+ 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 3.48, size = 212, normalized size = 1.41

$$\frac{105\left(c^4\cos\left(fx+e\right)^4+2\,c^4\cos\left(fx+e\right)^3+c^4\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4+2\,c^4\cos\left(fx+e\right)^3+c^4\cos\left(fx+e\right)^2\right)\log\left(-\sin\left(fx+e\right)+1\right)-2\left(164\,c^4\cos\left(fx+e\right)^3+229\,c^4\cos\left(fx+e\right)^2+30\,c^4\cos\left(fx+e\right)-3\,c^4\right)\sin\left(fx+e\right)}{12\left(a^2f\cos\left(fx+e\right)^4+2\,a^2f\cos\left(fx+e\right)^3+a^2f\cos\left(fx+e\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*
log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^
4*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(164*c^4*cos(f*x + e)^3 + 229*
c^4*cos(f*x + e)^2 + 30*c^4*cos(f*x + e) - 3*c^4)*sin(f*x + e))/(a^2*f*cos(
f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{6\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^5\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-4*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(6*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integra
l(-4*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.69, size = 140, normalized size = 0.93

$$\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}+\frac{6\left(13\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-11\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^2}-\frac{16\left(a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+9\,a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 105*c^4*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^2 + 6*(13*c^4*tan(1/2*f*x + 1/2*e)^3 - 11*c^4*tan(1
/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^2) - 16*(a^4*c^4*tan(1/2
*f*x + 1/2*e)^3 + 9*a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B]
time = 1.68, size = 136, normalized size = 0.91

$$\frac{13\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-11\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2\right)}-\frac{24\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}-\frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}+\frac{35\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (13*c^4*tan(e/2 + (f*x)/2)^3 - 11*c^4*tan(e/2 + (f*x)/2))/(f*(a^2*tan(e/2 +
 (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) - (24*c^4*tan(e/2 + (f*x)/
2))/(a^2*f) - (8*c^4*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) + (35*c^4*atanh(tan(e/
2 + (f*x)/2)))/(a^2*f)
```

3.44 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2} dx$

Optimal. Leaf size=119

$$\frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{5c^3\tan(e+fx)}{a^2f}+\frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{10(c^3-c^3\sec(e+fx))\tan(}{3f\left(a^2+a^2\sec(e+fx\right.}$$

```
[Out] 5*c^3*arctanh(sin(f*x+e))/a^2/f-5*c^3*tan(f*x+e)/a^2/f+2/3*c*(c-c*sec(f*x+e
))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-10/3*(c^3-c^3*sec(f*x+e))*tan(f*x+e)/f
/(a^2+a^2*sec(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4042, 3872, 3855, 3852, 8}

$$-\frac{5c^3\tan(e+fx)}{a^2f}+\frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{10\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (5*c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (5*c^3*Tan[e + f*x])/(a^2*f) + (2*c
*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (10*(c
^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx &= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(5c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,d}{3a}\\ &= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10(c^3-c^3\sec(e+fx))\tan(e}{3f\left(a^2+a^2\sec(e+fx\right)}\\ &= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10(c^3-c^3\sec(e+fx))\tan(e}{3f\left(a^2+a^2\sec(e+fx\right)}\\ &= \frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{1}{}\\ &= \frac{5c^3\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{5c^3\tan(e+fx)}{a^2f} + \frac{2c(c-c\sec(e+fx)}{3f(a+a\sec(e}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 485 vs. 2(119) = 238.
time = 4.38, size = 485, normalized size = 4.08

$$\frac{c^3(-1+\cos(e+fx))^3\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\left(-15\cos(e)\cot^5\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sec^2\left(\frac{e}{2}\right)+2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{e}{2}\right)\left(-\sin\left(\frac{e}{2}\right)+\sin\left(\frac{3e}{2}\right)\right)+20\cot^2\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-26\cot^4\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-\frac{1}{16}(-40+40\cos(e)+78\cos(fx)-80\cos(e+fx)-40\cos(2(e+fx))+66\cos(2e+fx)+23\cos(e+2fx)+17\cos(3e+2fx))\csc^5\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-\cot^3\left(\frac{1}{2}(e+fx)\right)\left(15\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-4\csc^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)\right)\left(-1+\tan^2\left(\frac{e}{2}\right)\right)\right)}{6a^2f(1+\cos(e+fx))^2\left(-1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(-1+\tan\left(\frac{e}{2}\right)\right)\left(1+\tan\left(\frac{e}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^3*(-1 + Cos[e + f*x])^3*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2*(-15*Cos[e]*
Cot[(e + f*x)/2]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]])*Sec[e/2]^2 + 2*Cot[(e + f*x)/2]*Csc[(e + f*x)
/2]^2*Sec[e/2]^3*(-Sin[e/2] + Sin[(3*e)/2]) + 20*Cot[(e + f*x)/2]^2*Csc[(e
+ f*x)/2]*Sec[e/2]*Sin[(f*x)/2] - 26*Cot[(e + f*x)/2]^4*Csc[(e + f*x)/2]*Se
c[e/2]*Sin[(f*x)/2] - ((-40 + 40*Cos[e] + 78*Cos[f*x] - 80*Cos[e + f*x] - 4
0*Cos[2*(e + f*x)] + 66*Cos[2*e + f*x] + 23*Cos[e + 2*f*x] + 17*Cos[3*e + 2
*f*x])*Csc[(e + f*x)/2]^5*Sec[e/2]^3*Sin[(f*x)/2])/16 - Cot[(e + f*x)/2]^3*
(15*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[
```

```
(e + f*x)/2]]) - 4*Csc[(e + f*x)/2]^2*Tan[e/2])*(-1 + Tan[e/2]^2)))/(6*a^2*
f*(1 + Cos[e + f*x])^2*(-1 + Cot[(e + f*x)/2])*(1 + Cot[(e + f*x)/2])*(-1 +
 Tan[e/2])*(1 + Tan[e/2]))
```

Maple [A]
time = 0.18, size = 95, normalized size = 0.80

method	result
derivativedivides	$\frac{4c^3\left(-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f\,a^2}$
default	$\frac{4c^3\left(-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f\,a^2}$
risch	$-\frac{2ic^3\left(12\,e^{4i(fx+e)}+51\,e^{3i(fx+e)}+41\,e^{2i(fx+e)}+57\,e^{i(fx+e)}+23\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3\left(e^{2i(fx+e)}+1\right)}+\frac{5c^3\ln\left(e^{i(fx+e)}+i\right)}{a^2 f}-\frac{5c^3\ln\left(e^{i(fx+e)}-i\right)}{a^2 f}$
norman	$\frac{-\frac{80c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{22c^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{4c^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{10c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{4c^3\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3 a}-\frac{5c^3\ln\left(\tan\left(\frac{f}{2}\right.\right.}{a^2 f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 4/f*c^3/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*f*x+1/2*e)+1/4/(tan(1/2*f*
x+1/2*e)+1)+5/4*ln(tan(1/2*f*x+1/2*e)+1)+1/4/(tan(1/2*f*x+1/2*e)-1)-5/4*ln(
tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(125) = 250$.
time = 0.28, size = 369, normalized size = 3.10

$$-\frac{c^3\left(\frac{\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\,\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+3\,c^3\left(\frac{\frac{9\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{3\,c^3\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c^3\left(\frac{3\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 3*c^3*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(
```

```
cos(f*x + e) + 1)^3)/a^2 - c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 3.36, size = 192, normalized size = 1.61

$$\frac{15\left(c^{3}\cos\left(fx+e\right)^{3}+2\,c^{3}\cos\left(fx+e\right)^{2}+c^{3}\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^{3}\cos\left(fx+e\right)^{3}+2\,c^{3}\cos\left(fx+e\right)^{2}+c^{3}\cos\left(fx+e\right)\right)\log\left(-\sin\left(fx+e\right)+1\right)-2\left(23\,c^{3}\cos\left(fx+e\right)^{2}+34\,c^{3}\cos\left(fx+e\right)+3\,c^{3}\right)\sin\left(fx+e\right)}{6\left(a^{2}f\cos\left(fx+e\right)^{3}+2\,a^{2}f\cos\left(fx+e\right)^{2}+a^{2}f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*log(
sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos
(f*x + e))*log(-sin(f*x + e) + 1) - 2*(23*c^3*cos(f*x + e)^2 + 34*c^3*cos(f
*x + e) + 3*c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)
^2 + a^2*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{3\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{3\sec^{3}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^{4}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-3*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.61, size = 121, normalized size = 1.02

$$\frac{\frac{15\,c^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^{2}}-\frac{15\,c^{3}\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^{2}}+\frac{6\,c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)a^{2}}-\frac{4\left(a^{4}c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+6\,a^{4}c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{6}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 15*c^3*log(abs(tan(1/2
*f*x + 1/2*e) - 1))/a^2 + 6*c^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)
^2 - 1)*a^2) - 4*(a^4*c^3*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^3*tan(1/2*f*x +
1/2*e))/a^6)/f
```

Mupad [B]
time = 1.65, size = 104, normalized size = 0.87

$$\frac{10\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}-\frac{4\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}-\frac{8\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f}+\frac{2\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (10*c^3*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (4*c^3*tan(e/2 + (f*x)/2)^3)/(
3*a^2*f) - (8*c^3*tan(e/2 + (f*x)/2))/(a^2*f) + (2*c^3*tan(e/2 + (f*x)/2))/
(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2))
```

3.45 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=88

$$\frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{2c^2\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))}+\frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] c^2*arctanh(sin(f*x+e))/a^2/f-2*c^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+2/3*(
c^2-c^2*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4042, 3855}

$$\frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f}-\frac{2c^2\tan(e+fx)}{f\,(a^2\sec(e+fx)+a^2)}+\frac{2\tan(e+fx)\,(c^2-c^2\sec(e+fx))}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*ArcTanh[Sin[e + f*x]])/(a^2*f) - (2*c^2*Tan[e + f*x])/(f*(a^2 + a^2*Se
c[e + f*x])) + (2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e
+ f*x])^2)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{a}$$
$$= -\frac{2c^2\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))} + \frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{c^2 \cdots}{\cdots}$$
$$= \frac{c^2\tanh^{-1}(\sin(e+fx))}{a^2f} - \frac{2c^2\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))} + \frac{2(c^2-c^2\sec(e \cdots}{3f(a+a \cdots}$$

Mathematica [A]
time = 0.09, size = 109, normalized size = 1.24

$$\frac{c^2\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{4\tan\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{2\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{3f}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f - (4*Tan[(e + f*x)/2])/(3*f) - (2*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/(3*f)))/a^2
```

Maple [A]
time = 0.17, size = 65, normalized size = 0.74

method	result
derivativedivides	$\frac{2c^2\left(-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^2}$
default	$\frac{2c^2\left(-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^2}$
risch	$-\frac{8ic^2\left(3\,e^{i(fx+e)}+1\right)}{3f\,a^2\left(e^{i(fx+e)}+1\right)^3}+\frac{c^2\ln\left(e^{i(fx+e)}+i\right)}{a^2f}-\frac{c^2\ln\left(e^{i(fx+e)}-i\right)}{a^2f}$
norman	$\frac{-\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{10c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{2c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{2c^2\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2 a}+\frac{c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{a^2f}-\frac{c^2\ln\left(\tan \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 2/f*c^2/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-tan(1/2*f*x+1/2*e)-1/2*ln(tan(1/2*f*
x+1/2*e)-1)+1/2*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(93) = 186$.
time = 0.28, size = 212, normalized size = 2.41

$$-\frac{c^{2}\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}{a^{2}}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^{2}}\right)+\frac{2c^{2}\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}}-\frac{c^{2}\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/(cos
(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 2.71, size = 148, normalized size = 1.68

$$\frac{3\left(c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\cos\left(fx+e\right)+c^{2}\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\cos\left(fx+e\right)+c^{2}\right)\log\left(-\sin\left(fx+e\right)+1\right)-8\left(c^{2}\cos\left(fx+e\right)+2\,c^{2}\right)\sin\left(fx+e\right)}{6\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+a^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(sin(f*x + e) + 1
) - 3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(-sin(f*x + e) + 1
) - 8*(c^2*cos(f*x + e) + 2*c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^
2*f*cos(f*x + e) + a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^{2}\left(\int\frac{\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{2\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^{3}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-2*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.56, size = 89, normalized size = 1.01

$$\frac{\frac{3\,c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{3\,c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}-\frac{2\left(a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 3*c^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/a^2 - 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 + 3*a^4*c^2*tan(1
/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B]
time = 1.61, size = 46, normalized size = 0.52

$$-\frac{2\,c^2\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] -(2*c^2*(3*tan(e/2 + (f*x)/2) - 3*atanh(tan(e/2 + (f*x)/2)) + tan(e/2 + (f*
x)/2)^3))/(3*a^2*f)
```

3.46 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$\frac{(c-c\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] 1/3*(c-c*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{4035\}$

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{(c-c\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A]
time = 0.09, size = 23, normalized size = 0.64

$$-\frac{c\tan^3\left(\frac{1}{2}(e+fx)\right)}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(c*Tan[(e + f*x)/2]^3)/(a^2*f)
```

Maple [A]
time = 0.16, size = 21, normalized size = 0.58

method	result	size
derivativedivides	$-\frac{c\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f\,a^{2}}$	21
default	$-\frac{c\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f\,a^{2}}$	21
risch	$\frac{2ic\left(3\,\mathrm{e}^{2i(fx+e)}+1\right)}{3f\,a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}$	37
norman	$\frac{\frac{c\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{c\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{a\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/f*c/a^2*tan(1/2*f*x+1/2*e)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(38) = 76$.
time = 0.27, size = 102, normalized size = 2.83

$$-\frac{\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}}-\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 3.41, size = 57, normalized size = 1.58

$$\frac{(c\cos(fx+e)-c)\sin(fx+e)}{3\left(a^{2}f\cos(fx+e)^{2}+2\,a^{2}f\cos(fx+e)+a^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(c*cos(f*x + e) - c)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f
*x + e) + a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.60, size = 20, normalized size = 0.56

$$-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}{3\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/3*c*tan(1/2*f*x + 1/2*e)^3/(a^2*f)
```

Mupad [B]
time = 1.57, size = 20, normalized size = 0.56

$$-\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{3\,a^{2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] -(c*tan(e/2 + (f*x)/2)^3)/(3*a^2*f)
```

3.47 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=59

$$\frac{\cot^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}$$

```
[Out] 1/3*cot(f*x+e)^3/a^2/c/f+csc(f*x+e)/a^2/c/f-1/3*csc(f*x+e)^3/a^2/c/f
```

Rubi [A]
time = 0.10, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4043, 2686, 2687, 30}

$$\frac{\cot^3(e+fx)}{3a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] Cot[e + f*x]^3/(3*a^2*c*f) + Csc[e + f*x]/(a^2*c*f) - Csc[e + f*x]^3/(3*a^2
*c*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
```

```
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx &= \frac{\int (c\cot^3(e+fx)\csc(e+fx)-c\cot^2(e+fx)\csc^2(e+fx)}{a^2c^2}\\ &= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2c}-\frac{\int \cot^2(e+fx)\csc^2(e+f}{a^2c}\\ &= -\frac{\mathrm{Subst}\left(\int x^2\,dx,x,-\cot(e+fx)\right)}{a^2cf}-\frac{\mathrm{Subst}\left(\int (-1+x^2)\,d\right.}{a^2c}\\ &= \frac{\cot^3(e+fx)}{3a^2cf}+\frac{\csc(e+fx)}{a^2cf}-\frac{\csc^3(e+fx)}{3a^2cf}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 83, normalized size = 1.41

$$-\frac{\csc(e)\csc^3(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)(-6\sin(e)+2\sin(fx)+10\sin(e+fx)+5\sin(2(e+fx))-6\sin(2e+fx)-2\sin(e+2fx))}{6a^2cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/6*(Csc[e]*Csc[e + f*x]^3*Sin[(e + f*x)/2]^2*(-6*Sin[e] + 2*Sin[f*x] + 10
*Sin[e + f*x] + 5*Sin[2*(e + f*x)] - 6*Sin[2*e + f*x] - 2*Sin[e + 2*f*x]))/
(a^2*c*f)
```

Maple [A]
time = 0.14, size = 48, normalized size = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^2c}$	48
default	$\frac{-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^2c}$	48
norman	$\frac{\frac{1}{4acf}+\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{2acf}-\frac{\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)}{12acf}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	72
risch	$\frac{2i\left(3\,e^{3i(fx+e)}+3\,e^{2i(fx+e)}+e^{i(fx+e)}-1\right)}{3f\,a^2c\left(e^{i(fx+e)}+1\right)^3\left(e^{i(fx+e)}-1\right)}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a^2/c*(-1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+1/tan(1/2*f*x+1
/2*e))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 1.39

$$\frac{\frac{\frac{6\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c}+\frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*((6*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/(a^2*c) + 3*(cos(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

Fricas [A]
time = 2.82, size = 53, normalized size = 0.90

$$-\frac{\cos(fx+e)^2-2\cos(fx+e)-2}{3(a^2cf\cos(fx+e)+a^2cf)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/3*(cos(f*x + e)^2 - 2*cos(f*x + e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f
)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec(e+fx)}{\sec^3(e+fx)+\sec^2(e+fx)-\sec(e+fx)-1}\,dx}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) -
1), x)/(a**2*c)
```

Giac [A]
time = 0.53, size = 69, normalized size = 1.17

$$\frac{\frac{3}{a^2 c \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)} - \frac{a^4 c^2 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 - 6 a^4 c^2 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)}{a^6 c^3}}{12 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/12*(3/(a^2*c*tan(1/2*f*x + 1/2*e)) - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 6*
a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

Mupad [B]
time = 1.62, size = 61, normalized size = 1.03

$$-\frac{4 \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^4 - 8 \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^2 + 1}{12 a^2 c f \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{f x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)
```

```
[Out] -(4*cos(e/2 + (f*x)/2)^4 - 8*cos(e/2 + (f*x)/2)^2 + 1)/(12*a^2*c*f*cos(e/2
+ (f*x)/2)^3*sin(e/2 + (f*x)/2))
```

3.48 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{\csc(e+fx)}{a^2c^2f}-\frac{\csc^3(e+fx)}{3a^2c^2f}$$

[Out] csc(f*x+e)/a^2/c^2/f-1/3*csc(f*x+e)^3/a^2/c^2/f

Rubi [A]
time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4043, 2686}

$$\frac{\csc(e+fx)}{a^2c^2f}-\frac{\csc^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]

[Out] Csc[e + f*x]/(a^2*c^2*f) - Csc[e + f*x]^3/(3*a^2*c^2*f)

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx &= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2c^2}\\ &= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(e+fx)\right)}{a^2c^2f}\\ &= \frac{\csc(e+fx)}{a^2c^2f}-\frac{\csc^3(e+fx)}{3a^2c^2f}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 33, normalized size = 0.87

$$\frac{\frac{\csc(e+fx)}{f}-\frac{\csc^3(e+fx)}{3f}}{a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
[Out] (Csc[e + f*x]/f - Csc[e + f*x]^3/(3*f))/(a^2*c^2)
```

Maple [A]
time = 0.21, size = 66, normalized size = 1.74

method	result	size
default	$\frac{-\frac{\cos^4(fx+e)}{3\sin(fx+e)^3}+\frac{\cos^4(fx+e)}{3\sin(fx+e)}+\frac{\left(2+\cos^2(fx+e)\right)\sin(fx+e)}{3}}{a^2c^2f}$	66
risch	$\frac{2i\left(3\,\mathrm{e}^{5i(fx+e)}-2\,\mathrm{e}^{3i(fx+e)}+3\,\mathrm{e}^{i(fx+e)}\right)}{3f\,a^2c^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3\left(\mathrm{e}^{i(fx+e)}-1\right)^3}$	73
norman	$\frac{-\frac{1}{24acf}+\frac{3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}+\frac{3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}-\frac{\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)}{24acf}}{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	97
derivativedivides	`error in RationalFunction: argument is not a rational function\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
[Out] 1/a^2/c^2/f*(-1/3/sin(f*x+e)^3*cos(f*x+e)^4+1/3/sin(f*x+e)*cos(f*x+e)^4+1/3
*(2+cos(f*x+e)^2)*sin(f*x+e))
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.87

$$\frac{3\,\sin\left(fx+e\right)^2-1}{3\,a^2c^2f\sin\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
[Out] 1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

Fricas [A]
time = 2.39, size = 53, normalized size = 1.39

$$\frac{3\,\cos\left(fx+e\right)^2-2}{3\left(a^2c^2f\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(3*cos(f*x + e)^2 - 2)/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{\sec^4(e+fx)-2\sec^2(e+fx)+1}\,dx}{a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c
**2)
```

Giac [A]
time = 0.54, size = 31, normalized size = 0.82

$$\frac{3\sin(fx+e)^2-1}{3\,a^2c^2f\sin(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

Mupad [B]
time = 1.57, size = 28, normalized size = 0.74

$$\frac{\sin(e+f\,x)^2-\frac{1}{3}}{a^2\,c^2\,f\sin(e+f\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (sin(e + f*x)^2 - 1/3)/(a^2*c^2*f*sin(e + f*x)^3)
```

3.49 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=80

$$\frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}$$

```
[Out] 1/5*cot(f*x+e)^5/a^2/c^3/f+csc(f*x+e)/a^2/c^3/f-2/3*csc(f*x+e)^3/a^2/c^3/f+
1/5*csc(f*x+e)^5/a^2/c^3/f
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4043, 2686, 200, 2687, 30}

$$\frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Cot[e + f*x]^5/(5*a^2*c^3*f) + Csc[e + f*x]/(a^2*c^3*f) - (2*Csc[e + f*x]^3
)/(3*a^2*c^3*f) + Csc[e + f*x]^5/(5*a^2*c^3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx &= -\frac{\int \left(a\cot^5(e+fx)\csc(e+fx)+a\cot^4(e+fx)\csc^2(e+f\right.}{a^3c^3}\\ &= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^2c^3}-\frac{\int \cot^4(e+fx)\csc^2(e+}{a^2c^3}\\ &= -\frac{\operatorname{Subst}\left(\int x^4\,dx,x,-\cot(e+fx)\right)}{a^2c^3f}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\right.}{a^2c^3}\\ &= \frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(e+fx)\right)}{a^2c^3f}\\ &= \frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc^5(e+f}{5a^2c^3f}\end{aligned}$$

Mathematica [A]
time = 0.99, size = 147, normalized size = 1.84

$$-\frac{\csc(e)\csc^2\left(\frac{1}{2}(e+fx)\right)\csc^3(e+fx)(-200\sin(e)+104\sin(fx)+534\sin(e+fx)-178\sin(2(e+fx))-178\sin(3(e+fx))+89\sin(4(e+fx))+40\sin(2e+fx)-168\sin(e+2fx)+120\sin(3e+2fx)+72\sin(2e+3fx)-120\sin(4e+3fx)+24\sin(3e+4fx))}{1920a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] -1/1920*(Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]^3*(-200*Sin[e] + 104*Sin[f*
x] + 534*Sin[e + f*x] - 178*Sin[2*(e + f*x)] - 178*Sin[3*(e + f*x)] + 89*Si
n[4*(e + f*x)] + 40*Sin[2*e + f*x] - 168*Sin[e + 2*f*x] + 120*Sin[3*e + 2*f
*x] + 72*Sin[2*e + 3*f*x] - 120*Sin[4*e + 3*f*x] + 24*Sin[3*e + 4*f*x]))/(a
^2*c^3*f)
```

Maple [A]
time = 0.17, size = 76, normalized size = 0.95

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{16f\,c^{3}a^{2}}$	76
default	$\frac{-\frac{\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{16f\,c^{3}a^{2}}$	76
risch	$\frac{2i\left(15\,e^{7i(fx+e)}-15\,e^{6i(fx+e)}-5\,e^{5i(fx+e)}+25\,e^{4i(fx+e)}+13\,e^{3i(fx+e)}-21\,e^{2i(fx+e)}+9\,e^{i(fx+e)}+3\right)}{15f\,c^{3}a^{2}\left(e^{i(fx+e)}+1\right)^{3}\left(e^{i(fx+e)}-1\right)^{5}}$	118
norman	$\frac{\frac{1}{80acf}-\frac{\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}{12acf}+\frac{3\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}+\frac{\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)}{4acf}-\frac{\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)}{48acf}}{a\,c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/16/f/c^3/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3+4*tan(1/2*f*x+1/2*e)+6/tan(1/2*f*
x+1/2*e)-4/3/tan(1/2*f*x+1/2*e)^3+1/5/tan(1/2*f*x+1/2*e)^5)
```

Maxima [A]

time = 0.30, size = 131, normalized size = 1.64

$$\frac{\frac{5\left(\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}c^{3}}-\frac{\left(\frac{20\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{90\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-3\right)(\cos(fx+e)+1)^{5}}{a^{2}c^{3}\sin(fx+e)^{5}}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/240*(5*(12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/(a^2*c^3) - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 90*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^
5))/f
```

Fricas [A]

time = 2.85, size = 117, normalized size = 1.46

$$\frac{3\cos\left(fx+e\right)^{4}+12\cos\left(fx+e\right)^{3}-12\cos\left(fx+e\right)^{2}-8\cos\left(fx+e\right)+8}{15\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}-a^{2}c^{3}f\cos\left(fx+e\right)^{2}-a^{2}c^{3}f\cos\left(fx+e\right)+a^{2}c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(3*cos(f*x + e)^4 + 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 - 8*cos(f*x
+ e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f
*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec{\left(e+fx \right)}}{\sec^{5}{\left(e+fx \right)}-\sec^{4}{\left(e+fx \right)}-2\sec^{3}{\left(e+fx \right)}+2\sec^{2}{\left(e+fx \right)}+\sec{\left(e+fx \right)}-1}\, dx}{a^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)*
*3 + 2*sec(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

Giac [A]
time = 0.50, size = 96, normalized size = 1.20

$$\frac{\frac{90\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-20\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+3}{a^{2}c^{3}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}}-\frac{5\left(a^{4}c^{6}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-12\,a^{4}c^{6}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{6}c^{9}}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/240*((90*tan(1/2*f*x + 1/2*e)^4 - 20*tan(1/2*f*x + 1/2*e)^2 + 3)/(a^2*c^3
*tan(1/2*f*x + 1/2*e)^5) - 5*(a^4*c^6*tan(1/2*f*x + 1/2*e)^3 - 12*a^4*c^6*t
an(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

Mupad [B]
time = 2.01, size = 76, normalized size = 0.95

$$\frac{-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+60\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+90\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3}{240\,a^2\,c^3\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (90*tan(e/2 + (f*x)/2)^4 - 20*tan(e/2 + (f*x)/2)^2 + 60*tan(e/2 + (f*x)/2)^
6 - 5*tan(e/2 + (f*x)/2)^8 + 3)/(240*a^2*c^3*f*tan(e/2 + (f*x)/2)^5)
```

3.50 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=98

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}$$

```
[Out] -2/7*cot(f*x+e)^7/a^2/c^4/f+csc(f*x+e)/a^2/c^4/f-4/3*csc(f*x+e)^3/a^2/c^4/f
+csc(f*x+e)^5/a^2/c^4/f-2/7*csc(f*x+e)^7/a^2/c^4/f
```

Rubi [A]
time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4043, 2686, 200, 2687, 30, 276}

$$-\frac{2\cot^7(e+fx)}{7a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (-2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + Csc[e + f*x]/(a^2*c^4*f) - (4*Csc[e + f
*x]^3)/(3*a^2*c^4*f) + Csc[e + f*x]^5/(a^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a
^2*c^4*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx &= \frac{\int \left(a^2\cot^7(e+fx)\csc(e+fx)+2a^2\cot^6(e+fx)\csc^2(e+\ldots\right.}{a^4c^4} \\
&= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^2c^4}+\frac{\int \cot^5(e+fx)\csc^3(e+fx\ldots}{a^2c^4} \\
&= -\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\csc(e+fx)\right)}{a^2c^4f}-\frac{\text{Subst}\left(\int (-\ldots\right.}{\ldots} \\
&= -\frac{2\cot^7(e+fx)}{7a^2c^4f}-\frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,c\ldots\right.}{a^2c^4f} \\
&= -\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc^5(e\ldots}{a^2c\ldots}
\end{aligned}$$

Mathematica [A]
time = 0.93, size = 179, normalized size = 1.83

$$\frac{\csc(e)\csc^4\left(\frac{1}{2}(e+fx)\right)\csc^3(e+fx)(42\sin(e)-28\sin(fx)-182\sin(e+fx)+104\sin(2(e+fx))+39\sin(3(e+fx))-52\sin(4(e+fx))+13\sin(5(e+fx))-56\sin(2e+fx)+76\sin(e+2fx)-28\sin(3e+2fx)-24\sin(2e+3fx)+42\sin(4e+3fx)-3\sin(3e+4fx)-21\sin(5e+4fx)+6\sin(4e+5fx))}{1344a^2c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^4*Csc[e + f*x]^3*(42*Sin[e] - 28*Sin[f*x] - 182*Si
n[e + f*x] + 104*Sin[2*(e + f*x)] + 39*Sin[3*(e + f*x)] - 52*Sin[4*(e + f*x
)] + 13*Sin[5*(e + f*x)] - 56*Sin[2*e + f*x] + 76*Sin[e + 2*f*x] - 28*Sin[3
```

```
*e + 2*f*x] - 24*Sin[2*e + 3*f*x] + 42*Sin[4*e + 3*f*x] - 3*Sin[3*e + 4*f*x
] - 21*Sin[5*e + 4*f*x] + 6*Sin[4*e + 5*f*x]))/(1344*a^2*c^4*f)
```

Maple [A]
time = 0.19, size = 87, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{10}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{10}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{32f\,a^{2}c^{4}}$
default	$\frac{-\frac{\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{10}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{10}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{32f\,a^{2}c^{4}}$
risch	$\frac{2i\left(21\,e^{9i(fx+e)}-42\,e^{8i(fx+e)}+28\,e^{7i(fx+e)}+56\,e^{6i(fx+e)}-42\,e^{5i(fx+e)}-28\,e^{4i(fx+e)}+76\,e^{3i(fx+e)}-24\,e^{2i(fx+e)}-3\,e^{i(\cdots}}{21f\,a^{2}c^{4}\left(e^{i(fx+e)}-1\right)^{7}\left(e^{i(fx+e)}+1\right)^{3}}$
norman	$\frac{-\frac{1}{224acf}+\frac{\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}{32acf}-\frac{5\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{48acf}+\frac{5\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16acf}+\frac{5\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32acf}-\frac{\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)}{96acf}}{a\,c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/32/f/a^2/c^4*(-1/3*tan(1/2*f*x+1/2*e)^3+5*tan(1/2*f*x+1/2*e)+10/tan(1/2*f
*x+1/2*e)-10/3/tan(1/2*f*x+1/2*e)^3-1/7/tan(1/2*f*x+1/2*e)^7+1/tan(1/2*f*x+
1/2*e)^5)
```

Maxima [A]
time = 0.31, size = 152, normalized size = 1.55

$$\frac{\frac{7\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}c^{4}}+\frac{\left(\frac{21\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{70\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{210\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-3\right)(\cos(fx+e)+1)^{7}}{a^{2}c^{4}\sin(fx+e)^{7}}}{672\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/672*(7*(15*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/(a^2*c^4) + (21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 210*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(
cos(f*x + e) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

Fricas [A]
time = 2.05, size = 129, normalized size = 1.32

$$\frac{6\cos\left(fx+e\right)^{5}+9\cos\left(fx+e\right)^{4}-24\cos\left(fx+e\right)^{3}+4\cos\left(fx+e\right)^{2}+16\cos\left(fx+e\right)-8}{21\left(a^{2}c^{4}f\cos\left(fx+e\right)^{4}-2\,a^{2}c^{4}f\cos\left(fx+e\right)^{3}+2\,a^{2}c^{4}f\cos\left(fx+e\right)-a^{2}c^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/21*(6*cos(f*x + e)^5 + 9*cos(f*x + e)^4 - 24*cos(f*x + e)^3 + 4*cos(f*x +
 e)^2 + 16*cos(f*x + e) - 8)/((a^2*c^4*f*cos(f*x + e)^4 - 2*a^2*c^4*f*cos(f
*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{\sec^6(e+fx)-2\sec^5(e+fx)-\sec^4(e+fx)+4\sec^3(e+fx)-\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**
4 + 4*sec(e + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**
4)
```

Giac [A]
time = 0.51, size = 109, normalized size = 1.11

$$\frac{\frac{210\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+21\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3}{a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}-\frac{7\left(a^4c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-15a^4c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6c^{12}}}{672f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/672*((210*tan(1/2*f*x + 1/2*e)^6 - 70*tan(1/2*f*x + 1/2*e)^4 + 21*tan(1/2
*f*x + 1/2*e)^2 - 3)/(a^2*c^4*tan(1/2*f*x + 1/2*e)^7) - 7*(a^4*c^8*tan(1/2*
f*x + 1/2*e)^3 - 15*a^4*c^8*tan(1/2*f*x + 1/2*e))/(a^6*c^12))/f
```

Mupad [B]
time = 2.67, size = 89, normalized size = 0.91

$$\frac{-7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+105\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+210\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-70\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-3}{672\,a^2\,c^4\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)
```

```
[Out] (21*tan(e/2 + (f*x)/2)^2 - 70*tan(e/2 + (f*x)/2)^4 + 210*tan(e/2 + (f*x)/2)
^6 + 105*tan(e/2 + (f*x)/2)^8 - 7*tan(e/2 + (f*x)/2)^10 - 3)/(672*a^2*c^4*f
*tan(e/2 + (f*x)/2)^7)
```

3.51 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5} dx$

Optimal. Leaf size=141

$$\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}$$

```
[Out] 1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+csc(f*x+e)/a^2/c^5/f-
7/3*csc(f*x+e)^3/a^2/c^5/f+3*csc(f*x+e)^5/a^2/c^5/f-13/7*csc(f*x+e)^7/a^2/c
^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f
```

Rubi [A]
time = 0.19, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {4043, 2686, 200, 2687, 30, 276, 14}

$$\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^9)/(9*a^2*c^5*f) + Csc[e + f
*x]/(a^2*c^5*f) - (7*Csc[e + f*x]^3)/(3*a^2*c^5*f) + (3*Csc[e + f*x]^5)/(a^
2*c^5*f) - (13*Csc[e + f*x]^7)/(7*a^2*c^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^
5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx &= -\frac{\int \left(a^3\cot^9(e+fx)\csc(e+fx)+3a^3\cot^8(e+fx)\csc^2(e-\right.}{} \\
&= -\frac{\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^2c^5}-\frac{\int \cot^6(e+fx)\csc^4(e+}{a^2c^5} \\
&= \frac{\mathrm{Subst}\left(\int \left(-1+x^2\right)^4\,dx,x,\csc(e+fx)\right)}{a^2c^5f}-\frac{\mathrm{Subst}\left(\int x^6(1+\right.}{} \\
&= \frac{\cot^9(e+fx)}{3a^2c^5f}+\frac{\mathrm{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,\right.}{a^2c^5f} \\
&= \frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+}{3a^2c^5f}
\end{aligned}
$$

Mathematica [A]
time = 1.25, size = 211, normalized size = 1.50

$$
-\frac{\csc(e)\csc^6\left(\frac{1}{2}(e+fx)\right)\csc^3(e+fx)\left(-9408\sin(e)+9792\sin(fx)+36252\sin(e+fx)-27189\sin(2(e+fx))-2014\sin(3(e+fx))+12084\sin(4(e+fx))-6042\sin(5(e+fx))+1007\sin(6(e+fx))+12096\sin(2e+fx)-14400\sin(e+2fx)-2016\sin(3e+2fx)+7520\sin(2e+3fx)-8736\sin(4e+3fx)+1248\sin(3e+4fx)+6048\sin(5e+4fx)-1632\sin(4e+5fx)-2016\sin(6e+5fx)+608\sin(5e+6fx)\right)}{516096a^2c^5f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] -1/516096*(Csc[e]*Csc[(e + f*x)/2]^6*Csc[e + f*x]^3*(-9408*Sin[e] + 9792*Si
n[f*x] + 36252*Sin[e + f*x] - 27189*Sin[2*(e + f*x)] - 2014*Sin[3*(e + f*x)
] + 12084*Sin[4*(e + f*x)] - 6042*Sin[5*(e + f*x)] + 1007*Sin[6*(e + f*x)]
+ 12096*Sin[2*e + f*x] - 14400*Sin[e + 2*f*x] - 2016*Sin[3*e + 2*f*x] + 752
0*Sin[2*e + 3*f*x] - 8736*Sin[4*e + 3*f*x] + 1248*Sin[3*e + 4*f*x] + 6048*S
in[5*e + 4*f*x] - 1632*Sin[4*e + 5*f*x] - 2016*Sin[6*e + 5*f*x] + 608*Sin[5
*e + 6*f*x]))/(a^2*c^5*f)
```

Maple [A]
time = 0.19, size = 102, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{3}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3}+6 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)-\frac{20}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}+\frac{1}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}+\frac{15}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}-\frac{6}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}}{64 f\, c^{5} a^{2}}$
default	$\frac{-\frac{\left(\tan^{3}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3}+6 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)-\frac{20}{3 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}+\frac{1}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}+\frac{15}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}-\frac{6}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}}{64 f\, c^{5} a^{2}}$
risch	$\frac{2 i \left(63 \,{\mathrm e}^{11 i (f x+e)}-189 \,{\mathrm e}^{10 i (f x+e)}+273 \,{\mathrm e}^{9 i (f x+e)}+63 \,{\mathrm e}^{8 i (f x+e)}-378 \,{\mathrm e}^{7 i (f x+e)}+294 \,{\mathrm e}^{6 i (f x+e)}+306 \,{\mathrm e}^{5 i (f x+e)}-450 \,{\mathrm e}^{4 i (f x}}{63 f\, c^{5} a^{2} \left({\mathrm e}^{i (f x+e)}-1\right)^{9} \left({\mathrm e}^{i (f x+e)}+1\right)^{3}}$
norman	$\frac{\frac{1}{576 a c f}-\frac{3 \left(\tan^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{224 a c f}+\frac{3 \left(\tan^{4}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{64 a c f}-\frac{5 \left(\tan^{6}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{48 a c f}+\frac{15 \left(\tan^{8}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{64 a c f}+\frac{3 \left(\tan^{10}\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{32 a c f}-\frac{\tan^{12}\left(\frac{f x}{2}+\frac{e}{2}\right)}{192 a c f}}{a\, c^{4} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/64/f/c^5/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3+6*tan(1/2*f*x+1/2*e)-20/3/tan(1/2
*f*x+1/2*e)^3+3/tan(1/2*f*x+1/2*e)^5+1/9/tan(1/2*f*x+1/2*e)^9+15/tan(1/2*f*
x+1/2*e)-6/7/tan(1/2*f*x+1/2*e)^7)
```

Maxima [A]
time = 0.29, size = 175, normalized size = 1.24

$$\frac{\frac{21 \left(\frac{18 \sin\left(f x+e\right)}{\cos\left(f x+e\right)+1}-\frac{\sin\left(f x+e\right)^{3}}{\left(\cos\left(f x+e\right)+1\right)^{3}}\right)}{a^{2} c^{5}}-\frac{\left(\frac{54 \sin\left(f x+e\right)^{2}}{\left(\cos\left(f x+e\right)+1\right)^{2}}-\frac{189 \sin\left(f x+e\right)^{4}}{\left(\cos\left(f x+e\right)+1\right)^{4}}+\frac{420 \sin\left(f x+e\right)^{6}}{\left(\cos\left(f x+e\right)+1\right)^{6}}-\frac{945 \sin\left(f x+e\right)^{8}}{\left(\cos\left(f x+e\right)+1\right)^{8}}-7\right) \left(\cos\left(f x+e\right)+1\right)^{9}}{a^{2} c^{5} \sin\left(f x+e\right)^{9}}}{4032 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/4032*(21*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a^2*c^5) - (54*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 9
45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(a^2*c^5*s
in(f*x + e)^9))/f
```

Fricas [A]
time = 2.11, size = 175, normalized size = 1.24

$$\frac{19\cos(fx+e)^6+6\cos(fx+e)^5-66\cos(fx+e)^4+56\cos(fx+e)^3+24\cos(fx+e)^2-48\cos(fx+e)+16}{63\left(a^2c^5f\cos(fx+e)^5-3a^2c^5f\cos(fx+e)^4+2a^2c^5f\cos(fx+e)^3+2a^2c^5f\cos(fx+e)^2-3a^2c^5f\cos(fx+e)+a^2c^5f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/63*(19*cos(f*x + e)^6 + 6*cos(f*x + e)^5 - 66*cos(f*x + e)^4 + 56*cos(f*x
 + e)^3 + 24*cos(f*x + e)^2 - 48*cos(f*x + e) + 16)/((a^2*c^5*f*cos(f*x + e
)^5 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5*f
*cos(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec(e+fx)}{\sec^7(e+fx)-3\sec^6(e+fx)+\sec^5(e+fx)+5\sec^4(e+fx)-5\sec^3(e+fx)-\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{a^2c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)*
*5 + 5*sec(e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*
x) - 1), x)/(a**2*c**5)
```

Giac [A]
time = 0.57, size = 122, normalized size = 0.87

$$\frac{\frac{945\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-420\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+189\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-54\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7}{a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}-\frac{21\left(a^4c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-18a^4c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6c^{15}}}{4032f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/4032*((945*tan(1/2*f*x + 1/2*e)^8 - 420*tan(1/2*f*x + 1/2*e)^6 + 189*tan(
1/2*f*x + 1/2*e)^4 - 54*tan(1/2*f*x + 1/2*e)^2 + 7)/(a^2*c^5*tan(1/2*f*x +
```

```
1/2*e)^9) - 21*(a^4*c^10*tan(1/2*f*x + 1/2*e)^3 - 18*a^4*c^10*tan(1/2*f*x +
 1/2*e))/(a^6*c^15))/f
```

Mupad [B]
time = 4.24, size = 102, normalized size = 0.72

$$\frac{-21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+378\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+945\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-420\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+189\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-54\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+7}{4032\,a^2\,c^5\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (189*tan(e/2 + (f*x)/2)^4 - 54*tan(e/2 + (f*x)/2)^2 - 420*tan(e/2 + (f*x)/2
)^6 + 945*tan(e/2 + (f*x)/2)^8 + 378*tan(e/2 + (f*x)/2)^10 - 21*tan(e/2 + (
f*x)/2)^12 + 7)/(4032*a^2*c^5*f*tan(e/2 + (f*x)/2)^9)
```

3.52 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=215

$$-\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f}+\frac{924c^6\tan(e+fx)}{5a^3f}-\frac{693c^6\sec(e+fx)\tan(e+fx)}{10a^3f}-\frac{22c^2(c-c\sec(e+fx))^4\,\mathrm{t}}{15af(a+a\sec(e+}$$

```
[Out] -231/2*c^6*arctanh(sin(f*x+e))/a^3/f+924/5*c^6*tan(f*x+e)/a^3/f-693/10*c^6*
sec(f*x+e)*tan(f*x+e)/a^3/f-22/15*c^2*(c-c*sec(f*x+e))^4*tan(f*x+e)/a/f/(a+
a*sec(f*x+e))^2+2/5*c*(c-c*sec(f*x+e))^5*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+66
/5*(c^2-c^2*sec(f*x+e))^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+77/5*c^6*tan(f*
x+e)^3/a^3/f
```

Rubi [A]
time = 0.25, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4042, 3876, 3855, 3852, 8, 3853}

$$\frac{77c^6\tan^3(e+fx)}{5a^3f}+\frac{924c^6\tan(e+fx)}{5a^3f}-\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{693c^6\tan(e+fx)\sec(e+fx)}{10a^3f}+\frac{66\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^3}{5f\left(a^3\sec(e+fx)+a^3\right)}-\frac{22c^2\tan(e+fx)(c-c\sec(e+fx))^4}{15af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-231*c^6*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (924*c^6*Tan[e + f*x])/(5*a^3*
f) - (693*c^6*Sec[e + f*x]*Tan[e + f*x])/(10*a^3*f) - (22*c^2*(c - c*Sec[e
+ f*x])^4*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e
 + f*x])^5*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (66*(c^2 - c^2*Sec[
e + f*x])^3*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) + (77*c^6*Tan[e +
f*x]^3)/(5*a^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(11c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}}{5a}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\,\mathrm{ta}}{5f(a+a\sec(e+f}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\,\mathrm{ta}}{5f(a+a\sec(e+f}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\,\mathrm{ta}}{5f(a+a\sec(e+f}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\,\mathrm{ta}}{5f(a+a\sec(e+f}$$

$$= -\frac{231c^6\tanh^{-1}(\sin(e+fx))}{5a^3f} - \frac{693c^6\sec(e+fx)\tan(e+fx)}{10a^3f} - \frac{2\!2}{}$$

$$= -\frac{231c^6\tanh^{-1}(\sin(e+fx))}{2a^3f} + \frac{924c^6\tan(e+fx)}{5a^3f} - \frac{693c^6\sec(e+}{1\!0}$$

Mathematica [A]
time = 2.12, size = 406, normalized size = 1.89

$$\frac{c^6\cos\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\left(887040\cos^5\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\sec\left(\frac{e}{2}\right)\sec(e)\sec^3(e+fx)\left(-65436\sin\left(\frac{fx}{2}\right)+127498\sin\left(\frac{3fx}{2}\right)-130340\sin\left(e-\frac{fx}{2}\right)+75600\sin\left(e+\frac{fx}{2}\right)-120176\sin\left(2e+\frac{fx}{2}\right)-34230\sin\left(e+\frac{3fx}{2}\right)+82278\sin\left(2e+\frac{3fx}{2}\right)-79450\sin\left(3e+\frac{3fx}{2}\right)+91670\sin\left(e+\frac{5fx}{2}\right)-14730\sin\left(2e+\frac{5fx}{2}\right)+61920\sin\left(3e+\frac{5fx}{2}\right)-44480\sin\left(4e+\frac{5fx}{2}\right)+53593\sin\left(2e+\frac{7fx}{2}\right)-1735\sin\left(3e+\frac{7fx}{2}\right)+38123\sin\left(4e+\frac{7fx}{2}\right)-17205\sin\left(5e+\frac{7fx}{2}\right)+23735\sin\left(3e+\frac{9fx}{2}\right)+2455\sin\left(4e+\frac{9fx}{2}\right)+17785\sin\left(5e+\frac{9fx}{2}\right)-3495\sin\left(6e+\frac{9fx}{2}\right)+5446\sin\left(4e+\frac{11fx}{2}\right)+1190\sin\left(5e+\frac{11fx}{2}\right)+4256\sin\left(6e+\frac{11fx}{2}\right)\right)\right)}{960a^3f(1+\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^6*Cos[(e + f*x)/2]*Sec[e + f*x]^3*(887040*Cos[(e + f*x)/2]^5*(Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])
+ Sec[e/2]*Sec[e]*Sec[e + f*x]^3*(-65436*Sin[(f*x)/2] + 127498*Sin[(3*f*x)/
2] - 130340*Sin[e - (f*x)/2] + 75600*Sin[e + (f*x)/2] - 120176*Sin[2*e + (f
*x)/2] - 34230*Sin[e + (3*f*x)/2] + 82278*Sin[2*e + (3*f*x)/2] - 79450*Sin[
3*e + (3*f*x)/2] + 91670*Sin[e + (5*f*x)/2] - 14730*Sin[2*e + (5*f*x)/2] +
61920*Sin[3*e + (5*f*x)/2] - 44480*Sin[4*e + (5*f*x)/2] + 53593*Sin[2*e + (
7*f*x)/2] - 1735*Sin[3*e + (7*f*x)/2] + 38123*Sin[4*e + (7*f*x)/2] - 17205*
Sin[5*e + (7*f*x)/2] + 23735*Sin[3*e + (9*f*x)/2] + 2455*Sin[4*e + (9*f*x)/
2] + 17785*Sin[5*e + (9*f*x)/2] - 3495*Sin[6*e + (9*f*x)/2] + 5446*Sin[4*e
+ (11*f*x)/2] + 1190*Sin[5*e + (11*f*x)/2] + 4256*Sin[6*e + (11*f*x)/2])))/
(960*a^3*f*(1 + Sec[e + f*x])^3)
```

Maple [A]
time = 0.21, size = 168, normalized size = 0.78

method	result
derivativedivides	$\frac{16c^6\left(-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{5}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{89}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{231\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}+\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{4\left(\tan^3\right.}{}\right.}{f\,a}$
default	$\frac{16c^6\left(-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{5}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{89}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{231\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}+\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{4\left(\tan^3\right.}{}\right.}{f\,a}$
risch	$\frac{ic^6\left(3495\,e^{10i(fx+e)}+17205\,e^{9i(fx+e)}+44480\,e^{8i(fx+e)}+79450\,e^{7i(fx+e)}+120176\,e^{6i(fx+e)}+130340\,e^{5i(fx+e)}+127498\,e^{4}\right.}{15a^3f\left(e^{2i(fx+e)}+1\right)^3\left(e^{i(fx+e)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/f*c^6/a^3*(-1/48/(tan(1/2*f*x+1/2*e)-1)^3-5/16/(tan(1/2*f*x+1/2*e)-1)^2-
89/32/(tan(1/2*f*x+1/2*e)-1)+231/32*ln(tan(1/2*f*x+1/2*e)-1)+1/5*tan(1/2*f*
x+1/2*e)^5+4/3*tan(1/2*f*x+1/2*e)^3+10*tan(1/2*f*x+1/2*e)-1/48/(tan(1/2*f*x
+1/2*e)+1)^3+5/16/(tan(1/2*f*x+1/2*e)+1)^2-89/32/(tan(1/2*f*x+1/2*e)+1)-231
/32*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(218) = 436$.

time = 0.31, size = 1015, normalized size = 4.72

$$\frac{c^{6}\left(\frac{20\left(\frac{33\sin(fx+e)}{\cos(fx+e)+1}-\frac{76\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{51\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}-\frac{3a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{3a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}-\frac{a^{3}\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}}+\frac{\frac{735\sin(fx+e)}{\cos(fx+e)+1}+\frac{50\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}{a^{3}}-\frac{690\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{3}}+\frac{690\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^{3}}\right)+6c^{6}\left(\frac{60\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{7\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{3}-\frac{2a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a^{3}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}}+\frac{\frac{465\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}{a^{3}}-\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{3}}+\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^{3}}\right)+45c^{6}\left(\frac{40\sin(fx+e)}{\left(a^{3}-\frac{a^{3}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}\right)(\cos(fx+e)+1)}+\frac{\frac{85\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}{a^{3}}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{3}}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^{3}}\right)+20c^{6}\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}}{a^{3}}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{3}}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^{3}}\right)+\frac{15c^{6}\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}}+\frac{c^{6}\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}}-\frac{18c^{6}\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*x +
e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 690
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^6*(60*(5*sin(f*x + e)/
(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 - 2*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/
a^3) + 45*c^6*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6
0*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a^3) + 20*c^6*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5)/a^3 + c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 18*c^6*
(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3)/f
```

Fricas [A]
time = 2.34, size = 283, normalized size = 1.32

$$-\frac{3465\left(c^{6}\cos(fx+e)^{6}+3c^{6}\cos(fx+e)^{5}+3c^{6}\cos(fx+e)^{4}+c^{6}\cos(fx+e)^{3}\right)\log(\sin(fx+e)+1)-3465\left(c^{6}\cos(fx+e)^{6}+3c^{6}\cos(fx+e)^{5}+3c^{6}\cos(fx+e)^{4}+c^{6}\cos(fx+e)^{3}\right)\log(-\sin(fx+e)+1)-2\left(5446c^{6}\cos(fx+e)^{5}+12843c^{6}\cos(fx+e)^{4}+8711c^{6}\cos(fx+e)^{3}+815c^{6}\cos(fx+e)^{2}-105c^{6}\cos(fx+e)+10c^{6}\right)\sin(fx+e)}{60\left(a^{3}f\cos(fx+e)^{6}+3a^{3}f\cos(fx+e)^{5}+3a^{3}f\cos(fx+e)^{4}+a^{3}f\cos(fx+e)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/60*(3465*(c^6*cos(f*x + e)^6 + 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)
^4 + c^6*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 3465*(c^6*cos(f*x + e)^6 +
```

```
 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)^4 + c^6*cos(f*x + e)^3)*log(-sin
(f*x + e) + 1) - 2*(5446*c^6*cos(f*x + e)^5 + 12843*c^6*cos(f*x + e)^4 + 87
11*c^6*cos(f*x + e)^3 + 815*c^6*cos(f*x + e)^2 - 105*c^6*cos(f*x + e) + 10*
c^6)*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3*a^3*f
*cos(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^6\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{6\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int \frac{15\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{20\sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int \frac{15\sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{6\sec^6(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int \frac{\sec^7(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**6*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*sec(e + f*x)**3/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-20*sec(e +
f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + In
tegral(15*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-6*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e + f*
x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**7/(sec(e + f*x)**3
 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.65, size = 176, normalized size = 0.82

$$-\frac{\frac{3465\,c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{3465\,c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{10\left(267\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-472\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+213\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^3}-\frac{32\left(3\,a^{12}c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+20\,a^{12}c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+150\,a^{12}c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{30\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(3465*c^6*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 3465*c^6*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/a^3 + 10*(267*c^6*tan(1/2*f*x + 1/2*e)^5 - 472*c^
6*tan(1/2*f*x + 1/2*e)^3 + 213*c^6*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/
2*e)^2 - 1)^3*a^3) - 32*(3*a^12*c^6*tan(1/2*f*x + 1/2*e)^5 + 20*a^12*c^6*ta
n(1/2*f*x + 1/2*e)^3 + 150*a^12*c^6*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.65, size = 193, normalized size = 0.90

$$\frac{160\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}-\frac{89\,c^6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5-\frac{472\,c^6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{3}+71\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-a^3\right)}+\frac{64\,c^6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{3\,a^3\,f}+\frac{16\,c^6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{5\,a^3\,f}-\frac{231\,c^6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^6/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)

[Out] (160*c^6*tan(e/2 + (f*x)/2))/(a^3*f) - (89*c^6*tan(e/2 + (f*x)/2)^5 - (472*
c^6*tan(e/2 + (f*x)/2)^3)/3 + 71*c^6*tan(e/2 + (f*x)/2))/(f*(3*a^3*tan(e/2
+ (f*x)/2)^2 - 3*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^6 - a^3)
) + (64*c^6*tan(e/2 + (f*x)/2)^3)/(3*a^3*f) + (16*c^6*tan(e/2 + (f*x)/2)^5)
/(5*a^3*f) - (231*c^6*atanh(tan(e/2 + (f*x)/2)))/(a^3*f)
```

3.53 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=193

$$-\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f}+\frac{42c^5\tan(e+fx)}{a^3f}-\frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f}-\frac{6c^2(c-c\sec(e+fx))^3\tan(e}{5af(a+a\sec(e+fx))^2}$$

```
[Out] -63/2*c^5*arctanh(sin(f*x+e))/a^3/f+42*c^5*tan(f*x+e)/a^3/f-21/2*c^5*sec(f*
x+e)*tan(f*x+e)/a^3/f-6/5*c^2*(c-c*sec(f*x+e))^3*tan(f*x+e)/a/f/(a+a*sec(f*
x+e))^2+2/5*c*(c-c*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+42/5*c*(c^
2-c^2*sec(f*x+e))^2*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A]
time = 0.21, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4042, 3873, 3852, 8, 4131, 3855}

$$\frac{42c^5\tan(e+fx)}{a^3f}-\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{21c^5\tan(e+fx)\sec(e+fx)}{2a^3f}+\frac{42c\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{5f\left(a^3\sec(e+fx)+a^3\right)}-\frac{6c^2\tan(e+fx)(c-c\sec(e+fx))^3}{5af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-63*c^5*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (42*c^5*Tan[e + f*x])/(a^3*f) -
 (21*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^3*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e + f*x])
^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (42*c*(c^2 - c^2*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx &= \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{(9c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,d}{5a}\\
&=-\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2}+\frac{2c(c-c\sec(e+fx))^4\tan}{5f(a+a\sec(e+fx}\\
&=\frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}-\frac{6c^2(c-c\sec(e+fx))^3\tan}{5af(a+a\sec(e+fx}\\
&=\frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}-\frac{6c^2(c-c\sec(e+fx))^3\tan}{5af(a+a\sec(e+fx}\\
&=-\frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f}+\frac{42c^3(c-c\sec(e+fx))^2\tan(e+}{5f\left(a^3+a^3\sec(e+fx)\right)}\\
&=-\frac{63c^5\tanh^{-1}(\sin(e+fx))}{2a^3f}+\frac{42c^5\tan(e+fx)}{a^3f}-\frac{21c^5\sec(e+fx)}{2a^3}\end{aligned}$$

Mathematica [A]
time = 1.31, size = 380, normalized size = 1.97

$$\frac{\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)(c-c\sec(e+fx))^5\left(-40320\cos^2(e+fx)\cot^5\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\csc^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sec(e)\left(3465\sin\left(\frac{fx}{2}\right)-6115\sin\left(\frac{3fx}{2}\right)+7351\sin\left(e-\frac{fx}{2}\right)-5271\sin\left(e+\frac{fx}{2}\right)+5545\sin\left(2e+\frac{fx}{2}\right)+2205\sin\left(e+\frac{3fx}{2}\right)-4515\sin\left(2e+\frac{3fx}{2}\right)+3805\sin\left(3e+\frac{3fx}{2}\right)-4407\sin\left(e+\frac{5fx}{2}\right)+585\sin\left(2e+\frac{5fx}{2}\right)-3447\sin\left(3e+\frac{5fx}{2}\right)+1545\sin\left(4e+\frac{5fx}{2}\right)-2155\sin\left(2e+\frac{7fx}{2}\right)-75\sin\left(3e+\frac{7fx}{2}\right)-1755\sin\left(4e+\frac{7fx}{2}\right)+325\sin\left(5e+\frac{7fx}{2}\right)-496\sin\left(3e+\frac{9fx}{2}\right)-80\sin\left(4e+\frac{9fx}{2}\right)-416\sin\left(5e+\frac{9fx}{2}\right)\right)\right)}{5120a^3f(1+\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4*(c - c*Sec[e + f*x])^5*(-40320*Cos[e +
 f*x]^2*Cot[(e + f*x)/2]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + Csc[(e + f*x)/2]^5*Sec[e/2]*Sec[e]*
(3465*Sin[(f*x)/2] - 6115*Sin[(3*f*x)/2] + 7351*Sin[e - (f*x)/2] - 5271*Sin
[e + (f*x)/2] + 5545*Sin[2*e + (f*x)/2] + 2205*Sin[e + (3*f*x)/2] - 4515*Si
n[2*e + (3*f*x)/2] + 3805*Sin[3*e + (3*f*x)/2] - 4407*Sin[e + (5*f*x)/2] +
585*Sin[2*e + (5*f*x)/2] - 3447*Sin[3*e + (5*f*x)/2] + 1545*Sin[4*e + (5*f*
x)/2] - 2155*Sin[2*e + (7*f*x)/2] - 75*Sin[3*e + (7*f*x)/2] - 1755*Sin[4*e
+ (7*f*x)/2] + 325*Sin[5*e + (7*f*x)/2] - 496*Sin[3*e + (9*f*x)/2] - 80*Sin
[4*e + (9*f*x)/2] - 416*Sin[5*e + (9*f*x)/2])))/(5120*a^3*f*(1 + Sec[e + f*
x])^3)
```

Maple [A]
time = 0.22, size = 136, normalized size = 0.70

method	result
derivativedivides	$\frac{8c^5\left(\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{17}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{63\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}-\right.}{f\,a^3}$
default	$\frac{8c^5\left(\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{17}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{63\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16}-\right.}{f\,a^3}$
risch	$\frac{ic^5\left(325\,e^{8i(fx+e)}+1545\,e^{7i(fx+e)}+3805\,e^{6i(fx+e)}+5545\,e^{5i(fx+e)}+7351\,e^{4i(fx+e)}+6115\,e^{3i(fx+e)}+4407\,e^{2i(fx+e)}+215\right.}{5f\,a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{2i(fx+e)}+1\right)^2}$
norman	$\frac{\frac{8c^5\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}-\frac{63c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{294c^5\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2688c^5\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}+\frac{474c^5\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{193c^5\left(\tan^9\left(\right.\right.}{af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*c^5/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+tan(1/2*f*x+1/2*e)^3+6*tan(1/2*f*x+1/
2*e)+1/16/(tan(1/2*f*x+1/2*e)+1)^2-17/16/(tan(1/2*f*x+1/2*e)+1)-63/16*ln(ta
n(1/2*f*x+1/2*e)+1)-1/16/(tan(1/2*f*x+1/2*e)-1)^2-17/16/(tan(1/2*f*x+1/2*e)
-1)+63/16*ln(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(199) = 398$.
time = 0.30, size = 738, normalized size = 3.82

$$\frac{c^5\left(\frac{60\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^3-\frac{2a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{465\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+15c^5\left(\frac{40\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+10c^5\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+\frac{10c^5\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^5\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{15c^5\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^5*(40*sin(f*x + e)/((a^3 - a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^
3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*((105*sin(f*x
 + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*(15*
sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^5*(15*sin(f*x + e)/(cos(f*x
+ e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5)/a^3 - 15*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 2.23, size = 269, normalized size = 1.39

$$-\frac{315\left(c^5\cos\left(fx+e\right)^5+3\,c^5\cos\left(fx+e\right)^4+3\,c^5\cos\left(fx+e\right)^3+c^5\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-315\left(c^5\cos\left(fx+e\right)^5+3\,c^5\cos\left(fx+e\right)^4+3\,c^5\cos\left(fx+e\right)^3+c^5\cos\left(fx+e\right)^2\right)\log\left(-\sin\left(fx+e\right)+1\right)-2\left(496\,c^5\cos\left(fx+e\right)^4+1163\,c^5\cos\left(fx+e\right)^3+801\,c^5\cos\left(fx+e\right)^2+65\,c^5\cos\left(fx+e\right)-5\,c^5\right)\sin\left(fx+e\right)}{20\left(a^3f\cos\left(fx+e\right)^5+3\,a^3f\cos\left(fx+e\right)^4+3\,a^3f\cos\left(fx+e\right)^3+a^3f\cos\left(fx+e\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/20*(315*(c^5*cos(f*x + e)^5 + 3*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^
3 + c^5*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 3
*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f
*x + e) + 1) - 2*(496*c^5*cos(f*x + e)^4 + 1163*c^5*cos(f*x + e)^3 + 801*c^
5*cos(f*x + e)^2 + 65*c^5*cos(f*x + e) - 5*c^5)*sin(f*x + e))/(a^3*f*cos(f*
x + e)^5 + 3*a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x
+ e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^5\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{5\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{10\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{10\sec^4\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{5\sec^5\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^6\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

[Out] -c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(5*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**3/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e +
f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + I
ntegral(-5*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e + f*x)
2 + 3*sec(e + f*x) + 1), x))/a3

Giac [A]
time = 0.66, size = 159, normalized size = 0.82

$$-\frac{\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{10\,\left(17\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^3}-\frac{16\,\left(a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")

[Out] -1/10*(315*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 315*c^5*log(abs(tan
(1/2*f*x + 1/2*e) - 1))/a^3 + 10*(17*c^5*tan(1/2*f*x + 1/2*e)^3 - 15*c^5*ta
n(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^3) - 16*(a^12*c^5*tan
(1/2*f*x + 1/2*e)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*tan(1
/2*f*x + 1/2*e))/a^15)/f

Mupad [B]
time = 1.64, size = 159, normalized size = 0.82

$$\frac{48\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}-\frac{17\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-15\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^3\right)}+\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{a^3\,f}+\frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}-\frac{63\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - c/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)

[Out] (48*c^5*tan(e/2 + (f*x)/2))/(a^3*f) - (17*c^5*tan(e/2 + (f*x)/2)^3 - 15*c^5
*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (f*x)/2)^4 - 2*a^3*tan(e/2 + (f*x)/2
)^2 + a^3)) + (8*c^5*tan(e/2 + (f*x)/2)^3)/(a^3*f) + (8*c^5*tan(e/2 + (f*x)
/2)^5)/(5*a^3*f) - (63*c^5*atanh(tan(e/2 + (f*x)/2)))/(a^3*f)

3.54 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=164

$$-\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{7c^4\tan(e+fx)}{a^3f}+\frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{14(c^2-c^2\sec(e+fx))^2}{15af(a+a\sec(e+}$$

```
[Out] -7*c^4*arctanh(sin(f*x+e))/a^3/f+7*c^4*tan(f*x+e)/a^3/f+2/5*c*(c-c*sec(f*x+
e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^3-14/15*(c^2-c^2*sec(f*x+e))^2*tan(f*x+
e)/a/f/(a+a*sec(f*x+e))^2+14/3*(c^4-c^4*sec(f*x+e))*tan(f*x+e)/f/(a^3+a^3*s
ec(f*x+e))
```

Rubi [A]

time = 0.19, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = $\{4042, 3872, 3855, 3852, 8\}$

$$\frac{7c^4\tan(e+fx)}{a^3f}-\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{14\tan(e+fx)\left(c^4-c^4\sec(e+fx)\right)}{3f\left(a^3\sec(e+fx)+a^3\right)}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{15af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-7*c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) + (7*c^4*Tan[e + f*x])/(a^3*f) + (2*
c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (14*(
c^2 - c^2*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (
14*(c^4 - c^4*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(7c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e}{15af(a+a\sec(e+fx))}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e}{15af(a+a\sec(e+fx))}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e}{15af(a+a\sec(e+fx))}$$

$$= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - $$

$$= -\frac{7c^4\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{7c^4\tan(e+fx)}{a^3f} + \frac{2c(c-c\sec(e+fx}{5f(a+a\sec(e}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 826 vs. 2(164) = 328.
time = 6.33, size = 826, normalized size = 5.04

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (7*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*Log[Cos[e/2 + (f*
x)/2] - Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec[e + f*x
])^3) - (7*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*Log[Cos[e
```

```
/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c - c*Sec[e + f*x])^4)/(2*f*(a + a*Sec
[e + f*x])^3) + (76*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^5*Csc[e/2 + (f*x)/2]^3*
Sec[e/2]*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(15*f*(a + a*Sec[e + f*x])^3)
 + (8*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^3*Csc[e/2 + (f*x)/2]^5*Sec[e/2]*(c -
c*Sec[e + f*x])^4*Sin[(f*x)/2])/(15*f*(a + a*Sec[e + f*x])^3) + (2*Cos[e +
f*x]*Cot[e/2 + (f*x)/2]*Csc[e/2 + (f*x)/2]^7*Sec[e/2]*(c - c*Sec[e + f*x])^
4*Sin[(f*x)/2])/(5*f*(a + a*Sec[e + f*x])^3) + (Cos[e + f*x]*Cot[e/2 + (f*x
)/2]^6*Csc[e/2 + (f*x)/2]^2*(c - c*Sec[e + f*x])^4*Sin[(f*x)/2])/(2*f*(a +
a*Sec[e + f*x])^3*(Cos[e/2] - Sin[e/2])*(Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*
x)/2])) + (Cos[e + f*x]*Cot[e/2 + (f*x)/2]^6*Csc[e/2 + (f*x)/2]^2*(c - c*Se
c[e + f*x])^4*Sin[(f*x)/2])/(2*f*(a + a*Sec[e + f*x])^3*(Cos[e/2] + Sin[e/2
])*(Cos[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2])) + (8*Cos[e + f*x]*Cot[e/2 + (
f*x)/2]^4*Csc[e/2 + (f*x)/2]^4*(c - c*Sec[e + f*x])^4*Tan[e/2])/(15*f*(a +
a*Sec[e + f*x])^3) + (2*Cos[e + f*x]*Cot[e/2 + (f*x)/2]^2*Csc[e/2 + (f*x)/2
]^6*(c - c*Sec[e + f*x])^4*Tan[e/2])/(5*f*(a + a*Sec[e + f*x])^3)
```

Maple [A]
time = 0.20, size = 108, normalized size = 0.66

method	result
derivativedivides	$\frac{4c^4\left(\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{7\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\cdots}{f\,a^3}$
default	$\frac{4c^4\left(\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{7\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\cdots}{f\,a^3}$
risch	$\frac{2ic^4\left(120\,e^{6i(fx+e)}+495\,e^{5i(fx+e)}+1235\,e^{4i(fx+e)}+1270\,e^{3i(fx+e)}+1342\,e^{2i(fx+e)}+715\,e^{i(fx+e)}+167\right)}{15f\,a^3\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}+1\right)^5}+\frac{7c^4\ln\left(e^{i(f\cdots}{a^3 f\cdots}$
norman	$\frac{\frac{14c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{154c^4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{1022c^4\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15af}-\frac{186c^4\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}+\frac{92c^4\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15af}-\frac{8c^4\left(\tan^{11}\cdots\right)}{15\cdots}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 4/f*c^4/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+2/3*tan(1/2*f*x+1/2*e)^3+3*tan(1/2*f*
x+1/2*e)-1/4/(tan(1/2*f*x+1/2*e)-1)+7/4*ln(tan(1/2*f*x+1/2*e)-1)-1/4/(tan(1
/2*f*x+1/2*e)+1)-7/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(172) = 344.
time = 0.29, size = 510, normalized size = 3.11

$$\frac{3\,c^4\left(\frac{40\,\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+4\,c^4\left(\frac{\frac{105\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+\frac{6\,c^4\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^4\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{12\,c^4\left(\frac{5\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*c^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^3) + 4*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^3) + 6*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) +
10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 12*c^4*(5*s
in(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/
f
```

Fricas [A]
time = 2.39, size = 249, normalized size = 1.52

$$-\frac{105\left(c^4\cos\left(fx+e\right)^4+3\,c^4\cos\left(fx+e\right)^3+3\,c^4\cos\left(fx+e\right)^2+c^4\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4+3\,c^4\cos\left(fx+e\right)^3+3\,c^4\cos\left(fx+e\right)^2+c^4\cos\left(fx+e\right)\right)\log\left(-\sin\left(fx+e\right)+1\right)-2\left(167\,c^4\cos\left(fx+e\right)^3+381\,c^4\cos\left(fx+e\right)^2+277\,c^4\cos\left(fx+e\right)+15\,c^4\right)\sin\left(fx+e\right)}{30\left(a^3f\cos\left(fx+e\right)^4+3\,a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+a^3f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(105*(c^4*cos(f*x + e)^4 + 3*c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^
2 + c^4*cos(f*x + e))*log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 3*c
^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*log(-sin(f*x +
 e) + 1) - 2*(167*c^4*cos(f*x + e)^3 + 381*c^4*cos(f*x + e)^2 + 277*c^4*cos
(f*x + e) + 15*c^4)*sin(f*x + e))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x +
 e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^4\left(\int\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{6\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^4\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^5\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-4*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x
)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*
x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
```

```
gral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1), x))/a**3
```

Giac [A]
time = 0.61, size = 141, normalized size = 0.86

$$-\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{30\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3}-\frac{4\left(3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+10\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+45\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/15*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 105*c^4*log(abs(tan
(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^4*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1
/2*e)^2 - 1)*a^3) - 4*(3*a^12*c^4*tan(1/2*f*x + 1/2*e)^5 + 10*a^12*c^4*tan(
1/2*f*x + 1/2*e)^3 + 45*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.63, size = 126, normalized size = 0.77

$$\frac{12\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}+\frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f}+\frac{4\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}-\frac{14\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}-\frac{2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (12*c^4*tan(e/2 + (f*x)/2))/(a^3*f) + (8*c^4*tan(e/2 + (f*x)/2)^3)/(3*a^3*f
) + (4*c^4*tan(e/2 + (f*x)/2)^5)/(5*a^3*f) - (14*c^4*atanh(tan(e/2 + (f*x)/
2)))/(a^3*f) - (2*c^4*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (f*x)/2)^2 - a^
3))
```

3.55 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{2c^3\tan(e+fx)}{f\left(a^3+a^3\sec(e+fx)\right)}+\frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{2(c^3-c^3\sec(e+fx}{3af(a+a\sec(}$$

```
[Out] -c^3*arctanh(sin(f*x+e))/a^3/f+2*c^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+2/5*
c*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3-2/3*(c^3-c^3*sec(f*x+e
))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4042, 3855}

$$-\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{2c^3\tan(e+fx)}{f\left(a^3\sec(e+fx)+a^3\right)}-\frac{2\tan(e+fx)\left(c^3-c^3\sec(e+fx)\right)}{3af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*ArcTanh[Sin[e + f*x]])/(a^3*f)) + (2*c^3*Tan[e + f*x])/(f*(a^3 + a^3
*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec
[e + f*x])^3) - (2*(c^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*a*f*(a + a*Sec
[e + f*x])^2)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx}{a}$$
$$= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{2(c^3-c^3\sec(e+fx))\tan(e}{3af(a+a\sec(e+fx))}$$
$$= \frac{2c^3\tan(e+fx)}{f\,(a^3+a^3\sec(e+fx))} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{2(}{}$$
$$= -\frac{c^3\tanh^{-1}(\sin(e+fx))}{a^3f} + \frac{2c^3\tan(e+fx)}{f\,(a^3+a^3\sec(e+fx))} + \frac{2c(c-c\sec(}{5f(a+}$$

Mathematica [A]
time = 0.12, size = 139, normalized size = 1.06

$$-\frac{c^3\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{26\tan\left(\frac{1}{2}(e+fx)\right)}{15f}+\frac{2\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{15f}-\frac{2\sec^4\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{5f}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
[Out] -((c^3*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]]/f - (26*Tan[(e + f*x)/2])/(15*f) + (2*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/(15*f) - (2*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])/(5*f
)))/a^3)
```

Maple [A]
time = 0.20, size = 76, normalized size = 0.58

method	result
derivativedivides	$\frac{2c^3\left(\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^3}$
default	$\frac{2c^3\left(\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^3}$
risch	$\frac{4ic^3\left(15\,e^{4i(fx+e)}+30\,e^{3i(fx+e)}+100\,e^{2i(fx+e)}+50\,e^{i(fx+e)}+13\right)}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}+\frac{c^3\ln\left(e^{i(fx+e)}-i\right)}{a^3f}-\frac{c^3\ln\left(e^{i(fx+e)}+i\right)}{a^3f}$
norman	$\frac{\frac{16c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}-\frac{22c^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}+\frac{6c^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}-\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{8c^3\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15af}+\frac{2c^3\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 2/f*c^3/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+1/3*tan(1/2*f*x+1/2*e)^3+tan(1/2*f*x+
1/2*e)+1/2*ln(tan(1/2*f*x+1/2*e)-1)-1/2*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(138) = 276$.
time = 0.28, size = 330, normalized size = 2.52

$$\frac{c^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+\frac{3c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{9c^3\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x
 + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
- 1)/a^3) + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^3*(15*
sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^3*(5*sin(f*x + e)/(cos(f*x
 + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 3.27, size = 206, normalized size = 1.57

$$-\frac{15\left(c^3\cos\left(fx+e\right)^3+3c^3\cos\left(fx+e\right)^2+3c^3\cos\left(fx+e\right)+c^3\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+3c^3\cos\left(fx+e\right)^2+3c^3\cos\left(fx+e\right)+c^3\right)\log\left(-\sin\left(fx+e\right)+1\right)-4\left(13c^3\cos\left(fx+e\right)^2+24c^3\cos\left(fx+e\right)+23c^3\right)\sin\left(fx+e\right)}{30\left(a^3f\cos\left(fx+e\right)^3+3a^3f\cos\left(fx+e\right)^2+3a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) +
 c^3)*log(sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2
 + 3*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 4*(13*c^3*cos(f*x + e
)^2 + 24*c^3*cos(f*x + e) + 23*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3
*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3\left(\int\left(-\frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{3\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}dx+\int\left(-\frac{3\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{\sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f
*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x
)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.58, size = 109, normalized size = 0.83

$$-\frac{\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{2\left(3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^{15}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/15*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 15*c^3*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 + 5*a^12*c
^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.65, size = 61, normalized size = 0.47

$$\frac{2\,c^3\,\left(15\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-15\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\right)}{15\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (2*c^3*(15*tan(e/2 + (f*x)/2) - 15*atanh(tan(e/2 + (f*x)/2)) + 5*tan(e/2 +
(f*x)/2)^3 + 3*tan(e/2 + (f*x)/2)^5))/(15*a^3*f)
```

3.56 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=38

$$\frac{(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

```
[Out] 1/5*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{4035\}$

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

Mathematica [A]
time = 0.11, size = 25, normalized size = 0.66

$$\frac{c^2\tan^5\left(\frac{1}{2}(e+fx)\right)}{5a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (c^2*Tan[(e + f*x)/2]^5)/(5*a^3*f)
```

Maple [A]
time = 0.17, size = 23, normalized size = 0.61

method	result	size
derivativedivides	$\frac{c^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5f\,a^{3}}$	23
default	$\frac{c^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5f\,a^{3}}$	23
risch	$\frac{2ic^{2}\left(5\,\mathrm{e}^{4i(fx+e)}+10\,\mathrm{e}^{2i(fx+e)}+1\right)}{5f\,a^{3}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}}$	50
norman	$\frac{\frac{c^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}-\frac{2c^{2}\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}+\frac{c^{2}\left(\tan^{9}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5af}}{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}a^{2}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/5/f*c^2/a^3*tan(1/2*f*x+1/2*e)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(40) = 80$.
time = 0.28, size = 201, normalized size = 5.29

$$\frac{\frac{c^{2}\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\,\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}}+\frac{c^{2}\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\,\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{3\,\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}}-\frac{6\,c^{2}\left(\frac{5\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*x
+ e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6*c^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(40) = 80$.
time = 2.39, size = 88, normalized size = 2.32

$$\frac{\left(c^{2}\cos\left(fx+e\right)^{2}-2\,c^{2}\cos\left(fx+e\right)+c^{2}\right)\sin\left(fx+e\right)}{5\,\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/5*(c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) + c^2)*sin(f*x + e)/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{c^2\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{2\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int \frac{\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-2*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)*
*3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.65, size = 22, normalized size = 0.58

$$\frac{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}{5\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/5*c^2*tan(1/2*f*x + 1/2*e)^5/(a^3*f)
```

Mupad [B]
time = 1.59, size = 22, normalized size = 0.58

$$\frac{c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c^2*tan(e/2 + (f*x)/2)^5)/(5*a^3*f)
```

3.57 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3} dx$

Optimal. Leaf size=76

$$\frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(c-c\sec(e+fx))\tan(e+fx)}{15af(a+a\sec(e+fx))^2}$$

```
[Out] 1/5*(c-c*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(c-c*sec(f*x+e))*
tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4036, 4035}

$$\frac{\tan(e+fx)(c-c\sec(e+fx))}{15af(a\sec(e+fx)+a)^2}+\frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - c*
Sec[e + f*x])*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx}{5a}$$
$$= \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-c\sec(e+fx))\tan(e+fx)}{15af(a+a\sec(e+fx))^2}$$

Mathematica [A]
time = 0.35, size = 87, normalized size = 1.14

$$\frac{c\sec\left(\frac{e}{2}\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\left(25\sin\left(\frac{fx}{2}\right)-15\sin\left(e+\frac{fx}{2}\right)+5\sin\left(e+\frac{3fx}{2}\right)-15\sin\left(2e+\frac{3fx}{2}\right)+4\sin\left(2e+\frac{5fx}{2}\right)\right)}{240a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
[Out] (c*Sec[e/2]*Sec[(e + f*x)/2]^5*(25*Sin[(f*x)/2] - 15*Sin[e + (f*x)/2] + 5*S
in[e + (3*f*x)/2] - 15*Sin[2*e + (3*f*x)/2] + 4*Sin[2*e + (5*f*x)/2]))/(240
*a^3*f)
```

Maple [A]
time = 0.18, size = 37, normalized size = 0.49

method	result	size
derivativedivides	$\frac{c\left(\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}\right)}{2f\,a^3}$	37
default	$\frac{c\left(\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}\right)}{2f\,a^3}$	37
risch	$\frac{2ic\left(15\,e^{4i(fx+e)}+15\,e^{3i(fx+e)}+25\,e^{2i(fx+e)}+5\,e^{i(fx+e)}+4\right)}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}$	70
norman	$\frac{\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}-\frac{4c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15af}+\frac{c\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)a^2}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/2/f*c/a^3*(1/5*tan(1/2*f*x+1/2*e)^5-1/3*tan(1/2*f*x+1/2*e)^3)
```

Maxima [A]
time = 0.29, size = 125, normalized size = 1.64

$$\frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{3c\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 2.57, size = 85, normalized size = 1.12

$$\frac{\left(4\,c\cos\left(fx+e\right)^{2}-3\,c\cos\left(fx+e\right)-c\right)\sin\left(fx+e\right)}{15\,\left(a^{3}f\,\cos\left(fx+e\right)^{3}+3\,a^{3}f\,\cos\left(fx+e\right)^{2}+3\,a^{3}f\,\cos\left(fx+e\right)+a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*c*cos(f*x + e)^2 - 3*c*cos(f*x + e) - c)*sin(f*x + e)/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.57, size = 37, normalized size = 0.49

$$\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}}{30\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 - 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

Mupad [B]
time = 1.58, size = 35, normalized size = 0.46

$$\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-5\right)}{30\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
[Out] (c*tan(e/2 + (f*x)/2)^3*(3*tan(e/2 + (f*x)/2)^2 - 5))/(30*a^3*f)
```

3.58 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=78

$$-\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{\csc(e+fx)}{a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}$$

```
[Out] -2/5*cot(f*x+e)^5/a^3/c/f+csc(f*x+e)/a^3/c/f-csc(f*x+e)^3/a^3/c/f+2/5*csc(f
*x+e)^5/a^3/c/f
```

Rubi [A]
time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {4043, 2686, 200, 2687, 30, 14}

$$-\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{\csc(e+fx)}{a^3cf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] (-2*Cot[e + f*x]^5)/(5*a^3*c*f) + Csc[e + f*x]/(a^3*c*f) - Csc[e + f*x]^3/(
a^3*c*f) + (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \left(c^2\cot^5(e+fx)\csc(e+fx)-2c^2\cot^4(e+fx)\csc^2(e+\right.}{a^3c^3}$$

$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c} - \frac{\int \cot^3(e+fx)\csc^3(e+f}{a^3c}$$

$$= \frac{\operatorname{Subst}\left(\int x^2(-1+x^2)\,dx, x, \csc(e+fx)\right)}{a^3cf} + \frac{\operatorname{Subst}\Big(\int (-1+}{}$$

$$= -\frac{2\cot^5(e+fx)}{5a^3cf} + \frac{\operatorname{Subst}\left(\int (1-2x^2+x^4)\,dx, x, \csc(e+fx\right.}{a^3cf}$$

$$= -\frac{2\cot^5(e+fx)}{5a^3cf} + \frac{\csc(e+fx)}{a^3cf} - \frac{\csc^3(e+fx)}{a^3cf} + \frac{2\csc^5(e+}{5a^3cf}$$

Mathematica [**A**]
time = 0.80, size = 109, normalized size = 1.40

$$-\frac{\csc(e)\csc^5(e+fx)\sin^4\left(\frac{1}{2}(e+fx)\right)(-40\sin(e)+65\sin(e+fx)+52\sin(2(e+fx))+13\sin(3(e+fx))-40\sin(2e+fx)-12\sin(e+2fx)-20\sin(3e+2fx)-8\sin(2e+3fx))}{20a^3cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/20*(Csc[e]*Csc[e + f*x]^5*Sin[(e + f*x)/2]^4*(-40*Sin[e] + 65*Sin[e + f*
x] + 52*Sin[2*(e + f*x)] + 13*Sin[3*(e + f*x)] - 40*Sin[2*e + f*x] - 12*Sin
[e + 2*f*x] - 20*Sin[3*e + 2*f*x] - 8*Sin[2*e + 3*f*x]))/(a^3*c*f)
```

Maple [**A**]
time = 0.15, size = 61, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8f\,a^3c}$	61
default	$\frac{\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8f\,a^3c}$	61
risch	$\frac{2i\left(5\,\mathrm{e}^{5i(fx+e)}+10\,\mathrm{e}^{4i(fx+e)}+10\,\mathrm{e}^{3i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}-2\right)}{5f\,a^3c\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(\mathrm{e}^{i(fx+e)}-1\right)}$	85
norman	$\frac{\frac{1}{8acf}+\frac{3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}-\frac{\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)}{8acf}+\frac{\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)}{40acf}}{a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/a^3/c*(1/5*tan(1/2*f*x+1/2*e)^5-tan(1/2*f*x+1/2*e)^3+3*tan(1/2*f*x+1/
2*e)+1/tan(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.28, size = 103, normalized size = 1.32

$$\frac{\frac{\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{5\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}}{a^3c}+\frac{5\left(\cos\left(fx+e\right)+1\right)}{a^3c\sin\left(fx+e\right)}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/40*((15*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) + 5*(cos(f*x + e) + 1
)/(a^3*c*sin(f*x + e)))/f
```

Fricas [A]
time = 2.69, size = 82, normalized size = 1.05

$$-\frac{2\,\cos\left(fx+e\right)^3-\cos\left(fx+e\right)^2-4\,\cos\left(fx+e\right)-2}{5\left(a^3cf\cos\left(fx+e\right)^2+2\,a^3cf\cos\left(fx+e\right)+a^3cf\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/5*(2*cos(f*x + e)^3 - cos(f*x + e)^2 - 4*cos(f*x + e) - 2)/((a^3*c*f*cos
(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec(e+fx)}{\sec^4(e+fx)+2\sec^3(e+fx)-2\sec(e+fx)-1}\,dx}{a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x
) - 1), x)/(a**3*c)
```

Giac [A]
time = 0.53, size = 87, normalized size = 1.12

$$\frac{\frac{5}{a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}+\frac{a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-5\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}c^5}}{40\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/40*(5/(a^3*c*tan(1/2*f*x + 1/2*e)) + (a^12*c^4*tan(1/2*f*x + 1/2*e)^5 - 5
*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e))/(a^15*
c^5))/f
```

Mupad [B]
time = 1.62, size = 74, normalized size = 0.95

$$-\frac{16\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-28\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+8\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}{40\,a^3\,c\,f\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)
```

```
[Out] -(8*cos(e/2 + (f*x)/2)^2 - 28*cos(e/2 + (f*x)/2)^4 + 16*cos(e/2 + (f*x)/2)^
6 - 1)/(40*a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2))
```

3.59 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=80

$$-\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}$$

```
[Out] -1/5*cot(f*x+e)^5/a^3/c^2/f+csc(f*x+e)/a^3/c^2/f-2/3*csc(f*x+e)^3/a^3/c^2/f
+1/5*csc(f*x+e)^5/a^3/c^2/f
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4043, 2686, 200, 2687, 30}

$$-\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -1/5*Cot[e + f*x]^5/(a^3*c^2*f) + Csc[e + f*x]/(a^3*c^2*f) - (2*Csc[e + f*x
]^3)/(3*a^3*c^2*f) + Csc[e + f*x]^5/(5*a^3*c^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx &= -\frac{\int \left(c\cot^5(e+fx)\csc(e+fx)-c\cot^4(e+fx)\csc^2(e+f\cdots\right.}{a^3c^3} \\
&= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c^2}+\frac{\int \cot^4(e+fx)\csc^2(e+\cdots}{a^3c^2} \\
&= \frac{\operatorname{Subst}\left(\int x^4\,dx,x,-\cot(e+fx)\right)}{a^3c^2f}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx\cdots\right.}{a^3c^2f} \\
&= -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(e+fx\cdots\right.}{a^3c^2f} \\
&= -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc^5(e+\cdots}{5a^3c^2\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.80, size = 147, normalized size = 1.84

$$\frac{\csc(e)\csc^5(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)(200\sin(e)+104\sin(fx)-534\sin(e+fx)-178\sin(2(e+fx))+178\sin(3(e+fx))+89\sin(4(e+fx))+40\sin(2e+fx)+168\sin(e+2fx)-120\sin(3e+2fx)+72\sin(2e+3fx)-120\sin(4e+3fx)-24\sin(3e+4fx))}{480a^3c^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] (Csc[e]*Csc[e + f*x]^5*Sin[(e + f*x)/2]^2*(200*Sin[e] + 104*Sin[f*x] - 534*
Sin[e + f*x] - 178*Sin[2*(e + f*x)] + 178*Sin[3*(e + f*x)] + 89*Sin[4*(e +
f*x)] + 40*Sin[2*e + f*x] + 168*Sin[e + 2*f*x] - 120*Sin[3*e + 2*f*x] + 72*
Sin[2*e + 3*f*x] - 120*Sin[4*e + 3*f*x] - 24*Sin[3*e + 4*f*x]))/(480*a^3*c^
2*f)
```

Maple [A]
time = 0.17, size = 76, normalized size = 0.95

---

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^2a^3}$	76
default	$\frac{\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{4\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,c^2a^3}$	76
risch	$\frac{2i\left(15\,e^{7i(fx+e)}+15\,e^{6i(fx+e)}-5\,e^{5i(fx+e)}-25\,e^{4i(fx+e)}+13\,e^{3i(fx+e)}+21\,e^{2i(fx+e)}+9\,e^{i(fx+e)}-3\right)}{15f\,c^2a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{i(fx+e)}-1\right)^3}$	118
norman	$\frac{-\frac{1}{48acf}+\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{4acf}+\frac{3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8acf}-\frac{\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)}{12acf}+\frac{\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)}{80acf}}{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/16/f/c^2/a^3*(1/5*tan(1/2*f*x+1/2*e)^5-4/3*tan(1/2*f*x+1/2*e)^3+6*tan(1/2
*f*x+1/2*e)-1/3/tan(1/2*f*x+1/2*e)^3+4/tan(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.29, size = 130, normalized size = 1.62

$$\frac{\frac{\frac{90\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c^2}+\frac{5\left(\frac{12\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{a^3c^2\sin(fx+e)^3}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*((90*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) + 5*(12*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e
)^3))/f
```

Fricas [A]
time = 4.18, size = 117, normalized size = 1.46

$$-\frac{3\,\cos\left(fx+e\right)^4-12\,\cos\left(fx+e\right)^3-12\,\cos\left(fx+e\right)^2+8\,\cos\left(fx+e\right)+8}{15\left(a^3c^2f\cos\left(fx+e\right)^3+a^3c^2f\cos\left(fx+e\right)^2-a^3c^2f\cos\left(fx+e\right)-a^3c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/15*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 + 8*cos(f*x
 + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*
f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec{(e+fx)}}{\sec^5{(e+fx)}+\sec^4{(e+fx)}-2\sec^3{(e+fx)}-2\sec^2{(e+fx)}+\sec{(e+fx)}+1}\,dx}{a^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**
3 - 2*sec(e + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

Giac [A]
time = 0.64, size = 103, normalized size = 1.29

$$\frac{\frac{5\left(12\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}{a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}+\frac{3a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-20a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+90a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^{10}}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/240*(5*(12*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2*tan(1/2*f*x + 1/2*e)^3) +
 (3*a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 20*a^12*c^8*tan(1/2*f*x + 1/2*e)^3 +
90*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f
```

Mupad [B]
time = 1.71, size = 111, normalized size = 1.39

$$\frac{48\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-192\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+168\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-32\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3}{240\,a^3\,c^2\,f\,\left(\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (168*cos(e/2 + (f*x)/2)^4 - 32*cos(e/2 + (f*x)/2)^2 - 192*cos(e/2 + (f*x)/2
)^6 + 48*cos(e/2 + (f*x)/2)^8 + 3)/(240*a^3*c^2*f*(cos(e/2 + (f*x)/2)^5*sin
(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)^7*sin(e/2 + (f*x)/2)))
```

3.60 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal. Leaf size=59

$$\frac{\csc(e+fx)}{a^3c^3f}-\frac{2\csc^3(e+fx)}{3a^3c^3f}+\frac{\csc^5(e+fx)}{5a^3c^3f}$$

```
[Out] csc(f*x+e)/a^3/c^3/f-2/3*csc(f*x+e)^3/a^3/c^3/f+1/5*csc(f*x+e)^5/a^3/c^3/f
```

Rubi [A]
time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4043, 2686, 200}

$$\frac{\csc^5(e+fx)}{5a^3c^3f}-\frac{2\csc^3(e+fx)}{3a^3c^3f}+\frac{\csc(e+fx)}{a^3c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Csc[e + f*x]/(a^3*c^3*f) - (2*Csc[e + f*x]^3)/(3*a^3*c^3*f) + Csc[e + f*x]^
5/(5*a^3*c^3*f)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c^3}$$
$$= \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{a^3c^3f}$$
$$= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(e+fx)\right)}{a^3c^3f}$$
$$= \frac{\csc(e+fx)}{a^3c^3f} - \frac{2\csc^3(e+fx)}{3a^3c^3f} + \frac{\csc^5(e+fx)}{5a^3c^3f}$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.85

$$-\frac{-\frac{\csc(e+fx)}{f}+\frac{2\csc^3(e+fx)}{3f}-\frac{\csc^5(e+fx)}{5f}}{a^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
[Out] -((-(Csc[e + f*x]/f) + (2*Csc[e + f*x]^3)/(3*f) - Csc[e + f*x]^5/(5*f))/(a^
3*c^3))
```

Maple [A]
time = 0.22, size = 95, normalized size = 1.61

method	result	size
default	$-\frac{-\frac{\cos^6(fx+e)}{5\sin(fx+e)^5}+\frac{\cos^6(fx+e)}{15\sin(fx+e)^3}-\frac{\cos^6(fx+e)}{5\sin(fx+e)}-\frac{\left(\frac{8}{3}+\cos^4(fx+e)+\frac{4\left(\cos^2(fx+e)\right)}{3}\right)\sin(fx+e)}{5}}{c^3a^3f}$	95
risch	$\frac{2i\left(15\,e^{9i(fx+e)}-20\,e^{7i(fx+e)}+58\,e^{5i(fx+e)}-20\,e^{3i(fx+e)}+15\,e^{i(fx+e)}\right)}{15f\,c^3a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{i(fx+e)}-1\right)^5}$	95
norman	$\frac{\frac{1}{160acf}-\frac{5\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96acf}+\frac{5\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16acf}+\frac{5\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16acf}-\frac{5\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96acf}+\frac{\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)}{160acf}}{a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	141
derivativedivides	`error in RationalFunction: argument is not a rational function\`	N/A

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
[Out] -1/c^3/a^3/f*(-1/5/sin(f*x+e)^5*cos(f*x+e)^6+1/15/sin(f*x+e)^3*cos(f*x+e)^6
-1/5/sin(f*x+e)*cos(f*x+e)^6-1/5*(8/3+cos(f*x+e)^4+4/3*cos(f*x+e)^2)*sin(f*
x+e))
```

Maxima [A]
time = 0.32, size = 44, normalized size = 0.75

$$\frac{15\,\sin\left(fx+e\right)^{4}-10\,\sin\left(fx+e\right)^{2}+3}{15\,a^{3}c^{3}f\sin\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5)
```

Fricas [A]
time = 4.67, size = 81, normalized size = 1.37

$$\frac{15\,\cos\left(fx+e\right)^{4}-20\,\cos\left(fx+e\right)^{2}+8}{15\,\left(a^{3}c^{3}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{3}f\cos\left(fx+e\right)^{2}+a^{3}c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)/((a^3*c^3*f*cos(f*x + e)^4
 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec\left(e+fx\right)}{\sec^{6}\left(e+fx\right)-3\sec^{4}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)-1}\,dx}{a^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x
)**2 - 1), x)/(a**3*c**3)
```

Giac [A]
time = 0.58, size = 41, normalized size = 0.69

$$\frac{15\,\sin\left(fx+e\right)^{4}-10\,\sin\left(fx+e\right)^{2}+3}{15\,a^{3}c^{3}f\sin\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5)
```

Mupad [B]
time = 1.63, size = 38, normalized size = 0.64

$$\frac{\sin\left(e+f\,x\right)^4-\frac{2\,\sin\left(e+f\,x\right)^2}{3}+\frac{1}{5}}{a^3\,c^3\,f\,\sin\left(e+f\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)
[Out] (sin(e + f*x)^4 - (2*sin(e + f*x)^2)/3 + 1/5)/(a^3*c^3*f*sin(e + f*x)^5)
```

3.61 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

Optimal. Leaf size=99

$$-\frac{\cot^7(e+fx)}{7a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}$$

```
[Out] -1/7*cot(f*x+e)^7/a^3/c^4/f+csc(f*x+e)/a^3/c^4/f-csc(f*x+e)^3/a^3/c^4/f+3/5
*csc(f*x+e)^5/a^3/c^4/f-1/7*csc(f*x+e)^7/a^3/c^4/f
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4043, 2686, 200, 2687, 30}

$$-\frac{\cot^7(e+fx)}{7a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -1/7*Cot[e + f*x]^7/(a^3*c^4*f) + Csc[e + f*x]/(a^3*c^4*f) - Csc[e + f*x]^3
/(a^3*c^4*f) + (3*Csc[e + f*x]^5)/(5*a^3*c^4*f) - Csc[e + f*x]^7/(7*a^3*c^4
*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\int (a\cot^7(e+fx)\csc(e+fx)+a\cot^6(e+fx)\csc^2(e+fx)}{a^4c^4}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^3c^4} + \frac{\int \cot^6(e+fx)\csc^2(e+fx}{a^3c^4}$$

$$= \frac{\mathrm{Subst}\left(\int x^6\,dx, x, -\cot(e+fx)\right)}{a^3c^4f} - \frac{\mathrm{Subst}\left(\int (-1+x^2)^3\,dx\right.}{a^3c^4f}$$

$$= -\frac{\cot^7(e+fx)}{7a^3c^4f} - \frac{\mathrm{Subst}\left(\int (-1+3x^2-3x^4+x^6)\,dx, x, \csc\right.}{a^3c^4f}$$

$$= -\frac{\cot^7(e+fx)}{7a^3c^4f} + \frac{\csc(e+fx)}{a^3c^4f} - \frac{\csc^3(e+fx)}{a^3c^4f} + \frac{3\csc^5(e+}{5a^3c^4f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 211 vs. 2(99) = 198.
time = 1.26, size = 211, normalized size = 2.13

$$\frac{\csc(e)\csc^2\left(\frac{1}{2}(e+fx)\right)\csc^5(e+fx)(2912\sin(e)+416\sin(fx)-7620\sin(e+fx)+1905\sin(2(e+fx))+3810\sin(3(e+fx))-1524\sin(4(e+fx))-762\sin(5(e+fx))+381\sin(6(e+fx))-2016\sin(2e+fx)+2080\sin(e+2fx)-1680\sin(3e+2fx)+240\sin(2e+3fx)+560\sin(4e+3fx)-880\sin(3e+4fx)+560\sin(5e+4fx)+400\sin(4e+5fx)-560\sin(6e+5fx)+80\sin(5e+6fx))}{35840a^3c^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (Csc[e]*Csc[(e + f*x)/2]^2*Csc[e + f*x]^5*(2912*Sin[e] + 416*Sin[f*x] - 762
0*Sin[e + f*x] + 1905*Sin[2*(e + f*x)] + 3810*Sin[3*(e + f*x)] - 1524*Sin[4
*(e + f*x)] - 762*Sin[5*(e + f*x)] + 381*Sin[6*(e + f*x)] - 2016*Sin[2*e +
f*x] + 2080*Sin[e + 2*f*x] - 1680*Sin[3*e + 2*f*x] + 240*Sin[2*e + 3*f*x] +
 560*Sin[4*e + 3*f*x] - 880*Sin[3*e + 4*f*x] + 560*Sin[5*e + 4*f*x] + 400*S
in[4*e + 5*f*x] - 560*Sin[6*e + 5*f*x] + 80*Sin[5*e + 6*f*x]))/(35840*a^3*c
^4*f)
```

Maple [A]
time = 0.19, size = 102, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-2\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{6}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{20}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{5}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}}{64f\,c^{4}a^{3}}$
default	$\frac{\frac{\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-2\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{6}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{20}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{5}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}}{64f\,c^{4}a^{3}}$
risch	$\frac{2i\left(35\,e^{11i(fx+e)}-35\,e^{10i(fx+e)}-35\,e^{9i(fx+e)}+105\,e^{8i(fx+e)}+126\,e^{7i(fx+e)}-182\,e^{6i(fx+e)}+26\,e^{5i(fx+e)}+130\,e^{4i(fx+e}\right.}{35f\,c^{4}a^{3}\left(e^{i(fx+e)}+1\right)^{5}\left(e^{i(fx+e)}-1\right)^{7}}$
norman	$\frac{-\frac{1}{448acf}+\frac{3\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{160acf}-\frac{5\left(\tan^{4}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64acf}+\frac{5\left(\tan^{6}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{16acf}+\frac{15\left(\tan^{8}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64acf}-\frac{\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)}{32acf}+\frac{\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)}{320acf}}{c^{3}a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/64/f/c^4/a^3*(1/5*tan(1/2*f*x+1/2*e)^5-2*tan(1/2*f*x+1/2*e)^3+15*tan(1/2*
f*x+1/2*e)+6/5/tan(1/2*f*x+1/2*e)^5+20/tan(1/2*f*x+1/2*e)-1/7/tan(1/2*f*x+1
/2*e)^7-5/tan(1/2*f*x+1/2*e)^3)
```

Maxima [A]
time = 0.28, size = 173, normalized size = 1.75

$$\frac{\frac{7\left(\frac{75\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}+\frac{\sin(fx+e)^{5}}{(\cos(fx+e)+1)^{5}}\right)}{a^{3}c^{4}}+\frac{\left(\frac{42\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{175\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}+\frac{700\sin(fx+e)^{6}}{(\cos(fx+e)+1)^{6}}-5\right)(\cos(fx+e)+1)^{7}}{a^{3}c^{4}\sin(fx+e)^{7}}}{2240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/2240*(7*(75*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) + (42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 700
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin
(f*x + e)^7))/f
```

Fricas [A]
time = 3.69, size = 175, normalized size = 1.77

$$\frac{5\cos\left(fx+e\right)^{6}+30\cos\left(fx+e\right)^{5}-30\cos\left(fx+e\right)^{4}-40\cos\left(fx+e\right)^{3}+40\cos\left(fx+e\right)^{2}+16\cos\left(fx+e\right)-16}{35\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}-a^{3}c^{4}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{4}f\cos\left(fx+e\right)^{3}+2\,a^{3}c^{4}f\cos\left(fx+e\right)^{2}+a^{3}c^{4}f\cos\left(fx+e\right)-a^{3}c^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/35*(5*cos(f*x + e)^6 + 30*cos(f*x + e)^5 - 30*cos(f*x + e)^4 - 40*cos(f*x
 + e)^3 + 40*cos(f*x + e)^2 + 16*cos(f*x + e) - 16)/((a^3*c^4*f*cos(f*x + e
)^5 - a^3*c^4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*c
os(f*x + e)^2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{\sec^7(e+fx)-\sec^6(e+fx)-3\sec^5(e+fx)+3\sec^4(e+fx)+3\sec^3(e+fx)-3\sec^2(e+fx)-\sec(e+fx)+1}\,dx}{a^3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**
5 + 3*sec(e + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x
) + 1), x)/(a**3*c**4)
```

Giac [A]
time = 0.66, size = 128, normalized size = 1.29

$$\frac{\frac{700\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-175\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+42\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-5}{a^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}+\frac{7\left(a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-10a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+75a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{15}c^{20}}}{2240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/2240*((700*tan(1/2*f*x + 1/2*e)^6 - 175*tan(1/2*f*x + 1/2*e)^4 + 42*tan(1
/2*f*x + 1/2*e)^2 - 5)/(a^3*c^4*tan(1/2*f*x + 1/2*e)^7) + 7*(a^12*c^16*tan(
1/2*f*x + 1/2*e)^5 - 10*a^12*c^16*tan(1/2*f*x + 1/2*e)^3 + 75*a^12*c^16*tan
(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

Mupad [B]
time = 2.54, size = 129, normalized size = 1.30

$$\frac{\left(2\sin\left(\frac{e}{4}+\frac{fx}{4}\right)^2-1\right)\left(\frac{235\sin(e+fx)^2}{16}-\frac{45\sin(2e+2fx)^2}{8}+\frac{19\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{2}+\frac{5\sin(3e+3fx)^2}{16}-\frac{5\sin\left(\frac{3e}{2}+\frac{3fx}{2}\right)^2}{4}+\frac{15\sin\left(\frac{5e}{2}+\frac{5fx}{2}\right)^2}{4}-5\right)}{2240a^3c^4f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)
```

```
[Out] ((2*sin(e/4 + (f*x)/4)^2 - 1)*((19*sin(e/2 + (f*x)/2)^2)/2 - (45*sin(2*e +
2*f*x)^2)/8 + (5*sin(3*e + 3*f*x)^2)/16 - (5*sin((3*e)/2 + (3*f*x)/2)^2)/4
+ (15*sin((5*e)/2 + (5*f*x)/2)^2)/4 + (235*sin(e + f*x)^2)/16 - 5))/(2240*a
^3*c^4*f*sin(e/2 + (f*x)/2)^7*(sin(e/2 + (f*x)/2)^2 - 1)^3)
```

3.62 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

Optimal. Leaf size=120

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}$$

```
[Out] 2/9*cot(f*x+e)^9/a^3/c^5/f+csc(f*x+e)/a^3/c^5/f-5/3*csc(f*x+e)^3/a^3/c^5/f+
9/5*csc(f*x+e)^5/a^3/c^5/f-csc(f*x+e)^7/a^3/c^5/f+2/9*csc(f*x+e)^9/a^3/c^5/
f
```

Rubi [A]
time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4043, 2686, 200, 2687, 30, 276}

$$\frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (2*Cot[e + f*x]^9)/(9*a^3*c^5*f) + Csc[e + f*x]/(a^3*c^5*f) - (5*Csc[e + f*
x]^3)/(3*a^3*c^5*f) + (9*Csc[e + f*x]^5)/(5*a^3*c^5*f) - Csc[e + f*x]^7/(a^
3*c^5*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx &= -\frac{\int \left(a^2\cot^9(e+fx)\csc(e+fx)+2a^2\cot^8(e+fx)\csc^2(e\right.}{a^5c^5}\\ &= -\frac{\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^3c^5}-\frac{\int \cot^7(e+fx)\csc^3(e+}{a^3c^5}\\ &= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^3\,dx,x,\csc(e+fx)\right)}{a^3c^5f}+\frac{\text{Subst}\left(\int (-\right.}{}\\ &= \frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,\right.}{a^3c^5f}\\ &= \frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{9\csc^5(e}{5a^3c}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(120) = 240.
time = 1.50, size = 257, normalized size = 2.14

$$-\frac{\csc(e)\sec^7(e+fx)(-33024\sin(e)+6144\sin(fx)+76455\sin(e+fx)-33980\sin(2(e+fx))-32281\sin(3(e+fx))+27184\sin(4(e+fx))+1699\sin(5(e+fx))-6796\sin(6(e+fx))+1699\sin(7(e+fx))+22656\sin(2e+fx)-17216\sin(e+2fx)+4416\sin(3e+2fx)+3200\sin(2e+3fx)-15360\sin(4e+3fx)+12160\sin(3e+4fx)-1920\sin(5e+4fx)-5120\sin(4e+5fx)+5760\sin(6e+5fx)+320\sin(5e+6fx)-2880\sin(7e+6fx)+640\sin(6e+7fx))\tan(e+fx)}{184320a^3c^5f(-1+\sec(e+fx))^5(1+\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] -1/184320*(Csc[e]*Sec[e + f*x]^7*(-33024*Sin[e] + 6144*Sin[f*x] + 76455*Sin
[e + f*x] - 33980*Sin[2*(e + f*x)] - 32281*Sin[3*(e + f*x)] + 27184*Sin[4*(
```

```
e + f*x)] + 1699*Sin[5*(e + f*x)] - 6796*Sin[6*(e + f*x)] + 1699*Sin[7*(e +
 f*x)] + 22656*Sin[2*e + f*x] - 17216*Sin[e + 2*f*x] + 4416*Sin[3*e + 2*f*x
] + 3200*Sin[2*e + 3*f*x] - 15360*Sin[4*e + 3*f*x] + 12160*Sin[3*e + 4*f*x]
 - 1920*Sin[5*e + 4*f*x] - 5120*Sin[4*e + 5*f*x] + 5760*Sin[6*e + 5*f*x] +
320*Sin[5*e + 6*f*x] - 2880*Sin[7*e + 6*f*x] + 640*Sin[6*e + 7*f*x])*Tan[e
+ f*x])/(a^3*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^3)
```

Maple [A]
time = 0.19, size = 115, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{7\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{21}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{35}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{}{3\tan(}}{128f\,c^5a^3}$
default	$\frac{\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{7\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{21}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{35}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{}{3\tan(}}{128f\,c^5a^3}$
risch	$\frac{2i\left(45\,\mathrm{e}^{13i(fx+e)}-90\,\mathrm{e}^{12i(fx+e)}+30\,\mathrm{e}^{11i(fx+e)}+240\,\mathrm{e}^{10i(fx+e)}-69\,\mathrm{e}^{9i(fx+e)}-354\,\mathrm{e}^{8i(fx+e)}+516\,\mathrm{e}^{7i(fx+e)}+96\,\mathrm{e}^{6i(fx+e)}\right.}{45f\,c^5a^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(\mathrm{e}^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{1}{1152acf}-\frac{\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}{128acf}+\frac{21\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{640acf}-\frac{35\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{384acf}+\frac{35\left(\tan^8\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{128acf}+\frac{21\left(\tan^{10}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{128acf}-\frac{7\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{384acf}}{a^2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/128/f/c^5/a^3*(1/5*tan(1/2*f*x+1/2*e)^5-7/3*tan(1/2*f*x+1/2*e)^3+21*tan(1
/2*f*x+1/2*e)+21/5/tan(1/2*f*x+1/2*e)^5+1/9/tan(1/2*f*x+1/2*e)^9-1/tan(1/2*
f*x+1/2*e)^7+35/tan(1/2*f*x+1/2*e)-35/3/tan(1/2*f*x+1/2*e)^3)
```

Maxima [A]
time = 0.32, size = 197, normalized size = 1.64

$$\frac{\frac{3\left(\frac{315\sin(fx+e)}{\cos(fx+e)+1}-\frac{35\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^5}-\frac{\left(\frac{45\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{189\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{525\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1575\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-5\right)(\cos(fx+e)+1)^9}{a^3c^5\sin(fx+e)^9}}{5760\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/5760*(3*(315*sin(f*x + e)/(cos(f*x + e) + 1) - 35*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - (45*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
525*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1575*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

Fricas [A]
time = 3.67, size = 204, normalized size = 1.70

$$\frac{10\cos(fx+e)^7+25\cos(fx+e)^6-60\cos(fx+e)^5-10\cos(fx+e)^4+80\cos(fx+e)^3-24\cos(fx+e)^2-32\cos(fx+e)+16}{45\left(a^3c^5f\cos(fx+e)^6-2a^3c^5f\cos(fx+e)^5-a^3c^5f\cos(fx+e)^4+4a^3c^5f\cos(fx+e)^3-a^3c^5f\cos(fx+e)^2-2a^3c^5f\cos(fx+e)+a^3c^5f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/45*(10*cos(f*x + e)^7 + 25*cos(f*x + e)^6 - 60*cos(f*x + e)^5 - 10*cos(f*
x + e)^4 + 80*cos(f*x + e)^3 - 24*cos(f*x + e)^2 - 32*cos(f*x + e) + 16)/((
a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^3*c^5*f*cos(f*x +
 e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e)^2 - 2*a^3*c^5*f
*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec(e+fx)}{\sec^8(e+fx)-2\sec^7(e+fx)-2\sec^6(e+fx)+6\sec^5(e+fx)-6\sec^3(e+fx)+2\sec^2(e+fx)+2\sec(e+fx)-1}\,dx}{a^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*x
)**6 + 6*sec(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(e
+ f*x) - 1), x)/(a**3*c**5)
```

Giac [A]
time = 0.74, size = 142, normalized size = 1.18

$$\frac{\frac{1575\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-525\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+189\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+5}{a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}+\frac{3\left(3a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-35a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+315a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^{15}c^{25}}}{5760f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/5760*((1575*tan(1/2*f*x + 1/2*e)^8 - 525*tan(1/2*f*x + 1/2*e)^6 + 189*tan
(1/2*f*x + 1/2*e)^4 - 45*tan(1/2*f*x + 1/2*e)^2 + 5)/(a^3*c^5*tan(1/2*f*x +
 1/2*e)^9) + 3*(3*a^12*c^20*tan(1/2*f*x + 1/2*e)^5 - 35*a^12*c^20*tan(1/2*f
*x + 1/2*e)^3 + 315*a^12*c^20*tan(1/2*f*x + 1/2*e))/(a^15*c^25))/f
```

Mupad [B]
time = 2.88, size = 109, normalized size = 0.91

$$\frac{\frac{145\cos(3e+3fx)}{32}-\frac{169\cos(2e+2fx)}{32}-\frac{129\cos(e+fx)}{32}+\frac{55\cos(4e+4fx)}{16}-\frac{85\cos(5e+5fx)}{32}+\frac{25\cos(6e+6fx)}{32}+\frac{5\cos(7e+7fx)}{32}+\frac{129}{16}}{5760a^3c^5f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)
```

```
[Out] ((145*cos(3*e + 3*f*x))/32 - (169*cos(2*e + 2*f*x))/32 - (129*cos(e + f*x))
/32 + (55*cos(4*e + 4*f*x))/16 - (85*cos(5*e + 5*f*x))/32 + (25*cos(6*e + 6
*f*x))/32 + (5*cos(7*e + 7*f*x))/32 + 129/16)/(5760*a^3*c^5*f*cos(e/2 + (f*
x)/2)^5*sin(e/2 + (f*x)/2)^9)
```

3.63 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

Optimal. Leaf size=162

$$-\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\csc(e+fx)}{a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{17\csc^9(\dots}{9a^3\dots}$$

```
[Out] -1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(f*x+e)^11/a^3/c^6/f+csc(f*x+e)/a^3/c^6
/f-8/3*csc(f*x+e)^3/a^3/c^6/f+22/5*csc(f*x+e)^5/a^3/c^6/f-4*csc(f*x+e)^7/a^
3/c^6/f+17/9*csc(f*x+e)^9/a^3/c^6/f-4/11*csc(f*x+e)^11/a^3/c^6/f
```

Rubi [A]
time = 0.19, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {4043, 2686, 200, 2687, 30, 276, 14}

$$-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}+\frac{\csc(e+fx)}{a^3c^6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] -1/9*Cot[e + f*x]^9/(a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + Csc[
e + f*x]/(a^3*c^6*f) - (8*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (22*Csc[e + f*x]^
5)/(5*a^3*c^6*f) - (4*Csc[e + f*x]^7)/(a^3*c^6*f) + (17*Csc[e + f*x]^9)/(9*
a^3*c^6*f) - (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx &= \frac{\int \left(a^3\cot^{11}(e+fx)\csc(e+fx)+3a^3\cot^{10}(e+fx)\csc^2(e+\right.}{} \\
&= \frac{\int \cot^{11}(e+fx)\csc(e+fx)\,dx}{a^3c^6}+\frac{\int \cot^8(e+fx)\csc^4(e+f}{a^3c^6} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^5\,dx,x,\csc(e+fx)\right)}{a^3c^6f}+\frac{\operatorname{Subst}\left(\int x^8(1\right.}{} \\
&= -\frac{3\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\operatorname{Subst}\left(\int \left(-1+5x^2-10x^4+10x^6-5\right.\right.}{a^3c^6f} \\
&= -\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\csc(e+fx)}{a^3c^6f}-\frac{8\csc^3(e}{3a^3c}
\end{aligned}
$$

Mathematica [A]
time = 2.11, size = 289, normalized size = 1.78

$$
\frac{\csc(e)\sec^8(e+fx)(1119360\sin(e)-260480\sin(fx)-3440690\sin(e+fx)+2064414\sin(2(e+fx))+1063486\sin(3(e+fx))-1563950\sin(4(e+fx))+312790\sin(5(e+fx))+312790\sin(6(e+fx))-187674\sin(7(e+fx))+31279\sin(8(e+fx))-1499520\sin(2e+fx)+1051776\sin(e+2fx)+4224\sin(3e+2fx)-85376\sin(2e+3fx)+629376\sin(4e+3fx)-483200\sin(3e+4fx)-316800\sin(5e+4fx)+392320\sin(4e+5fx)-232320\sin(6e+5fx)-30080\sin(5e+6fx)+190080\sin(7e+6fx)-32640\sin(6e+7fx)-63360\sin(8e+7fx)+16000\sin(7e+8fx))\tan(e+fx)}{8110080a^3c^6f(-1+\sec(e+fx))^6(1+\sec(e+fx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] (Csc[e]*Sec[e + f*x]^8*(1119360*Sin[e] - 260480*Sin[f*x] - 3440690*Sin[e +
f*x] + 2064414*Sin[2*(e + f*x)] + 1063486*Sin[3*(e + f*x)] - 1563950*Sin[4*
(e + f*x)] + 312790*Sin[5*(e + f*x)] + 312790*Sin[6*(e + f*x)] - 187674*Sin
[7*(e + f*x)] + 31279*Sin[8*(e + f*x)] - 1499520*Sin[2*e + f*x] + 1051776*S
in[e + 2*f*x] + 4224*Sin[3*e + 2*f*x] - 85376*Sin[2*e + 3*f*x] + 629376*Sin
[4*e + 3*f*x] - 483200*Sin[3*e + 4*f*x] - 316800*Sin[5*e + 4*f*x] + 392320*
Sin[4*e + 5*f*x] - 232320*Sin[6*e + 5*f*x] - 30080*Sin[5*e + 6*f*x] + 19008
0*Sin[7*e + 6*f*x] - 32640*Sin[6*e + 7*f*x] - 63360*Sin[8*e + 7*f*x] + 1600
0*Sin[7*e + 8*f*x])*Tan[e + f*x])/(8110080*a^3*c^6*f*(-1 + Sec[e + f*x])^6*
(1 + Sec[e + f*x])^3)
```

Maple [A]
time = 0.19, size = 128, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{8\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+28\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{70}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{56}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{56}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{}{11\,\mathrm{t}}}{256 f a^3 c^6}$
default	$\frac{\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{8\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+28\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{70}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{56}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{56}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{}{11\,\mathrm{t}}}{256 f a^3 c^6}$
risch	$\frac{2i\left(495\,\mathrm{e}^{15i(fx+e)}-1485\,\mathrm{e}^{14i(fx+e)}+1815\,\mathrm{e}^{13i(fx+e)}+2475\,\mathrm{e}^{12i(fx+e)}-4917\,\mathrm{e}^{11i(fx+e)}-33\,\mathrm{e}^{10i(fx+e)}+11715\,\mathrm{e}^{9i(fx+e)}\right.}{495 f a^3 c^6\left(\mathrm{e}^{i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/256/f/a^3/c^6*(1/5*tan(1/2*f*x+1/2*e)^5-8/3*tan(1/2*f*x+1/2*e)^3+28*tan(1
/2*f*x+1/2*e)-70/3/tan(1/2*f*x+1/2*e)^3+56/5/tan(1/2*f*x+1/2*e)^5+56/tan(1/
2*f*x+1/2*e)-4/tan(1/2*f*x+1/2*e)^7-1/11/tan(1/2*f*x+1/2*e)^11+8/9/tan(1/2*
f*x+1/2*e)^9)
```

Maxima [A]
time = 0.28, size = 218, normalized size = 1.35

$$\frac{\frac{33\left(\frac{420\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6}+\frac{\left(\frac{440\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1980\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5544\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{11550\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{27720\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-45\right)(\cos(fx+e)+1)^{11}}{a^3c^6\sin(fx+e)^{11}}}{126720\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/126720*(33*(420*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) + (440*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1980*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 5544*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 11550*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 27720*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 45)*(cos(f*x
 + e) + 1)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

Fricas [A]
time = 4.14, size = 233, normalized size = 1.44

$$\frac{125 \cos \left(fx+e\right)^{8}+120 \cos \left(fx+e\right)^{7}-680 \cos \left(fx+e\right)^{6}+400 \cos \left(fx+e\right)^{5}+720 \cos \left(fx+e\right)^{4}-832 \cos \left(fx+e\right)^{3}-64 \cos \left(fx+e\right)^{2}+384 \cos \left(fx+e\right)-128}{495 \left(a^{3} c^{6} f \cos \left(fx+e\right)^{7}-3 a^{3} c^{6} f \cos \left(fx+e\right)^{6}+a^{3} c^{6} f \cos \left(fx+e\right)^{5}+5 a^{3} c^{6} f \cos \left(fx+e\right)^{4}-5 a^{3} c^{6} f \cos \left(fx+e\right)^{3}-a^{3} c^{6} f \cos \left(fx+e\right)^{2}+3 a^{3} c^{6} f \cos \left(fx+e\right)-a^{3} c^{6} f\right) \sin \left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/495*(125*cos(f*x + e)^8 + 120*cos(f*x + e)^7 - 680*cos(f*x + e)^6 + 400*c
os(f*x + e)^5 + 720*cos(f*x + e)^4 - 832*cos(f*x + e)^3 - 64*cos(f*x + e)^2
 + 384*cos(f*x + e) - 128)/((a^3*c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x
 + e)^6 + a^3*c^6*f*cos(f*x + e)^5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6
*f*cos(f*x + e)^3 - a^3*c^6*f*cos(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a
^3*c^6*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec \left(e+fx\right)}{\sec^{9}\left(e+fx\right)-3\sec^{8}\left(e+fx\right)+8\sec^{6}\left(e+fx\right)-6\sec^{5}\left(e+fx\right)-6\sec^{4}\left(e+fx\right)+8\sec^{3}\left(e+fx\right)-3\sec\left(e+fx\right)+1}\,dx}{a^{3} c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x)
**6 - 6*sec(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e +
 f*x) + 1), x)/(a**3*c**6)
```

Giac [A]
time = 0.68, size = 155, normalized size = 0.96

$$\frac{\frac{27720 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{10}-11550 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{8}+5544 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{6}-1980 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{4}+440 \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{2}-45}{a^{3} c^{6} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{11}}+\frac{33 \left(3 a^{12} c^{24} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{5}-40 a^{12} c^{24} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)^{3}+420 a^{12} c^{24} \tan\left(\frac{1}{2} fx+\frac{1}{2} e\right)\right)}{a^{15} c^{30}}}{126720 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/126720*((27720*tan(1/2*f*x + 1/2*e)^10 - 11550*tan(1/2*f*x + 1/2*e)^8 + 5
544*tan(1/2*f*x + 1/2*e)^6 - 1980*tan(1/2*f*x + 1/2*e)^4 + 440*tan(1/2*f*x
+ 1/2*e)^2 - 45)/(a^3*c^6*tan(1/2*f*x + 1/2*e)^11) + 33*(3*a^12*c^24*tan(1/
2*f*x + 1/2*e)^5 - 40*a^12*c^24*tan(1/2*f*x + 1/2*e)^3 + 420*a^12*c^24*tan(
1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

Mupad [B]
time = 3.11, size = 120, normalized size = 0.74

$$-\frac{\frac{605\cos(e+fx)}{8}+\frac{1023\cos(2e+2fx)}{16}-\frac{349\cos(3e+3fx)}{8}-\frac{325\cos(4e+4fx)}{32}+\frac{305\cos(5e+5fx)}{8}-\frac{215\cos(6e+6fx)}{16}+\frac{15\cos(7e+7fx)}{8}+\frac{125\cos(8e+8fx)}{128}-\frac{8745}{128}}{126720\,a^3\,c^6\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)
```

```
[Out] -((605*cos(e + f*x))/8 + (1023*cos(2*e + 2*f*x))/16 - (349*cos(3*e + 3*f*x)
)/8 - (325*cos(4*e + 4*f*x))/32 + (305*cos(5*e + 5*f*x))/8 - (215*cos(6*e +
 6*f*x))/16 + (15*cos(7*e + 7*f*x))/8 + (125*cos(8*e + 8*f*x))/128 - 8745/1
28)/(126720*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^11)
```

3.64 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=163

$$-\frac{256c^4(a+a\sec(e+fx))\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{105f}-\frac{8c^2(a+$$

```
[Out] -8/21*c^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/9*c*(a+a*s
ec(f*x+e))*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/315*c^4*(a+a*sec(f*x+e))
*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/105*c^3*(a+a*sec(f*x+e))*(c-c*sec(f
*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4040, 4038}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)}{315f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{105f}-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{21f}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x]
]) - (64*c^3*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(1
05*f) - (8*c^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(21*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(9*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
```

```
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,\mathrm{t}}{9f}$$
$$= -\frac{8c^2(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}{21f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}{105f}$$
$$= -\frac{256c^4(a+a\sec(e+fx))\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{64c}{}$$

Mathematica [A]
time = 0.85, size = 86, normalized size = 0.53

$$\frac{ac^3\cos^2\left(\frac{1}{2}(e+fx)\right)(-782+1617\cos(e+fx)-642\cos(2(e+fx))+319\cos(3(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a*c^3*Cos[(e + f*x)/2]^2*(-782 + 1617*Cos[e + f*x] - 642*Cos[2*(e + f*x)]
+ 319*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[c - c*Sec[e +
f*x]])/(315*f)
```

Maple [A]
time = 2.04, size = 83, normalized size = 0.51

method	result	size
default	$\frac{2a\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}\left(\sin^3(fx+e)\right)\left(319\left(\cos^3(fx+e)\right)-321\left(\cos^2(fx+e)\right)+165\cos(fx+e)-35\right)}{315f(-1+\cos(fx+e))^5\cos(fx+e)}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/315*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*sin(f*x+e)^3*(319*cos(f*x+e)
^3-321*cos(f*x+e)^2+165*cos(f*x+e)-35)/(-1+cos(f*x+e))^5/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
maxima")
```

```
[Out] -2/315*(315*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(5*(a*c^3*f*cos(2*f*x + 2*e)^4 + a*c^3*f*sin(2*f*x + 2*e)^4 + 4*a
*c^3*f*cos(2*f*x + 2*e)^3 + 6*a*c^3*f*cos(2*f*x + 2*e)^2 + 4*a*c^3*f*cos(2*
f*x + 2*e) + a*c^3*f + 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*f*x
+ 2*e) + a*c^3*f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x
 + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*
f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f
*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*
f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f
*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*
f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f
*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x +
 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*
x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)))/((cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4*cos(2*f*x + 2*e)^5
+ (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2*f*x + 2*e)^4 + 6*co
s(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x +
 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^
2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(
cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*
e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^3 +
(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4
+ sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x
+ 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4 + 8*cos(2*f*x + 2*e)
^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*
(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2
*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos(2*f*x + 2*e)^2
 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x
 + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)
*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*
```

```
e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 + 4*cos(2*
f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos(2*f*x +
 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e)^2 + cos
(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^
5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + 2*
(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^4 + 4*
cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4 + 4*cos
(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x
+ 2*e))*sin(4*f*x + 4*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))^2 + (cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4*cos(2*f*x + 2*e)^5
 + (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2*f*x + 2*e)^4 + 6*c
os(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x
+ 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)
^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*
(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^3 +
 (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x +
2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4
 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x
 + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x ...
```

Fricas [A]
time = 3.44, size = 128, normalized size = 0.79

$$\frac{2\left(319\,ac^{3}\cos\left(fx+e\right)^{5}+317\,ac^{3}\cos\left(fx+e\right)^{4}-158\,ac^{3}\cos\left(fx+e\right)^{3}-26\,ac^{3}\cos\left(fx+e\right)^{2}+95\,ac^{3}\cos\left(fx+e\right)-35\,ac^{3}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{315\,f\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
fricas")
```

```
[Out] 2/315*(319*a*c^3*cos(f*x + e)^5 + 317*a*c^3*cos(f*x + e)^4 - 158*a*c^3*cos(
f*x + e)^3 - 26*a*c^3*cos(f*x + e)^2 + 95*a*c^3*cos(f*x + e) - 35*a*c^3)*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [A]
time = 1.03, size = 107, normalized size = 0.66

$$\frac{32\sqrt{2}\left(105\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{3}c^{2}+189\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{2}c^{3}+135\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)c^{4}+35c^{5}\right)ac^{3}}{315\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
giac")
```

```
[Out] 32/315*sqrt(2)*(105*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^2 + 189*(c*tan(1/2*f
*x + 1/2*e)^2 - c)^2*c^3 + 135*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 35*c^5)
*a*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

Mupad [B]
time = 9.13, size = 483, normalized size = 2.96

$$\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a\,c^3\,2i}{f}+\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,638i}{315\,f}\right)}{e^{e\,1i+f\,x\,1i}-1}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a\,c^3\,32i}{9\,f}+\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,32i}{9\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^4}+\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a\,c^3\,96i}{7\,f}+\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,32i}{63\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^3}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a\,c^3\,64i}{5\,f}-\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,736i}{105\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^2}+\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a\,c^3\,8i}{3\,f}-\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,1256i}{315\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*2i)/
f + (a*c^3*exp(e*1i + f*x*1i)*638i)/(315*f)))/(exp(e*1i + f*x*1i) - 1) - ((
c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*32i)/(
9*f) + (a*c^3*exp(e*1i + f*x*1i)*32i)/(9*f)))/((exp(e*1i + f*x*1i) - 1)*(ex
p(e*2i + f*x*2i) + 1)^4) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a*c^3*96i)/(7*f) + (a*c^3*exp(e*1i + f*x*1i)*32i)/(63*f)))
/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*64i)/(5*f) - (a*c^3*ex
p(e*1i + f*x*1i)*736i)/(105*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*
2i) + 1)^2) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2
)*((a*c^3*8i)/(3*f) - (a*c^3*exp(e*1i + f*x*1i)*1256i)/(315*f)))/((exp(e*1i
 + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
```

3.65 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=122

$$-\frac{64c^3(a+a\sec(e+fx))\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{35f}-\frac{2c(a+a$$

```
[Out] -2/7*c*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/105*c^3*(a+a
*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/35*c^2*(a+a*sec(f*x+e))
*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4040, 4038}

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)}{105f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{35f}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[c - c*Sec[e + f*x]]
) - (16*c^2*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(35
*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(7
*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,\mathrm{ta}}{7f}$$
$$= -\frac{16c^2(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,\mathrm{t}}{35f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a}{}$$

Mathematica [A]
time = 0.47, size = 76, normalized size = 0.62

$$\frac{2ac^2\cos^2\left(\frac{1}{2}(e+fx)\right)(101-108\cos(e+fx)+71\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*a*c^2*Cos[(e + f*x)/2]^2*(101 - 108*Cos[e + f*x] + 71*Cos[2*(e + f*x)])*
Cot[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[c - c*Sec[e + f*x]])/(105*f)
```

Maple [A]
time = 2.32, size = 73, normalized size = 0.60

method	result	size
default	$\frac{2a\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}\left(\sin^3(fx+e)\right)\left(71\left(\cos^2(fx+e)\right)-54\cos(fx+e)+15\right)}{105f(-1+\cos(fx+e))^4\cos(fx+e)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/105*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*sin(f*x+e)^3*(71*cos(f*x+e)^
2-54*cos(f*x+e)+15)/(-1+cos(f*x+e))^4/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")

[Out] -2/105*(105*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(3/4)*(3*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a
*c^2*f*cos(2*f*x + 2*e) + a*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x
+ 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f
*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*
x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f
*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*
x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f
*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*
x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x +
2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2
*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*
e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 +
 cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 +
cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*
e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x
+ 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2
*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*
x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*
e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*
```

```
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6
*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/4)), x) - 5*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*
x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2*e) + a*c^2*f)*integrate((((cos(6*f*x +
 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x +
2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x +
6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2
*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4
*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*
e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6
*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e...
```

Fricas [A]
time = 3.77, size = 113, normalized size = 0.93

$$\frac{2\,\left(71\,ac^2\cos\left(fx+e\right)^4+88\,ac^2\cos\left(fx+e\right)^3-22\,ac^2\cos\left(fx+e\right)^2-24\,ac^2\cos\left(fx+e\right)+15\,ac^2\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{105\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] 2/105*(71*a*c^2*cos(f*x + e)^4 + 88*a*c^2*cos(f*x + e)^3 - 22*a*c^2*cos(f*x
 + e)^2 - 24*a*c^2*cos(f*x + e) + 15*a*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int\left(-c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)\right)dx+\int\left(-c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{3}\left(e+fx\right)\right)dx+\int c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{4}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-c**
2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-c**2*sqrt(-c*se
c(e + f*x) + c)*sec(e + f*x)**3, x) + Integral(c**2*sqrt(-c*sec(e + f*x) +
c)*sec(e + f*x)**4, x))
```

Giac [A]
time = 1.08, size = 83, normalized size = 0.68

$$\frac{16\,\sqrt{2}\,\left(35\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^2+42\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^3+15\,c^4\right)ac^2}{105\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] 16/105*sqrt(2)*(35*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 + 42*(c*tan(1/2*f*x
 + 1/2*e)^2 - c)*c^3 + 15*c^4)*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*
f)
```

Mupad [B]
time = 6.00, size = 384, normalized size = 3.15

$$\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a\,c^2\,2\mathrm{i}}{f}+\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,142\mathrm{i}}{105\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}+\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a\,c^2\,16\mathrm{i}}{7\,f}-\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}-\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a\,c^2\,8\mathrm{i}}{5\,f}-\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,184\mathrm{i}}{35\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}-\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a\,c^2\,4\mathrm{i}}{3\,f}+\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,244\mathrm{i}}{105\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*2i)/
f + (a*c^2*exp(e*1i + f*x*1i)*142i)/(105*f)))/(exp(e*1i + f*x*1i) - 1) + ((
c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*16i)/(
7*f) - (a*c^2*exp(e*1i + f*x*1i)*16i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(ex
p(e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a*c^2*8i)/(5*f) - (a*c^2*exp(e*1i + f*x*1i)*184i)/(35*f)))
/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) - ((c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*4i)/(3*f) + (a*c^2*exp
(e*1i + f*x*1i)*244i)/(105*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2
i) + 1))
```

3.66 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=81

$$-\frac{8c^2(a+a\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5f}$$

```
[Out] -8/15*c^2*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/5*c*(a+a*s
ec(f*x+e))*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4040, 4038}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f*x]])
- (2*c*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\ \text{ta}}{5f}$$
$$= -\frac{8c^2(a+a\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a-}{}$$

Mathematica [A]
time = 0.26, size = 64, normalized size = 0.79

$$\frac{4ac\cos^2\left(\frac{1}{2}(e+fx)\right)(-3+7\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*a*c*Cos[(e + f*x)/2]^2*(-3 + 7*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e + f*
x]^2*Sqrt[c - c*Sec[e + f*x]])/(15*f)
```

Maple [A]
time = 2.41, size = 63, normalized size = 0.78

method	result	size
default	$\frac{2a(7\cos(fx+e)-3)(\sin^3(fx+e))\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}{15f(-1+\cos(fx+e))^3\cos(fx+e)}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/15*a/f*(7*cos(f*x+e)-3)*sin(f*x+e)^3*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
/(-1+cos(f*x+e))^3/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] -2/15*(15*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*((a*c*f*cos(2*f*x + 2*e)^2 + a*c*f*sin(2*f*x + 2*e)^2 + 2*a*c*f*cos
```

```
(2*f*x + 2*e) + a*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos
(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(
2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*co
s(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(
6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2
*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(
4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*
f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f
*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos
(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*
e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e
))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*
f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x +
2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*si
n(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2
+ (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*
cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
 + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) +
cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*
```

```
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1))^2), x) - 2*(a*c*f*cos(2*f*x + 2*e)^2 + a*c*f*sin(2*f*x + 2*e)^2 + 2*a*c
*f*cos(2*f*x + 2*e) + a*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)
^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e
)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)
*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e
)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)
*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (
cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + c
os(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))
/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x ...
```

Fricas [A]
time = 2.89, size = 89, normalized size = 1.10

$$\frac{2\left(7\,ac\cos\left(fx+e\right)^{3}+11\,ac\cos\left(fx+e\right)^{2}+ac\cos\left(fx+e\right)-3\,ac\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] 2/15*(7*a*c*cos(f*x + e)^3 + 11*a*c*cos(f*x + e)^2 + a*c*cos(f*x + e) - 3*a
*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{3}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-c*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**3, x))
```

Giac [A]
time = 0.88, size = 56, normalized size = 0.69

$$\frac{8\sqrt{2}\left(5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^3+3c^4\right)a}{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] 8/15*sqrt(2)*(5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + 3*c^4)*a/((c*tan(1/2*f
*x + 1/2*e)^2 - c)^(5/2)*f)
```

Mupad [B]
time = 5.36, size = 120, normalized size = 1.48

$$-\frac{2\,a\,c\left(e^{e\,1i+f\,x\,1i}\,1i+1i\right)^3\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(7+7\,e^{e\,2i+f\,x\,2i}-6\,e^{e\,1i+f\,x\,1i}\right)}{15\,f\,\left(e^{e\,1i+f\,x\,1i}-1\right)\,\left(e^{e\,2i+f\,x\,2i}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] -(2*a*c*(exp(e*1i + f*x*1i)*1i + 1i)^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp
(e*1i + f*x*1i)/2))^(1/2)*(7*exp(e*2i + f*x*2i) - 6*exp(e*1i + f*x*1i) + 7)
)/(15*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

3.67 $\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=39

$$-\frac{2c(a+a\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/3*c*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{4038\}$

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx=-\frac{2c(a+a\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.18, size = 51, normalized size = 1.31

$$\frac{4a\cos^2\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
```

[Out] (4*a*Cos[(e + f*x)/2]^2*Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(3*f)

Maple [A]
time = 2.31, size = 53, normalized size = 1.36

method	result	size
default	$\frac{2a\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,(\sin^3(fx+e))}{3f\cos(fx+e)(-1+\cos(fx+e))^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/3*a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^3/cos(f*x+e)/(-1+cos(f*x+e))^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] 2/3*(3*(a*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2

```
+ (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x +
 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(
2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e)
 + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x
 + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/4)), x) - a*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*
cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*s
in(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*s
in(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*si
n(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*s
in(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*si
n(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos
(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(
2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((
(2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6
*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*
f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) +
 cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x +
 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(
6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*
e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)), x))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(3/4)*sqrt(c) - (3*a*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1))*sin(2*f*x + 2*e) - (3*a*cos(2*f*x + 2*e) + a)*sin
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(c))/((cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4)*f)
```

Fricas [A]
time = 3.31, size = 71, normalized size = 1.82

$$\frac{2\left(a\cos\left(fx+e\right)^{2}+2\,a\cos\left(fx+e\right)+a\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] 2/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{-c\sec{(e+fx)}+c}\,\sec{(e+fx)}\,dx+\int\sqrt{-c\sec{(e+fx)}+c}\,\sec^{2}{(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(sqrt(-c*s
ec(e + f*x) + c)*sec(e + f*x)**2, x))
```

Giac [A]
time = 0.80, size = 31, normalized size = 0.79

$$\frac{4\sqrt{2}\,ac^{2}}{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] 4/3*sqrt(2)*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*f)
```

Mupad [B]
time = 2.75, size = 87, normalized size = 2.23

$$\frac{2\,a\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{3\,f\,\left(8\,\cos\left(2\,e+2\,f\,x\right)-12\,\cos\left(e+f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a*(c - c/cos(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(3
*f*(8*cos(2*e + 2*f*x) - 12*cos(e + f*x) - 4*cos(3*e + 3*f*x) + cos(4*e + 4
*f*x) + 7))
```

3.68 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{2\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)/
f/c^(1/2)+2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4041, 3880, 209}

$$\frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}-\frac{2\sqrt{2}\,a\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x
]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx$$

$$= \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{(4a)\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+f}}\right.}{f}$$

$$= -\frac{2\sqrt{2}\,a\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f} + \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.64, size = 132, normalized size = 1.71

$$-\frac{i\sqrt{2}\,a\left(-1+e^{i(e+fx)}\right)\left(\sqrt{2}\left(1+e^{i(e+fx)}\right)-2\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)\right)}{\left(1+e^{2i(e+fx)}\right)f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*a*(-1 + E^(I*(e + f*x)))*(Sqrt[2]*(1 + E^(I*(e + f*x))) - 2*S
qrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1
+ E^((2*I)*(e + f*x))])]))/((1 + E^((2*I)*(e + f*x)))*f*Sqrt[c - c*Sec[e +
f*x]])
```

Maple [A]
time = 2.18, size = 85, normalized size = 1.10

method	result	size
default	$-\frac{2a\left(\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-1\right)\sin(fx+e)}{f\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\cos(fx+e)}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*a/f*(arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)-1)*sin(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/cos(f*x
+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Fricas [A]
time = 3.54, size = 294, normalized size = 3.82

$$\left[\frac{\sqrt{2}\,ac\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)-2\left(a\cos(fx+e)+a\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{cf\sin(fx+e)}, \frac{2\left(\sqrt{2}\,a\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)-\left(a\cos(fx+e)+a\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\right)}{cf\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [(sqrt(2)*a*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*sin
(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(a*cos(f*x +
 e) + a)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e)), 2*(sq
rt(2)*a*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(
f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - (a*cos(f*x + e) + a)*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx+\int\frac{\sec^2(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f
*x)**2/sqrt(-c*sec(e + f*x) + c), x))
```

Giac [A]
time = 1.13, size = 61, normalized size = 0.79

$$\frac{2\,a\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] 2*a*(sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c) + s
qrt(2)/sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(e+f\,x)}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.69 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] 1/2*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(3/2)
/f*2^(1/2)-a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4042, 3880, 209}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(Sqrt
[2]*c^(3/2)*f) - (a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}\\ &= -\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}+\frac{a\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx,x,\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+f}}\right.}{cf}\\ &= \frac{a\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.12, size = 246, normalized size = 3.24

$$\frac{a\left(\frac{i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^3\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{3/2}}-4\csc\left(\frac{e}{2}\right)\sec^2(e+fx)\sin\left(\frac{fx}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)+4\cot\left(\frac{e}{2}\right)\sec^2(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)-8\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)\sec^2(e+fx)\sin^3\left(\frac{1}{2}(e+fx)\right)+8\sec^2(e+fx)\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^3\left(\frac{1}{2}(e+fx)\right)\right)}{2cf(-1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]

[Out] (a*((I*Sqrt[2]*(-1 + E^(I*(e + f*x)))^3*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt
[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*I)*(e + f*x)))^(3/2) - 4*Cs
c[e/2]*Sec[e + f*x]^2*Sin[(f*x)/2]*Sin[(e + f*x)/2] + 4*Cot[e/2]*Sec[e + f*
x]^2*Sin[(e + f*x)/2]^2 - 8*Cos[e/2]*Cos[(f*x)/2]*Sec[e + f*x]^2*Sin[(e + f
*x)/2]^3 + 8*Sec[e + f*x]^2*Sin[e/2]*Sin[(f*x)/2]*Sin[(e + f*x)/2]^3))/(2*c
*f*(-1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(66) = 132$.
time = 2.25, size = 164, normalized size = 2.16

method	result
default	$\frac{2a(-1+\cos(fx+e))^2\left(\cos(fx+e)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)+\cos(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right.\right.}{f\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)^3\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*a/f*(-1+cos(f*x+e))^2*(cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))
^(1/2))+cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-arctan(1/(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2))+(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))/(c*(-1+co
s(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(70) = 140$.
time = 3.57, size = 370, normalized size = 4.87

$$\left[\frac{\sqrt{2}\,(ac\cos(fx+e)-ac)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\,(\cos(fx+e)^2+\cos(fx+e))\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}+(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+4\,(a\cos(fx+e)^2+a\cos(fx+e))\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{4\,(c^2f\cos(fx+e)-c^2f)\sin(fx+e)},\ -\frac{\frac{\sqrt{2}\,(ac\cos(fx+e)-ac)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}}-2\,(a\cos(fx+e)^2+a\cos(fx+e))\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\,(c^2f\cos(fx+e)-c^2f)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*c*cos(f*x + e) - a*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos(f*x +
 e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) +
(3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f
*x + e) + 4*(a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), -1/2*(sqrt(2)*(a
*c*cos(f*x + e) - a*c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sqrt(c) - 2*(a*cos(f*x
 + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*
cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{\sec^2(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqr
t(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**2/(-c*sqrt(-c*sec(e +
f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A]
time = 1.11, size = 72, normalized size = 0.95

$$-\frac{\sqrt{2}\left(\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}\right)a}{2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)) +
sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/tan(1/2*f*x + 1/2*e)^2)*a/(c^2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(e+f\,x)}}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.70 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}\,c^{5/2}f}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}+\frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}$$

```
[Out] 1/16*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(5/2
)/f*2^(1/2)-1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+1/8*a*tan(f*x+e)/c/f/
(c-c*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4042, 3881, 3880, 209}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}\,c^{5/2}f}+\frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]])/(8*Sq
rt[2]*c^(5/2)*f) - (a*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(5/2)) + (a*T
an[e + f*x])/(8*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{a\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c}$$

$$= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} - \frac{a\int \frac{}{\sqrt{c-}}$$

$$= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} + \frac{a\mathrm{Subst}\Big(\int}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}\,c^{5/2}f} - \frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{}{8}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.13, size = 309, normalized size = 2.73

$$\frac{a\left(-\frac{i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^5\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{5/2}}+16\csc\left(\frac{e}{2}\right)\sec^3(e+fx)\sin\left(\frac{fx}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)-16\cot\left(\frac{e}{2}\right)\sec^3(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)-56\csc\left(\frac{e}{2}\right)\sec^3(e+fx)\sin\left(\frac{fx}{2}\right)\sin^3\left(\frac{1}{2}(e+fx)\right)+56\cot\left(\frac{e}{2}\right)\sec^3(e+fx)\sin^4\left(\frac{1}{2}(e+fx)\right)-48\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)\sec^3(e+fx)\sin^5\left(\frac{1}{2}(e+fx)\right)+48\sec^3(e+fx)\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^5\left(\frac{1}{2}(e+fx)\right)\right)}{16c^2f(-1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(((-I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^5*ArcTanh[(1 + E^(I*(e + f*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*I)*(e + f*x)))^(5/2) + 1
6*Csc[e/2]*Sec[e + f*x]^3*Sin[(f*x)/2]*Sin[(e + f*x)/2] - 16*Cot[e/2]*Sec[e
 + f*x]^3*Sin[(e + f*x)/2]^2 - 56*Csc[e/2]*Sec[e + f*x]^3*Sin[(f*x)/2]*Sin[
(e + f*x)/2]^3 + 56*Cot[e/2]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^4 - 48*Cos[e/2
```

```
]*Cos[(f*x)/2]*Sec[e + f*x]^3*Sin[(e + f*x)/2]^5 + 48*Sec[e + f*x]^3*Sin[e/
2]*Sin[(f*x)/2]*Sin[(e + f*x)/2]^5))/(16*c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(94) = 188$.
time = 1.96, size = 308, normalized size = 2.73

method	result
default	$-\frac{a(-1+\cos(fx+e))^3\left((\cos^2(fx+e))\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}+4\cos(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}-(\cos^2(fx+e))\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-(\cos$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2*a/f*(-1+cos(f*x+e))^3*(cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/
2)+4*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)-cos(f*x+e)^2*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2))+3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+2*cos(f*x+e)*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)+2*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2))-(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-arctan(1/(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)^5/(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2), x)
```

Fricas [**A**]
time = 4.37, size = 439, normalized size = 3.88

$$\left[-\frac{\sqrt{2}\left(a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)+a\right)\sqrt{-c}\,\log\left(-\frac{2\,\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}-(3\,c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin\left(fx+e\right)-4\left(3\,a\cos\left(fx+e\right)^3+4\,a\cos\left(fx+e\right)^2+a\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{32\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)},\ -\frac{\sqrt{2}\left(a\cos\left(fx+e\right)^2-2\,a\cos\left(fx+e\right)+a\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin\left(fx+e\right)-2\left(3\,a\cos\left(fx+e\right)^3+4\,a\cos\left(fx+e\right)^2+a\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{16\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/32*(sqrt(2)*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-c)*log(-(2*
sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e)) - (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*si
n(f*x + e)))*sin(f*x + e) - 4*(3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)
^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), -1/16*(sqrt(2)*(a*cos(f*x
 + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(
3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3
*f)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec{\left(e+fx \right)}}{c^{2}\sqrt{-c\sec{\left(e+fx \right)}+c}\sec^{2}{\left(e+fx \right)}-2c^{2}\sqrt{-c\sec{\left(e+fx \right)}+c}\sec{\left(e+fx \right)}+c^{2}\sqrt{-c\sec{\left(e+fx \right)}+c}}\, dx+\int \frac{\sec^{2}{\left(e+fx \right)}}{c^{2}\sqrt{-c\sec{\left(e+fx \right)}+c}\sec^{2}{\left(e+fx \right)}-2c^{2}\sqrt{-c\sec{\left(e+fx \right)}+c}\sec{\left(e+fx \right)}+c^{2}\sqrt{-c\sec{\left(e+fx \right)}+c}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 -
2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) +
 c)), x) + Integral(sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e +
 f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec
(e + f*x) + c)), x))
```

Giac [**A**]
time = 1.23, size = 104, normalized size = 0.92

$$-\frac{\sqrt{2}\left(a\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{\sqrt{c}}\right)+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}ac-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\,ac^{2}}{c^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}}\right)}{16\,c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/16*sqrt(2)*(a*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))
 + ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a*c - sqrt(c*tan(1/2*f*x + 1/2*e)^
2 - c)*a*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))/(c^3*f)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(e+fx)}}{\cos(e+fx)\left(c-\frac{c}{\cos(e+fx)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.71 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=171

$$-\frac{256c^4(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{231f}-\frac{8c^2(a-$$

```
[Out] -8/33*c^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/11*c*(a+
a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/1155*c^4*(a+a*sec(f
*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/231*c^3*(a+a*sec(f*x+e))^2*
(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.31, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4040, 4038}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{231f}-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{33f}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(231*f) - (8*c^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(33*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)*Ta
n[e + f*x])/(11*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
```

```
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/}}{11f}$$
$$= -\frac{8c^2(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}{33f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx}}{231f}$$
$$= -\frac{256c^4(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\sqrt{c-c\sec(e+fx)}} - \frac{64}{}$$

Mathematica [A]
time = 1.66, size = 88, normalized size = 0.51

$$\frac{2a^2c^3\cos^4\left(\frac{1}{2}(e+fx)\right)(-1930+3419\cos(e+fx)-1510\cos(2(e+fx))+533\cos(3(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\sqrt{c-c\sec(e+fx)}}{1155f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*a^2*c^3*Cos[(e + f*x)/2]^4*(-1930 + 3419*Cos[e + f*x] - 1510*Cos[2*(e +
f*x)] + 533*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Se
c[e + f*x]])/(1155*f)
```

Maple [A]
time = 2.47, size = 85, normalized size = 0.50

method	result	size
default	$-\frac{2a^2\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}(\sin^5(fx+e))(533(\cos^3(fx+e))-755(\cos^2(fx+e))+455\cos(fx+e)-105)}{1155f(-1+\cos(fx+e))^6\cos(fx+e)^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/1155*a^2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*sin(f*x+e)^5*(533*cos(f*
x+e)^3-755*cos(f*x+e)^2+455*cos(f*x+e)-105)/(-1+cos(f*x+e))^6/cos(f*x+e)^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 4.05, size = 157, normalized size = 0.92

$$\frac{2\left(533\,a^{2}c^{3}\cos\left(fx+e\right)^{6}+844\,a^{2}c^{3}\cos\left(fx+e\right)^{5}-211\,a^{2}c^{3}\cos\left(fx+e\right)^{4}-472\,a^{2}c^{3}\cos\left(fx+e\right)^{3}+295\,a^{2}c^{3}\cos\left(fx+e\right)^{2}+140\,a^{2}c^{3}\cos\left(fx+e\right)-105\,a^{2}c^{3}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{1155\,f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] 2/1155*(533*a^2*c^3*cos(f*x + e)^6 + 844*a^2*c^3*cos(f*x + e)^5 - 211*a^2*c
^3*cos(f*x + e)^4 - 472*a^2*c^3*cos(f*x + e)^3 + 295*a^2*c^3*cos(f*x + e)^2
 + 140*a^2*c^3*cos(f*x + e) - 105*a^2*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5987 deep
```

Giac [A]
time = 1.13, size = 109, normalized size = 0.64

$$-\frac{64\,\sqrt{2}\left(231\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{3}c^{3}+495\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c^{4}+385\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{5}+105\,c^{6}\right)a^{2}c^{3}}{1155\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{11}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] -64/1155*sqrt(2)*(231*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^3 + 495*(c*tan(1/2
*f*x + 1/2*e)^2 - c)^2*c^4 + 385*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 105*c
^6)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11/2)*f)
```

Mupad [B]
time = 14.40, size = 606, normalized size = 3.54

$$\frac{\left(\frac{a^2\,c^3\,2{}\mathrm{i}}{f}+\frac{a^2\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1066{}\mathrm{i}}{1155\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1}+\frac{\left(\frac{a^2\,c^3\,64{}\mathrm{i}}{11\,f}-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,64{}\mathrm{i}}{11\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^5}-\frac{\left(\frac{a^2\,c^3\,32{}\mathrm{i}}{3\,f}-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,608{}\mathrm{i}}{33\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^4}-\frac{\left(\frac{a^2\,c^3\,4{}\mathrm{i}}{f}+\frac{a^2\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,2932{}\mathrm{i}}{1155\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}+\frac{\left(\frac{a^2\,c^3\,16{}\mathrm{i}}{5\,f}+\frac{a^2\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,4272{}\mathrm{i}}{385\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^2}+\frac{\left(\frac{a^2\,c^3\,32{}\mathrm{i}}{7\,f}-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,4640{}\mathrm{i}}{231\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] (((a^2*c^3*2i)/f + (a^2*c^3*exp(e*1i + f*x*1i)*1066i)/(1155*f))*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1
) + (((a^2*c^3*64i)/(11*f) - (a^2*c^3*exp(e*1i + f*x*1i)*64i)/(11*f))*(c -
c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*
1i) - 1)*(exp(e*2i + f*x*2i) + 1)^5) - (((a^2*c^3*32i)/(3*f) - (a^2*c^3*exp
(e*1i + f*x*1i)*608i)/(33*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^4) - (
((a^2*c^3*4i)/f + (a^2*c^3*exp(e*1i + f*x*1i)*2932i)/(1155*f))*(c - c/(exp(
- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1
)*(exp(e*2i + f*x*2i) + 1)) + (((a^2*c^3*16i)/(5*f) + (a^2*c^3*exp(e*1i + f
*x*1i)*4272i)/(385*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^2*c
^3*32i)/(7*f) - (a^2*c^3*exp(e*1i + f*x*1i)*4640i)/(231*f))*(c - c/(exp(- e
*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(
exp(e*2i + f*x*2i) + 1)^3)
```

3.72 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{64c^3(a+a\sec(e+fx))^2\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{63f}-\frac{2c(a+a$$

```
[Out] -2/9*c*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/315*c^3*(a
+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/63*c^2*(a+a*sec(f*x
+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.23, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4040, 4038}

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2}{315f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{63f}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(63*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(9*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/}}{9f}$$
$$= -\frac{16c^2(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx}}{63f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^2\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{16}{}$$

Mathematica [A]
time = 1.26, size = 78, normalized size = 0.61

$$\frac{4a^2c^2\cos^4\left(\frac{1}{2}(e+fx)\right)(177-220\cos(e+fx)+107\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}}{315f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*a^2*c^2*Cos[(e + f*x)/2]^4*(177 - 220*Cos[e + f*x] + 107*Cos[2*(e + f*x)
])*Cot[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(315*f)
```

Maple [A]
time = 2.19, size = 75, normalized size = 0.59

method	result	size
default	$-\frac{2a^2\left(107\left(\cos^2(fx+e)\right)-110\cos(fx+e)+35\right)\left(\sin^5(fx+e)\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}}{315f(-1+\cos(fx+e))^5\cos(fx+e)^2}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315*a^2/f*(107*cos(f*x+e)^2-110*cos(f*x+e)+35)*sin(f*x+e)^5*(c*(-1+cos(f
*x+e))/cos(f*x+e))^(5/2)/(-1+cos(f*x+e))^5/cos(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")

[Out] -2/315*(315*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*((a^2*c^2*f*cos(2*f*x + 2*e)^4 + a^2*c^2*f*sin(2*f*x + 2*e)^4 + 4
*a^2*c^2*f*cos(2*f*x + 2*e)^3 + 6*a^2*c^2*f*cos(2*f*x + 2*e)^2 + 4*a^2*c^2*
f*cos(2*f*x + 2*e) + a^2*c^2*f + 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + 2*a^2*c^
2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*sin(2*f*x + 2*e)^2)*integrate((((cos(8*f*
x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f
*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*
x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f
*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*
x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f
*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*
f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f
*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f
*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*
x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f
*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4
+ (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e
)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(
2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
 + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*
f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*
f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e
)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*
```

```
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*
x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + si
n(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x +
 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(
2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2
*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*
f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f
*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
+ cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + ...
```

Fricas [A]
time = 4.13, size = 140, normalized size = 1.09

$$\frac{2\left(107\,a^{2}c^{2}\cos\left(fx+e\right)^{5}+211\,a^{2}c^{2}\cos\left(fx+e\right)^{4}+26\,a^{2}c^{2}\cos\left(fx+e\right)^{3}-118\,a^{2}c^{2}\cos\left(fx+e\right)^{2}-5\,a^{2}c^{2}\cos\left(fx+e\right)+35\,a^{2}c^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{315\,f\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2/315*(107*a^2*c^2*cos(f*x + e)^5 + 211*a^2*c^2*cos(f*x + e)^4 + 26*a^2*c^2
*cos(f*x + e)^3 - 118*a^2*c^2*cos(f*x + e)^2 - 5*a^2*c^2*cos(f*x + e) + 35*
a^2*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x
+ e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int\left(-2c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{3}\left(e+fx\right)\right)dx+\int c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{5}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-
2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3, x) + Integral(c**2*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x)**5, x))
```

Giac [A]
time = 1.01, size = 85, normalized size = 0.66

$$-\frac{32\,\sqrt{2}\left(63\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c^{3}+90\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{4}+35\,c^{5}\right)a^{2}c^{2}}{315\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] -32/315*sqrt(2)*(63*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^3 + 90*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^4 + 35*c^5)*a^2*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/
2)*f)
```

Mupad [B]
time = 7.99, size = 503, normalized size = 3.93

$$\frac{\left(\frac{a^2\,c^2\,2\mathrm{i}}{f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,214\mathrm{i}}{315\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}+\frac{\left(\frac{a^2\,c^2\,32\mathrm{i}}{9\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4}-\frac{\left(\frac{a^2\,c^2\,64\mathrm{i}}{7\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,320\mathrm{i}}{63\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}+\frac{\left(\frac{a^2\,c^2\,48\mathrm{i}}{5\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,368\mathrm{i}}{105\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2}-\frac{\left(\frac{a^2\,c^2\,16\mathrm{i}}{3\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,208\mathrm{i}}{315\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] (((a^2*c^2*2i)/f + (a^2*c^2*exp(e*1i + f*x*1i)*214i)/(315*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1)
+ (((a^2*c^2*32i)/(9*f) + (a^2*c^2*exp(e*1i + f*x*1i)*32i)/(9*f))*(c - c/(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i)
- 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^2*c^2*64i)/(7*f) + (a^2*c^2*exp(e*1
i + f*x*1i)*320i)/(63*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3) + (((a^
2*c^2*48i)/(5*f) + (a^2*c^2*exp(e*1i + f*x*1i)*368i)/(105*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)
*(exp(e*2i + f*x*2i) + 1)^2) - (((a^2*c^2*16i)/(3*f) + (a^2*c^2*exp(e*1i +
f*x*1i)*208i)/(315*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
```

3.73 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=85

$$-\frac{8c^2(a+a\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{7f}$$

```
[Out] -8/35*c^2*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/7*c*(a+a
*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.15, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4040, 4038}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(7*f
)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\ \text{ta}}{7f}$$
$$= -\frac{8c^2(a+a\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a}{}$$

Mathematica [A]
time = 0.78, size = 66, normalized size = 0.78

$$\frac{8a^2c\cos^4\left(\frac{1}{2}(e+fx)\right)(-5+9\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{35f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*c*Cos[(e + f*x)/2]^4*(-5 + 9*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e +
f*x]^3*Sqrt[c - c*Sec[e + f*x]])/(35*f)
```

Maple [A]
time = 2.32, size = 65, normalized size = 0.76

method	result	size
default	$-\frac{2a^2(9\cos(fx+e)-5)(\sin^5(fx+e))\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}{35f(-1+\cos(fx+e))^4\cos(fx+e)^2}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/35*a^2/f*(9*cos(f*x+e)-5)*sin(f*x+e)^5*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3
/2)/(-1+cos(f*x+e))^4/cos(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] 2/35*(35*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(3/4)*((a^2*c*f*cos(2*f*x + 2*e)^2 + a^2*c*f*sin(2*f*x + 2*e)^2 + 2*a^2*c*
```

```
f*cos(2*f*x + 2*e) + a^2*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
+ 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
 + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6
*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x +
2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6
*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x +
2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8
*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x +
4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
 + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8
*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2
 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos
(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x +
4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2
 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)
 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos
(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) +
 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) +
 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x +
 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(
2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3
 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (
cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x +
6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos
```

```
(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)
^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*c
os(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x
+ 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*
e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*c
os(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)
^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*c
os(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e)
 + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(s
in(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x +...
```

Fricas [**A**]
time = 3.88, size = 113, normalized size = 1.33

$$\frac{2\left(9\,a^{2}c\cos\left(fx+e\right)^{4}+22\,a^{2}c\cos\left(fx+e\right)^{3}+12\,a^{2}c\cos\left(fx+e\right)^{2}-6\,a^{2}c\cos\left(fx+e\right)-5\,a^{2}c\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{35\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/35*(9*a^2*c*cos(f*x + e)^4 + 22*a^2*c*cos(f*x + e)^3 + 12*a^2*c*cos(f*x +
 e)^2 - 6*a^2*c*cos(f*x + e) - 5*a^2*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)\,dx+\int c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)\,dx+\int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{3}\left(e+fx\right)\right)\,dx+\int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{4}\left(e+fx\right)\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(c*sq
rt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-c*sqrt(-c*sec(e + f
```

```
*x) + c)*sec(e + f*x)**3, x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e
+ f*x)**4, x))
```

Giac [A]
time = 1.03, size = 58, normalized size = 0.68

$$-\frac{16\,\sqrt{2}\,\left(7\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^4+5\,c^5\right)a^2}{35\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -16/35*sqrt(2)*(7*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 5*c^5)*a^2/((c*tan(1
/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

Mupad [B]
time = 6.14, size = 384, normalized size = 4.52

$$\frac{\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left(\frac{a^2\,c\,2{}\mathrm{i}}{f}+\frac{a^2\,c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,18{}\mathrm{i}}{35\,f}\right)}{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1}-\frac{\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left(\frac{a^2\,c\,16{}\mathrm{i}}{7\,f}-\frac{a^2\,c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,16{}\mathrm{i}}{7\,f}\right)}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^3}-\frac{\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left(\frac{a^2\,c\,4{}\mathrm{i}}{f}-\frac{a^2\,c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,44{}\mathrm{i}}{35\,f}\right)}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}+\frac{\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left(\frac{a^2\,c\,24{}\mathrm{i}}{5\,f}-\frac{a^2\,c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,72{}\mathrm{i}}{35\,f}\right)}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*2i)/
f + (a^2*c*exp(e*1i + f*x*1i)*18i)/(35*f)))/(exp(e*1i + f*x*1i) - 1) - ((c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*16i)/(7*
f) - (a^2*c*exp(e*1i + f*x*1i)*16i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(
e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*((a^2*c*4i)/f - (a^2*c*exp(e*1i + f*x*1i)*44i)/(35*f)))/((exp(
e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i - f*x*1
i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*24i)/(5*f) - (a^2*c*exp(e*1i +
f*x*1i)*72i)/(35*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

3.74 $\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2c(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/5*c*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {4038}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.46, size = 55, normalized size = 1.34

$$\frac{8a^2\cos^4\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (8*a^2*Cos[(e + f*x)/2]^4*Cot[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[c - c*Sec[e
+ f*x]])/(5*f)
```

Maple [A]
time = 2.20, size = 55, normalized size = 1.34

method	result	size
default	$-\frac{2a^2\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,(\sin^5(fx+e))}{5f\cos(fx+e)^2(-1+\cos(fx+e))^3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5*a^2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^5/cos(f*x+e)^2/(
-1+cos(f*x+e))^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] 2/5*(5*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*(3*(a^2*f*cos(2*f*x + 2*e)^2 + a^2*f*sin(2*f*x + 2*e)^2 + 2*a^2*f*cos(
2*f*x + 2*e) + a^2*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*co
s(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos
(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*si
n(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*
cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*si
n(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*s
in(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*si
n(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*s
in(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*si
n(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*s
in(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (co
s(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*c
os(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*si
```

```
n(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/
(((2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x
 + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*co
s(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x
+ 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*si
n(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 +
6*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2
*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 +
 (2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x
+ 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos
(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x +
 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin
(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6
*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6
*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*
f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x
) - 2*(a^2*f*cos(2*f*x + 2*e)^2 + a^2*f*sin(2*f*x + 2*e)^2 + 2*a^2*f*cos(2*
f*x + 2*e) + a^2*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(
6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2
*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(
2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*co
s(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(
8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin
(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(
2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin
(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(
2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(
8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos
(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(
2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((
(2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x +
 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(
6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x +
4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(
4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*
(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*
```

```
e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f
*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (
2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x +
8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6
*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(40) = 80$.
time = 2.43, size = 91, normalized size = 2.22

$$\frac{2\left(a^{2}\cos\left(fx+e\right)^{3}+3\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)+a^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int 2\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)dx+\int\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{3}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**3, x))
```

Giac [**A**]
time = 0.95, size = 33, normalized size = 0.80

$$-\frac{8\sqrt{2}\,a^{2}c^{3}}{5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -8/5*sqrt(2)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*f)
```

Mupad [B]
time = 5.73, size = 93, normalized size = 2.27

$$\frac{2\,a^2\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}+1{}\mathrm{i}\right)}^5\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{5\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a^2*(exp(e*1i + f*x*1i)*1i + 1i)^5*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(
e*1i + f*x*1i)/2))^(1/2))/(5*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i)
 + 1)^2)
```

3.75 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=117

$$-\frac{4\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{16a^2\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\sqrt{c-c\sec(e+fx)}\,\tan(e+f}{3cf}$$

```
[Out] -4*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/f/c^(1/2)+16/3*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/3*a^2*(c-c*sec(f
*x+e))^(1/2)*tan(f*x+e)/c/f
```

Rubi [A]
time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4041, 3880, 209}

$$-\frac{4\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-4*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (4*a^2*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
```

```
x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(}{\sqrt{c-c\sec(e+}$$
$$= \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (4a^2$$
$$= \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{(8a^2}{}$$
$$= -\frac{4\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f} + \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.92, size = 173, normalized size = 1.48

$$\frac{4a^2e^{-\frac{1}{2}i(e+fx)}\sec(e+fx)\left(-3\sqrt{2}\,e^{-\frac{1}{2}i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\,\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)+\cos\left(\frac{1}{2}(e+fx)\right)(7+\sec(e+fx))\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^2*Sec[e + f*x]*((-3*Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 +
 E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/E^((I/2)*(e + f
*x)) + Cos[(e + f*x)/2]*(7 + Sec[e + f*x]))*(Cos[(e + f*x)/2] + I*Sin[(e +
f*x)/2])*Sin[(e + f*x)/2])/(3*E^((I/2)*(e + f*x))*f*Sqrt[c - c*Sec[e + f*x]
])
```

Maple [A]
time = 2.24, size = 145, normalized size = 1.24

method	result

default	$\frac{2a^2\left(3\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\cos(fx+e)+3\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}+7\cos(}{3f\cos(fx+e)^2\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3*a^2/f*(3*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(3/2)*cos(f*x+e)+3*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+7*cos(f*x+e)+1)*sin(f*x+e)/cos(
f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Fricas [A]
time = 3.21, size = 371, normalized size = 3.17

$$\left[\frac{2\left(3\sqrt{2}a^2c\sqrt{-\frac{1}{c}}\cos(fx+e)\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)-\left(7a^2\cos(fx+e)^2+8a^2\cos(fx+e)+a^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\right)}{3cf\cos(fx+e)\sin(fx+e)}, \frac{2\left(6\sqrt{2}a^2\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\cos(fx+e)\sin(fx+e)-\left(7a^2\cos(fx+e)^2+8a^2\cos(fx+e)+a^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\right)}{3cf\cos(fx+e)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a^2*c*sqrt(-1/c)*cos(f*x + e)*log(-(2*sqrt(2)*(cos(f*x + e)
^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*
cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x
+ e) - (7*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)*sin(f*x + e)), 2/3*(6*sqrt(2)*a^2*
sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - (7*a^2*cos(f*x + e)^2
```

```
+ 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*c
os(f*x + e)*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{2\sec^2(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{\sec^3(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(2*sec(
e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**3/sqrt(-
c*sec(e + f*x) + c), x))
```

Giac [**A**]
time = 1.17, size = 82, normalized size = 0.70

$$\frac{4\,a^2\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,c\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 4/3*a^2*(3*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(
c) + sqrt(2)*(3*c*tan(1/2*f*x + 1/2*e)^2 - 4*c)/(c*tan(1/2*f*x + 1/2*e)^2 -
 c)^(3/2))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^2}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.76 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}} dx$

Optimal. Leaf size=113

$$\frac{3\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{2a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{2a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 3*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)
/c^(3/2)/f-2*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-2*a^2*tan(f*x+e)/c/f/(
c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4042, 4041, 3880, 209}

$$\frac{3\sqrt{2}\,a^2\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (3*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*
x]])])/(c^(3/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[
e + f*x])^(3/2)) - (3*a^2*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
```

```
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(3a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}}{2c}$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \cdots$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} + \cdots$$

$$= \frac{3\sqrt{2}\,a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{(a^2+a^2\sec(e+fx)\cdots}{f(c-c\sec(e+\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.45, size = 184, normalized size = 1.63

$$\frac{a^2e^{-2i(e+fx)}\left(4\left(1+e^{3i(e+fx)}\right)-3\sqrt{2}\left(-1+e^{i(e+fx)}\right)^2\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\right)\sec^2(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)}{2cf(-1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] (a^2*(4*(1 + E^((3*I)*(e + f*x))) - 3*Sqrt[2]*(-1 + E^(I*(e + f*x)))^2*Sqrt
[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E
^((2*I)*(e + f*x))])])*Sec[e + f*x]^2*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2
```

```
])*Sin[(e + f*x)/2])/(2*c*E^((2*I)*(e + f*x))*f*(-1 + Sec[e + f*x])*Sqrt[c
- c*Sec[e + f*x]])
```

Maple [A]
time = 2.40, size = 144, normalized size = 1.27

method	result
default	$\frac{a^2\left(3\cos(fx+e)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-3\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-4\,\mathrm{cc}\right.}{f\cos(fx+e)^2\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] a^2/f*(3*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)-3*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-4*cos(f*x+e)+2)*sin(f*x+e)/cos(f*x+
e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Fricas [A]
time = 2.87, size = 400, normalized size = 3.54

$$\left[\frac{3\sqrt{2}\left(a^2c\cos(fx+e)-a^2c\right)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}+(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+4\left(2a^2\cos(fx+e)^2+a^2\cos(fx+e)-a^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}, -\frac{\frac{3\sqrt{2}\left(a^2c\cos(fx+e)-a^2c\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}}-2\left(2a^2\cos(fx+e)^2+a^2\cos(fx+e)-a^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/2*(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos
(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1
/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))
*sin(f*x + e) + 4*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
, -(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*arctan(sqrt(2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sq
rt(c) - 2*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{2\sec^2(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{\sec^3(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*
sqrt(-c*sec(e + f*x) + c)), x) + Integral(2*sec(e + f*x)**2/(-c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integral(s
ec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e
 + f*x) + c)), x))
```

Giac [A]
time = 2.57, size = 109, normalized size = 0.96

$$-\frac{a^2\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{2}\left(3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}+\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c\right)c}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -a^2*(3*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3/2)
+ sqrt(2)*(3*c*tan(1/2*f*x + 1/2*e)^2 - c)/(((c*tan(1/2*f*x + 1/2*e)^2 - c)
^(3/2) + sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c)*c))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.77 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{3a^2\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}-\frac{a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{5/2}}+\frac{5a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -3/8*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(5
/2)/f*2^(1/2)-a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+5/4*a^2*tan(f*x+e)/c/
f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4042, 3880, 209}

$$-\frac{3a^2\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,c^{5/2}f}+\frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-3*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(2*f*(c - c
*Sec[e + f*x])^(5/2)) + (3*a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(3
/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
```

```
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{(3a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,d}{4c}$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} +$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} -$$

$$= -\frac{3a^2\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} - \frac{(a^2+a^2\sec(e+fx))\tan}{2f(c-c\sec(e+fx}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.49, size = 359, normalized size = 3.07

$$-\frac{a^2e^{-\frac{1}{2}i(e+fx)}\csc\left(\frac{e}{2}\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\,(1+\sec(e+fx))^2\left(\frac{e^{-\frac{3ie}{2}}(-1+e^{ie})\left(\cos\left(\frac{fx}{2}\right)+i\sin\left(\frac{fx}{2}\right)\right)\left(-9ie^{ie}(1+e^{ie})\cos\left(\frac{fx}{2}\right)+i(1+e^{3ie})\cos\left(\frac{3fx}{2}\right)-9e^{ie}\sin\left(\frac{fx}{2}\right)+9e^{2ie}\sin\left(\frac{fx}{2}\right)+\sin\left(\frac{3fx}{2}\right)-e^{3ie}\sin\left(\frac{3fx}{2}\right)\right)}{16\sqrt{\sec(e+fx)}}+3\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\sin\left(\frac{e}{2}\right)\sin^4\left(\frac{1}{2}(e+fx)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{4c^2f(-1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] -1/4*(a^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])
^2*(((-1 + E^(I*e))*(Cos[(f*x)/2] + I*Sin[(f*x)/2])*((-9*I)*E^(I*e)*(1 + E^
(I*e))*Cos[(f*x)/2] + I*(1 + E^((3*I)*e))*Cos[(3*f*x)/2] - 9*E^(I*e)*Sin[(f
*x)/2] + 9*E^((2*I)*e)*Sin[(f*x)/2] + Sin[(3*f*x)/2] - E^((3*I)*e)*Sin[(3*f
*x)/2]))/(16*E^(((3*I)/2)*e)*Sqrt[Sec[e + f*x]]) + 3*Sqrt[E^(I*(e + f*x))/(
1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(
e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sin[e/2]*Sin[(e + f*x)/
2]^4)*Tan[(e + f*x)/2])/(c^2*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^2*Sq
rt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(100) = 200.
time = 2.43, size = 230, normalized size = 1.97

method	result
default	$-\frac{a^{2}\left(-1+\cos\left(fx+e\right)\right)^{3}\left(\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\frac{2\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}+3\left(\cos^{2}\left(fx+e\right)\right)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}}\right)-4\cos\left(fx+e\right)\sqrt{-\frac{}{c}}\right.}{f\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\sin\left(fx\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -a^2/f*(-1+cos(f*x+e))^3*(cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
+3*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-4*cos(f*x+e)
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-6*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2))-5*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*arctan(1/(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/s
in(f*x+e)^5/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2),
x)
```

Fricas [A]
time = 3.24, size = 463, normalized size = 3.96

$$\left[-\frac{3\sqrt{2}\left(a^{2}\cos\left(fx+e\right)^{2}-2a^{2}\cos\left(fx+e\right)+a^{2}\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+4\left(a^{2}\cos\left(fx+e\right)^{3}-4a^{2}\cos\left(fx+e\right)^{2}-5a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{16\left(c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)},\ \frac{3\sqrt{2}\left(a^{2}\cos\left(fx+e\right)^{2}-2a^{2}\cos\left(fx+e\right)+a^{2}\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\sqrt{c}\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\left(a^{2}\cos\left(fx+e\right)^{3}-4a^{2}\cos\left(fx+e\right)^{2}-5a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{8\left(c^{3}f\cos\left(fx+e\right)^{2}-2c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-c)*
log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 4*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x +
```

```
e)^2 - 5*a^2*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f
*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/8*(3*sqrt(
2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(c)*arctan(sqrt(2)*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))
*sin(f*x + e) - 2*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x + e)^2 - 5*a^2*cos(f*
x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2
*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{\sec(e+fx)}{c^{2}\sqrt{-c\sec(e+fx)+c}\sec^{2}(e+fx)-2c^{2}\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^{2}\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{2\sec^{2}(e+fx)}{c^{2}\sqrt{-c\sec(e+fx)+c}\sec^{2}(e+fx)-2c^{2}\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^{2}\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{\sec^{3}(e+fx)}{c^{2}\sqrt{-c\sec(e+fx)+c}\sec^{2}(e+fx)-2c^{2}\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^{2}\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2
 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x
) + c)), x) + Integral(2*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-
c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**3/(c**2*sqrt(-c*sec(e + f
*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) +
c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A]
time = 1.39, size = 106, normalized size = 0.91

$$\frac{\sqrt{2}\left(3\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{\sqrt{c}}\right)+\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}c+5\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\,c^{2}}{c^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}}\right)a^{2}}{8c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/8*sqrt(2)*(3*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)) +
 (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 5*sqrt(c*tan(1/2*f*x + 1/2*e)^
2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))*a^2/(c^3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^{2}}{\cos\left(e+f\,x\right)\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.78 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=164

$$-\frac{a^2\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f}-\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}-16c$$

```
[Out] -1/32*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(
7/2)/f*2^(1/2)-1/3*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)
+1/4*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)-1/16*a^2*tan(f*x+e)/c^2/f/(c
-c*sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.20, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4042, 3881, 3880, 209}

$$-\frac{a^2\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f}-\frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)\,(a^2\sec(e+fx)+a^2)}{3f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/16*(a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])
])/(Sqrt[2]*c^(7/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c -
c*Sec[e + f*x])^(7/2)) + (a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(5/
2)) - (a^2*Tan[e + f*x])/(16*c^2*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
```

```
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx}{2c}$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} +$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} -$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} -$$

$$= -\frac{a^2\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}\,c^{7/2}f} - \frac{(a^2+a^2\sec(e+fx))\,\mathrm{ta}}{3f(c-c\sec(e+f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.74, size = 398, normalized size = 2.43

$$\frac{a^2e^{-\frac{1}{2}i(e+fx)}\csc\left(\frac{e}{2}\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{3}{2}}(e+fx)(1+\sec(e+fx))^2\left(-3\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\sin\left(\frac{e}{2}\right)\sin^6\left(\frac{1}{2}(e+fx)\right)+e^{\frac{ie}{2}}\sqrt{\sec(e+fx)}\left(-4i+4ie^{ifx}-\frac{1}{8}e^{\frac{ifx}{2}}\cos\left(\frac{e}{2}\right)(-57+36\cos(e+fx)-43\cos(2(e+fx)))\sin\left(\frac{1}{2}(e+fx)\right)-\frac{7}{2}e^{\frac{ifx}{2}}\csc\left(\frac{fx}{2}\right)\sin(e)\sin(fx)\sin^6\left(\frac{1}{2}(e+fx)\right)+e^{\frac{ifx}{2}}\sin\left(\frac{fx}{2}\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\left(34-43\sin^2\left(\frac{1}{2}(e+fx)\right)+14\sin^2\left(\frac{e}{2}\right)\sin^4\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{24c^3f(-1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),
x]

[Out] (a^2*Csc[e/2]*Sec[(e + f*x)/2]^3*Sec[e + f*x]^(3/2)*(1 + Sec[e + f*x])^2*(-
3*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*
```

```
x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]
*Sin[e/2]*Sin[(e + f*x)/2]^6 + E^((I/2)*e)*Sqrt[Sec[e + f*x]]*(-4*I + (4*I)
*E^(I*f*x) - (E^((I/2)*f*x)*Cos[e/2]*(-57 + 36*Cos[e + f*x] - 43*Cos[2*(e +
 f*x)])*Sin[(e + f*x)/2])/8 - (7*E^((I/2)*f*x)*Csc[(f*x)/2]*Sin[e]*Sin[f*x]
*Sin[(e + f*x)/2]^6)/2 + E^((I/2)*f*x)*Sin[(f*x)/2]*Sin[(e + f*x)/2]^2*(34
- 43*Sin[(e + f*x)/2]^2 + 14*Sin[e/2]^2*Sin[(e + f*x)/2]^4)))*Tan[(e + f*x)
/2])/(24*c^3*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e +
 f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(141) = 282$.
time = 2.48, size = 402, normalized size = 2.45

method	result
default	$-\frac{a^2(-1+\cos(fx+e))^4\left(5\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}(\cos^3(fx+e))+15(\cos^2(fx+e))\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}+3\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^3(fx+e)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*a^2/f*(-1+cos(f*x+e))^4*(5*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)*cos(f*
x+e)^3+15*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+3*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3+3*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(1/2))*cos(f*x+e)^3+27*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)-
9*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-9*cos(f*x+e)^2*arctan(1
/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+17*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3
/2)+9*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+9*cos(f*x+e)*arctan(1
/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)-3*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos
(f*x+e))^(7/2)/sin(f*x+e)^7/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.64, size = 557, normalized size = 3.40

$$\left[-\frac{3\sqrt{2}\left(a^{2}\cos\left(fx+e\right)^{3}-3\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)-a^{2}\right)\sqrt{-c}\,\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin\left(fx+e\right)-4\left(7\,a^{2}\cos\left(fx+e\right)^{4}+29\,a^{2}\cos\left(fx+e\right)^{3}+25\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{192\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-c^{4}f\right)\sin\left(fx+e\right)},\ \frac{3\sqrt{2}\left(a^{2}\cos\left(fx+e\right)^{3}-3\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)-a^{2}\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin\left(fx+e\right)+2\left(7\,a^{2}\cos\left(fx+e\right)^{4}+29\,a^{2}\cos\left(fx+e\right)^{3}+25\,a^{2}\cos\left(fx+e\right)^{2}+3\,a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{96\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-c^{4}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f
*x + e) - a^2)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt
(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f
*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*(7*a^2*cos(f*x
 + e)^4 + 29*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*
f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/96*(3*sqr
t(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)
*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e
)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*(7*a^2*cos(f*x + e)^4 + 29*a^2*c
os(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 +
 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{\sec\left(e+fx\right)}{-c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\left(e+fx\right)+3c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)-3c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^{3}\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{2\sec^{2}\left(e+fx\right)}{-c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\left(e+fx\right)+3c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)-3c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^{3}\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{\sec^{3}\left(e+fx\right)}{-c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\left(e+fx\right)+3c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)-3c^{3}\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^{3}\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**
3 + 3*c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e
 + f*x) + c)*sec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x) + Integral(
2*sec(e + f*x)**2/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*c**3
*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e + f*x) +
c)*sec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*
x)**3/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*c**3*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f
*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A]
time = 1.26, size = 139, normalized size = 0.85

$$\frac{\sqrt{2}\left(3\,a^{2}\sqrt{c}\,\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{\sqrt{c}}\right)+\frac{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}a^{2}c+8\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}a^{2}c^{2}-3\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,a^{2}c^{3}}{c^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}\right)}{96\,c^{4}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] 1/96*sqrt(2)*(3*a^2*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(
c)) + (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*a^2*c + 8*(c*tan(1/2*f*x + 1/
2*e)^2 - c)^(3/2)*a^2*c^2 - 3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^2*c^3)/(
c^3*tan(1/2*f*x + 1/2*e)^6))/(c^4*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)), x)
```

3.79 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=171

$$-\frac{256c^4(a+a\sec(e+fx))^3\tan(e+fx)}{3003f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{429f}-\frac{24c^2(}{}$$

```
[Out] -24/143*c^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/13*c*(
a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/3003*c^4*(a+a*sec
(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/429*c^3*(a+a*sec(f*x+e))^
3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]

time = 0.30, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4040, 4038}

$$-\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^3}{3003f\sqrt{c-c\sec(e+fx)}}-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{429f}-\frac{24c^2\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{143f}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(429*f) - (24*c^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[
e + f*x])/(143*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)*
Tan[e + f*x])/(13*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
```

```
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = -\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}{13f}$$
$$= -\frac{24c^2(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/}}{143f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}{429f}$$
$$= -\frac{256c^4(a+a\sec(e+fx))^3\tan(e+fx)}{3003f\sqrt{c-c\sec(e+fx)}} - \frac{64c}{}$$

Mathematica [A]
time = 2.55, size = 88, normalized size = 0.51

$$\frac{4a^3c^3\cos^6\left(\frac{1}{2}(e+fx)\right)(-3766+6285\cos(e+fx)-2842\cos(2(e+fx))+835\cos(3(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^6(e+fx)\sqrt{c-c\sec(e+fx)}}{3003f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (4*a^3*c^3*Cos[(e + f*x)/2]^6*(-3766 + 6285*Cos[e + f*x] - 2842*Cos[2*(e +
f*x)] + 835*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^6*Sqrt[c - c*Se
c[e + f*x]])/(3003*f)
```

Maple [A]
time = 2.40, size = 85, normalized size = 0.50

method	result	size
default	$\frac{2a^3(835(\cos^3(fx+e))-1421(\cos^2(fx+e))+945\cos(fx+e)-231)(\sin^7(fx+e))\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}}{3003f(-1+\cos(fx+e))^7\cos(fx+e)^3}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3003*a^3/f*(835*cos(f*x+e)^3-1421*cos(f*x+e)^2+945*cos(f*x+e)-231)*sin(f*
x+e)^7*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/(-1+cos(f*x+e))^7/cos(f*x+e)^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.68, size = 174, normalized size = 1.02

$$\frac{2\left(835\,a^3c^3\cos\left(fx+e\right)^7+1919\,a^3c^3\cos\left(fx+e\right)^6+271\,a^3c^3\cos\left(fx+e\right)^5-1637\,a^3c^3\cos\left(fx+e\right)^4-103\,a^3c^3\cos\left(fx+e\right)^3+973\,a^3c^3\cos\left(fx+e\right)^2+21\,a^3c^3\cos\left(fx+e\right)-231\,a^3c^3\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3003\,f\cos\left(fx+e\right)^6\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] 2/3003*(835*a^3*c^3*cos(f*x + e)^7 + 1919*a^3*c^3*cos(f*x + e)^6 + 271*a^3*
c^3*cos(f*x + e)^5 - 1637*a^3*c^3*cos(f*x + e)^4 - 103*a^3*c^3*cos(f*x + e)
^3 + 973*a^3*c^3*cos(f*x + e)^2 + 21*a^3*c^3*cos(f*x + e) - 231*a^3*c^3)*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^6*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7317 deep
```

Giac [A]
time = 1.26, size = 109, normalized size = 0.64

$$\frac{128\,\sqrt{2}\left(429\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3c^4+1001\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^5+819\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^6+231\,c^7\right)a^3c^3}{3003\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{13}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] 128/3003*sqrt(2)*(429*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^4 + 1001*(c*tan(1/
2*f*x + 1/2*e)^2 - c)^2*c^5 + 819*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^6 + 231*
c^7)*a^3*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(13/2)*f)
```

Mupad [B]
time = 14.67, size = 710, normalized size = 4.15

$$\frac{\left(\frac{a^3\,c^3\,2{}\mathrm{i}}{f}+\frac{a^3\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1670{}\mathrm{i}}{3003\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1}+\frac{\left(\frac{a^3\,c^3\,128{}\mathrm{i}}{13\,f}+\frac{a^3\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,128{}\mathrm{i}}{13\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^6}-\frac{\left(\frac{a^3\,c^3\,384{}\mathrm{i}}{11\,f}+\frac{a^3\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,3456{}\mathrm{i}}{143\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^5}-\frac{\left(\frac{a^3\,c^3\,8{}\mathrm{i}}{f}+\frac{a^3\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,2168{}\mathrm{i}}{3003\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}+\frac{\left(\frac{a^3\,c^3\,24{}\mathrm{i}}{f}+\frac{a^3\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,5464{}\mathrm{i}}{1001\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^2}+\frac{\left(\frac{a^3\,c^3\,160{}\mathrm{i}}{3\,f}+\frac{a^3\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,11360{}\mathrm{i}}{429\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^4}-\frac{\left(\frac{a^3\,c^3\,320{}\mathrm{i}}{7\,f}+\frac{a^3\,c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,46400{}\mathrm{i}}{3003\,f}\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] (((a^3*c^3*2i)/f + (a^3*c^3*exp(e*1i + f*x*1i)*1670i)/(3003*f))*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1
) + (((a^3*c^3*128i)/(13*f) + (a^3*c^3*exp(e*1i + f*x*1i)*128i)/(13*f))*(c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*
x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^6) - (((a^3*c^3*384i)/(11*f) + (a^3*c^3
*exp(e*1i + f*x*1i)*3456i)/(143*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^
5) - (((a^3*c^3*8i)/f + (a^3*c^3*exp(e*1i + f*x*1i)*2168i)/(3003*f))*(c - c
/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1
i) - 1)*(exp(e*2i + f*x*2i) + 1)) + (((a^3*c^3*24i)/f + (a^3*c^3*exp(e*1i +
 f*x*1i)*5464i)/(1001*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^
3*c^3*160i)/(3*f) + (a^3*c^3*exp(e*1i + f*x*1i)*11360i)/(429*f))*(c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) -
 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^3*c^3*320i)/(7*f) + (a^3*c^3*exp(e*1
i + f*x*1i)*46400i)/(3003*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

3.80 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=128

$$-\frac{64c^3(a+a\sec(e+fx))^3\tan(e+fx)}{693f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{99f}-\frac{2c(a+}{}$$

```
[Out] -2/11*c*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/693*c^3*(
a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/99*c^2*(a+a*sec(f*
x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.22, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4040, 4038}

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3}{693f\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{99f}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{11f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(99*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(11*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}{11f}$$
$$= -\frac{16c^2(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}{99f}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^3\tan(e+fx)}{693f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2}{}$$

Mathematica [A]
time = 1.56, size = 78, normalized size = 0.61

$$\frac{8a^3c^2\cos^6\left(\frac{1}{2}(e+fx)\right)(277-364\cos(e+fx)+151\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\sqrt{c-c\sec(e+fx)}}{693f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
[Out] (8*a^3*c^2*Cos[(e + f*x)/2]^6*(277 - 364*Cos[e + f*x] + 151*Cos[2*(e + f*x)
])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Sec[e + f*x]])/(693*f)
```

Maple [A]
time = 2.46, size = 75, normalized size = 0.59

method	result	size
default	$\frac{2a^3(151(\cos^2(fx+e))-182\cos(fx+e)+63)(\sin^7(fx+e))\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}}{693f(-1+\cos(fx+e))^6\cos(fx+e)^3}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
[Out] 2/693*a^3/f*(151*cos(f*x+e)^2-182*cos(f*x+e)+63)*sin(f*x+e)^7*(c*(-1+cos(f*
x+e))/cos(f*x+e))^(5/2)/(-1+cos(f*x+e))^6/cos(f*x+e)^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.34, size = 157, normalized size = 1.23

$$\frac{2\left(151\,a^3c^2\cos\left(fx+e\right)^6+422\,a^3c^2\cos\left(fx+e\right)^5+241\,a^3c^2\cos\left(fx+e\right)^4-236\,a^3c^2\cos\left(fx+e\right)^3-199\,a^3c^2\cos\left(fx+e\right)^2+70\,a^3c^2\cos\left(fx+e\right)+63\,a^3c^2\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{693\,f\cos\left(fx+e\right)^5\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2/693*(151*a^3*c^2*cos(f*x + e)^6 + 422*a^3*c^2*cos(f*x + e)^5 + 241*a^3*c^
2*cos(f*x + e)^4 - 236*a^3*c^2*cos(f*x + e)^3 - 199*a^3*c^2*cos(f*x + e)^2
+ 70*a^3*c^2*cos(f*x + e) + 63*a^3*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [A]
time = 1.18, size = 85, normalized size = 0.66

$$\frac{64\,\sqrt{2}\left(99\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^4+154\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^5+63\,c^6\right)a^3c^2}{693\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{11}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 64/693*sqrt(2)*(99*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^4 + 154*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^5 + 63*c^6)*a^3*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11
/2)*f)
```

Mupad [B]
time = 13.71, size = 607, normalized size = 4.74

$$\frac{\left(\frac{a^3\,c^2\,2\mathrm{i}}{f}+\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,302\mathrm{i}}{693\,f}\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}-\frac{\left(\frac{a^3\,c^2\,64\mathrm{i}}{11\,f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,64\mathrm{i}}{11\,f}\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^5}+\frac{\left(\frac{a^3\,c^2\,16\mathrm{i}}{f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,944\mathrm{i}}{231\,f}\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}+\frac{\left(\frac{a^3\,c^2\,160\mathrm{i}}{9\,f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1120\mathrm{i}}{99\,f}\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}-\frac{\left(\frac{a^3\,c^2\,20\mathrm{i}}{3\,f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,844\mathrm{i}}{693\,f}\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}-\frac{\left(\frac{a^3\,c^2\,160\mathrm{i}}{7\,f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,6880\mathrm{i}}{693\,f}\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] (((a^3*c^2*2i)/f + (a^3*c^2*exp(e*1i + f*x*1i)*302i)/(693*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1)
- (((a^3*c^2*64i)/(11*f) - (a^3*c^2*exp(e*1i + f*x*1i)*64i)/(11*f))*(c - c/
(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i
) - 1)*(exp(e*2i + f*x*2i) + 1)^5) + (((a^3*c^2*16i)/f - (a^3*c^2*exp(e*1i
+ f*x*1i)*944i)/(231*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i
)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^3
*c^2*160i)/(9*f) - (a^3*c^2*exp(e*1i + f*x*1i)*1120i)/(99*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)
*(exp(e*2i + f*x*2i) + 1)^4) - (((a^3*c^2*20i)/(3*f) - (a^3*c^2*exp(e*1i +
f*x*1i)*844i)/(693*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) - (((a^3*c^2
*160i)/(7*f) - (a^3*c^2*exp(e*1i + f*x*1i)*6880i)/(693*f))*(c - c/(exp(- e*
1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(e
xp(e*2i + f*x*2i) + 1)^3)
```

3.81 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=85

$$-\frac{8c^2(a+a\sec(e+fx))^3\tan(e+fx)}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{9f}$$

```
[Out] -8/63*c^2*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/9*c*(a+a
*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{4040, 4038\}$

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(9*f
)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,\mathrm{ta}}{9f}$$

$$= -\frac{8c^2(a+a\sec(e+fx))^3\tan(e+fx)}{63f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a}{}$$

Mathematica [A]
time = 1.04, size = 66, normalized size = 0.78

$$\frac{16a^3c\cos^6\left(\frac{1}{2}(e+fx)\right)(-7+11\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}}{63f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (16*a^3*c*Cos[(e + f*x)/2]^6*(-7 + 11*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e
+ f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(63*f)
```

Maple [A]
time = 2.30, size = 65, normalized size = 0.76

method	result	size
default	$\frac{2a^3(11\cos(fx+e)-7)(\sin^7(fx+e))\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}{63f(-1+\cos(fx+e))^5\cos(fx+e)^3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/63*a^3/f*(11*cos(f*x+e)-7)*sin(f*x+e)^7*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3
/2)/(-1+cos(f*x+e))^5/cos(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] 2/63*(63*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(1/4)*(3*(a^3*c*f*cos(2*f*x + 2*e)^4 + a^3*c*f*sin(2*f*x + 2*e)^4 + 4*a^3*
```

```
c*f*cos(2*f*x + 2*e)^3 + 6*a^3*c*f*cos(2*f*x + 2*e)^2 + 4*a^3*c*f*cos(2*f*x
 + 2*e) + a^3*c*f + 2*(a^3*c*f*cos(2*f*x + 2*e)^2 + 2*a^3*c*f*cos(2*f*x + 2
*e) + a^3*c*f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(10*f*x + 10*e)*cos(2*f*x
+ 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x
 + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10
*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin
(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(
2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos
(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*
cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - c
os(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) -
6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*
sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e)
 + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e
) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*
e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2
*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x +
 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x
+ 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x
 + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*
x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 +
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(10*f
*x + 10*e)^2 + 16*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(8*f*x + 8*e)^2 + 36*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 16*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x
+ 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(10*f*x + 10*e)^2 + 16*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 36*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 16*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (
2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(
2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 4*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + 6*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6
*e) + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(10*f*x + 10
*e) + 8*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 6*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6
*e) + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
```

```
cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e
) + 12*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 4*(cos(2
*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*
e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 8*(cos(2*f
*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 +
cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*
e)^3 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + 8*(sin(2*f*x + 2*e)
^3 + 6*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 12*(sin(2*f*x + 2*e)^3 + 4*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)
+ (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + 8*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 ...
```

Fricas [A]
time = 3.23, size = 128, normalized size = 1.51

$$\frac{2\left(11\,a^{3}c\cos\left(fx+e\right)^{5}+37\,a^{3}c\cos\left(fx+e\right)^{4}+38\,a^{3}c\cos\left(fx+e\right)^{3}+2\,a^{3}c\cos\left(fx+e\right)^{2}-17\,a^{3}c\cos\left(fx+e\right)-7\,a^{3}c\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{63\,f\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/63*(11*a^3*c*cos(f*x + e)^5 + 37*a^3*c*cos(f*x + e)^4 + 38*a^3*c*cos(f*x
+ e)^3 + 2*a^3*c*cos(f*x + e)^2 - 17*a^3*c*cos(f*x + e) - 7*a^3*c)*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)dx+\int 2c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)dx+\int\left(-2c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{4}\left(e+fx\right)\right)dx+\int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{5}\left(e+fx\right)\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*c*
sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-2*c*sqrt(-c*sec(e
```

```
 + f*x) + c)*sec(e + f*x)**4, x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**5, x))
```

Giac [A]
time = 0.98, size = 58, normalized size = 0.68

$$\frac{32\sqrt{2}\left(9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+7c^6\right)a^3}{63\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 32/63*sqrt(2)*(9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 7*c^6)*a^3/((c*tan(1/
2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

Mupad [B]
time = 9.19, size = 471, normalized size = 5.54

$$\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,c\,2\mathrm{i}}{f}+\frac{a^3\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,22\mathrm{i}}{63\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}-\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,c\,32\mathrm{i}}{9\,f}+\frac{a^3\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}-\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,c\,8\mathrm{i}}{3\,f}-\frac{a^3\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,200\mathrm{i}}{63\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,c\,32\mathrm{i}}{7\,f}+\frac{a^3\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,608\mathrm{i}}{63\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}-\frac{a^3\,c\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,160\mathrm{i}}{21\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*2i)/
f + (a^3*c*exp(e*1i + f*x*1i)*22i)/(63*f)))/(exp(e*1i + f*x*1i) - 1) - ((c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*32i)/(9*
f) + (a^3*c*exp(e*1i + f*x*1i)*32i)/(9*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(
e*2i + f*x*2i) + 1)^4) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*((a^3*c*8i)/(3*f) - (a^3*c*exp(e*1i + f*x*1i)*200i)/(63*f)))/(
(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i -
f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*32i)/(7*f) + (a^3*c*exp(e*
1i + f*x*1i)*608i)/(63*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) +
 1)^3) - (a^3*c*exp(e*1i + f*x*1i)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1
i + f*x*1i)/2))^(1/2)*160i)/(21*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*
2i) + 1)^2)
```

3.82 $\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=41

$$-\frac{2c(a+a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/7*c*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {4038}

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3}{7f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.74, size = 55, normalized size = 1.34

$$\frac{16a^3\cos^6\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (16*a^3*Cos[(e + f*x)/2]^6*Cot[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[c - c*Sec[e
 + f*x]])/(7*f)
```

Maple [A]
time = 2.31, size = 55, normalized size = 1.34

method	result	size
default	$\frac{2a^3\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,(\sin^7(fx+e))}{7f\cos(fx+e)^3(-1+\cos(fx+e))^4}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/7*a^3/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*sin(f*x+e)^7/cos(f*x+e)^3/(-
1+cos(f*x+e))^4
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] 2/7*(7*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
3/4)*(5*(a^3*f*cos(2*f*x + 2*e)^2 + a^3*f*sin(2*f*x + 2*e)^2 + 2*a^3*f*cos(
2*f*x + 2*e) + a^3*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*
cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4
*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e
)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*
e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*
e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x +
 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x +
 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x
 + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*
f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2
*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(
8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos
```

(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) + 5*(a^3*f*cos(2*f*x + 2*e)^2 + a^3*f*sin(2*f*x + 2*e)^2 + 2*a^3*f*cos(2*f*x + 2*e) + a^3*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*

```
e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10
*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x +
6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*
e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*co...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(40) = 80$.
time = 2.99, size = 105, normalized size = 2.56

$$\frac{2\left(a^{3}\cos\left(fx+e\right)^{4}+4\,a^{3}\cos\left(fx+e\right)^{3}+6\,a^{3}\cos\left(fx+e\right)^{2}+4\,a^{3}\cos\left(fx+e\right)+a^{3}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{7\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x +
 e)^3*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)\,dx+\int3\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)\,dx+\int3\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\left(e+fx\right)\,dx+\int\sqrt{-c\sec\left(e+fx\right)+c}\sec^{4}\left(e+fx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(3*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(3*sqrt(-c*sec(e + f*x)
 + c)*sec(e + f*x)**3, x) + Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)
**4, x))
```

Giac [A]
time = 1.00, size = 33, normalized size = 0.80

$$\frac{16\,\sqrt{2}\,a^{3}c^{4}}{7\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{7}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 16/7*sqrt(2)*a^3*c^4/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

Mupad [B]
time = 5.62, size = 375, normalized size = 9.15

$$\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,2\mathrm{i}}{f}+\frac{a^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2\mathrm{i}}{7\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1}-\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,8\mathrm{i}}{f}+\frac{a^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,8\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^2}+\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,4\mathrm{i}}{f}+\frac{a^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,36\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{a^3\,16\mathrm{i}}{7\,f}-\frac{a^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*2i)/f
+ (a^3*exp(e*1i + f*x*1i)*2i)/(7*f)))/(exp(e*1i + f*x*1i) - 1) - ((c - c/(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*8i)/f + (a^3*exp
(e*1i + f*x*1i)*8i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) +
 1)^2) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a
^3*4i)/f + (a^3*exp(e*1i + f*x*1i)*36i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(
exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a^3*16i)/(7*f) - (a^3*exp(e*1i + f*x*1i)*16i)/(7*f)))/((ex
p(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

3.83 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=164

$$-\frac{8\sqrt{2}\,a^3\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -8*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/f/c^(1/2)+8*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+2/5*a*(a+a*sec(f*x+e)
)^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+4/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)
/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4041, 3880, 209}

$$-\frac{8\sqrt{2}\,a^3\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f}+\frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{4\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\sqrt{c-c\sec(e+fx)}}+\frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-8*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (8*a^3*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]]) + (
4*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
```

```
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec}{\sqrt{c-c\sec(e+}} \\
&= \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4(a^3+a^3\sec(e+fx))\tan(e}{3f\sqrt{c-c\sec(e+fx)}} \\
&= \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4(a^3}{} \\
&= \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4(a^3}{} \\
&= -\frac{8\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f} + \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.55, size = 185, normalized size = 1.13

$$\frac{4a^3e^{-\frac{1}{2}i(e+fx)}\sec(e+fx)\left(-30\sqrt{2}\,e^{-\frac{1}{2}i(e+fx)}\sqrt{1+e^{2i(e+fx)}}\,\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)+\cos\left(\frac{1}{2}(e+fx)\right)\left(73+16\sec(e+fx)+3\sec^2(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)}{15f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^3*Sec[e + f*x]*((-30*Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1
+ E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/E^((I/2)*(e +
f*x)) + Cos[(e + f*x)/2]*(73 + 16*Sec[e + f*x] + 3*Sec[e + f*x]^2))*(Cos[(e
 + f*x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/(15*E^((I/2)*(e + f*x))*
f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.33, size = 206, normalized size = 1.26

method	result

default	$-\frac{2a^{3}\left(15\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}\left(\cos^{2}(fx+e)\right)+30\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}}{15f\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15*a^3/f*(15*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)^2+30*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)+15*arctan(1/(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)-73
*cos(f*x+e)^2-16*cos(f*x+e)-3)*sin(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1
/2)/cos(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Fricas [A]
time = 4.11, size = 407, normalized size = 2.48

$$\left[\frac{2\left(30\sqrt{2}a^{3}c\sqrt{-\frac{1}{c}}\cos(fx+e)^{2}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)-(73a^{3}\cos(fx+e)^{3}+89a^{3}\cos(fx+e)^{2}+19a^{3}\cos(fx+e)+3a^{3})\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\right)}{15cf\cos(fx+e)^{2}\sin(fx+e)}, \frac{2\left(60\sqrt{2}a^{3}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\cos(fx+e)^{2}\sin(fx+e)-(73a^{3}\cos(fx+e)^{3}+89a^{3}\cos(fx+e)^{2}+19a^{3}\cos(fx+e)+3a^{3})\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\right)}{15cf\cos(fx+e)^{2}\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*a^3*c*sqrt(-1/c)*cos(f*x + e)^2*log(-(2*sqrt(2)*(cos(f*x
+ e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) -
 (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(
f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos(f*x
+ e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)^2*
sin(f*x + e)), 2/15*(60*sqrt(2)*a^3*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x
```

```
+ e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)^2
*sin(f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos
(f*x + e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x +
e)^2*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{\sec (e+fx)}{\sqrt{-c\sec (e+fx)+c}}\,dx+\int \frac{3\sec ^{2}(e+fx)}{\sqrt{-c\sec (e+fx)+c}}\,dx+\int \frac{3\sec ^{3}(e+fx)}{\sqrt{-c\sec (e+fx)+c}}\,dx+\int \frac{\sec ^{4}(e+fx)}{\sqrt{-c\sec (e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(
e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(e + f*x)**3/sqrt
(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**4/sqrt(-c*sec(e + f*x) +
 c), x))
```

Giac [A]
time = 2.10, size = 111, normalized size = 0.68

$$\frac{8\,a^{3}\left(\frac{15\,\sqrt{2}\,\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\left(15\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}-5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c+3\,c^{2}\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}}\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 8/15*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqr
t(c) + sqrt(2)*(15*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 - 5*(c*tan(1/2*f*x + 1/
2*e)^2 - c)*c + 3*c^2)/(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos (e+f\,x)}\right)}^{3}}{\cos (e+f\,x)\,\sqrt{c-\frac{c}{\cos (e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.84 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=168

$$\frac{10\sqrt{2}\,a^3\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-$$

```
[Out] 10*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/c^(3/2)/f-a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-10*a^3
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(1/2)-5/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/
c/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4042, 4041, 3880, 209}

$$\frac{10\sqrt{2}\,a^3\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f}-\frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}-\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (10*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(c^(3/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec
[e + f*x])^(3/2)) - (10*a^3*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]]) -
(5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
```

```
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(5a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}}{2c}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{5(a^3+a^3\sec(e+fx))\tan(}{3cf\sqrt{c-c\sec(e+f}}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5}{}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5}{}$$

$$= \frac{10\sqrt{2}\,a^3\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{a(a+a\sec(e+fx}{f(c-c\sec(e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.94, size = 324, normalized size = 1.93

$$-\frac{a^3e^{-\frac{1}{2}i(e+fx)}\csc\left(\frac{e}{2}\right)\sec^2\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left(-\frac{ie^{\frac{ifx}{2}}(-1+e^{ie})\left(19-24e^{i(e+fx)}+34e^{2i(e+fx)}-24e^{3i(e+fx)}+19e^{4i(e+fx)}\right)\sqrt{\sec(e+fx)}}{2\left(-1+e^{i(e+fx)}\right)^2\left(1+e^{2i(e+fx)}\right)}-15\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\sec\left(\frac{1}{2}(e+fx)\right)\sin\left(\frac{e}{2}\right)\right)\tan^3\left(\frac{1}{2}(e+fx)\right)}{3cf(-1+\sec(e+fx))\sec^{\frac{3}{2}}(e+fx)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] -1/3*(a^3*Csc[e/2]*Sec[(e + f*x)/2]^2*(1 + Sec[e + f*x])^3*(((-1/2*I)*E^((I
/2)*f*x)*(-1 + E^(I*e))*(19 - 24*E^(I*(e + f*x)) + 34*E^((2*I)*(e + f*x)) -
 24*E^((3*I)*(e + f*x)) + 19*E^((4*I)*(e + f*x)))*Sqrt[Sec[e + f*x]])/((-1
+ E^(I*(e + f*x)))^2*(1 + E^((2*I)*(e + f*x)))) - 15*Sqrt[E^(I*(e + f*x))/(
1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(
e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sec[(e + f*x)/2]*Sin[e/
2])*Tan[(e + f*x)/2]^3)/(c*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])*Sec[e
+ f*x]^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.37, size = 157, normalized size = 0.93

method	result
default	$-\frac{a^{3}\left(15\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\left(\cos^{2}(fx+e)\right)-15\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}+3}{3f\cos(fx+e)^{3}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/3*a^3/f*(15*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(3/2)*cos(f*x+e)^2-15*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+38*cos(f*x+e)^2-24*cos(f*x
+e)-2)*sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Fricas [A]
time = 3.67, size = 466, normalized size = 2.77

$$\left[\frac{15\sqrt{2}\left(a^{3}c\cos(fx+e)^{2}-a^{3}c\cos(fx+e)\right)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}+(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+2\left(19a^{3}\cos(fx+e)^{3}+7a^{3}\cos(fx+e)^{2}-13a^{3}\cos(fx+e)-a^{3}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\left(c^{2}f\cos(fx+e)^{2}-c^{2}f\cos(fx+e)\right)\sin(fx+e)}, -\frac{2\left(\frac{15\sqrt{2}\left(a^{3}c\cos(fx+e)^{2}-a^{3}c\cos(fx+e)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}}-\left(19a^{3}\cos(fx+e)^{3}+7a^{3}\cos(fx+e)^{2}-13a^{3}\cos(fx+e)-a^{3}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\right)}{3\left(c^{2}f\cos(fx+e)^{2}-c^{2}f\cos(fx+e)\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/3*(15*sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*sqrt(-1/c)*log
((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*sqrt(-1/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1
)*sin(f*x + e)))*sin(f*x + e) + 2*(19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x +
e)^2 - 13*a^3*cos(f*x + e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/
((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e)), -2/3*(15*sqrt(2
)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*arctan(sqrt(2)*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)
/sqrt(c) - (19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x + e)^2 - 13*a^3*cos(f*x +
 e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e)^2
- c^2*f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{3\sec^{2}\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{3\sec^{3}\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx+\int \frac{\sec^{4}\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*
sqrt(-c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**2/(-c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integral(3
*sec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec
(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(-c*sqrt(-c*sec(e + f*x) + c
)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A]
time = 1.18, size = 124, normalized size = 0.74

$$-\frac{2\,a^{3}\left(\frac{15\,\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\,\sqrt{2}\left(6\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-7\,c\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}c}+\frac{3\,\sqrt{2}\,\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -2/3*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(
3/2) + 2*sqrt(2)*(6*c*tan(1/2*f*x + 1/2*e)^2 - 7*c)/((c*tan(1/2*f*x + 1/2*e
)^2 - c)^(3/2)*c) + 3*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^2*tan(1
/2*f*x + 1/2*e)^2))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^3}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.85 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{15a^3\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f}-\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}+\frac{5(a^3+a^3\sec(e+fx))\tan}{4cf(c-c\sec(e+fx)}$$

```
[Out] -15/4*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(
5/2)/f*2^(1/2)-1/2*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)
+5/4*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(3/2)+15/4*a^3*ta
n(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4042, 4041, 3880, 209}

$$-\frac{15a^3\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f}+\frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}}+\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{4cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-15*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(2*f*(c -
c*Sec[e + f*x])^(5/2)) + (5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(4*c*f*(
c - c*Sec[e + f*x])^(3/2)) + (15*a^3*Tan[e + f*x])/(4*c^2*f*Sqrt[c - c*Sec[
e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
```

```
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{(5a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}}{4c}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5(a^3+a^3\sec(e+fx))\tan(e}{4cf(c-c\sec(e+fx))^3}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5(a^3+a^3\sec(e+fx))\tan(e}{4cf(c-c\sec(e+fx))^3}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5(a^3+a^3\sec(e+fx))\tan(e}{4cf(c-c\sec(e+fx))^3}$$

$$= -\frac{15a^3\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}\,c^{5/2}f} - \frac{a(a+a\sec(e+fx))^2\,t}{2f(c-c\sec(e+f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.59, size = 263, normalized size = 1.51

$$-\frac{a^3e^{-\frac{1}{2}i(2e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left(120e^{\frac{ie}{2}}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)+\left(25\cos\left(\frac{3}{2}(e+fx)\right)-9\cos\left(\frac{5}{2}(e+fx)\right)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\left(\cos\left(e+\frac{fx}{2}\right)+i\sin\left(e+\frac{fx}{2}\right)\right)\right)\tan^5\left(\frac{1}{2}(e+fx)\right)}{32c^2f(-1+\sec(e+fx))^2\sqrt{\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] -1/32*(a^3*Sec[(e + f*x)/2]*(1 + Sec[e + f*x])^3*(120*E^((I/2)*e)*Sqrt[E^(I
*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTan
h[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])] + (25*Cos[
(3*(e + f*x))/2] - 9*Cos[(5*(e + f*x))/2])*Csc[(e + f*x)/2]^4*Sqrt[Sec[e +
f*x]]*(Cos[e + (f*x)/2] + I*Sin[e + (f*x)/2]))*Tan[(e + f*x)/2]^5)/(c^2*E^(
(I/2)*(2*e + f*x))*f*(-1 + Sec[e + f*x])^2*Sqrt[Sec[e + f*x]]*Sqrt[c - c*Se
c[e + f*x]])
```

Maple [A]
time = 2.52, size = 206, normalized size = 1.18

method	result
default	$-\frac{a^{3}\left(15\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^{2}(fx+e))-30\cos(fx+e)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\cdots}\right.}{4f\cos(fx+e)^{3}\left(\frac{c(-1\cdots}{c\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/4*a^3/f*(15*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-2*cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-30*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2))*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-18*cos(f*x+e)^2
+15*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)+34*cos(f*x+e)-8)*sin(f*x+e)/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/co
s(f*x+e))^(5/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.43, size = 475, normalized size = 2.73

$$\left[-\frac{15\sqrt{2}\left(a^{3}\cos(fx+e)^{2}-2a^{3}\cos(fx+e)+a^{3}\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+4\left(9a^{3}\cos(fx+e)^{3}-8a^{3}\cos(fx+e)^{2}-13a^{3}\cos(fx+e)+4a^{3}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{8\left(c^{3}f\cos(fx+e)^{2}-2c^{3}f\cos(fx+e)+c^{3}f\right)\sin(fx+e)},\ \frac{15\sqrt{2}\left(a^{3}\cos(fx+e)^{2}-2a^{3}\cos(fx+e)+a^{3}\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)-2\left(9a^{3}\cos(fx+e)^{3}-8a^{3}\cos(fx+e)^{2}-13a^{3}\cos(fx+e)+4a^{3}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{4\left(c^{3}f\cos(fx+e)^{2}-2c^{3}f\cos(fx+e)+c^{3}f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(-c)*
log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 4*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x
+ e)^2 - 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1
/4*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(c)*arct
an(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*si
n(f*x + e)))*sin(f*x + e) - 2*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2
- 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c
^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\,\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{3\sec^2(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\,\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{3\sec^3(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\,\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}}\,dx+\int \frac{\sec^4(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\,\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2
 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x
) + c)), x) + Integral(3*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-
c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**3/(c**2*sqrt(-c*sec(e +
 f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)
+ c**2*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(c**2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

Giac [A]
time = 1.18, size = 133, normalized size = 0.76

$$\frac{a^3\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{8\sqrt{2}}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2}+\frac{7\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}+9\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c}{c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/4*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(5
/2) + 8*sqrt(2)/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2) + (7*sqrt(2)*(c*ta
n(1/2*f*x + 1/2*e)^2 - c)^(3/2) + 9*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 -
 c)*c)/(c^4*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^3}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.86 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=142

$$\frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}}+\frac{32c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5af}+\frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af}+\frac{2c}{}$$

```
[Out] 12/5*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^(5/2)*t
an(f*x+e)/f/(a+a*sec(f*x+e))+128/5*c^4*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2
)+32/5*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f
```

Rubi [A]
time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4039, 3878, 3877}

$$\frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}}+\frac{32c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5af}+\frac{12c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (128*c^4*Tan[e + f*x])/(5*a*f*Sqrt[c - c*Sec[e + f*x]]) + (32*c^3*Sqrt[c -
c*Sec[e + f*x]]*Tan[e + f*x])/(5*a*f) + (12*c^2*(c - c*Sec[e + f*x])^(3/2)*
Tan[e + f*x])/(5*a*f) + (2*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(a
 + a*Sec[e + f*x]))
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
```

```
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(6c)\int \sec(e+fx)(c-cs}{a}$$
$$= \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af} + \frac{2c(c-c\sec(e+fx))^{5/}}{f(a+a\sec(e+}$$
$$= \frac{32c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5af} + \frac{12c^2(c-c\sec(e+fx))^3}{5af}$$
$$= \frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}} + \frac{32c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5af}$$

Mathematica [A]
time = 0.77, size = 86, normalized size = 0.61

$$-\frac{c^3(90+245\cos(e+fx)+86\cos(2(e+fx))+91\cos(3(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{10af(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
[Out] -1/10*(c^3*(90 + 245*Cos[e + f*x] + 86*Cos[2*(e + f*x)] + 91*Cos[3*(e + f*x
)])*Cot[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1 + Cos
[e + f*x]))
```

Maple [A]
time = 2.32, size = 83, normalized size = 0.58

method	result	size
default	$-\frac{2\left(91\left(\cos^3(fx+e)\right)+43\left(\cos^2(fx+e)\right)-7\cos(fx+e)+1\right)\cos(fx+e)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}}{5af\sin(fx+e)(-1+\cos(fx+e))^3}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/5/a/f*(91*cos(f*x+e)^3+43*cos(f*x+e)^2-7*cos(f*x+e)+1)*cos(f*x+e)*(c*(-1
+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)/(-1+cos(f*x+e))^3
```

Maxima [A]
time = 0.51, size = 175, normalized size = 1.23

$$\frac{8\left(16\sqrt{2}\,c^{\frac{7}{2}}-\frac{56\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{70\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{35\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{5\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)}{5\,af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 8/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2)
*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8)/(a*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)
*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(7/2))
```

Fricas [A]
time = 2.74, size = 95, normalized size = 0.67

$$-\frac{2\left(91\,c^3\cos\left(fx+e\right)^3+43\,c^3\cos\left(fx+e\right)^2-7\,c^3\cos\left(fx+e\right)+c^3\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\,af\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2/5*(91*c^3*cos(f*x + e)^3 + 43*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) +
c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)^2*sin(f*x +
e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [A]
time = 0.96, size = 108, normalized size = 0.76

$$-\frac{8\,\sqrt{2}\,c^{3}\left(\frac{5\,\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{a}-\frac{15\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c+5\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{2}+c^{3}}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}a}\right)}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -8/5*sqrt(2)*c^3*(5*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/a - (15*(c*tan(1/2*f
*x + 1/2*e)^2 - c)^2*c + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2 + c^3)/((c*ta
n(1/2*f*x + 1/2*e)^2 - c)^(5/2)*a))/f
```

Mupad [B]
time = 6.33, size = 164, normalized size = 1.15

$$-\frac{2\,c^{3}\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,86\mathrm{i}+{\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,245\mathrm{i}+{\mathrm{e}}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,180\mathrm{i}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,245\mathrm{i}+{\mathrm{e}}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,86\mathrm{i}+{\mathrm{e}}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,91\mathrm{i}+91\mathrm{i}\right)}{5\,a\,f\,\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(2*c^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(
e*1i + f*x*1i)*86i + exp(e*2i + f*x*2i)*245i + exp(e*3i + f*x*3i)*180i + ex
p(e*4i + f*x*4i)*245i + exp(e*5i + f*x*5i)*86i + exp(e*6i + f*x*6i)*91i + 9
1i))/(5*a*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

3.87 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}}+\frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3af}+\frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] 2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))+32/3*c^3*tan(f*x+e
)/a/f/(c-c*sec(f*x+e))^(1/2)+8/3*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f
```

Rubi [A]

time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4039, 3878, 3877}

$$\frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}}+\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (32*c^3*Tan[e + f*x])/(3*a*f*Sqrt[c - c*Sec[e + f*x]]) + (8*c^2*Sqrt[c - c*
Sec[e + f*x]]*Tan[e + f*x])/(3*a*f) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
```

```
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(4c)\int \sec(e+fx)(c-cs}{a}$$
$$= \frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3af} + \frac{2c(c-c\sec(e+fx))^{3/2}\,t}{f(a+a\sec(e+f}$$
$$= \frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}} + \frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3af}$$

Mathematica [A]
time = 0.43, size = 74, normalized size = 0.69

$$-\frac{c^2(21+20\cos(e+fx)+23\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{3af(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -1/3*(c^2*(21 + 20*Cos[e + f*x] + 23*Cos[2*(e + f*x)])*Cot[(e + f*x)/2]*Sec
[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1 + Cos[e + f*x]))
```

Maple [A]
time = 2.05, size = 73, normalized size = 0.68

method	result	size
default	$-\frac{2\left(23\left(\cos^2(fx+e)\right)+10\cos(fx+e)-1\right)\cos(fx+e)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}}{3af\sin(fx+e)(-1+\cos(fx+e))^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3/a/f*(23*cos(f*x+e)^2+10*cos(f*x+e)-1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/co
s(f*x+e))^(5/2)/sin(f*x+e)/(-1+cos(f*x+e))^2
```

Maxima [A]
time = 0.53, size = 147, normalized size = 1.36

$$-\frac{4\left(8\sqrt{2}\,c^{\frac{5}{2}}-\frac{20\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{3\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{3\,af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -4/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*sqrt(2)*
c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a*f*(sin(f*x + e)/(cos(f*x +
e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Fricas [A]
time = 2.67, size = 83, normalized size = 0.77

$$-\frac{2\left(23\,c^2\cos\left(fx+e\right)^2+10\,c^2\cos\left(fx+e\right)-c^2\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,af\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2/3*(23*c^2*cos(f*x + e)^2 + 10*c^2*cos(f*x + e) - c^2)*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int\frac{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x
) + Integral(-2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f*x
) + 1), x) + Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3/(sec(e
 + f*x) + 1), x))/a
```

Giac [A]
time = 0.89, size = 84, normalized size = 0.78

$$-\frac{4\sqrt{2}\,c^2\left(\frac{3\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{a}-\frac{6\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c+c^2}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -4/3*sqrt(2)*c^2*(3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/a - (6*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c + c^2)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a))/f
```

Mupad [B]
time = 4.23, size = 125, normalized size = 1.16

$$\frac{2\,c^2\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(e+f\,x\right)-44\,\sin\left(2\,e+2\,f\,x\right)+25\,\sin\left(3\,e+3\,f\,x\right)-26\,\sin\left(4\,e+4\,f\,x\right)+23\,\sin\left(5\,e+5\,f\,x\right)\right)}{3\,a\,f\,\left(\cos\left(3\,e+3\,f\,x\right)-2\,\cos\left(e+f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)+\cos\left(5\,e+5\,f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (2*c^2*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(2*sin(e + f*x) - 44*sin
(2*e + 2*f*x) + 25*sin(3*e + 3*f*x) - 26*sin(4*e + 4*f*x) + 23*sin(5*e + 5*
f*x)))/(3*a*f*(cos(3*e + 3*f*x) - 2*cos(e + f*x) - 2*cos(4*e + 4*f*x) + cos
(5*e + 5*f*x) + 2))
```

3.88 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] 4*c^2*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2)+2*c*(c-c*sec(f*x+e))^(1/2)*tan(
f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {4039, 3877}

$$\frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}}+\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (4*c^2*Tan[e + f*x])/(a*f*Sqrt[c - c*Sec[e + f*x]]) + (2*c*Sqrt[c - c*Sec[e
 + f*x]]*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{2c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(2c)\int\sec(e+fx)\sqrt{c-c\text{:}}}{a}$$
$$= \frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f(a+a\sec(e+fx))}$$

Mathematica [A]
time = 0.24, size = 54, normalized size = 0.75

$$-\frac{2c(1+3\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{af(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
[Out] (-2*c*(1 + 3*Cos[e + f*x])*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*x]])/(a*f*
(1 + Cos[e + f*x]))
```

Maple [A]
time = 2.46, size = 63, normalized size = 0.88

method	result	size
default	$-\frac{2(3\cos(fx+e)+1)\cos(fx+e)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}{af\sin(fx+e)(-1+\cos(fx+e))}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERB
OSE)
[Out] -2/a/f*(3*cos(f*x+e)+1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin
(f*x+e)/(-1+cos(f*x+e))
```

Maxima [A]
time = 0.52, size = 118, normalized size = 1.64

$$\frac{2\left(2\sqrt{2}\,c^{\frac{3}{2}} - \frac{3\sqrt{2}\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{\sqrt{2}\,c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 2*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a*f*(sin(f*x + e)
/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

Fricas [A]
time = 3.72, size = 54, normalized size = 0.75

$$-\frac{2\,(3\,c\cos\left(fx+e\right)+c)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{af\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2*(3*c*cos(f*x + e) + c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*sin(
f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\,\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x) +
 Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f*x) + 1),
x))/a
```

Giac [A]
time = 0.81, size = 60, normalized size = 0.83

$$-\frac{2\sqrt{2}\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,c}{a}-\frac{c^{2}}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -2*sqrt(2)*(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c/a - c^2/(sqrt(c*tan(1/2*f*
x + 1/2*e)^2 - c)*a))/f
```

Mupad [B]
time = 2.37, size = 77, normalized size = 1.07

$$-\frac{c\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(e+f\,x\right)+6\,\sin\left(2\,e+2\,f\,x\right)+2\,\sin\left(3\,e+3\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)\right)}{a\,f\,{\sin\left(2\,e+2\,f\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(c*(c - c/cos(e + f*x))^(1/2)*(2*sin(e + f*x) + 6*sin(2*e + 2*f*x) + 2*sin
(3*e + 3*f*x) + 3*sin(4*e + 4*f*x)))/(a*f*sin(2*e + 2*f*x)^2)
```

3.89 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=39

$$\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {4038}

$$\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.14, size = 29, normalized size = 0.74

$$-\frac{2\cot(e+fx)\sqrt{c-c\sec(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-2*Cot[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f)
```

Maple [A]
time = 2.55, size = 43, normalized size = 1.10

method	result	size
default	$-\frac{2\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\cos(fx+e)}{af\sin(fx+e)}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/a/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(40) = 80$.
time = 0.53, size = 90, normalized size = 2.31

$$-\frac{\sqrt{2}\,\sqrt{c}-\frac{\sqrt{2}\,\sqrt{c}\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}{af\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\,\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -(sqrt(2)*sqrt(c) - sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/(a
*f*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e
) + 1) - 1))
```

Fricas [A]
time = 3.39, size = 49, normalized size = 1.26

$$-\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\,\cos(fx+e)}{af\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(a*f*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x)/a
```

Giac [A]
time = 0.77, size = 59, normalized size = 1.51

$$-\frac{\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}(\cos(fx+e))}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*sgn(tan(1/2*f*x + 1/2*e)^3 + ta
n(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a*f)
```

Mupad [B]
time = 1.77, size = 40, normalized size = 1.03

$$-\frac{\sin(2\,e+2\,f\,x)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{a\,f\,\sin(e+f\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(sin(2*e + 2*f*x)*(c - c/cos(e + f*x))^(1/2))/(a*f*sin(e + f*x)^2)
```

3.90 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}+\frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/f*2^(1
/2)/c^(1/2)+tan(f*x+e)/f/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4045, 3880, 209}

$$\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -(ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2
]*a*Sqrt[c]*f)) + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f
*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
```

```
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}}{2a}$$

$$= \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{2c+x^2}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\,a\sqrt{c}\,f} + \frac{\tan(e}{f(a+a\sec(e+fx)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.50, size = 155, normalized size = 1.74

$$-\frac{i\left(-1+e^{2i(e+fx)}\right)\left(2\left(1+e^{2i(e+fx)}\right)-\sqrt{2}\left(1+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\right)}{2a\left(1+e^{2i(e+fx)}\right)^2 f(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] ((-1/2*I)*(-1 + E^((2*I)*(e + f*x)))*(2*(1 + E^((2*I)*(e + f*x))) - Sqrt[2]
*(1 + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e +
 f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])]))/(a*(1 + E^((2*I)*(e + f*
x)))^2*f*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.29, size = 107, normalized size = 1.20

method	result	size
default	$-\frac{(-1+\cos(fx+e))\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}+\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\right)}{af\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/a/f*(-1+cos(f*x+e))*((-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+arctan(1/(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(
f*x+e)/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)), x)
```

Fricas [**A**]
time = 3.78, size = 291, normalized size = 3.27

$$\left[\frac{\sqrt{2}\,c\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)-4\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{4\,acf\sin(fx+e)},\ \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)-2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{2\,acf\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*s
in(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt
(2)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x
+ e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\,\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec
(e + f*x) + c)), x)/a
```

Giac [A]
time = 0.58, size = 64, normalized size = 0.72

$$\frac{\sqrt{2}\left(\frac{\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{c}\right)}{2\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] 1/2*sqrt(2)*(arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c) - s
qrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/c)/(a*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2)), x)
```

3.91 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}-\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -3/8*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/c^(3/2
)/f*2^(1/2)-3/4*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(3/2)+tan(f*x+e)/f/(a+a*sec
(f*x+e))/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.15, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4045, 3881, 3880, 209}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f}-\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) - (3*Tan[e + f*x])/(4*a*f*(c - c*Sec[e + f*x])^(3/2)) +
 Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

```
&& IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}} + \frac{3\int \frac{\sec(e+f}{(c-c\sec(e+}}{2a}$$

$$= -\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-cs}$$

$$= -\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-cs}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,ac^{3/2}f} - \frac{3\tan(e+f}{4af(c-c\sec(e+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.41, size = 183, normalized size = 1.50

$$\frac{e^{-\frac{1}{2}i(e+fx)}\left(\frac{6\sqrt{2}\,e^{-i(e+fx)}\left(-1+e^{i(e+fx)}\right)^2\left(1+e^{i(e+fx)}\right)\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)}{\sqrt{1+e^{2i(e+fx)}}}-8(-3+\cos(e+fx))\right)\csc(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)}{32acf\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (((6*Sqrt[2]*(-1 + E^(I*(e + f*x)))^2*(1 + E^(I*(e + f*x)))*ArcTanh[(1 + E^
(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(E^(I*(e + f*x))*S
qrt[1 + E^((2*I)*(e + f*x))]) - 8*(-3 + Cos[e + f*x]))*Csc[e + f*x]*(Cos[(e
 + f*x)/2] + I*Sin[(e + f*x)/2]))/(32*a*c*E^((I/2)*(e + f*x))*f*Sqrt[c - c*
Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(105) = 210$.
time = 2.23, size = 266, normalized size = 2.18

method	result
default	$\frac{(-1+\cos(fx+e))^2 \left(\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}} \cos(fx+e) + \left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}} + \cos(fx+e) \left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}} - \left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}} - 3\cos(f \right.}{2af\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/a/f*(-1+cos(f*x+e))^2*((-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)+
(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)+cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+
1))^(3/2)-(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)-3*cos(f*x+e)*(-2*cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-3*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2))+3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*arctan(1/(-2*cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)),
x)
```

Fricas [A]
time = 4.09, size = 357, normalized size = 2.93

$$\left[-\frac{3\sqrt{2}\sqrt{-c}(\cos(fx+e)-1)\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+4\left(\cos(fx+e)^2-3\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{16\left(ac^2f\cos(fx+e)-ac^2f\right)\sin(fx+e)}, \frac{3\sqrt{2}\sqrt{c}(\cos(fx+e)-1)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)-2\left(\cos(fx+e)^2-3\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{8\left(ac^2f\cos(fx+e)-ac^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) - 1)*log((2*sqrt(2)*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*c
os(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x +
 e) + 4*(cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/8*(3*sqrt(2)*sqr
t(c)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(cos(f*x + e)^2 -
 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^2*f*cos(f*x
 + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqr
t(-c*sec(e + f*x) + c)), x)/a
```

Giac [A]
time = 0.63, size = 97, normalized size = 0.80

$$\frac{\sqrt{2}\left(3\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-2\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}-\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}\right)}{8ac^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] 1/8*sqrt(2)*(3*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)) -
 2*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/
tan(1/2*f*x + 1/2*e)^2)/(a*c^2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2)), x)
```

3.92 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{15\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}-\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+f}$$

```
[Out] -15/64*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/c^(5
/2)/f*2^(1/2)-5/8*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(5/2)+tan(f*x+e)/f/(a+a*s
ec(f*x+e))/(c-c*sec(f*x+e))^(5/2)-15/32*tan(f*x+e)/a/c/f/(c-c*sec(f*x+e))^(
3/2)
```

Rubi [A]
time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4045, 3881, 3880, 209}

$$-\frac{15\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f}-\frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}}-\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-15*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) - (5*Tan[e + f*x])/(8*a*f*(c - c*Sec[e + f*x])^(5/2))
 + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)) - (15*T
an[e + f*x])/(32*a*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}} + \frac{5\int \frac{\sec(e+f}{(c-c\sec(e+}}{2a}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-cs}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-cs}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-cs}$$

$$= -\frac{15\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}\,ac^{5/2}f} - \frac{5\tan(e+}{8af(c-c\sec(e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.01, size = 306, normalized size = 1.96

$$\frac{e^{-\frac{1}{2}i(e+fx)}\cos\left(\frac{1}{2}(e+fx)\right)\sec^{\frac{7}{2}}(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)\left(-\frac{1}{32}e^{-\frac{5}{2}i(e+fx)}\left(3+40e^{i(e+fx)}-51e^{2i(e+fx)}+80e^{3i(e+fx)}-51e^{4i(e+fx)}+40e^{5i(e+fx)}+3e^{6i(e+fx)}\right)\sqrt{\sec(e+fx)}-\frac{15}{2}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\sin^3\left(\frac{1}{2}(e+fx)\right)\sin(e+fx)\right)}{4ac^2f(-1+\sec(e+fx))^2(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e + f*x]^(7/2)*Sin[(e + f*x)/2]*(-1/32*((3 + 40*E^(I*
(e + f*x)) - 51*E^((2*I)*(e + f*x)) + 80*E^((3*I)*(e + f*x)) - 51*E^((4*I)*
(e + f*x)) + 40*E^((5*I)*(e + f*x)) + 3*E^((6*I)*(e + f*x)))*Sqrt[Sec[e + f
```

```
*x]])/E^(((5*I)/2)*(e + f*x)) - (15*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e +
 f*x)))]*Sqrt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[
2]*Sqrt[1 + E^((2*I)*(e + f*x))])]*Sin[(e + f*x)/2]^3*Sin[e + f*x])/2))/(4*
a*c^2*E^((I/2)*(e + f*x))*f*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x])*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(135) = 270$.
time = 2.26, size = 471, normalized size = 3.02

method	result
default	$-\frac{(-1+\cos(fx+e))^3\left(5(\cos^2(fx+e))\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}+4\cos(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}+3(\cos^2(fx+e))\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}-\left(-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/8/a/f*(-1+cos(f*x+e))^3*(5*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
7/2)+4*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)+3*cos(f*x+e)^2*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(5/2)-(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)-6*(-2*
cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)-5*cos(f*x+e)^2*(-2*cos(f*x+e)/(
cos(f*x+e)+1))^(3/2)+3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)+10*cos(f*x+e)*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+15*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)+15*cos(f*x+e)^2*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
)-5*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)-30*cos(f*x+e)*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)-30*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2
))+15*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+15*arctan(1/(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)^5/(-2*co
s(f*x+e)/(cos(f*x+e)+1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(5/2)),
x)
```

Fricas [A]

time = 3.21, size = 435, normalized size = 2.79

$$\left[-\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^{2}-2\cos\left(fx+e\right)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-4\left(3\cos\left(fx+e\right)^{3}+20\cos\left(fx+e\right)^{2}-15\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{128\left(ac^{3}f\cos\left(fx+e\right)^{2}-2ac^{3}f\cos\left(fx+e\right)+ac^{3}f\right)\sin\left(fx+e\right)},\ \frac{15\sqrt{2}\left(\cos\left(fx+e\right)^{2}-2\cos\left(fx+e\right)+1\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\sqrt{c}\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+2\left(3\cos\left(fx+e\right)^{3}+20\cos\left(fx+e\right)^{2}-15\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{64\left(ac^{3}f\cos\left(fx+e\right)^{2}-2ac^{3}f\cos\left(fx+e\right)+ac^{3}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/128*(15*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*s
qrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin
(f*x + e)))*sin(f*x + e) - 4*(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^
2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)), 1/64*(15*sqrt(2)*(cos(
f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*
(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) +
a*c^3*f)*sin(f*x + e))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\left(e+fx\right)-c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)-c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c^{2}\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c**
2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - c**2*sqrt(-c*sec(e + f*x) + c
)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x)/a
```

Giac [A]

time = 0.68, size = 128, normalized size = 0.82

$$\frac{\sqrt{2}\left(15\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{\sqrt{c}}\right)-8\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}-\frac{9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}c+7\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\,c^{2}}{c^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}}\right)}{64\,ac^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] 1/64*sqrt(2)*(15*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))
 - 8*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c) - (9*(c*tan(1/2*f*x + 1/2*e)^2 - c)
^(3/2)*c + 7*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2
*e)^4))/(a*c^3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2)), x)
```

3.93 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{16c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3a^2f}-\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))}+$$

```
[Out] -4*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+2/3*c*(c-c*
sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-64/3*c^4*tan(f*x+e)/a^2/f
/(c-c*sec(f*x+e))^(1/2)-16/3*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a^2/f
```

Rubi [A]
time = 0.22, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4039, 3878, 3877}

$$-\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3a^2f}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f\,(a^2\sec(e+fx)+a^2)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-64*c^4*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (16*c^3*Sqrt[c
- c*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f) - (4*c^2*(c - c*Sec[e + f*x])^(3/
2)*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^(
5/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
```

```
x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx &= \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx}{a+a\sec(e+fx)}}{a} \\ &= -\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^{5/}}{3f(a+a\sec(e-} \\ &= -\frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3a^2f} - \frac{4c^2(c-c\sec(e+fx))}{f\left(a^2+a^2\sec(e\right.} \\ &= -\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+}{3a^2f}\end{aligned}$$

Mathematica [A]
time = 0.93, size = 84, normalized size = 0.54

$$\frac{c^3(134+195\cos(e+fx)+138\cos(2(e+fx))+45\cos(3(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{6a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (c^3*(134 + 195*Cos[e + f*x] + 138*Cos[2*(e + f*x)] + 45*Cos[3*(e + f*x)])*
Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(6*a^2*f*(1 + Cos[e
 + f*x])^2)
```

Maple [A]
time = 2.21, size = 85, normalized size = 0.55

method	result	size
default	$-\frac{2(3\cos(fx+e)+1)\left(15\left(\cos^2(fx+e)\right)+18\cos(fx+e)-1\right)\left(\cos^2(fx+e)\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}}{3a^2f\sin(fx+e)^3(-1+\cos(fx+e))^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3/a^2/f*(3*cos(f*x+e)+1)*(15*cos(f*x+e)^2+18*cos(f*x+e)-1)*cos(f*x+e)^2*
(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)^3/(-1+cos(f*x+e))^2
```

Maxima [A]
time = 0.51, size = 202, normalized size = 1.30

$$-\frac{4\left(16\sqrt{2}\,c^{\frac{7}{2}}-\frac{56\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{70\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{35\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{4\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)}{3\,a^2 f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -4/3*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2
)*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*sqrt(2)*c^(7/2)*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 + sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)^(7/2))
```

Fricas [A]
time = 2.81, size = 111, normalized size = 0.72

$$\frac{2\left(45\,c^3\cos(fx+e)^3+69\,c^3\cos(fx+e)^2+15\,c^3\cos(fx+e)-c^3\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\left(a^2f\cos(fx+e)^2+a^2f\cos(fx+e)\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(45*c^3*cos(f*x + e)^3 + 69*c^3*cos(f*x + e)^2 + 15*c^3*cos(f*x + e) -
c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e)^2 + a^2*f
*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [A]
time = 1.04, size = 121, normalized size = 0.78

$$-\frac{4\sqrt{2}\,c^3\left(\frac{9\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c+c^2}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a^2}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a^4c^2+9\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,a^4c^3}{a^6c^3}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -4/3*sqrt(2)*c^3*((9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + c^2)/((c*tan(1/2*f*
x + 1/2*e)^2 - c)^(3/2)*a^2) - ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a^4*c^
2 + 9*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^4*c^3)/(a^6*c^3))/f
```

Mupad [B]
time = 6.02, size = 188, normalized size = 1.21

$$\frac{2\,c^3\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,138{}\mathrm{i}+{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,195{}\mathrm{i}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,268{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,195{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,138{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,45{}\mathrm{i}+45{}\mathrm{i}\right)}{3\,a^2\,f\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^3\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e
*1i + f*x*1i)*138i + exp(e*2i + f*x*2i)*195i + exp(e*3i + f*x*3i)*268i + ex
p(e*4i + f*x*4i)*195i + exp(e*5i + f*x*5i)*138i + exp(e*6i + f*x*6i)*45i +
45i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1)^3*(exp(e*1i + f*x*1i) - exp(e*2i +
f*x*2i) + exp(e*3i + f*x*3i) - 1))
```

3.94 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}+\frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] 2/3*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-16/3*c^3*tan(f
*x+e)/a^2/f/(c-c*sec(f*x+e))^(1/2)-8/3*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e
)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A]
time = 0.19, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4039, 3877}

$$-\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-16*c^3*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (8*c^2*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c
*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(4c)\int \frac{\sec(e+fx)(c-c\sec(e+fx}{a+a\sec(e+fx)}}{3a}$$
$$= -\frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^{3/2}}{3f(a+a\sec(e+}$$
$$= -\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+f}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

Mathematica [A]
time = 0.42, size = 68, normalized size = 0.55

$$\frac{c^2(17+36\cos(e+fx)+11\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{3a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (c^2*(17 + 36*Cos[e + f*x] + 11*Cos[2*(e + f*x)])*Cot[(e + f*x)/2]*Sqrt[c -
 c*Sec[e + f*x]])/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A]
time = 2.21, size = 75, normalized size = 0.61

method	result	size
default	$-\frac{2\left(11\left(\cos^2(fx+e)\right)+18\cos(fx+e)+3\right)\left(\cos^2(fx+e)\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}}{3a^2f\sin(fx+e)^3(-1+\cos(fx+e))}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3/a^2/f*(11*cos(f*x+e)^2+18*cos(f*x+e)+3)*cos(f*x+e)^2*(c*(-1+cos(f*x+e)
)/cos(f*x+e))^(5/2)/sin(f*x+e)^3/(-1+cos(f*x+e))
```

Maxima [A]
time = 0.51, size = 175, normalized size = 1.42

$$\frac{2\left(8\sqrt{2}\,c^{\frac{5}{2}} - \frac{20\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{2\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}\right)}{3\,a^2f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*sqrt(2)*c
^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - sqrt(2)*c^(5/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Fricas [A]
time = 2.74, size = 88, normalized size = 0.72

$$\frac{2\left(11\,c^{2}\cos\left(fx+e\right)^{2}+18\,c^{2}\cos\left(fx+e\right)+3\,c^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(11*c^2*cos(f*x + e)^2 + 18*c^2*cos(f*x + e) + 3*c^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.93, size = 99, normalized size = 0.80

$$-\frac{2\sqrt{2}\,c^{2}\left(\frac{3\,c}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,a^{2}}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}a^{4}c^{2}+6\,\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,a^{4}c^{3}}{a^{6}c^{3}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -2/3*sqrt(2)*c^2*(3*c/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^2) - ((c*tan(1/
2*f*x + 1/2*e)^2 - c)^(3/2)*a^4*c^2 + 6*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*
a^4*c^3)/(a^6*c^3))/f
```

Mupad [B]
time = 5.54, size = 136, normalized size = 1.11

$$\frac{2\,c^2\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,36{}\mathrm{i}+{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,34{}\mathrm{i}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,36{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,11{}\mathrm{i}+11{}\mathrm{i}\right)}{3\,a^2\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c^2*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e
*1i + f*x*1i)*36i + exp(e*2i + f*x*2i)*34i + exp(e*3i + f*x*3i)*36i + exp(e
*4i + f*x*4i)*11i + 11i))/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x
*1i) + 1)^3)
```

3.95 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=89

$$-\frac{4c^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] -4/3*c^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)+2/3*c*(c-
c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.15, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4039, 4038}

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2}-\frac{4c^2\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-4*c^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]
]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^
2)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(2c)\int \frac{\sec(e+fx)\sqrt{c-c\sec(}{a+a\sec(e+fx)}}{3a}$$

$$= -\frac{4c^2\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))\,\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(}{3f(a+}$$

Mathematica [A]
time = 0.26, size = 60, normalized size = 0.67

$$\frac{2c\cos(e+fx)(3+\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{3a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (2*c*Cos[e + f*x]*(3 + Cos[e + f*x])*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*
x]])/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A]
time = 2.46, size = 53, normalized size = 0.60

method	result	size
default	$-\frac{2(\cos(fx+e)+3)\left(\cos^2(fx+e)\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}{3a^2f\sin(fx+e)^3}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3/a^2/f*(cos(f*x+e)+3)*cos(f*x+e)^2*(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)
/sin(f*x+e)^3
```

Maxima [A]
time = 0.51, size = 118, normalized size = 1.33

$$-\frac{2\sqrt{2}\,c^{\frac{3}{2}} - \frac{3\sqrt{2}\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{\sqrt{2}\,c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}{3\,a^2f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -1/3*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + sqrt(2)*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^2*f*(sin(f*x
 + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(
3/2))
```

Fricas [A]
time = 4.02, size = 78, normalized size = 0.88

$$\frac{2\left(c\cos\left(fx+e\right)^{2}+3\,c\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec
(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/
(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.81, size = 60, normalized size = 0.67

$$\frac{\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}}{a^{2}}+\frac{3\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,c}{a^{2}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)/a^2 + 3*sqrt(2)*sqrt(c*ta
n(1/2*f*x + 1/2*e)^2 - c)*c/a^2)/f
```

Mupad [B]
time = 5.23, size = 134, normalized size = 1.51

$$\frac{2\,c\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,6\mathrm{i}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,6\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e*1
i + f*x*1i)*6i + exp(e*2i + f*x*2i)*2i + exp(e*3i + f*x*3i)*6i + exp(e*4i +
 f*x*4i)*1i + 1i))/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) +
1)^3)
```

3.96 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=41

$$\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2/3*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = $\{4038\}$

$$\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.15, size = 55, normalized size = 1.34

$$-\frac{\cos^2(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{6a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/6*(Cos[e + f*x]^2*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e +
 f*x]])/(a^2*f)
```

Maple [A]
time = 2.49, size = 53, normalized size = 1.29

method	result	size
default	$\frac{2(-1+\cos(fx+e))\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,(\cos^2(fx+e))}{3a^2 f\sin(fx+e)^3}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3/a^2/f*(-1+cos(f*x+e))*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)^2
/sin(f*x+e)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(40) = 80$.
time = 0.53, size = 117, normalized size = 2.85

$$-\frac{\sqrt{2}\sqrt{c} - \frac{2\sqrt{2}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{\sqrt{2}\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{6\,a^2 f\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\,\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -1/6*(sqrt(2)*sqrt(c) - 2*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a^2*f*sqrt(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Fricas [A]
time = 3.11, size = 65, normalized size = 1.59

$$-\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\,\cos(fx+e)^2}{3\,(a^2 f\cos(fx+e)+a^2 f)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] -2/3*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/((a^2*f*cos(f*x
 + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x)/a**2
```

Giac [A]
time = 0.76, size = 62, normalized size = 1.51

$$\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}(\cos\left(fx+e\right))}{6\,a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/6*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^3
 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a^2*c*f)
```

Mupad [B]
time = 5.30, size = 94, normalized size = 2.29

$$\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^2\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] ((exp(e*2i + f*x*2i)*1i + 1i)^2*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2))^(1/2)*2i)/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^3)
```

3.97 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=138

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{2f(a^2+a^2\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/4*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/f*2^
(1/2)/c^(1/2)+1/3*tan(f*x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+1/
2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4045, 3880, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}+\frac{\tan(e+fx)}{2f(a^2\sec(e+fx)+a^2)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqr
t[2]*a^2*Sqrt[c]*f) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*S
ec[e + f*x]]) + Tan[e + f*x]/(2*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e
 + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
```

```
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{}{(a+a\sec(e+f}}{}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{}{2f\left(a^2+a^2 s\right.}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{}{2f\left(a^2+a^2 s\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} + \frac{\tan}{3f(a+a\sec(e+f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.16, size = 259, normalized size = 1.88

$$\frac{2e^{-\frac{1}{2}i(e+fx)}\cos\left(\frac{1}{2}(e+fx)\right)\left(-3\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\sqrt{1+e^{2i(e+fx)}}\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)\cos^3\left(\frac{1}{2}(e+fx)\right)+\frac{1}{8}e^{-\frac{3}{2}i(e+fx)}\left(5+6e^{i(e+fx)}+10e^{2i(e+fx)}+6e^{3i(e+fx)}+5e^{4i(e+fx)}\right)\sqrt{\sec(e+fx)}\right)\sec^{\frac{5}{2}}(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)}{3a^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-3*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqr
t[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 +
E^((2*I)*(e + f*x))])]*Cos[(e + f*x)/2]^3 + ((5 + 6*E^(I*(e + f*x)) + 10*E^
((2*I)*(e + f*x)) + 6*E^((3*I)*(e + f*x)) + 5*E^((4*I)*(e + f*x)))*Sqrt[Sec
[e + f*x]])/(8*E^(((3*I)/2)*(e + f*x))))*Sec[e + f*x]^(5/2)*Sin[(e + f*x)/2
])/(3*a^2*E^((I/2)*(e + f*x))*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x
]])
```

Maple [A]
time = 2.30, size = 131, normalized size = 0.95

method	result	size

default	$\frac{(-1+\cos(fx+e))\left(\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}-3\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-3\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)\right)}{6a^2f\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6/a^2/f*(-1+cos(f*x+e))*((-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)-3*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)-3*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/(-2*cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*sqrt(-c*sec(f*x + e) + c)),
x)
```

Fricas [A]
time = 3.38, size = 359, normalized size = 2.60

$$\left[-\frac{3\sqrt{2}\sqrt{-c}\left(\cos\left(fx+e\right)+1\right)\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin\left(fx+e\right)+4\left(5\cos\left(fx+e\right)^2+3\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{24\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right)}, \frac{3\sqrt{2}\sqrt{c}\left(\cos\left(fx+e\right)+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin\left(fx+e\right)-2\left(5\cos\left(fx+e\right)^2+3\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{12\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) + 1)*log((2*sqrt(2)*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*c
os(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x +
 e) + 4*(5*cos(f*x + e)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/12*(3*sqrt(2)*
sqrt(c)*(cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x
```

```
 + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e
)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c*f*co
s(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + 2*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

Giac [A]
time = 0.70, size = 93, normalized size = 0.67

$$\frac{\sqrt{2}\left(\frac{3\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^4-3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^5}{c^6}\right)}{12\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
[Out] 1/12*sqrt(2)*(3*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c)
+ ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^4 - 3*sqrt(c*tan(1/2*f*x + 1/2*e)
^2 - c)*c^5)/c^6)/(a^2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^2\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2)), x)
```

3.98 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}-\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+$$

```
[Out] -5/16*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/c^(
3/2)/f*2^(1/2)-5/8*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(3/2)+1/3*tan(f*x+e)/f
/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2)+5/6*tan(f*x+e)/f/(a^2+a^2*sec(f*
x+e))/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.24, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4045, 3881, 3880, 209}

$$-\frac{5\text{ArcTan}\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f}-\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}}+\frac{5\tan(e+fx)}{6f(a^2\sec(e+fx)+a^2)(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-5*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) - (5*Tan[e + f*x])/(8*a^2*f*(c - c*Sec[e + f*x])^(3/2
)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)) +
 (5*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{5\int \frac{\ldots}{(a+a\sec\ldots}}{\ldots}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{\ldots}{6f\,(a^2+a\ldots}$$

$$= -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+f\ldots}{3f(a+a\sec(e+fx))^2(c-\ldots}$$

$$= -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+f\ldots}{3f(a+a\sec(e+fx))^2(c-\ldots}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}\,a^2c^{3/2}f} - \frac{5\tan(e+\ldots}{8a^2f(c-c\sec(e\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.39, size = 365, normalized size = 2.16

$$\frac{-\frac{15i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^3\left(1+e^{i(e+fx)}\right)^4\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{7/2}} - 48\cos^4\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{e}{2}\right)\sec^4(e+fx)\sin\left(\frac{fx}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right) + 48\cos^4\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{e}{2}\right)\sec^4(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right) + 32\cos\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sin^3\left(\frac{1}{2}(e+fx)\right) + 416\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sin^3\left(\frac{1}{2}(e+fx)\right) - 416\csc\left(\frac{1}{2}(e+fx)\right)\csc^4(2(e+fx))\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin^8(e+fx) - 40\sec(e+fx)\tan^3(e+fx)}{48a^2cf(-1+\sec(e+fx))(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),
x]
```

```
[Out] (((-15*I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^3*(1 + E^(I*(e + f*x)))^4*ArcTanh[
(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])])/(1 + E^((2*
```

```
I)*(e + f*x)))^(7/2) - 48*Cos[(e + f*x)/2]^4*Csc[e/2]*Sec[e + f*x]^4*Sin[(f
*x)/2]*Sin[(e + f*x)/2] + 48*Cos[(e + f*x)/2]^4*Cot[e/2]*Sec[e + f*x]^4*Sin
[(e + f*x)/2]^2 + 32*Cos[(e + f*x)/2]*Sec[e + f*x]^4*Sin[(e + f*x)/2]^3 + 4
16*Cos[e/2]*Cos[(f*x)/2]*Cos[(e + f*x)/2]^4*Sec[e + f*x]^4*Sin[(e + f*x)/2]
^3 - 416*Csc[(e + f*x)/2]*Csc[2*(e + f*x)]^4*Sin[e/2]*Sin[(f*x)/2]*Sin[e +
f*x]^8 - 40*Sec[e + f*x]*Tan[e + f*x]^3)/(48*a^2*c*f*(-1 + Sec[e + f*x])*(1
 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(146) = 292$.
time = 2.46, size = 320, normalized size = 1.89

method	result
default	$-\frac{(-1+\cos(fx+e))^2\left(3\cos(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}+3\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}+3\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}\cos(fx+e)-3\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/12/a^2/f*(-1+cos(f*x+e))^2*(3*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(7/2)+3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)+3*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(5/2)*cos(f*x+e)-3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)-5*cos(f*x+e)*(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+5*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+15
*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+15*cos(f*x+e)*arctan(1/(-2
*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-15*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
15*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f
*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(3/2))
, x)
```

Fricas [A]
time = 4.57, size = 399, normalized size = 2.36

$$\left[-\frac{15 \sqrt{2} \left(\cos\left(fx+e\right)^{2}-1\right) \sqrt{-c} \log\left(\frac{2 \sqrt{2} \left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right) \sqrt{-c} \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3 c \cos\left(fx+e\right)+c\right) \sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right) \sin\left(fx+e\right)}\right) \sin\left(fx+e\right)+4 \left(13 \cos\left(fx+e\right)^{3}-10 \cos\left(fx+e\right)^{2}-15 \cos\left(fx+e\right)\right) \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{96 \left(a^{2} c^{2} f \cos\left(fx+e\right)^{2}-a^{2} c^{2} f\right) \sin\left(fx+e\right)}, \frac{15 \sqrt{2} \left(\cos\left(fx+e\right)^{2}-1\right) \sqrt{c} \arctan\left(\frac{\sqrt{2} \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}} \cos\left(fx+e\right)}{\sqrt{c} \sin\left(fx+e\right)}\right) \sin\left(fx+e\right)-2 \left(13 \cos\left(fx+e\right)^{3}-10 \cos\left(fx+e\right)^{2}-15 \cos\left(fx+e\right)\right) \sqrt{\frac{c \cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{48 \left(a^{2} c^{2} f \cos\left(fx+e\right)^{2}-a^{2} c^{2} f\right) \sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/96*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x +
e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*
c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*
x + e) + 4*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*
sin(f*x + e)), 1/48*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(c)*arctan(sqrt(2)
*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)
))*sin(f*x + e) - 2*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^
2*c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{3}\left(e+fx\right)-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c*sqr
t(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqrt(-c*sec(e + f*x) + c)*sec(e
+ f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

Giac [A]
time = 0.68, size = 129, normalized size = 0.76

$$\frac{\sqrt{2}\left(15\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{\sqrt{c}}\right)-\frac{3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}+\frac{2\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)^{\frac{3}{2}}c^{2}-6\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c}\,c^{3}\right)}{c^{3}}\right)}{48\,a^{2}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/48*sqrt(2)*(15*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))
 - 3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/tan(1/2*f*x + 1/2*e)^2 + 2*((c*tan(
1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^2 - 6*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^
3)/c^3)/(a^2*c^2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+fx)\,\left(a+\frac{a}{\cos(e+fx)}\right)^2\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2)), x)
```

3.99 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{35\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+}$$

```
[Out] -35/128*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/c
^(5/2)/f*2^(1/2)-35/48*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(5/2)+1/3*tan(f*x+
e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2)+7/6*tan(f*x+e)/f/(a^2+a^2*se
c(f*x+e))/(c-c*sec(f*x+e))^(5/2)-35/64*tan(f*x+e)/a^2/c/f/(c-c*sec(f*x+e))^
(3/2)
```

Rubi [A]
time = 0.27, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4045, 3881, 3880, 209}

$$-\frac{35\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f}-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{7\tan(e+fx)}{6f\left(a^2\sec(e+fx)+a^2\right)(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-35*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) - (35*Tan[e + f*x])/(48*a^2*f*(c - c*Sec[e + f*x])^
(5/2)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2
)) + (7*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5
/2)) - (35*Tan[e + f*x])/(64*a^2*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
```

```
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{7\int \frac{\cdots}{(a+a\,s\cdots}}{\cdots}$$

$$= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{\cdots}{6f\,(a^2+\cdots}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+\cdots}{3f(a+a\sec(e+fx))^2(\cdots}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+\cdots}{3f(a+a\sec(e+fx))^2(\cdots}$$

$$= -\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+\cdots}{3f(a+a\sec(e+fx))^2(\cdots}$$

$$= -\frac{35\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}\,a^2c^{5/2}f} - \frac{35\tan(\cdots}{48a^2f(c-c\,\mathrm{se}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.41, size = 434, normalized size = 2.14

$$\frac{\cot^4(e+fx)\left(\frac{105i\sqrt{2}\left(-1+e^{i(e+fx)}\right)^5\left(1+e^{i(e+fx)}\right)^4\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\sqrt{1+e^{2i(e+fx)}}}\right)}{\left(1+e^{2i(e+fx)}\right)^{9/2}}+192\cos^4\left(\frac{1}{2}(e+fx)\right)\csc\left(\frac{e}{2}\right)\sec^5(e+fx)\sin\left(\frac{fx}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)-192\cos^4\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{e}{2}\right)\sec^5(e+fx)\sin^2\left(\frac{1}{2}(e+fx)\right)+256\cos\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\sin^5\left(\frac{1}{2}(e+fx)\right)+2752\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\sin^5\left(\frac{1}{2}(e+fx)\right)-13312\csc^5(2(e+fx))\sin^2\left(\frac{1}{2}(e+fx)\right)\sin^8(e+fx)-3648\csc\left(\frac{e}{2}\right)\csc\left(\frac{1}{2}(e+fx)\right)\csc^5(2(e+fx))\sin\left(\frac{fx}{2}\right)\sin^9(e+fx)-5504\csc^5(2(e+fx))\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)\sin^9(e+fx)+114\cot\left(\frac{e}{2}\right)\sec(e+fx)\tan^4(e+fx)\right)}{384a^2c^2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),
x]
```

```
[Out] (Cot[e + f*x]^4*(((105*I)*Sqrt[2]*(-1 + E^(I*(e + f*x)))^5*(1 + E^(I*(e + f
*x)))^4*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))
])])/(1 + E^((2*I)*(e + f*x)))^(9/2) + 192*Cos[(e + f*x)/2]^4*Csc[e/2]*Sec[
e + f*x]^5*Sin[(f*x)/2]*Sin[(e + f*x)/2] - 192*Cos[(e + f*x)/2]^4*Cot[e/2]*
Sec[e + f*x]^5*Sin[(e + f*x)/2]^2 + 256*Cos[(e + f*x)/2]*Sec[e + f*x]^5*Sin
[(e + f*x)/2]^5 + 2752*Cos[e/2]*Cos[(f*x)/2]*Cos[(e + f*x)/2]^4*Sec[e + f*x
]^5*Sin[(e + f*x)/2]^5 - 13312*Csc[2*(e + f*x)]^5*Sin[(e + f*x)/2]^2*Sin[e
+ f*x]^8 - 3648*Csc[e/2]*Csc[(e + f*x)/2]*Csc[2*(e + f*x)]^5*Sin[(f*x)/2]*S
in[e + f*x]^9 - 5504*Csc[2*(e + f*x)]^5*Sin[e/2]*Sin[(f*x)/2]*Sin[(e + f*x)
/2]*Sin[e + f*x]^9 + 114*Cot[e/2]*Sec[e + f*x]*Tan[e + f*x]^4))/(384*a^2*c^
2*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(176) = 352$.
time = 2.42, size = 551, normalized size = 2.71

method	result
default	$\frac{(-1+\cos(fx+e))^3\left(21\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{9}{2}}(\cos^2(fx+e))+12\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{9}{2}}\cos(fx+e)-9\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{9}{2}}+15(\cos^2(fx+e))\left(-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/48/a^2/f*(-1+cos(f*x+e))^3*(21*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*cos(f
*x+e)^2+12*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*cos(f*x+e)-9*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(9/2)+15*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)
-30*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)+15*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(7/2)-21*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)+42*(-
2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)-21*(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(5/2)+35*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)-70*cos(f*x+
e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+35*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
3/2)-105*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-105*cos(f*x+e)^2
*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+210*cos(f*x+e)*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)+210*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)
+1))^(1/2))-105*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-105*arctan(1/(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+
e)^5/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(5/2))
, x)
```

Fricas [A]
time = 3.96, size = 527, normalized size = 2.60

$$\left[-\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{-c}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(3c\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+4\left(43\cos\left(fx+e\right)^{4}-161\cos\left(fx+e\right)^{3}-35\cos\left(fx+e\right)^{2}+105\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{768\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}-a^{2}c^{3}f\cos\left(fx+e\right)^{2}-a^{2}c^{3}f\cos\left(fx+e\right)+a^{2}c^{3}f\right)\sin\left(fx+e\right)}, \frac{105\sqrt{2}\left(\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)+1\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{\sqrt{c}\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\left(43\cos\left(fx+e\right)^{4}-161\cos\left(fx+e\right)^{3}-35\cos\left(fx+e\right)^{2}+105\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{384\left(a^{2}c^{3}f\cos\left(fx+e\right)^{3}-a^{2}c^{3}f\cos\left(fx+e\right)^{2}-a^{2}c^{3}f\cos\left(fx+e\right)+a^{2}c^{3}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/768*(105*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*s
qrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f
*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(43*cos(f*x + e)^4 - 161*cos(f
*x + e)^3 - 35*cos(f*x + e)^2 + 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2
*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)), 1/384*(105*sqrt(2)*(cos(f*x
 + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f
*x + e) - 2*(43*cos(f*x + e)^4 - 161*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 1
05*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f
*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*
sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(e+fx\right)}{c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{4}\left(e+fx\right)-2c^{2}\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)+c^{2}\sqrt{-c\sec\left(e+fx\right)+c}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c
**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c**2*sqrt(-c*sec(e + f*x) +
 c)), x)/a**2
```

Giac [A]
time = 0.69, size = 160, normalized size = 0.79

$$\frac{\sqrt{2}\left(105\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{8\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^2-9\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^3\right)}{c^3}-\frac{3\left(13\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+11\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^2\right)}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{384\,a^2c^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/384*sqrt(2)*(105*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c
)) + 8*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^2 - 9*sqrt(c*tan(1/2*f*x + 1
/2*e)^2 - c)*c^3)/c^3 - 3*(13*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 11*s
qrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))/(a^2*c
^3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2)), x)
```

3.100 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=169

$$\frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}}+\frac{16c^3\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}-\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2}+2$$

```
[Out] -4/5*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+2/5*c*(c-
c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+32/5*c^4*tan(f*x+e)/a^3
/f/(c-c*sec(f*x+e))^(1/2)+16/5*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a^3
+a^3*sec(f*x+e))
```

Rubi [A]
time = 0.28, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4039, 3877}

$$\frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}}+\frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f\left(a^3\sec(e+fx)+a^3\right)}-\frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (32*c^4*Tan[e + f*x])/(5*a^3*f*Sqrt[c - c*Sec[e + f*x]]) + (16*c^3*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) - (4*c^2*(c -
 c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*
(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(6c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}}{5a}$$
$$= -\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{5/2}}{5f(a+a\sec(e+}$$
$$= \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a^3+a^3\sec(e+fx))} - \frac{4c^2(c-c\sec(e+fx))^{3/2}}{5af(a+a\sec(e+}$$
$$= \frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}} + \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a^3+a^3\sec(e+fx))}$$

Mathematica [A]
time = 0.62, size = 78, normalized size = 0.46

$$-\frac{c^3(130+249\cos(e+fx)+110\cos(2(e+fx))+23\cos(3(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{10a^3f(1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,
x]
[Out] -1/10*(c^3*(130 + 249*Cos[e + f*x] + 110*Cos[2*(e + f*x)] + 23*Cos[3*(e + f
*x)])*Cot[(e + f*x)/2]*Sqrt[c - c*Sec[e + f*x]])/(a^3*f*(1 + Cos[e + f*x])^
3)
```

Maple [A]
time = 2.66, size = 85, normalized size = 0.50

method	result	size
default	$-\frac{2\left(23\left(\cos^3(fx+e)\right)+55\left(\cos^2(fx+e)\right)+45\cos(fx+e)+5\right)\left(\cos^3(fx+e)\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}}{5a^3f\sin(fx+e)^5(-1+\cos(fx+e))}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNVE
RBOSE)
[Out] -2/5/a^3/f*(23*cos(f*x+e)^3+55*cos(f*x+e)^2+45*cos(f*x+e)+5)*cos(f*x+e)^3*(
c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)/sin(f*x+e)^5/(-1+cos(f*x+e))
```

Maxima [A]
time = 0.53, size = 230, normalized size = 1.36

$$\frac{2\left(16\sqrt{2}\,c^{\frac{7}{2}} - \frac{56\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{70\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{35\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{5\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - \frac{\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}} + \frac{\sqrt{2}\,c^{\frac{7}{2}}\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}\right)}{5\,a^3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] 2/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2)
*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8 - sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) +
 1)^10 + sqrt(2)*c^(7/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)/(a^3*f*(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(cos(f*x + e) + 1) -
1)^(7/2))
```

Fricas [A]
time = 1.98, size = 117, normalized size = 0.69

$$-\frac{2\left(23\,c^{3}\cos\left(fx+e\right)^{3}+55\,c^{3}\cos\left(fx+e\right)^{2}+45\,c^{3}\cos\left(fx+e\right)+5\,c^{3}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{5\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/5*(23*c^3*cos(f*x + e)^3 + 55*c^3*cos(f*x + e)^2 + 45*c^3*cos(f*x + e) +
 5*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*
a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [A]
time = 1.11, size = 126, normalized size = 0.75

$$\frac{2\sqrt{2}\,c^{3}\left(\frac{5\,c}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,a^{3}}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}a^{12}c^{8}+5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}a^{12}c^{9}+15\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,a^{12}c^{10}}{a^{15}c^{10}}\right)}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] 2/5*sqrt(2)*c^3*(5*c/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^3) - ((c*tan(1/2
*f*x + 1/2*e)^2 - c)^(5/2)*a^12*c^8 + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2
)*a^12*c^9 + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^12*c^10)/(a^15*c^10))/
f
```

Mupad [B]
time = 10.24, size = 492, normalized size = 2.91

$$-\frac{\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left(\frac{c^3\,46{}\mathrm{i}}{5\,a^3\,f}+\frac{c^3\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,4{}\mathrm{i}}{a^3\,f}+\frac{c^3\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,46{}\mathrm{i}}{5\,a^3\,f}\right)}{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}-\frac{c^3\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,16{}\mathrm{i}}{5\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^2}-\frac{c^3\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,48{}\mathrm{i}}{5\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^3}+\frac{c^3\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,128{}\mathrm{i}}{5\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^4}-\frac{c^3\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,64{}\mathrm{i}}{5\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2)*128i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^4) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*
1i + f*x*1i) + 1)^2) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f
*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*48i)/(5*a^3*f*(exp(e*1i + f*x*1i) -
 1)*(exp(e*1i + f*x*1i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1
i + f*x*1i)/2))^(1/2)*((c^3*46i)/(5*a^3*f) + (c^3*exp(e*1i + f*x*1i)*4i)/(a
^3*f) + (c^3*exp(e*2i + f*x*2i)*46i)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1)*
(exp(e*1i + f*x*1i) + 1)) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(e*1i + f*x*
1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

3.101 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=135

$$\frac{16c^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{2c(c-c\sec(e+fx)}{5f(a+a\sec(e}$$

```
[Out] 2/5*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+16/15*c^3*tan(
f*x+e)/f/(a^3+a^3*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)-8/15*c^2*(c-c*sec(f*x+
e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A]

time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4039, 4038}

$$\frac{16c^3\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{15af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (16*c^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]]) - (8*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(15*a*f*(a + a*Sec[e +
f*x])^2) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e
+ f*x])^3)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(4c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3}}{(a+a\sec(e+fx))^2}}{5a}$$

$$= -\frac{8c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{3/2}\,\mathrm{t}}{5f(a+a\sec(e+f}$$

$$= \frac{16c^3\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))\,\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e}}{15af(a+a}$$

Mathematica [A]
time = 0.35, size = 74, normalized size = 0.55

$$-\frac{c^2\cos(e+fx)(37+20\cos(e+fx)+7\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{15a^3f(1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] -1/15*(c^2*Cos[e + f*x]*(37 + 20*Cos[e + f*x] + 7*Cos[2*(e + f*x)])*Cot[(e
+ f*x)/2]*Sqrt[c - c*Sec[e + f*x]])/(a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A]
time = 2.54, size = 65, normalized size = 0.48

method	result	size
default	$-\frac{2\left(7\left(\cos^2(fx+e)\right)+10\cos(fx+e)+15\right)\left(\cos^3(fx+e)\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}}{15a^3f\sin(fx+e)^5}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15/a^3/f*(7*cos(f*x+e)^2+10*cos(f*x+e)+15)*cos(f*x+e)^3*(c*(-1+cos(f*x+e
))/cos(f*x+e))^(5/2)/sin(f*x+e)^5
```

Maxima [A]
time = 0.50, size = 203, normalized size = 1.50

$$-\frac{8\sqrt{2}\,c^{\frac{5}{2}} - \frac{20\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{5\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - \frac{3\sqrt{2}\,c^{\frac{5}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}}{15\,a^3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/15*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sqrt(2)
*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(5/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8 - 3*sqrt(2)*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)^(5/2))
```

Fricas [A]
time = 2.31, size = 112, normalized size = 0.83

$$-\frac{2\left(7\,c^{2}\cos\left(fx+e\right)^{3}+10\,c^{2}\cos\left(fx+e\right)^{2}+15\,c^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/15*(7*c^2*cos(f*x + e)^3 + 10*c^2*cos(f*x + e)^2 + 15*c^2*cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*co
s(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.98, size = 90, normalized size = 0.67

$$-\frac{\frac{15\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,c^{2}}{a^{3}}+\frac{3\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}+10\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}c}{a^{3}}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/15*(15*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2/a^3 + (3*sqrt(2)*(
c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 10*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2
- c)^(3/2)*c)/a^3)/f
```

Mupad [B]
time = 7.31, size = 456, normalized size = 3.38

$$-\frac{c^2\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,14{}\mathrm{i}}{15\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}+\frac{c^2\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,16{}\mathrm{i}}{15\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^2}-\frac{c^2\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,112{}\mathrm{i}}{15\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^3}+\frac{c^2\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,64{}\mathrm{i}}{5\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^4}-\frac{c^2\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,32{}\mathrm{i}}{5\,a^3\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2)*16i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^2) - (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(1/2)*14i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e
*1i + f*x*1i) + 1)) - (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*
x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*112i)/(15*a^3*f*(exp(e*1i + f*x*1i)
- 1)*(exp(e*1i + f*x*1i) + 1)^3) + (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(e*1i
 + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^4) - (c^2*(exp(e*2i + f*x*2i) + 1)
*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*32i)/(5*a^3*
f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

3.102 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=88

$$-\frac{4c^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

```
[Out] -4/15*c^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+2/5*c*(c
-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A]
time = 0.16, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4039, 4038}

$$\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3}-\frac{4c^2\tan(e+fx)}{15af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-4*c^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x
]]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])
^3)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(2c)\int \frac{\sec(e+fx)\sqrt{c-c\sec(e}{(a+a\sec(e+fx))^2}}{5a}$$
$$= -\frac{4c^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e}{5f(a+a}$$

Mathematica [A]
time = 0.30, size = 60, normalized size = 0.68

$$-\frac{2c(-5+\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{15a^3f(1+\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] (-2*c*(-5 + Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e + f*x]*Sqrt[c - c*Sec[e +
f*x]])/(15*a^3*f*(1 + Sec[e + f*x])^3)
```

Maple [A]
time = 2.42, size = 63, normalized size = 0.72

method	result	size
default	$-\frac{2\left(5+\cos^2(fx+e)-6\cos(fx+e)\right)\left(\cos^3(fx+e)\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}{15a^3f\sin(fx+e)^5}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15/a^3/f*(5+cos(f*x+e)^2-6*cos(f*x+e))*cos(f*x+e)^3*(c*(-1+cos(f*x+e))/c
os(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(86) = 172.
time = 0.50, size = 175, normalized size = 1.99

$$-\frac{2\sqrt{2}c^{\frac{3}{2}} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{7\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}{30a^3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/30*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7*sqrt(2)*c
^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)
*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

Fricas [A]
time = 3.29, size = 95, normalized size = 1.08

$$-\frac{2\left(c\cos\left(fx+e\right)^{3}-5\,c\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/15*(c*cos(f*x + e)^3 - 5*c*cos(f*x + e)^2)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x +
 e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec
(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) +
 1), x))/a**3
```

Giac [A]
time = 0.89, size = 58, normalized size = 0.66

$$-\frac{\sqrt{2}\left(3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}+5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}c\right)}{30\,a^{3}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/30*sqrt(2)*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 5*(c*tan(1/2*f*x +
1/2*e)^2 - c)^(3/2)*c)/(a^3*c*f)
```

Mupad [B]
time = 7.72, size = 446, normalized size = 5.07

$$-\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}+\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,28\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^2}-\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,76\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^3}+\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,32\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^4}-\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*28i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i)
+ 1)^2) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(
e*1i + f*x*1i)/2))^(1/2)*2i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i +
 f*x*1i) + 1)) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2
 + exp(e*1i + f*x*1i)/2))^(1/2)*76i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(ex
p(e*1i + f*x*1i) + 1)^3) + (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i -
 f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*32i)/(5*a^3*f*(exp(e*1i + f*x*1i)
 - 1)*(exp(e*1i + f*x*1i) + 1)^4) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i
+ f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

3.103 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=41

$$\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2/5*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {4038}

$$\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.16, size = 55, normalized size = 1.34

$$-\frac{\cos^3(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{20a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -1/20*(Cos[e + f*x]^3*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^5*Sqrt[c - c*Sec[e
+ f*x]])/(a^3*f)
```

Maple [A]
time = 2.52, size = 55, normalized size = 1.34

method	result	size
default	$-\frac{2(-1+\cos(fx+e))^2(\cos^3(fx+e))\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}}{5a^3f\sin(fx+e)^5}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/5/a^3/f*(-1+cos(f*x+e))^2*cos(f*x+e)^3*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1
/2)/sin(f*x+e)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(40) = 80$.
time = 0.53, size = 146, normalized size = 3.56

$$-\frac{\sqrt{2}\sqrt{c}-\frac{3\sqrt{2}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{2}\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{\sqrt{2}\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}{20\,a^3f\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/20*(sqrt(2)*sqrt(c) - 3*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - sqrt(2)*sqrt(
c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^3*f*sqrt(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Fricas [A]
time = 2.66, size = 80, normalized size = 1.95

$$-\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^3}{5\left(a^3f\cos(fx+e)^2+2\,a^3f\cos(fx+e)+a^3f\right)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/5*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^3/((a^3*f*cos(f*x
 + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{-c\sec\left(e+fx\right)+c}\,\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x)/a**3
```

Giac [A]
time = 1.27, size = 62, normalized size = 1.51

$$-\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}(\cos\left(fx+e\right))}{20\,a^3c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/20*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*sgn(tan(1/2*f*x + 1/2*e)
^3 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a^3*c^2*f)
```

Mupad [B]
time = 7.59, size = 441, normalized size = 10.76

$$-\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}+\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,8\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^2}-\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^3}+\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^4}-\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,8\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*8i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)
^2) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2))^(1/2)*2i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f
```

```
*x*1i) + 1)^3) + ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
 exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e
*1i + f*x*1i) + 1)^4) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*
1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*8i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*
(exp(e*1i + f*x*1i) + 1)^5)
```

3.104 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=181

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,a^3\sqrt{c}\,f}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e}{6af(a+a\sec(e+fx}$$

```
[Out] -1/8*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/f*2^
(1/2)/c^(1/2)+1/5*tan(f*x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2)+1/
6*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+1/4*tan(f*x+e)/f
/(a^3+a^3*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4045, 3880, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,a^3\sqrt{c}\,f}+\frac{\tan(e+fx)}{4f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{6af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqr
t[2]*a^3*Sqrt[c]*f) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*S
ec[e + f*x]]) + Tan[e + f*x]/(6*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e
 + f*x]]) + Tan[e + f*x]/(4*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f
*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
```

```
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{}{(a+a\sec(e+f}}{}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{}{6af(a+a\,\mathrm{se}}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{}{6af(a+a\,\mathrm{se}}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{}{6af(a+a\,\mathrm{se}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}\,a^3\sqrt{c}\,f} + \frac{\tan}{5f(a+a\sec(e+f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.61, size = 225, normalized size = 1.24

$$\frac{2e^{-\frac{1}{2}i(e+fx)}\cos\left(\frac{1}{2}(e+fx)\right)\left(-15\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\sqrt{1+e^{2i(e+fx)}}\,\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)\cos^5\left(\frac{1}{2}(e+fx)\right)+\frac{e^{\frac{1}{2}i(e+fx)}(67+80\cos(e+fx)+37\cos(2(e+fx)))}{8\sqrt{\sec(e+fx)}}\right)\sec^{\frac{7}{2}}(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)}{15a^3f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(-15*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sq
rt[1 + E^((2*I)*(e + f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 +
 E^((2*I)*(e + f*x))])]*Cos[(e + f*x)/2]^5 + (E^((I/2)*(e + f*x))*(67 + 80*
Cos[e + f*x] + 37*Cos[2*(e + f*x)]))/(8*Sqrt[Sec[e + f*x]]))*Sec[e + f*x]^(
7/2)*Sin[(e + f*x)/2])/(15*a^3*E^((I/2)*(e + f*x))*f*(1 + Sec[e + f*x])^3*S
qrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.48, size = 155, normalized size = 0.86

method	result
default	$-\frac{(-1+\cos(fx+e))\left(3\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{5}{2}}-5\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{3}{2}}+15\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}\right)+15\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\right)}{60a^3f\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60/a^3/f*(-1+cos(f*x+e))*(3*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)-5*(-2*c
os(f*x+e)/(cos(f*x+e)+1))^(3/2)+15*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^
(1/2))+15*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))/(c*(-1+cos(f*x+e))/cos(f*x+
e))^(1/2)/sin(f*x+e)/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*sqrt(-c*sec(f*x + e) + c)),
x)
```

Fricas [A]
time = 3.15, size = 435, normalized size = 2.40

$$\left[-\frac{15\sqrt{2}\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+4\left(37\cos(fx+e)^3+40\cos(fx+e)^2+15\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{240\left(a^3cf\cos(fx+e)^2+2a^3cf\cos(fx+e)+a^3cf\right)\sin(fx+e)}, \frac{15\sqrt{2}\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)-2\left(37\cos(fx+e)^3+40\cos(fx+e)^2+15\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{120\left(a^3cf\cos(fx+e)^2+2a^3cf\cos(fx+e)+a^3cf\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/240*(15*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*s
qrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin
(f*x + e)))*sin(f*x + e) + 4*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)
^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)), 1/120*(15*sqrt(2)*(co
```

```
s(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) -
2*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e)
 + a^3*c*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+3\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x)**2 + 3*sqrt(-c*sec(e + f*x) + c)*sec(e + f
*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

Giac [A]
time = 0.67, size = 119, normalized size = 0.66

$$\frac{\sqrt{2}\left(\frac{15\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}c^{12}-5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^{13}+15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^{14}}{c^{15}}\right)}{120\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/120*sqrt(2)*(15*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c
) - (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^12 - 5*(c*tan(1/2*f*x + 1/2*e
)^2 - c)^(3/2)*c^13 + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^14)/c^15)/(a^
3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^3\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2)), x)
```

3.105 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{7\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f}-\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+}$$

```
[Out] -7/32*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/c^(
3/2)/f*2^(1/2)-7/16*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(3/2)+1/5*tan(f*x+e)/
f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2)+7/30*tan(f*x+e)/a/f/(a+a*sec(f*
x+e))^2/(c-c*sec(f*x+e))^(3/2)+7/12*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))/(c-c*
sec(f*x+e))^(3/2)
```

Rubi [A]

time = 0.33, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4045, 3881, 3880, 209}

$$-\frac{7\text{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f}-\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{12f\left(a^3\sec(e+fx)+a^3\right)(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{30af(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-7*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(16*
Sqrt[2]*a^3*c^(3/2)*f) - (7*Tan[e + f*x])/(16*a^3*f*(c - c*Sec[e + f*x])^(3
/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
 + (7*Tan[e + f*x])/(30*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/
2)) + (7*Tan[e + f*x])/(12*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*x])^
(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{7\int \ldots}{\ldots}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{\ldots}{30af(a \ldots}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{\ldots}{30af(a \ldots}$$

$$= -\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+ \ldots}{5f(a+a\sec(e+fx))^3 \ldots}$$

$$= -\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+ \ldots}{5f(a+a\sec(e+fx))^3 \ldots}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}\,a^3c^{3/2}f} - \frac{7\tan(e \ldots}{16a^3f(c-c\sec \ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.44, size = 398, normalized size = 1.88

$$\frac{7e^{-\frac{1}{2}i(e+fx)}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\sqrt{1+e^{2i(e+fx)}}\,\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^{\frac{9}{2}}(e+fx)\sin^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^5(e+fx)\left(-\frac{278\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)}{15f}-\frac{\cot\left(\frac{e}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{242\sec\left(\frac{e}{2}+\frac{fx}{2}\right)}{15f}-\frac{56\sec^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{15f}+\frac{2\sec^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}+\frac{\csc\left(\frac{e}{2}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{f}+\frac{278\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)}{15f}\right)\sin^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),
x]
```

```
[Out] (7*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e + f
*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))])
]*Cos[e/2 + (f*x)/2]^6*Sec[e + f*x]^(9/2)*Sin[e/2 + (f*x)/2]^3)/(E^((I/2)*(
e + f*x))*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)) + (Cos[e/2 +
 (f*x)/2]^6*Sec[e + f*x]^5*((-278*Cos[e/2]*Cos[(f*x)/2])/(15*f) - (Cot[e/2]
*Csc[e/2 + (f*x)/2])/f + (242*Sec[e/2 + (f*x)/2])/(15*f) - (56*Sec[e/2 + (f
*x)/2]^3)/(15*f) + (2*Sec[e/2 + (f*x)/2]^5)/(5*f) + (Csc[e/2]*Csc[e/2 + (f*
x)/2]^2*Sin[(f*x)/2])/f + (278*Sin[e/2]*Sin[(f*x)/2])/(15*f))*Sin[e/2 + (f*
x)/2]^3)/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
```

Maple [A]
time = 2.60, size = 370, normalized size = 1.75

method	result
default	$\frac{(-1+\cos(fx+e))^2 \left(15\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{9}{2}}\cos(fx+e)+15\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{9}{2}}+15\cos(fx+e)\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}}-15\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{7}{2}} \cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/120/a^3/f*(-1+cos(f*x+e))^2*(15*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*cos(
f*x+e)+15*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)+15*cos(f*x+e)*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(7/2)-15*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)-21*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)+21*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5
/2)+35*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)-35*(-2*cos(f*x+e)/(c
os(f*x+e)+1))^(3/2)-105*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-105
*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+105*(-2*cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)+105*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))
)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^3/(-2*cos(f*x+e)/(cos(f*x
+e)+1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^(3/2))
, x)

Fricas [A]
time = 4.31, size = 523, normalized size = 2.47

$$\left[-\frac{105\sqrt{2}\left(\cos(fx+e)^3+\cos(fx+e)^2-\cos(fx+e)-1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+4\left(139\cos(fx+e)^4+21\cos(fx+e)^3-175\cos(fx+e)^2-105\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{960\left(a^3c^2f\cos(fx+e)^3+a^3c^2f\cos(fx+e)^2-a^3c^2f\cos(fx+e)-a^3c^2f\right)\sin(fx+e)}, \frac{105\sqrt{2}\left(\cos(fx+e)^3+\cos(fx+e)^2-\cos(fx+e)-1\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)-2\left(139\cos(fx+e)^4+21\cos(fx+e)^3-175\cos(fx+e)^2-105\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{480\left(a^3c^2f\cos(fx+e)^3+a^3c^2f\cos(fx+e)^2-a^3c^2f\cos(fx+e)-a^3c^2f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/960*(105*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*s
qrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f
*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(139*cos(f*x + e)^4 + 21*cos(f
*x + e)^3 - 175*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^
3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)), 1/480*(105*sqrt(2)*(cos(f*
x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(c)*arctan(sqrt(2)*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(
f*x + e) - 2*(139*cos(f*x + e)^4 + 21*cos(f*x + e)^3 - 175*cos(f*x + e)^2 -
 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos
(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f
)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^4(e+fx)-2c\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+2c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c*s
qrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 2*c*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

Giac [A]
time = 0.76, size = 154, normalized size = 0.73

$$\frac{\sqrt{2}\left(105\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-\frac{15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}-\frac{2\left(3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}c^8-10\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^9+45\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^{10}\right)}{c^{10}}\right)}{480\,a^3c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/480*sqrt(2)*(105*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c
)) - 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/tan(1/2*f*x + 1/2*e)^2 - 2*(3*(c
*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^8 - 10*(c*tan(1/2*f*x + 1/2*e)^2 - c)^
(3/2)*c^9 + 45*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^10)/c^10)/(a^3*c^2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^3\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2)), x)
```

3.106 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{63\mathrm{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}-\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e$$

```
[Out] -63/256*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/c
^(5/2)/f*2^(1/2)-21/32*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(5/2)+1/5*tan(f*x+
e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2)+3/10*tan(f*x+e)/a/f/(a+a*sec
(f*x+e))^2/(c-c*sec(f*x+e))^(5/2)+21/20*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))/(
c-c*sec(f*x+e))^(5/2)-63/128*tan(f*x+e)/a^3/c/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.36, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4045, 3881, 3880, 209}

$$-\frac{63\mathrm{ArcTan}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f}-\frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}-\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{21\tan(e+fx)}{20f\left(a^3\sec(e+fx)+a^3\right)(c-c\sec(e+fx))^{5/2}}+\frac{3\tan(e+fx)}{10af(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-63*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) - (21*Tan[e + f*x])/(32*a^3*f*(c - c*Sec[e + f*x])
^(5/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/
2)) + (3*Tan[e + f*x])/(10*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^
(5/2)) + (21*Tan[e + f*x])/(20*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*
x])^(5/2)) - (63*Tan[e + f*x])/(128*a^3*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{9\int \frac{\cdots}{(a+a\sec\cdots}}{\cdots}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{\cdots}{10af(a+\cdots}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{\cdots}{10af(a+\cdots}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+\cdots}{5f(a+a\sec(e+fx))^3(c\cdots}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+\cdots}{5f(a+a\sec(e+fx))^3(c\cdots}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+\cdots}{5f(a+a\sec(e+fx))^3(c\cdots}$$

$$= -\frac{63\tan^{-1}\left(\frac{\sqrt{c}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}\,a^3c^{5/2}f} - \frac{21\tan(e\cdots}{32a^3f(c-c\sec\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.65, size = 468, normalized size = 1.90

$$-\frac{63e^{-\frac{1}{2}i(e+fx)}\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,\sqrt{1+e^{2i(e+fx)}}\,\tanh^{-1}\left(\frac{1+e^{i(e+fx)}}{\sqrt{2}\,\sqrt{1+e^{2i(e+fx)}}}\right)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^{\frac{11}{2}}(e+fx)\sin^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{4f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}+\frac{\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^6(e+fx)\left(\frac{257\cos\left(\frac{e}{2}\right)\cos\left(\frac{fx}{2}\right)}{10f}+\frac{23\cot\left(\frac{e}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}\right)}{4f}-\frac{\cot\left(\frac{e}{2}\right)\csc^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{2f}-\frac{124\sec\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}+\frac{22\sec^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}-\frac{2\sec^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{5f}-\frac{23\csc\left(\frac{e}{2}\right)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{4f}+\frac{\csc\left(\frac{e}{2}\right)\csc^4\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{2f}-\frac{257\sin\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)}{10f}\right)\sin^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),
x]
```

```
[Out] (-63*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*Sqrt[1 + E^((2*I)*(e +
 f*x))]*ArcTanh[(1 + E^(I*(e + f*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(e + f*x))
])]*Cos[e/2 + (f*x)/2]^6*Sec[e + f*x]^(11/2)*Sin[e/2 + (f*x)/2]^5)/(4*E^((I
/2)*(e + f*x))*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)) + (Cos[
e/2 + (f*x)/2]^6*Sec[e + f*x]^6*((257*Cos[e/2]*Cos[(f*x)/2])/(10*f) + (23*C
ot[e/2]*Csc[e/2 + (f*x)/2])/(4*f) - (Cot[e/2]*Csc[e/2 + (f*x)/2]^3)/(2*f) -
 (124*Sec[e/2 + (f*x)/2])/(5*f) + (22*Sec[e/2 + (f*x)/2]^3)/(5*f) - (2*Sec[
e/2 + (f*x)/2]^5)/(5*f) - (23*Csc[e/2]*Csc[e/2 + (f*x)/2]^2*Sin[(f*x)/2])/(
4*f) + (Csc[e/2]*Csc[e/2 + (f*x)/2]^4*Sin[(f*x)/2])/(2*f) - (257*Sin[e/2]*S
in[(f*x)/2])/(10*f))*Sin[e/2 + (f*x)/2]^5)/((a + a*Sec[e + f*x])^3*(c - c*S
ec[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(215) = 430$.
time = 2.87, size = 631, normalized size = 2.57

method	result
default	$-\frac{(-1+\cos(fx+e))^3\left(45\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{11}{2}}(\cos^2(fx+e))+20\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{11}{2}}\cos(fx+e)-25\left(-\frac{2\cos(fx+e)}{\cos(fx+e)+1}\right)^{\frac{11}{2}}+35\left(-\frac{2\cos(}{\cos(fx}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/160/a^3/f*(-1+cos(f*x+e))^3*(45*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(11/2)*co
s(f*x+e)^2+20*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(11/2)*cos(f*x+e)-25*(-2*cos(f
*x+e)/(cos(f*x+e)+1))^(11/2)+35*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*cos(f*
x+e)^2-70*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(9/2)*cos(f*x+e)+35*(-2*cos(f*x+e)
/(cos(f*x+e)+1))^(9/2)-45*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)
+90*cos(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(7/2)-45*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(7/2)+63*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)-126*(
-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)*cos(f*x+e)+63*(-2*cos(f*x+e)/(cos(f*x+e
)+1))^(5/2)-105*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)+210*cos(f
*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(3/2)-105*(-2*cos(f*x+e)/(cos(f*x+e)+1
))^(3/2)+315*cos(f*x+e)^2*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+315*cos(f*x+
e)^2*arctan(1/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-630*cos(f*x+e)*(-2*cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)-630*cos(f*x+e)*arctan(1/(-2*cos(f*x+e)/(cos(f*
x+e)+1))^(1/2))+315*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+315*arctan(1/(-2*c
```

```
os(f*x+e)/(cos(f*x+e)+1))^(1/2)))/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(
f*x+e)^5/(-2*cos(f*x+e)/(cos(f*x+e)+1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^(5/2))
, x)
```

Fricas [A]
time = 2.00, size = 499, normalized size = 2.03

$$\left[-\frac{315\sqrt{2}\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)+4\left(257\cos(fx+e)^5-354\cos(fx+e)^4-588\cos(fx+e)^3+210\cos(fx+e)^2+315\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2560\left(a^3c^3f\cos(fx+e)^4-2a^3c^3f\cos(fx+e)^2+a^3c^3f\right)\sin(fx+e)}, \frac{315\sqrt{2}\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)-2\left(257\cos(fx+e)^5-354\cos(fx+e)^4-588\cos(fx+e)^3+210\cos(fx+e)^2+315\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{1280\left(a^3c^3f\cos(fx+e)^4-2a^3c^3f\cos(fx+e)^2+a^3c^3f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/2560*(315*sqrt(2)*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-c)*log(
(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)
*sin(f*x + e)))*sin(f*x + e) + 4*(257*cos(f*x + e)^5 - 354*cos(f*x + e)^4 -
 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x
 + e)^2 + a^3*c^3*f)*sin(f*x + e)), 1/1280*(315*sqrt(2)*(cos(f*x + e)^4 - 2
*cos(f*x + e)^2 + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(257*cos(f*x
 + e)^5 - 354*cos(f*x + e)^4 - 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 31
5*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*
x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^5(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}\sec^4(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**5 + c**
2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c**2*sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**3 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*
*2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c))
, x)/a**3
```

Giac [A]
time = 0.94, size = 184, normalized size = 0.75

$$\frac{\sqrt{2}\left(315\,\sqrt{c}\,\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}}{\sqrt{c}}\right)-\frac{5\left(17\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}c+15\,\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,c^{2}\right)}{c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}-\frac{8\left(\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{5}{2}}c^{8}-5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{\frac{3}{2}}c^{9}+30\,\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c}\,c^{10}\right)}{c^{10}}\right)}{1280\,a^{3}c^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/1280*sqrt(2)*(315*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(
c)) - 5*(17*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 15*sqrt(c*tan(1/2*f*x
+ 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4) - 8*((c*tan(1/2*f*x + 1/2
*e)^2 - c)^(5/2)*c^8 - 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^9 + 30*sqrt
(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^10)/c^10)/(a^3*c^3*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^3\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2)), x)
```

3.107 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=43

$$\frac{a(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 1/3*a*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4038}

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\,dx = \frac{a(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(43) = 86.
time = 0.47, size = 87, normalized size = 2.02

$$\frac{c^2(5-6\cos(e+fx)+3\cos(2(e+fx)))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] (c^2*(5 - 6*Cos[e + f*x] + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*
x)/2]*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(
12*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(37) = 74$.
time = 2.52, size = 82, normalized size = 1.91

method	result	size
default	$-\frac{\sin(fx+e)\left(7\left(\cos^2(fx+e)\right)-4\cos(fx+e)+1\right)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}}{3f(-1+\cos(fx+e))^3}$	82
risch	$\frac{2ic^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(3\,e^{5i(fx+e)}-6\,e^{4i(fx+e)}+10\,e^{3i(fx+e)}-6\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\left(e^{2i(fx+e)}+1\right)^2 f}$	165

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/f*sin(f*x+e)*(7*cos(f*x+e)^2-4*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*
x+e))^(5/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(40) = 80$.
time = 0.56, size = 688, normalized size = 16.00

$$\frac{2\left(30c^2\cos(3fx+3e)\sin(2fx+2e)-9c^2\cos(2fx+2e)\sin(fx+e)-3c^2\sin(fx+e)-\left(3c^2\sin(5fx+5e)-6c^2\sin(4fx+4e)+10c^2\sin(3fx+3e)-6c^2\sin(2fx+2e)+3c^2\sin(fx+e)\right)\cos(6fx+6e)+9\left(c^2\sin(4fx+4e)+c^2\sin(2fx+2e)\right)\cos(5fx+5e)-3\left(10c^2\sin(3fx+3e)+3c^2\sin(fx+e)\right)\cos(4fx+4e)+\left(3c^2\cos(5fx+5e)-6c^2\cos(4fx+4e)+10c^2\cos(3fx+3e)-6c^2\cos(2fx+2e)+3c^2\cos(fx+e)\right)\sin(6fx+6e)-3\left(3c^2\cos(4fx+4e)+3c^2\cos(2fx+2e)+c^2\right)\sin(5fx+5e)+3\left(10c^2\cos(3fx+3e)+3c^2\cos(fx+e)+2c^2\right)\sin(4fx+4e)-10\left(3c^2\cos(2fx+2e)+c^2\right)\sin(3fx+3e)+3\left(3c^2\cos(fx+e)+2c^2\right)\sin(2fx+2e)\right)\sqrt{a}\sqrt{c}}{3\left(2\left(3\cos(4fx+4e)+3\cos(2fx+2e)+1\right)\cos(6fx+6e)+\cos(6fx+6e)^2+6\left(3\cos(2fx+2e)+1\right)\cos(4fx+4e)+9\cos(4fx+4e)^2+9\cos(2fx+2e)^2+6\left(\sin(4fx+4e)+\sin(2fx+2e)\right)\sin(6fx+6e)+\sin(6fx+6e)^2+9\sin(4fx+4e)^2+18\sin(4fx+4e)\sin(2fx+2e)+9\sin(2fx+2e)^2+6\cos(2fx+2e)+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(30*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*c^2*cos(2*f*x + 2*e)*sin(
f*x + e) - 3*c^2*sin(f*x + e) - (3*c^2*sin(5*f*x + 5*e) - 6*c^2*sin(4*f*x +
 4*e) + 10*c^2*sin(3*f*x + 3*e) - 6*c^2*sin(2*f*x + 2*e) + 3*c^2*sin(f*x +
e))*cos(6*f*x + 6*e) + 9*(c^2*sin(4*f*x + 4*e) + c^2*sin(2*f*x + 2*e))*cos(
5*f*x + 5*e) - 3*(10*c^2*sin(3*f*x + 3*e) + 3*c^2*sin(f*x + e))*cos(4*f*x +
 4*e) + (3*c^2*cos(5*f*x + 5*e) - 6*c^2*cos(4*f*x + 4*e) + 10*c^2*cos(3*f*x
 + 3*e) - 6*c^2*cos(2*f*x + 2*e) + 3*c^2*cos(f*x + e))*sin(6*f*x + 6*e) - 3
*(3*c^2*cos(4*f*x + 4*e) + 3*c^2*cos(2*f*x + 2*e) + c^2)*sin(5*f*x + 5*e) +
 3*(10*c^2*cos(3*f*x + 3*e) + 3*c^2*cos(f*x + e) + 2*c^2)*sin(4*f*x + 4*e)
```

```
- 10*(3*c^2*cos(2*f*x + 2*e) + c^2)*sin(3*f*x + 3*e) + 3*(3*c^2*cos(f*x + e
) + 2*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*co
s(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x
+ 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2
+ 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*
e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(
2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(40) = 80$.
time = 2.68, size = 101, normalized size = 2.35

$$\frac{\left(3\,c^2\cos\left(fx+e\right)^2-3\,c^2\cos\left(fx+e\right)+c^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*c^2*cos(f*x + e)^2 - 3*c^2*cos(f*x + e) + c^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^
2*sin(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(37) = 74$.
time = 1.73, size = 104, normalized size = 2.42

$$\frac{8\left(3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^2+3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^3+c^4\right)\sqrt{-ac}\,|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

[Out] 8/3*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 + 3*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + c^4)*sqrt(-a*c)*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^3*f)

Mupad [B]
time = 3.80, size = 136, normalized size = 3.16

$$\frac{2\,c^2\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(10\,\sin\left(e+f\,x\right)-12\,\sin\left(2\,e+2\,f\,x\right)+13\,\sin\left(3\,e+3\,f\,x\right)-6\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(5\,e+5\,f\,x\right)\right)}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)-\cos\left(6\,e+6\,f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)

[Out] (2*c^2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(10*sin(e + f*x) - 12*sin(2*e + 2*f*x) + 13*sin(3*e + 3*f*x) - 6*sin(4*e + 4*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*e + 2*f*x) - 2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))

3.108 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=43

$$\frac{a(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 1/2*a*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4038}

$$\frac{a\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}\,dx = \frac{a(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 0.31, size = 73, normalized size = 1.70

$$\frac{c(-1+2\cos(e+fx))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] (c*(-1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(4*f)
```

Maple [A]
time = 2.67, size = 72, normalized size = 1.67

method	result	size
default	$-\frac{\sin(fx+e)(3\cos(fx+e)-1)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}{2f(-1+\cos(fx+e))^{2}}$	72
risch	$\frac{2ic\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{3i(fx+e)}-\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}\right)}{\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\left(\mathrm{e}^{2i(fx+e)}+1\right)f}$	137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f*sin(f*x+e)*(3*cos(f*x+e)-1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(c*(
-1+cos(f*x+e))/cos(f*x+e))^(3/2)/(-1+cos(f*x+e))^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(40) = 80$.
time = 0.56, size = 324, normalized size = 7.53

$$\frac{2\left(2\,c\cos\left(3\,fx+3\,e\right)\sin\left(2\,fx+2\,e\right)-2\,c\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)-\left(c\sin\left(3\,fx+3\,e\right)-c\sin\left(2\,fx+2\,e\right)+c\sin\left(fx+e\right)\right)\cos\left(4\,fx+4\,e\right)+\left(c\cos\left(3\,fx+3\,e\right)-c\cos\left(2\,fx+2\,e\right)+c\cos\left(fx+e\right)\right)\sin\left(4\,fx+4\,e\right)-\left(2\,c\cos\left(2\,fx+2\,e\right)+c\right)\sin\left(3\,fx+3\,e\right)+\left(2\,c\cos\left(fx+e\right)+c\right)\sin\left(2\,fx+2\,e\right)-c\sin\left(fx+e\right)\right)\sqrt{a}\sqrt{c}}{\left(2\left(2\,\cos\left(2\,fx+2\,e\right)+1\right)\cos\left(4\,fx+4\,e\right)+\cos\left(4\,fx+4\,e\right)^{2}+4\,\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(4\,fx+4\,e\right)^{2}+4\,\sin\left(4\,fx+4\,e\right)\sin\left(2\,fx+2\,e\right)+4\,\sin\left(2\,fx+2\,e\right)^{2}+4\,\cos\left(2\,fx+2\,e\right)+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*c*cos(2*f*x + 2*e)*sin(f*x + e
) - (c*sin(3*f*x + 3*e) - c*sin(2*f*x + 2*e) + c*sin(f*x + e))*cos(4*f*x +
4*e) + (c*cos(3*f*x + 3*e) - c*cos(2*f*x + 2*e) + c*cos(f*x + e))*sin(4*f*x
 + 4*e) - (2*c*cos(2*f*x + 2*e) + c)*sin(3*f*x + 3*e) + (2*c*cos(f*x + e) +
 c)*sin(2*f*x + 2*e) - c*sin(f*x + e))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin
(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^
2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(40) = 80$.

time = 2.76, size = 85, normalized size = 1.98

$$\frac{(2\,c\cos(fx+e)-c)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\,f\cos(fx+e)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*c*cos(f*x + e) - c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\,(\sec(e+fx)+1)}\,(-c(\sec(e+fx)-1))^{\frac{3}{2}}\sec(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)*sec(e +
f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(37) = 74.
time = 1.57, size = 83, normalized size = 1.93

$$\frac{2\left(2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{3}+c^{4}\right)\sqrt{-ac}\,|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 2*(2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + c^4)*sqrt(-a*c)*abs(c)*sgn(tan(1/
2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*
c^2*f)
```

Mupad [B]
time = 2.64, size = 78, normalized size = 1.81

$$\frac{c\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,(\sin(e+f\,x)-\sin(2\,e+2\,f\,x)+\sin(3\,e+3\,f\,x))}{f\sin(2\,e+2\,f\,x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] (c*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*(
sin(e + f*x) - sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(f*sin(2*e + 2*f*x)^2)
```

3.109 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}$

Optimal. Leaf size=41

$$-\frac{c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4038\}$

$$-\frac{c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.18, size = 56, normalized size = 1.37

$$\frac{\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Se
c[e + f*x]])/(2*f)
```

Maple [A]
time = 2.58, size = 62, normalized size = 1.51

method	result	size
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sin(fx+e)}{f(-1+\cos(fx+e))}$	62
risch	$\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,e^{i(fx+e)}}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/
2)*sin(f*x+e)/(-1+cos(f*x+e))
```

Maxima [A]
time = 0.51, size = 59, normalized size = 1.44

$$\frac{2\sqrt{-a}\,\sqrt{c}}{f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-a)*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1))
```

Fricas [A]
time = 2.10, size = 61, normalized size = 1.49

$$\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{f\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x
), x)
```

Giac [A]
time = 1.40, size = 54, normalized size = 1.32

$$\frac{2\sqrt{-ac}\,|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 2*sqrt(-a*c)*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*
tan(1/2*f*x + 1/2*e)^2 - c)*f)
```

Mupad [B]
time = 1.94, size = 47, normalized size = 1.15

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}}{f\,\sin\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2))/(f
*sin(e + f*x))
```

3.110 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{a\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A]
time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4037}

$$\frac{a\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.85, size = 99, normalized size = 1.94

$$-\frac{i\left(-1+e^{i(e+fx)}\right)\left(2\log\left(1-e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sec(e+fx))}}{\left(1+e^{i(e+fx)}\right)f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] ((-I)*(-1 + E^(I*(e + f*x)))*(2*Log[1 - E^(I*(e + f*x))] - Log[1 + E^((2*I)
*(e + f*x))])*Sqrt[a*(1 + Sec[e + f*x])])/((1 + E^(I*(e + f*x)))*f*Sqrt[c -
 c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(47) = 94$.
time = 2.76, size = 141, normalized size = 2.76

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(2\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)\right)\cos(fx+e)\sqrt{\frac{c(-1}{c}}}{f\sin(fx+e)c}$
risch	$-\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{2i(fx+e)}+1\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(2*ln(-(-1+cos(f*x+e))/sin(f*x+e))
-ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-ln(-(cos(f*x+e)-1+sin(f*x+e))/s
in(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c
```

Maple [A]
time = 0.51, size = 98, normalized size = 1.92

$$-\frac{\frac{\sqrt{-a}\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{c}}+\frac{\sqrt{-a}\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}}-\frac{2\sqrt{-a}\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(c) + sqrt(-a)*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - 2*sqrt(-a)*log(sin(f*x + e)/
(cos(f*x + e) + 1))/sqrt(c))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/(
c*sec(f*x + e) - c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1)
), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

3.111 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -1/2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4035}

$$-\frac{\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{2f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}$$

Mathematica [A]
time = 0.25, size = 62, normalized size = 1.48

$$\frac{\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{cf(-1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-1 + Sec[e
 + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.80, size = 60, normalized size = 1.43

method	result	size
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\sin(fx+e)}{2f\cos(fx+e)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}$	60
risch	$\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\;e^{i(fx+e)}}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\;f}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*sin(f*x+e)/cos(f*x+e)/(c*(-1+cos
(f*x+e))/cos(f*x+e))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(39) = 78.
time = 0.57, size = 556, normalized size = 13.24

$$-\frac{2\left(\left(\sin\left(3fx+3e\right)+\sin\left(fx+e\right)\right)\cos\left(4fx+4e\right)-\left(\cos\left(3fx+3e\right)+\cos\left(fx+e\right)\right)\sin\left(4fx+4e\right)+\left(2\cos\left(2fx+2e\right)+1\right)\sin\left(3fx+3e\right)-2\cos\left(3fx+3e\right)\sin\left(2fx+2e\right)-2\cos\left(fx+e\right)\sin\left(2fx+2e\right)+2\cos\left(2fx+2e\right)\sin\left(fx+e\right)+\sin\left(fx+e\right)\right)\sqrt{a}\sqrt{c}}{\left(c^2\cos\left(4fx+4e\right)^2+4c^2\cos\left(3fx+3e\right)^2+4c^2\cos\left(2fx+2e\right)^2+4c^2\cos\left(fx+e\right)^2+c^2\sin\left(4fx+4e\right)^2+4c^2\sin\left(3fx+3e\right)^2+4c^2\sin\left(2fx+2e\right)^2-8c^2\sin\left(2fx+2e\right)\sin\left(fx+e\right)+4c^2\sin\left(fx+e\right)^2-4c^2\cos\left(fx+e\right)+c^2-2\left(2c^2\cos\left(3fx+3e\right)-2c^2\cos\left(2fx+2e\right)+2c^2\cos\left(fx+e\right)-c^2\right)\cos\left(4fx+4e\right)-4\left(2c^2\cos\left(2fx+2e\right)-2c^2\cos\left(fx+e\right)+c^2\right)\cos\left(3fx+3e\right)-4\left(2c^2\cos\left(fx+e\right)-c^2\right)\cos\left(2fx+2e\right)-4\left(c^2\sin\left(3fx+3e\right)-c^2\sin\left(2fx+2e\right)+c^2\sin\left(fx+e\right)\right)\sin\left(4fx+4e\right)-8\left(c^2\sin\left(2fx+2e\right)-c^2\sin\left(fx+e\right)\right)\sin\left(3fx+3e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -2*((sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) - (cos(3*f*x + 3*e)
+ cos(f*x + e))*sin(4*f*x + 4*e) + (2*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e
) - 2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*cos(f*x + e)*sin(2*f*x + 2*e) +
 2*cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*x + e))*sqrt(a)*sqrt(c)/((c^2*cos(
4*f*x + 4*e)^2 + 4*c^2*cos(3*f*x + 3*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^
2*cos(f*x + e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(3*f*x + 3*e)^2 + 4*c^
2*sin(2*f*x + 2*e)^2 - 8*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*sin(f*x
+ e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(3*f*x + 3*e) - 2*c^2*cos(2
*f*x + 2*e) + 2*c^2*cos(f*x + e) - c^2)*cos(4*f*x + 4*e) - 4*(2*c^2*cos(2*f
*x + 2*e) - 2*c^2*cos(f*x + e) + c^2)*cos(3*f*x + 3*e) - 4*(2*c^2*cos(f*x +
 e) - c^2)*cos(2*f*x + 2*e) - 4*(c^2*sin(3*f*x + 3*e) - c^2*sin(2*f*x + 2*e
```

```
) + c^2*sin(f*x + e))*sin(4*f*x + 4*e) - 8*(c^2*sin(2*f*x + 2*e) - c^2*sin(
f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(39) = 78$.
time = 3.33, size = 86, normalized size = 2.05

$$\frac{\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{\left(c^2f\cos\left(fx+e\right)-c^2f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*cos(f*x + e)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(
3/2), x)
```

Giac [A]
time = 1.78, size = 58, normalized size = 1.38

$$\frac{a^2\left(\frac{1}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}-1\right)}{2\,\sqrt{-ac}\,cf|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/2*a^2*(1/tan(1/2*f*x + 1/2*e)^2 - 1)/(sqrt(-a*c)*c*f*abs(a)*sgn(tan(1/2*f
*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 3.00, size = 118, normalized size = 2.81

$$-\frac{2\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(\sin\left(e+f\,x\right)-2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{c^2\,f\,\left(4\,\cos\left(e+f\,x\right)+4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] -(2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/cos
(e + f*x))^(1/2)*(sin(e + f*x) - 2*sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(c
^2*f*(4*cos(e + f*x) + 4*cos(2*e + 2*f*x) - 4*cos(3*e + 3*f*x) + cos(4*e +
4*f*x) - 5))
```

3.112 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$-\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4038\}$

$$-\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A]
time = 0.39, size = 69, normalized size = 1.60

$$-\frac{(-1+2\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{2c^2f(-1+\cos(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] -1/2*((-1 + 2*Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c
^2*f*(-1 + Cos[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.61, size = 70, normalized size = 1.63

method	result	size
default	$-\frac{\left(3 \cos \left(f x+e\right)-1\right) \sqrt{\frac{a \left(\cos \left(f x+e\right)+1\right)}{\cos \left(f x+e\right)}}\, \sin \left(f x+e\right)}{8 f \cos \left(f x+e\right)^{2} \left(\frac{c \left(-1+\cos \left(f x+e\right)\right)}{\cos \left(f x+e\right)}\right)^{\frac{5}{2}}}$	70
risch	$\frac{2 i \sqrt{\frac{a \left({\mathrm e}^{i \left(f x+e\right)}+1\right)^{2}}{{\mathrm e}^{2 i \left(f x+e\right)}+1}}\, \left({\mathrm e}^{3 i \left(f x+e\right)}-{\mathrm e}^{2 i \left(f x+e\right)}+{\mathrm e}^{i \left(f x+e\right)}\right)}{c^{2} \left({\mathrm e}^{i \left(f x+e\right)}+1\right) \left({\mathrm e}^{i \left(f x+e\right)}-1\right)^{3} \sqrt{\frac{c \left({\mathrm e}^{i \left(f x+e\right)}-1\right)^{2}}{{\mathrm e}^{2 i \left(f x+e\right)}+1}}\, f}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/f*(3*cos(f*x+e)-1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*sin(f*x+e)/cos(
f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. 2(40) = 80.
time = 0.64, size = 816, normalized size = 18.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*((sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) + cos(4*f*x + 4*e)*sin(2*f*x + 2*e) - (cos(4*f*x + 4*e) + 2*cos(2*f*x + 2
*e) + 1)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(4*f*x
+ 4*e) + 2*cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 3
6*c^3*cos(2*f*x + 2*e)^2 + 16*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 16*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
```

```
^2 + c^3*sin(4*f*x + 4*e)^2 + 12*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36
*c^3*sin(2*f*x + 2*e)^2 + 16*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 16*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 12*c^3*cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*
f*x + 4*e) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*
x + 2*e) + c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c
^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(40) = 80$.
time = 3.25, size = 115, normalized size = 2.67

$$\frac{\left(2\,\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f
*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(
5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.
time = 1.81, size = 86, normalized size = 2.00

$$\frac{a^{2}\left(\frac{2\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)a+a^{2}}{a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}-1\right)}{8\,\sqrt{-ac}\,c^{2}f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/8*a^2*((2*(a*tan(1/2*f*x + 1/2*e)^2 - a)*a + a^2)/(a^2*tan(1/2*f*x + 1/2*
e)^4) - 1)/(sqrt(-a*c)*c^2*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*
x + 1/2*e)))
```

Mupad [B]
time = 6.61, size = 203, normalized size = 4.72

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^3\,f}+\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^3\,f}-\frac{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^3\,f}\right)}{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,10{}\mathrm{i}-{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,8{}\mathrm{i}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((exp(e*3i + f*x*3i)*(a + a/cos(e + f*x))^(1/2)
*4i)/(c^3*f) + (exp(e*3i + f*x*3i)*cos(2*e + 2*f*x)*(a + a/cos(e + f*x))^(1
/2)*4i)/(c^3*f) - (cos(e + f*x)*exp(e*3i + f*x*3i)*(a + a/cos(e + f*x))^(1/
2)*4i)/(c^3*f)))/(exp(e*3i + f*x*3i)*sin(e + f*x)*10i - exp(e*3i + f*x*3i)*
sin(2*e + 2*f*x)*8i + exp(e*3i + f*x*3i)*sin(3*e + 3*f*x)*2i)
```

3.113 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=89

$$\frac{a^2(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{10f\sqrt{a+a\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{5f}$$

```
[Out] 1/10*a^2*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/5*a*(
c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4038}

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{10f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(10*f*Sqrt[a + a*Sec[e + f*x]
]) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(
5*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}}{5f}$$
$$= \frac{a^2(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{10f\sqrt{a+a\sec(e+fx)}} + \frac{a\sqrt{}}{}$$

Mathematica [A]
time = 1.06, size = 108, normalized size = 1.21

$$\frac{ac^3(7-10\cos(e+fx)+20\cos(2(e+fx))-10\cos(3(e+fx))+5\cos(4(e+fx)))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2
),x]
```

```
[Out] (a*c^3*(7 - 10*Cos[e + f*x] + 20*Cos[2*(e + f*x)] - 10*Cos[3*(e + f*x)] + 5
*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*
(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(80*f)
```

Maple [A]
time = 2.54, size = 103, normalized size = 1.16

method	result
default	$\frac{\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(13(\cos^3(fx+e))-16(\cos^2(fx+e))+9\cos(fx+e)-2)(\sin^3(fx+e))a}{10f(-1+\cos(fx+e))^5\cos(fx+e)}$
risch	$\frac{2iac^3\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(5e^{9i(fx+e)}-10e^{8i(fx+e)}+20e^{7i(fx+e)}-10e^{6i(fx+e)}+14e^{5i(fx+e)}-10e^{4i(fx+e)}+}{5(e^{i(fx+e)}+1)(e^{2i(fx+e)}+1)^4(e^{i(fx+e)}-1)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/10/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(
1/2)*(13*cos(f*x+e)^3-16*cos(f*x+e)^2+9*cos(f*x+e)-2)*sin(f*x+e)^3/(-1+cos(
f*x+e))^5/cos(f*x+e)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. 2(83) = 166.
time = 0.60, size = 1804, normalized size = 20.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/5*(100*a*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 25*a*c^3*cos(2*f*x + 2*e
)*sin(f*x + e) - 5*a*c^3*sin(f*x + e) - (5*a*c^3*sin(9*f*x + 9*e) - 10*a*c^
3*sin(8*f*x + 8*e) + 20*a*c^3*sin(7*f*x + 7*e) - 10*a*c^3*sin(6*f*x + 6*e)
+ 14*a*c^3*sin(5*f*x + 5*e) - 10*a*c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*
x + 3*e) - 10*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e))*cos(10*f*x + 1
0*e) + 25*(a*c^3*sin(8*f*x + 8*e) + 2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(
4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 5*(20*a*c^3*sin(7
*f*x + 7*e) + 10*a*c^3*sin(6*f*x + 6*e) + 14*a*c^3*sin(5*f*x + 5*e) + 10*a*
c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*x + 3*e) + 5*a*c^3*sin(f*x + e))*co
s(8*f*x + 8*e) + 100*(2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(4*f*x + 4*e) +
 a*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(14*a*c^3*sin(5*f*x + 5*e) +
 20*a*c^3*sin(3*f*x + 3*e) - 5*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e
))*cos(6*f*x + 6*e) + 70*(2*a*c^3*sin(4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e)
)*cos(5*f*x + 5*e) - 50*(4*a*c^3*sin(3*f*x + 3*e) - a*c^3*sin(2*f*x + 2*e)
+ a*c^3*sin(f*x + e))*cos(4*f*x + 4*e) + (5*a*c^3*cos(9*f*x + 9*e) - 10*a*c
^3*cos(8*f*x + 8*e) + 20*a*c^3*cos(7*f*x + 7*e) - 10*a*c^3*cos(6*f*x + 6*e)
 + 14*a*c^3*cos(5*f*x + 5*e) - 10*a*c^3*cos(4*f*x + 4*e) + 20*a*c^3*cos(3*f
*x + 3*e) - 10*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x + e))*sin(10*f*x +
10*e) - 5*(5*a*c^3*cos(8*f*x + 8*e) + 10*a*c^3*cos(6*f*x + 6*e) + 10*a*c^3*
cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(9*f*x + 9*e) + 5*(
20*a*c^3*cos(7*f*x + 7*e) + 10*a*c^3*cos(6*f*x + 6*e) + 14*a*c^3*cos(5*f*x
+ 5*e) + 10*a*c^3*cos(4*f*x + 4*e) + 20*a*c^3*cos(3*f*x + 3*e) + 5*a*c^3*co
s(f*x + e) + 2*a*c^3)*sin(8*f*x + 8*e) - 20*(10*a*c^3*cos(6*f*x + 6*e) + 10
*a*c^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(7*f*x + 7*e
) + 10*(14*a*c^3*cos(5*f*x + 5*e) + 20*a*c^3*cos(3*f*x + 3*e) - 5*a*c^3*cos
(2*f*x + 2*e) + 5*a*c^3*cos(f*x + e) + a*c^3)*sin(6*f*x + 6*e) - 14*(10*a*c
^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(5*f*x + 5*e) +
10*(20*a*c^3*cos(3*f*x + 3*e) - 5*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x
+ e) + a*c^3)*sin(4*f*x + 4*e) - 20*(5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(
3*f*x + 3*e) + 5*(5*a*c^3*cos(f*x + e) + 2*a*c^3)*sin(2*f*x + 2*e))*sqrt(a)
*sqrt(c)/((2*(5*cos(8*f*x + 8*e) + 10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e
) + 5*cos(2*f*x + 2*e) + 1)*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 10*
(10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e) + 25*cos(8*f*x + 8*e)^2 + 20*(10*cos(4*f*x + 4*e) + 5*cos(2*f*x
+ 2*e) + 1)*cos(6*f*x + 6*e) + 100*cos(6*f*x + 6*e)^2 + 20*(5*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + 100*cos(4*f*x + 4*e)^2 + 25*cos(2*f*x + 2*e)^2
+ 10*(sin(8*f*x + 8*e) + 2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 50*(2*sin(6*f*x + 6*e
) + 2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 25*sin(8*f*x
+ 8*e)^2 + 100*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 1
```

```
00*sin(6*f*x + 6*e)^2 + 100*sin(4*f*x + 4*e)^2 + 100*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 25*sin(2*f*x + 2*e)^2 + 10*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.63, size = 121, normalized size = 1.36

$$\frac{\left(10\,ac^{3}\cos\left(fx+e\right)^{4}-10\,ac^{3}\cos\left(fx+e\right)^{3}+5\,ac^{3}\cos\left(fx+e\right)-2\,ac^{3}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{10\,f\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/10*(10*a*c^3*cos(f*x + e)^4 - 10*a*c^3*cos(f*x + e)^3 + 5*a*c^3*cos(f*x +
 e) - 2*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.83, size = 132, normalized size = 1.48

$$-\frac{8\left(10\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{3}c^{3}+20\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c^{4}+15\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{5}+4\,c^{6}\right)\sqrt{-ac}\,ac|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{5\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{5}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] -8/5*(10*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^3 + 20*(c*tan(1/2*f*x + 1/2*e)^
2 - c)^2*c^4 + 15*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 4*c^6)*sqrt(-a*c)*a*
c*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x
 + 1/2*e)^2 - c)^5*f)
```

Mupad [B]
time = 6.15, size = 294, normalized size = 3.30

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,c^3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,28\mathrm{i}}{5\,f}-\frac{a\,c^3\,\cos(e+f\,x)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,8\mathrm{i}}{f}+\frac{a\,c^3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,16\mathrm{i}}{f}-\frac{a\,c^3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(3\,e+3\,f\,x)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,8\mathrm{i}}{f}+\frac{a\,c^3\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4\mathrm{i}}{f}\right)}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,6\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*c^3*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))
^(1/2)*28i)/(5*f) - (a*c^3*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f
*x))^(1/2)*8i)/f + (a*c^3*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a/cos(e
+ f*x))^(1/2)*16i)/f - (a*c^3*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x)*(a + a/co
s(e + f*x))^(1/2)*8i)/f + (a*c^3*exp(e*5i + f*x*5i)*cos(4*e + 4*f*x)*(a + a
/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*5i + f*x*5i)*sin(e + f*x)*4i + exp(e*5i
 + f*x*5i)*sin(3*e + 3*f*x)*6i + exp(e*5i + f*x*5i)*sin(5*e + 5*f*x)*2i)
```

3.114 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{a+a\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{4f}$$

```
[Out] 1/6*a^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/4*a*(c
-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4038}

$$\frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{6f\sqrt{a\sec(e+fx)+a}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[a + a*Sec[e + f*x]]
) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(4
*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}\,\mathrm{t}}{4f}$$
$$= \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{a+a\sec(e+fx)}} + \frac{a\sqrt{a}}{}$$

Mathematica [A]
time = 0.58, size = 97, normalized size = 1.09

$$\frac{ac^2(5\cos(e+fx)-3\cos(2(e+fx))+3\cos(3(e+fx)))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] (a*c^2*(5*Cos[e + f*x] - 3*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*Csc[(e +
f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]])/(24*f)
```

Maple [A]
time = 2.68, size = 93, normalized size = 1.04

method	result	size
default	$\frac{\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(11(\cos^2(fx+e))-10\cos(fx+e)+3)(\sin^3(fx+e))a}{12f(-1+\cos(fx+e))^4\cos(fx+e)}$	93
risch	$\frac{2iac^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(3\,e^{7i(fx+e)}-3\,e^{6i(fx+e)}+5\,e^{5i(fx+e)}+5\,e^{3i(fx+e)}-3\,e^{2i(fx+e)}+3\,e^{i(fx+e)})}{3(e^{i(fx+e)}+1)(e^{2i(fx+e)}+1)^3(e^{i(fx+e)}-1)f}$	177

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/12/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(
1/2)*(11*cos(f*x+e)^2-10*cos(f*x+e)+3)*sin(f*x+e)^3/(-1+cos(f*x+e))^4/cos(f
*x+e)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1187 vs. 2(83) = 166.
time = 0.57, size = 1187, normalized size = 13.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(20*a*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a*c^2*cos(2*f*x + 2*e)
*sin(f*x + e) - 3*a*c^2*sin(f*x + e) - (3*a*c^2*sin(7*f*x + 7*e) - 3*a*c^2*
sin(6*f*x + 6*e) + 5*a*c^2*sin(5*f*x + 5*e) + 5*a*c^2*sin(3*f*x + 3*e) - 3*
a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*a*c^
2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*c
os(7*f*x + 7*e) - 2*(10*a*c^2*sin(5*f*x + 5*e) + 9*a*c^2*sin(4*f*x + 4*e) +
 10*a*c^2*sin(3*f*x + 3*e) + 6*a*c^2*sin(f*x + e))*cos(6*f*x + 6*e) + 10*(3
*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 6*(5
*a*c^2*sin(3*f*x + 3*e) - 3*a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*
cos(4*f*x + 4*e) + (3*a*c^2*cos(7*f*x + 7*e) - 3*a*c^2*cos(6*f*x + 6*e) + 5
*a*c^2*cos(5*f*x + 5*e) + 5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*
e) + 3*a*c^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a*c^2*cos(6*f*x + 6*e) +
 6*a*c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(7*f*x + 7
*e) + (20*a*c^2*cos(5*f*x + 5*e) + 18*a*c^2*cos(4*f*x + 4*e) + 20*a*c^2*cos
(3*f*x + 3*e) + 12*a*c^2*cos(f*x + e) + 3*a*c^2)*sin(6*f*x + 6*e) - 5*(6*a*
c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(5*f*x + 5*e) +
 6*(5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*e) + 3*a*c^2*cos(f*x +
 e))*sin(4*f*x + 4*e) - 5*(4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(3*f*x + 3*
e) + 3*(4*a*c^2*cos(f*x + e) + a*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2
*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f
*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)
 + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*s
in(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 3.75, size = 121, normalized size = 1.36

$$\frac{\left(12\,ac^2\cos\left(fx+e\right)^3-6\,ac^2\cos\left(fx+e\right)^2-4\,ac^2\cos\left(fx+e\right)+3\,ac^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{12\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(12*a*c^2*cos(f*x + e)^3 - 6*a*c^2*cos(f*x + e)^2 - 4*a*c^2*cos(f*x +
e) + 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.75, size = 107, normalized size = 1.20

$$-\frac{4\left(6\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c^{3}+8\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{4}+3\,c^{5}\right)\sqrt{-ac}\,a|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{4}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -4/3*(6*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^3 + 8*(c*tan(1/2*f*x + 1/2*e)^2
- c)*c^4 + 3*c^5)*sqrt(-a*c)*a*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*
f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^4*f)
```

Mupad [B]
time = 5.50, size = 195, normalized size = 2.19

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,c^2\,\cos(e+f\,x)\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,20{}\mathrm{i}}{3\,f}-\frac{a\,c^2\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{f}+\frac{a\,c^2\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos(3\,e+3\,f\,x)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{f}\right)}{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,4{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*c^2*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/
cos(e + f*x))^(1/2)*20i)/(3*f) - (a*c^2*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a*c^2*exp(e*4i + f*x*4i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)
*4i + exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

3.115 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{c^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}-\frac{c(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{3f}$$

```
[Out] -1/3*c^2*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/3*c*(
a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4038}

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{3f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*
x]]) - (c*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(3*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

$$= -\frac{c^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{c(a}{}$$

Mathematica [A]
time = 0.42, size = 78, normalized size = 0.88

$$\frac{ac(1+3\cos(2(e+fx)))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (a*c*(1 + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x
]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(12*f)
```

Maple [A]
time = 2.71, size = 83, normalized size = 0.93

method	result	size
default	$\frac{\left(\sin^3(fx+e)\right)(2\cos(fx+e)-1)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}a}{3f(-1+\cos(fx+e))^3\cos(fx+e)}$	83
risch	$\frac{2iac\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(3\,e^{5i(fx+e)}+2\,e^{3i(fx+e)}+3\,e^{i(fx+e)}\right)}{3\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\right)^2\left(e^{i(fx+e)}-1\right)f}$	142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/f*sin(f*x+e)^3*(2*cos(f*x+e)-1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(c*
(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/(-1+cos(f*x+e))^3/cos(f*x+e)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(83) = 166.
time = 0.59, size = 596, normalized size = 6.70

$$\frac{2\left(6ac\cos(3fx+3e)\sin(2fx+2e)+9ac\cos(fx+e)\sin(2fx+2e)-9ac\cos(2fx+2e)\sin(fx+e)-3ac\sin(fx+e)-(3ac\sin(5fx+5e)+2ac\sin(3fx+3e)+3ac\sin(fx+e))\cos(6fx+6e)+9(ac\sin(4fx+4e)+ac\sin(2fx+2e))\cos(5fx+5e)-3(2ac\sin(3fx+3e)+3ac\sin(fx+e))\cos(4fx+4e)+(3ac\cos(5fx+5e)+2ac\cos(3fx+3e)+3ac\cos(fx+e))\sin(6fx+6e)-3(3ac\cos(4fx+4e)+3ac\cos(2fx+2e)+ac)\sin(5fx+5e)+3(2ac\cos(3fx+3e)+3ac\cos(fx+e))\sin(4fx+4e)-2(3ac\cos(2fx+2e)+ac)\sin(3fx+3e)\right)\sqrt{a}\sqrt{c}}{3\left(2(3\cos(4fx+4e)+3\cos(2fx+2e)+1)\cos(6fx+6e)+\cos(6fx+6e)^2+6(3\cos(2fx+2e)+1)\cos(4fx+4e)+9\cos(4fx+4e)^2+9\cos(2fx+2e)^2+6(\sin(4fx+4e)+\sin(2fx+2e))\sin(6fx+6e)+\sin(6fx+6e)^2+9\sin(4fx+4e)^2+18\sin(4fx+4e)\sin(2fx+2e)+9\sin(2fx+2e)^2+6\cos(2fx+2e)+1\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(6*a*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 9*a*c*cos(f*x + e)*sin(2*f*x
 + 2*e) - 9*a*c*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a*c*sin(f*x + e) - (3*a*c
*sin(5*f*x + 5*e) + 2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(6*f*x
+ 6*e) + 9*(a*c*sin(4*f*x + 4*e) + a*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) -
 3*(2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(4*f*x + 4*e) + (3*a*c*
cos(5*f*x + 5*e) + 2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(6*f*x +
 6*e) - 3*(3*a*c*cos(4*f*x + 4*e) + 3*a*c*cos(2*f*x + 2*e) + a*c)*sin(5*f*x
 + 5*e) + 3*(2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(4*f*x + 4*e)
- 2*(3*a*c*cos(2*f*x + 2*e) + a*c)*sin(3*f*x + 3*e))*sqrt(a)*sqrt(c)/((2*(3
*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x +
6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2
 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.34, size = 89, normalized size = 1.00

$$\frac{\left(3\,ac\cos\left(fx+e\right)^{2}-ac\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a*c*cos(f*x + e)^2 - a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 1.70, size = 86, normalized size = 0.97

$$-\frac{4\left(3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{4}+2\,c^{5}\right)\sqrt{-ac}\,a|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{3}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -4/3*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 2*c^5)*sqrt(-a*c)*a*abs(c)*sgn
(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2
- c)^3*c^2*f)
```

Mupad [B]
time = 3.14, size = 108, normalized size = 1.21

$$\frac{2\,a\,c\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(e+f\,x\right)+5\,\sin\left(3\,e+3\,f\,x\right)+3\,\sin\left(5\,e+5\,f\,x\right)\right)}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)-\cos\left(6\,e+6\,f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] (2*a*c*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/
2)*(2*sin(e + f*x) + 5*sin(3*e + 3*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*
e + 2*f*x) - 2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

3.116 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}$

Optimal. Leaf size=43

$$-\frac{c(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*c*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4038\}$

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -1/2*(c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]
])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.28, size = 73, normalized size = 1.70

$$\frac{a(1+2\cos(e+fx))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] (a*(1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(4*f)
```

Maple [A]
time = 2.64, size = 73, normalized size = 1.70

method	result	size
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\sin^3(fx+e))\,\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\;a}{2f\cos(fx+e)(-1+\cos(fx+e))^2}$	73
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{3i(fx+e)}+e^{2i(fx+e)}+e^{i(fx+e)}\right)}{\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$	135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*sin(f*x+e)^3*(c*(-1+cos(f*x+e))/c
os(f*x+e))^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))^2*a
```

Maxima [A]
time = 0.50, size = 60, normalized size = 1.40

$$-\frac{2\sqrt{-a}\,a\sqrt{c}}{f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*sqrt(-a)*a*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.
time = 2.90, size = 83, normalized size = 1.93

$$\frac{(2\,a\cos(fx+e)+a)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\,f\cos(fx+e)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*a*cos(f*x + e) + a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}\sec(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))*sec(e +
f*x), x)
```

Giac [A]
time = 1.59, size = 56, normalized size = 1.30

$$-\frac{2\sqrt{-ac}\,ac|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -2*sqrt(-a*c)*a*c*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))
/((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*f)
```

Mupad [B]
time = 2.59, size = 76, normalized size = 1.77

$$\frac{a\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,(\sin(e+f\,x)+\sin(2\,e+2\,f\,x)+\sin(3\,e+3\,f\,x))}{f\sin(2\,e+2\,f\,x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (a*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*(
sin(e + f*x) + sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(f*sin(2*e + 2*f*x)^2)
```

3.117 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2*a^2*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4037}

$$\frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)\sqrt{a+a}}{\sqrt{c-c\sec(e}}$$

$$= \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{a\sqrt{a+a\sec(e+}}{f\sqrt{c-c\sec}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.40, size = 174, normalized size = 1.83

$$\frac{\sqrt{2}\,a(1+\cos(e+fx)\left(4\log\left(1-e^{i(e+fx)}\right)-2\log\left(1+e^{2i(e+fx)}\right)\right))\sec^{\frac{3}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)}{\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]
],x]
```

```
[Out] (Sqrt[2]*a*(1 + Cos[e + f*x]*(4*Log[1 - E^(I*(e + f*x))] - 2*Log[1 + E^((2*
I)*(e + f*x))]))*Sec[e + f*x]^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*(Cos[(e + f*
x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/((1 + E^(I*(e + f*x)))*Sqrt[E
^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.93, size = 162, normalized size = 1.71

method	result
default	$\frac{\left(2\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)-4\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-\cos(f}{f\sin(fx+e)c}$
risch	$-\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}-e^{i(fx+e)}\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f\left(e^{2i(fx+e)}+1\right)}-\frac{4ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia\sqrt{\frac{a(e^{i(fx}}{e^{2i(fx}}}}{(e^{i(f}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*(2*cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln
(-(-1+cos(f*x+e))/sin(f*x+e))+2*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))-cos(f*x+e)-1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(c*(-1+cos(f*x
+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c*a
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(94) = 188$.
time = 0.57, size = 296, normalized size = 3.12

$$-\frac{2\left(a\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\sin\left(2fx+2e\right)+\left(a\cos\left(2fx+2e\right)^2+a\sin\left(2fx+2e\right)^2+2a\cos\left(2fx+2e\right)+a\right)\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)-2\left(a\cos\left(2fx+2e\right)^2+a\sin\left(2fx+2e\right)^2+2a\cos\left(2fx+2e\right)+a\right)\arctan\left(\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right),\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)-1\right)-\left(a\cos\left(2fx+2e\right)+a\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\right)\sqrt{a}\sqrt{c}}{\left(c\cos\left(2fx+2e\right)^2+c\sin\left(2fx+2e\right)^2+2c\cos\left(2fx+2e\right)+c\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e)
 + (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*(a*cos(2*f*x + 2*e)^2
+ a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 1) - (a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(2*f*x + 2*e)^2 + c*sin(2
*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*f)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt
(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/sqrt(-c*(sec(e + f*x) -
 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

3.118 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{a^2\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-a^2*ln(1-sec(
f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4039, 4037}

$$-\frac{a^2\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)))
 - (a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{a\int \frac{\sec(e+fx)\sqrt{a+a\sec(e}{\sqrt{c-c\sec(e+}}{c}$$

$$= -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{a^2\log(1-\sec(e+f}{cf\sqrt{a+a\sec(e+fx)}\,}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.66, size = 134, normalized size = 1.35

$$-\frac{a\left(2-2\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(2\log\left(1-e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)+\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{cf(-1+\cos(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((a*(2 - 2*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(2*Log[1 - E^(I*(e + f*
x))] - Log[1 + E^((2*I)*(e + f*x))]) + Log[1 + E^((2*I)*(e + f*x))])*Sqrt[a
*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-1 + Cos[e + f*x])*Sqrt[c - c*
Sec[e + f*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(91) = 182$.
time = 2.76, size = 249, normalized size = 2.52

method	result
default	$-\frac{(-1+\cos(fx+e))\left(\cos(fx+e)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)-2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(}{\sin(fx+e)}\right.\right.}{f\cos(fx+e)\left(\frac{c(-1-}{\mathrm{co}}\right.}$
risch	$\frac{4ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,e^{i(fx+e)}}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{ia\sqrt{\frac{a\left(e^{i(fx+e)}-\right.}{e^{2i(fx+e)}}}}{c\left(e^{i(fx+}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(-1+cos(f*x+e))*(cos(f*x+e)*ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))-
2*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln(-(cos(f*x+e)-1+s
in(f*x+e))/sin(f*x+e))-ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+2*ln(-(-1+
cos(f*x+e))/sin(f*x+e))-ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))-cos(f*x+e
```

```
)-1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/cos(
f*x+e))^(3/2)/sin(f*x+e)*a
```

Maxima [A]
time = 0.52, size = 130, normalized size = 1.31

$$\frac{\frac{\sqrt{-a}\,a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{-a}\,a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{2\sqrt{-a}\,a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{-a}\,a(\cos(fx+e)+1)^2}{c^{\frac{3}{2}}\sin(fx+e)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] (sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(3/2) + sqrt(-a)*a*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 2*sqrt(-a)*a*log(sin(f*x
+ e)/(cos(f*x + e) + 1))/c^(3/2) + sqrt(-a)*a*(cos(f*x + e) + 1)^2/(c^(3/2)
*sin(f*x + e)^2))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt(
-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sec(e+fx)}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/(-c*(sec(e + f*x) - 1))
**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x
)
```

3.119 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/4*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{4f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5/2
))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A]
time = 0.47, size = 63, normalized size = 1.50

$$\frac{a\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}\,\tan\left(\frac{1}{2}(e+fx)\right)}{c^3f(-1+\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (a*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]*Tan[(e
+ f*x)/2])/(c^3*f*(-1 + Sec[e + f*x])^3)
```

Maple [A]
time = 2.63, size = 73, normalized size = 1.74

method	result	size
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\sin^3(fx+e))a}{4f(-1+\cos(fx+e))\cos(fx+e)^2\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}}$	73
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{c^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*sin(f*x+e)^3/(-1+cos(f*x+e))/cos(
f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(39) = 78.
time = 0.58, size = 575, normalized size = 13.69

$$\frac{2\left(6a\cos(3fx+3e)\sin(2fx+2e)+6a\cos(fx+e)\sin(2fx+2e)-6a\cos(2fx+2e)\sin(fx+e)-(a\sin(3fx+3e)+a\sin(fx+e))\cos(4fx+4e)+(a\cos(3fx+3e)+a\cos(fx+e))\sin(4fx+4e)-(6a\cos(2fx+2e)+a)\sin(3fx+3e)-a\sin(fx+e)\right)\sqrt{a}\sqrt{c}}{\left(c^3\cos(4fx+4e)^2+16c^3\cos(3fx+3e)^2+36c^3\cos(2fx+2e)^2+16c^3\cos(fx+e)^2+c^3\sin(4fx+4e)^2+16c^3\sin(3fx+3e)^2+36c^3\sin(2fx+2e)^2-48c^3\sin(2fx+2e)\sin(fx+e)+16c^3\sin(fx+e)^2-8c^3\cos(fx+e)+c^3-2(4c^3\cos(3fx+3e)-6c^3\cos(2fx+2e)+4c^3\cos(fx+e)-c^3)\cos(4fx+4e)-8(6c^3\cos(2fx+2e)-4c^3\cos(fx+e)+c^3)\cos(3fx+3e)-12(4c^3\cos(fx+e)-c^3)\cos(2fx+2e)-4(2c^3\sin(3fx+3e)-3c^3\sin(2fx+2e)+2c^3\sin(fx+e))\sin(4fx+4e)-16(3c^3\sin(2fx+2e)-2c^3\sin(fx+e))\sin(3fx+3e)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*(6*a*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 6*a*cos(f*x + e)*sin(2*f*x + 2*e
) - 6*a*cos(2*f*x + 2*e)*sin(f*x + e) - (a*sin(3*f*x + 3*e) + a*sin(f*x + e
))*cos(4*f*x + 4*e) + (a*cos(3*f*x + 3*e) + a*cos(f*x + e))*sin(4*f*x + 4*e
) - (6*a*cos(2*f*x + 2*e) + a)*sin(3*f*x + 3*e) - a*sin(f*x + e))*sqrt(a)*s
qrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 16*c^3*cos(3*f*x + 3*e)^2 + 36*c^3*cos(2*
f*x + 2*e)^2 + 16*c^3*cos(f*x + e)^2 + c^3*sin(4*f*x + 4*e)^2 + 16*c^3*sin(
3*f*x + 3*e)^2 + 36*c^3*sin(2*f*x + 2*e)^2 - 48*c^3*sin(2*f*x + 2*e)*sin(f*
x + e) + 16*c^3*sin(f*x + e)^2 - 8*c^3*cos(f*x + e) + c^3 - 2*(4*c^3*cos(3*
f*x + 3*e) - 6*c^3*cos(2*f*x + 2*e) + 4*c^3*cos(f*x + e) - c^3)*cos(4*f*x +
 4*e) - 8*(6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(f*x + e) + c^3)*cos(3*f*x + 3
*e) - 12*(4*c^3*cos(f*x + e) - c^3)*cos(2*f*x + 2*e) - 4*(2*c^3*sin(3*f*x +
```

```
 3*e) - 3*c^3*sin(2*f*x + 2*e) + 2*c^3*sin(f*x + e))*sin(4*f*x + 4*e) - 16*
(3*c^3*sin(2*f*x + 2*e) - 2*c^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(39) = 78$.
time = 3.03, size = 103, normalized size = 2.45

$$\frac{a\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f
)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a(\sec\left(e+fx\right)+1)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/(-c*(sec(e + f*x) - 1))
**(5/2), x)
```

Giac [A]
time = 1.87, size = 61, normalized size = 1.45

$$\frac{\left(a-\frac{a}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}\right)a^{2}}{4\sqrt{-ac}\,c^{2}f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/4*(a - a/tan(1/2*f*x + 1/2*e)^4)*a^2/(sqrt(-a*c)*c^2*f*abs(a)*sgn(tan(1/2
*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 4.83, size = 165, normalized size = 3.93

$$-\frac{2\,a\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(6\,\sin\left(e+f\,x\right)-8\,\sin\left(2\,e+2\,f\,x\right)+7\,\sin\left(3\,e+3\,f\,x\right)-4\,\sin\left(4\,e+4\,f\,x\right)+\sin\left(5\,e+5\,f\,x\right)\right)}{c^3\,f\,\left(48\,\cos\left(e+f\,x\right)+15\,\cos\left(2\,e+2\,f\,x\right)-40\,\cos\left(3\,e+3\,f\,x\right)+26\,\cos\left(4\,e+4\,f\,x\right)-8\,\cos\left(5\,e+5\,f\,x\right)+\cos\left(6\,e+6\,f\,x\right)-42\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*a*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/c
os(e + f*x))^(1/2)*(6*sin(e + f*x) - 8*sin(2*e + 2*f*x) + 7*sin(3*e + 3*f*x
) - 4*sin(4*e + 4*f*x) + sin(5*e + 5*f*x)))/(c^3*f*(48*cos(e + f*x) + 15*co
s(2*e + 2*f*x) - 40*cos(3*e + 3*f*x) + 26*cos(4*e + 4*f*x) - 8*cos(5*e + 5*
f*x) + cos(6*e + 6*f*x) - 42))
```

3.120 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}-\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{24cf(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/6*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)-1/24*(a+a*s
ec(f*x+e))^(3/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.21, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4036, 4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{24cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/6*((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(7/2
)) - ((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(24*c*f*(c - c*Sec[e + f*x])
^(5/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}}{6c}$$
$$= -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}} - \frac{(a+a\sec(e+fx))^{3/2}\text{ta}}{24cf(c-c\sec(e+f}$$

Mathematica [A]
time = 0.58, size = 80, normalized size = 0.91

$$\frac{a(-4+3\cos(e+fx)-3\cos(2(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{6c^3f(-1+\cos(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7
/2),x]
```

```
[Out] (a*(-4 + 3*Cos[e + f*x] - 3*Cos[2*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(6*c^3*f*(-1 + Cos[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 3.00, size = 83, normalized size = 0.94

method	result	size
default	$\frac{(5\cos(fx+e)-1)\left(\sin^3(fx+e)\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,a}{24f(-1+\cos(fx+e))\cos(fx+e)^3\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}}$	83
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(3\,e^{5i(fx+e)}-3\,e^{4i(fx+e)}+8\,e^{3i(fx+e)}-3\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3c^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^5\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$	153

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/24/f*(5*cos(f*x+e)-1)*sin(f*x+e)^3*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(-
1+cos(f*x+e))/cos(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1676 vs. 2(82) = 164.
time = 1.26, size = 1676, normalized size = 19.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*(a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 3*(a*si
n(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) - 4*a*sin(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(2*a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) +
 15*a*sin(2*f*x + 2*e) + 3*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(a*sin(6*f
*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e))*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a*cos(4*f*x + 4*e) + a*cos(2*f*x +
2*e))*sin(6*f*x + 6*e) + 3*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x + 2*e) - 3*(a
*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) - 4*a*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 4*(2*a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x +
 4*e) + 15*a*cos(2*f*x + 2*e) + 3*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 2*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 3*(a*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) + a)*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^4*
cos(6*f*x + 6*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 225*c^4*cos(2*f*x + 2*e)^
2 + 36*c^4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^4*sin(6*f*x + 6*e)^2 + 225
*c^4*sin(4*f*x + 4*e)^2 + 450*c^4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*c
^4*sin(2*f*x + 2*e)^2 + 36*c^4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 400*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 36*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*c^4*c
os(2*f*x + 2*e) + c^4 + 2*(15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e
) + c^4)*cos(6*f*x + 6*e) + 30*(15*c^4*cos(2*f*x + 2*e) + c^4)*cos(4*f*x +
4*e) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*
x + 2*e) - 20*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*
c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*cos(6*f*x + 6*e) + 15*c
^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*
c^4*cos(2*f*x + 2*e) + c^4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 30*(c^4*sin(4*f*x + 4*e) + c^4*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) -
 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*
e) - 20*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x
 + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e),
```

```
 cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*
e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [A]
time = 2.14, size = 144, normalized size = 1.64

$$\frac{\left(6\,a\cos\left(fx+e\right)^{3}-3\,a\cos\left(fx+e\right)^{2}+a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{6\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-c^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f
*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*si
n(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 1.81, size = 89, normalized size = 1.01

$$\frac{\left(a-\frac{3\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)a^{3}+a^{4}}{a^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}\right)a^{2}}{24\,\sqrt{-ac}\,c^{3}f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] 1/24*(a - (3*(a*tan(1/2*f*x + 1/2*e)^2 - a)*a^3 + a^4)/(a^3*tan(1/2*f*x + 1
/2*e)^6))*a^2/(sqrt(-a*c)*c^3*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2
*f*x + 1/2*e)))
```

Mupad [B]
time = 7.03, size = 273, normalized size = 3.10

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^4\,f}-\frac{a\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,44{}\mathrm{i}}{3\,c^4\,f}+\frac{a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^4\,f}-\frac{a\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^4\,f}\right)}{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,28{}\mathrm{i}-{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,28{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,12{}\mathrm{i}-{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*4i + f*x*4i)*(a + a/cos(e + f*x))^(1/
2)*4i)/(c^4*f) - (a*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/cos(e + f*x))^(1
/2)*44i)/(3*c^4*f) + (a*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)*(a + a/cos(e +
f*x))^(1/2)*4i)/(c^4*f) - (a*exp(e*4i + f*x*4i)*cos(3*e + 3*f*x)*(a + a/cos
(e + f*x))^(1/2)*4i)/(c^4*f)))/(exp(e*4i + f*x*4i)*sin(e + f*x)*28i - exp(e
*4i + f*x*4i)*sin(2*e + 2*f*x)*28i + exp(e*4i + f*x*4i)*sin(3*e + 3*f*x)*12
i - exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

3.121 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}}+\frac{a^2\tan(e+fx)}{12cf\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}}$$

```
[Out] 1/12*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/4*a
*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)
```

Rubi [A]
time = 0.20, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4039, 4038}

$$\frac{a^2\tan(e+fx)}{12cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -1/4*(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(9/2
)) + (a^2*Tan[e + f*x])/(12*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x
])^(7/2))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}} - \frac{a\int \frac{\sec(e+fx)\sqrt{a+a\sec(e}{(c-c\sec(e+fx))^{7/2}}}{4c}$$

$$= -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}} + \frac{a^2\tan(e+}{12cf\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 0.82, size = 90, normalized size = 0.98

$$-\frac{a(-8+17\cos(e+fx)-6\cos(2(e+fx))+3\cos(3(e+fx)))\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{12c^4f(-1+\cos(e+fx))^4\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9
/2),x]
```

```
[Out] -1/12*(a*(-8 + 17*Cos[e + f*x] - 6*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*S
qrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c^4*f*(-1 + Cos[e + f*x])^4*Sq
rt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.58, size = 93, normalized size = 1.01

method	result	size
default	$\frac{\left(17\left(\cos^2(fx+e)\right)-6\cos(fx+e)+1\right)\left(\sin^3(fx+e)\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,a}{96f(-1+\cos(fx+e))\cos(fx+e)^4\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{9}{2}}}$	93
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(3\,e^{7i(fx+e)}-6\,e^{6i(fx+e)}+17\,e^{5i(fx+e)}-16\,e^{4i(fx+e)}+17\,e^{3i(fx+e)}-6\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3c^4\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^7\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$	175

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/96/f*(17*cos(f*x+e)^2-6*cos(f*x+e)+1)*sin(f*x+e)^3*(a*(cos(f*x+e)+1)/cos(
f*x+e))^(1/2)/(-1+cos(f*x+e))/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e))^(
9/2)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2804 vs. 2(86) = 172.
time = 5.95, size = 2804, normalized size = 30.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(28*a*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 28*a*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e) + 2*(3*a*sin(6*f*x + 6*e) + 8*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x
+ 2*e))*cos(8*f*x + 8*e) + (3*a*sin(8*f*x + 8*e) + 36*a*sin(6*f*x + 6*e) +
82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e) - 32*a*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17
*a*sin(8*f*x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140
*a*sin(2*f*x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a*sin(8*f*
x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140*a*sin(2*f*
x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a*sin(8*f*x + 8*e) + 3
6*a*sin(6*f*x + 6*e) + 82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(3*a*cos(6*f*x + 6*e) +
 8*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - 2*(14*a*co
s(4*f*x + 4*e) - 3*a)*sin(6*f*x + 6*e) + 4*(7*a*cos(2*f*x + 2*e) + 4*a)*sin
(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - (3*a*cos(8*f*x + 8*e) + 36*a*cos(6*f
*x + 6*e) + 82*a*cos(4*f*x + 4*e) + 36*a*cos(2*f*x + 2*e) - 32*a*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 3*a)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - (17*a*cos(8*f*x + 8*e) + 140*a*cos(6*f*x + 6*e) + 294*a*cos(
4*f*x + 4*e) + 140*a*cos(2*f*x + 2*e) + 32*a*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 17*a)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - (17*a*cos(8*f*x + 8*e) + 140*a*cos(6*f*x + 6*e) + 294*a*cos(4*f*
x + 4*e) + 140*a*cos(2*f*x + 2*e) + 32*a*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 17*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - (3*a*cos(8*f*x + 8*e) + 36*a*cos(6*f*x + 6*e) + 82*a*cos(4*f*x + 4*e
) + 36*a*cos(2*f*x + 2*e) + 3*a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*sqrt(a)*sqrt(c)/((c^5*cos(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x +
6*e)^2 + 4900*c^5*cos(4*f*x + 4*e)^2 + 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*
cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + c^5*sin(8*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*
e)^2 + 4900*c^5*sin(4*f*x + 4*e)^2 + 3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + 784*c^5*sin(2*f*x + 2*e)^2 + 64*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 3136*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 64*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c
^5*cos(2*f*x + 2*e) + c^5 + 2*(28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x +
```

```
4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*
f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*
cos(2*f*x + 2*e) + c^5)*cos(4*f*x + 4*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^
5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56
*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*
c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + c^5)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 112*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f
*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + c^5)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*
f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 28*(2*c^5*sin(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e)
 + 2*c^5*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) +
 2*c^5*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c
^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 5
6*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*s
in(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e...
```

Fricas [A]
time = 2.30, size = 171, normalized size = 1.86

$$\frac{\left(6\,a\cos\left(fx+e\right)^{4}-6\,a\cos\left(fx+e\right)^{3}+4\,a\cos\left(fx+e\right)^{2}-a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{6\,\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}+6\,c^{5}f\cos\left(fx+e\right)^{2}-4\,c^{5}f\cos\left(fx+e\right)+c^{5}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a*cos(f*x + e)^4 - 6*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 - a*cos(f
*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos
(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.15, size = 113, normalized size = 1.23

$$\frac{\left(a-\frac{6\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{2}a^{3}+4\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)a^{4}+a^{5}}{a^{4}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}}\right)a^{2}}{96\sqrt{-ac}\,c^{4}f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="giac")
```

```
[Out] 1/96*(a - (6*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*a^3 + 4*(a*tan(1/2*f*x + 1/2*
e)^2 - a)*a^4 + a^5)/(a^4*tan(1/2*f*x + 1/2*e)^8))*a^2/(sqrt(-a*c)*c^4*f*ab
s(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 7.62, size = 340, normalized size = 3.70

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,68{}\mathrm{i}}{3\,c^5\,f}-\frac{a\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,88{}\mathrm{i}}{3\,c^5\,f}+\frac{a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,80{}\mathrm{i}}{3\,c^5\,f}-\frac{a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,8{}\mathrm{i}}{c^5\,f}+\frac{a\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^5\,f}\right)}{{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,84{}\mathrm{i}-{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,96{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,54{}\mathrm{i}-{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,16{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(9/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))^(1/
2)*68i)/(3*c^5*f) - (a*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))
^(1/2)*88i)/(3*c^5*f) + (a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a/cos(e
 + f*x))^(1/2)*80i)/(3*c^5*f) - (a*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x)*(a +
 a/cos(e + f*x))^(1/2)*8i)/(c^5*f) + (a*exp(e*5i + f*x*5i)*cos(4*e + 4*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f)))/(exp(e*5i + f*x*5i)*sin(e + f*x)*
84i - exp(e*5i + f*x*5i)*sin(2*e + 2*f*x)*96i + exp(e*5i + f*x*5i)*sin(3*e
+ 3*f*x)*54i - exp(e*5i + f*x*5i)*sin(4*e + 4*f*x)*16i + exp(e*5i + f*x*5i)
*sin(5*e + 5*f*x)*2i)
```

3.122 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}}+\frac{a^2\tan(e+fx)}{20cf\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{9/2}}$$

```
[Out] 1/20*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(9/2)/(a+a*sec(f*x+e))^(1/2)-1/5*a
*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)
```

Rubi [A]
time = 0.19, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4039, 4038}

$$\frac{a^2\tan(e+fx)}{20cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{9/2}}-\frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(11/2),x
]
```

```
[Out] -1/5*(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(11/
2)) + (a^2*Tan[e + f*x])/(20*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(9/2))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}} - \frac{a\int \frac{\sec(e+fx)\sqrt{a+a\sec(e}{(c-c\sec(e+fx))^{9/}}}{5c}$$
$$= -\frac{a\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}} + \frac{a^2\tan(e}{20cf\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 1.20, size = 100, normalized size = 1.09

$$\frac{a(-51+75\cos(e+fx)-50\cos(2(e+fx))+15\cos(3(e+fx))-5\cos(4(e+fx)))\sqrt{a(1+\sec(e+fx))}\,\tan\left(\frac{1}{2}(e+fx)\right)}{40c^5f(-1+\cos(e+fx))^5\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(1
1/2),x]
```

```
[Out] (a*(-51 + 75*Cos[e + f*x] - 50*Cos[2*(e + f*x)] + 15*Cos[3*(e + f*x)] - 5*C
os[4*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(40*c^5*f*(-1
 + Cos[e + f*x])^5*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.63, size = 103, normalized size = 1.12

method	result
default	$\frac{\left(49\left(\cos^3(fx+e)\right)-23\left(\cos^2(fx+e)\right)+7\cos(fx+e)-1\right)\left(\sin^3(fx+e)\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\;a}{320f(-1+\cos(fx+e))\cos(fx+e)^5\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{11}{2}}}$
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(5\,e^{9i(fx+e)}-15\,e^{8i(fx+e)}+50\,e^{7i(fx+e)}-75\,e^{6i(fx+e)}+102\,e^{5i(fx+e)}-75\,e^{4i(fx+e)}+50\,e^{3i(fx+e)}-15\,e^{2i(f}\right.}{5c^5\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^9\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\;f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/320/f*(49*cos(f*x+e)^3-23*cos(f*x+e)^2+7*cos(f*x+e)-1)*sin(f*x+e)^3*(a*(c
os(f*x+e)+1)/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/cos(f*x+e)^5/(c*(-1+cos(f*x+
e))/cos(f*x+e))^(11/2)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4202 vs. 2(86) = 172.
time = 33.30, size = 4202, normalized size = 45.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="maxima")
```

```
[Out] -2/5*(225*a*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 225*a*cos(4*f*x + 4*e)*sin(
2*f*x + 2*e) - 15*(a*sin(8*f*x + 8*e) + 5*a*sin(6*f*x + 6*e) + 5*a*sin(4*f*
x + 4*e) + a*sin(2*f*x + 2*e))*cos(10*f*x + 10*e) - 225*(a*sin(6*f*x + 6*e)
 + a*sin(4*f*x + 4*e))*cos(8*f*x + 8*e) - 5*(a*sin(10*f*x + 10*e) + 15*a*si
n(8*f*x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*sin(2
*f*x + 2*e) - 20*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4
8*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*a*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 10*(5*a*sin(10*f*x + 10*e) + 45*a*sin(8*f*x + 8*e) +
 150*a*sin(6*f*x + 6*e) + 150*a*sin(4*f*x + 4*e) + 45*a*sin(2*f*x + 2*e) -
36*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*a*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 6*(17*a*sin(10*f*x + 10*e) + 135*a*sin(8*f*x + 8*e)
 + 420*a*sin(6*f*x + 6*e) + 420*a*sin(4*f*x + 4*e) + 135*a*sin(2*f*x + 2*e)
 + 60*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 40*a*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 50*(a*sin(10*f*x + 10*e) + 9*a*sin(8*f*x + 8*e)
+ 30*a*sin(6*f*x + 6*e) + 30*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) + 2*
a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*(a*sin(10*f*x + 10*e) + 15*a*sin(8*f*
x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*sin(2*f*x +
 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 15*(a*cos(8*f
*x + 8*e) + 5*a*cos(6*f*x + 6*e) + 5*a*cos(4*f*x + 4*e) + a*cos(2*f*x + 2*e
))*sin(10*f*x + 10*e) + 15*(15*a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) -
 a)*sin(8*f*x + 8*e) - 75*(3*a*cos(2*f*x + 2*e) + a)*sin(6*f*x + 6*e) - 75*
(3*a*cos(2*f*x + 2*e) + a)*sin(4*f*x + 4*e) - 15*a*sin(2*f*x + 2*e) + 5*(a*
cos(10*f*x + 10*e) + 15*a*cos(8*f*x + 8*e) + 60*a*cos(6*f*x + 6*e) + 60*a*c
os(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) - 20*a*cos(7/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 48*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 20*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*
sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*(5*a*cos(10*f*x +
 10*e) + 45*a*cos(8*f*x + 8*e) + 150*a*cos(6*f*x + 6*e) + 150*a*cos(4*f*x +
 4*e) + 45*a*cos(2*f*x + 2*e) - 36*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 10*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 5*a)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(17*a*cos(1
0*f*x + 10*e) + 135*a*cos(8*f*x + 8*e) + 420*a*cos(6*f*x + 6*e) + 420*a*cos
(4*f*x + 4*e) + 135*a*cos(2*f*x + 2*e) + 60*a*cos(3/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 40*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 17*a)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 50*
(a*cos(10*f*x + 10*e) + 9*a*cos(8*f*x + 8*e) + 30*a*cos(6*f*x + 6*e) + 30*a
```

```
*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) + 2*a*cos(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) + 5*(a*cos(10*f*x + 10*e) + 15*a*cos(8*f*x + 8*e) + 60*a*cos(6*f*x
+ 6*e) + 60*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 1
0*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 + 44100*c^6*cos(6*f*x + 6*e)^2 + 44100
*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*cos(5/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 + 2025*c^6*sin(8*f*x + 8*e)^2 + 44100*
c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*sin(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(7/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*sin(5/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) + c^6 + 2*(45*c^6*cos(8*f*x + 8*e) + 2
10*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e
) + c^6)*cos(10*f*x + 10*e) + 90*(210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*
f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(8*f*x + 8*e) + 420*(210*c^6
*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(6*f*x + 6*e) + 420*(
45*c^6*cos(2*f*x + 2*e) + c^6)*cos(4*f*x + 4*e) - 20*(c^6*cos(10*f*x + 10*e
) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x
+ 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 252*c^6*cos(5/2*arctan2(...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(86) = 172$.

time = 2.31, size = 199, normalized size = 2.16

$$\frac{\left(20\,a\cos\left(fx+e\right)^{5}-30\,a\cos\left(fx+e\right)^{4}+30\,a\cos\left(fx+e\right)^{3}-15\,a\cos\left(fx+e\right)^{2}+3\,a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{20\left(c^{6}f\cos\left(fx+e\right)^{5}-5\,c^{6}f\cos\left(fx+e\right)^{4}+10\,c^{6}f\cos\left(fx+e\right)^{3}-10\,c^{6}f\cos\left(fx+e\right)^{2}+5\,c^{6}f\cos\left(fx+e\right)-c^{6}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/20*(20*a*cos(f*x + e)^5 - 30*a*cos(f*x + e)^4 + 30*a*cos(f*x + e)^3 - 15*
a*cos(f*x + e)^2 + 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*
cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*
f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.70, size = 137, normalized size = 1.49

$$\frac{\left(a-\frac{10\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{3}a^{3}+10\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{2}a^{4}+5\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)a^{5}+a^{6}}{a^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}\right)a^{2}}{320\,\sqrt{-ac}\,c^{5}f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="giac")
```

```
[Out] 1/320*(a - (10*(a*tan(1/2*f*x + 1/2*e)^2 - a)^3*a^3 + 10*(a*tan(1/2*f*x + 1
/2*e)^2 - a)^2*a^4 + 5*(a*tan(1/2*f*x + 1/2*e)^2 - a)*a^5 + a^6)/(a^5*tan(1
/2*f*x + 1/2*e)^10))*a^2/(sqrt(-a*c)*c^5*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^
3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 7.68, size = 407, normalized size = 4.42

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,60{}\mathrm{i}}{c^6\,f}-\frac{a\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,608{}\mathrm{i}}{5\,c^6\,f}+\frac{a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,72{}\mathrm{i}}{c^6\,f}-\frac{a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,44{}\mathrm{i}}{c^6\,f}+\frac{a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,12{}\mathrm{i}}{c^6\,f}-\frac{a\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^6\,f}\right)}{{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,88{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(11/2)),x
)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(1/
2)*60i)/(c^6*f) - (a*cos(e + f*x)*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(
1/2)*608i)/(5*c^6*f) + (a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a/cos(e
+ f*x))^(1/2)*72i)/(c^6*f) - (a*exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*(a + a/
cos(e + f*x))^(1/2)*44i)/(c^6*f) + (a*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(
a + a/cos(e + f*x))^(1/2)*12i)/(c^6*f) - (a*exp(e*6i + f*x*6i)*cos(5*e + 5*
f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^6*f)))/(exp(e*6i + f*x*6i)*sin(e + f
*x)*264i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*si
n(3*e + 3*f*x)*220i - exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i + exp(e*6i +
f*x*6i)*sin(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

3.123 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal. Leaf size=134

$$\frac{a^3(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}}+\frac{2a^2\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f}+\frac{a(a+a\sec$$

```
[Out] 1/6*a*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f+1/15*a^3*(
c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a^2*(c-c*sec
(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.29, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4038}

$$\frac{a^3\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{15f\sqrt{a\sec(e+fx)+a}}+\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{7/2}}{15f}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]
]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x
])/(15*f) + (a*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2)*Tan[e
+ f*x])/(6*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$
$$= \frac{2a^2\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{7/}}{15f}$$
$$= \frac{a^3(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sqrt{}}{}$$

Mathematica [A]
time = 1.28, size = 113, normalized size = 0.84

$$\frac{a^2c^3(78\cos(e+fx)+5(-5+7\cos(3(e+fx))-3\cos(4(e+fx))+3\cos(5(e+fx))))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{480f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2
),x]
```

```
[Out] (a^2*c^3*(78*Cos[e + f*x] + 5*(-5 + 7*Cos[3*(e + f*x)] - 3*Cos[4*(e + f*x)]
 + 3*Cos[5*(e + f*x)]))*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^5*Sq
rt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(480*f)
```

Maple [A]
time = 2.92, size = 105, normalized size = 0.78

method	result
default	$-\frac{\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{7}{2}}\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(21(\cos^3(fx+e))-33(\cos^2(fx+e))+21\cos(fx+e)-5)(\sin^5(fx+e))a^2}{30f(-1+\cos(fx+e))^6\cos(fx+e)^2}$
risch	$\frac{2ia^2c^3\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(15e^{11i(fx+e)}-15e^{10i(fx+e)}+35e^{9i(fx+e)}+78e^{7i(fx+e)}-50e^{6i(fx+e)}+78e^{5i(fx+e}}{15(e^{i(fx+e)}+1)(e^{2i(fx+e)}+1)^5(e^{i(fx+e)}-1)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/30/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^
(1/2)*(21*cos(f*x+e)^3-33*cos(f*x+e)^2+21*cos(f*x+e)-5)*sin(f*x+e)^5/(-1+co
s(f*x+e))^6/cos(f*x+e)^2*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2620 vs. $2(125) = 250$.

time = 0.57, size = 2620, normalized size = 19.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(210*a^2*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 90*a^2*c^3*cos(2*f*x
+ 2*e)*sin(f*x + e) - 15*a^2*c^3*sin(f*x + e) - (15*a^2*c^3*sin(11*f*x + 11
*e) - 15*a^2*c^3*sin(10*f*x + 10*e) + 35*a^2*c^3*sin(9*f*x + 9*e) + 78*a^2*
c^3*sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f*x +
 5*e) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15*a^2*
c^3*sin(f*x + e))*cos(12*f*x + 12*e) + 15*(6*a^2*c^3*sin(10*f*x + 10*e) + 1
5*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^3*sin(4
*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(11*f*x + 11*e) - 3*(70*a^2*c^
3*sin(9*f*x + 9*e) + 75*a^2*c^3*sin(8*f*x + 8*e) + 156*a^2*c^3*sin(7*f*x +
7*e) + 156*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e) + 70*a^2*
c^3*sin(3*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(10*f*x + 10*e) + 35*(15
*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^3*sin(4*
f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 15*(78*a^2*c^3*
sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f*x + 5*e
) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15*a^2*c^3*
sin(f*x + e))*cos(8*f*x + 8*e) + 78*(20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c
^3*sin(4*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(15
6*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e) + 70*a^2*c^3*sin(3
*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(6*f*x + 6*e) + 234*(5*a^2*c^3*si
n(4*f*x + 4*e) + 2*a^2*c^3*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 75*(7*a^2*c
^3*sin(3*f*x + 3*e) - 3*a^2*c^3*sin(2*f*x + 2*e) + 3*a^2*c^3*sin(f*x + e))*
cos(4*f*x + 4*e) + (15*a^2*c^3*cos(11*f*x + 11*e) - 15*a^2*c^3*cos(10*f*x +
 10*e) + 35*a^2*c^3*cos(9*f*x + 9*e) + 78*a^2*c^3*cos(7*f*x + 7*e) - 50*a^2
*c^3*cos(6*f*x + 6*e) + 78*a^2*c^3*cos(5*f*x + 5*e) + 35*a^2*c^3*cos(3*f*x
+ 3*e) - 15*a^2*c^3*cos(2*f*x + 2*e) + 15*a^2*c^3*cos(f*x + e))*sin(12*f*x
+ 12*e) - 15*(6*a^2*c^3*cos(10*f*x + 10*e) + 15*a^2*c^3*cos(8*f*x + 8*e) +
20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2
*f*x + 2*e) + a^2*c^3)*sin(11*f*x + 11*e) + 3*(70*a^2*c^3*cos(9*f*x + 9*e)
+ 75*a^2*c^3*cos(8*f*x + 8*e) + 156*a^2*c^3*cos(7*f*x + 7*e) + 156*a^2*c^3*
cos(5*f*x + 5*e) + 75*a^2*c^3*cos(4*f*x + 4*e) + 70*a^2*c^3*cos(3*f*x + 3*e
) + 30*a^2*c^3*cos(f*x + e) + 5*a^2*c^3)*sin(10*f*x + 10*e) - 35*(15*a^2*c^
3*cos(8*f*x + 8*e) + 20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^2*c^3*cos(4*f*x + 4
*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(9*f*x + 9*e) + 15*(78*a^2*c
^3*cos(7*f*x + 7*e) - 50*a^2*c^3*cos(6*f*x + 6*e) + 78*a^2*c^3*cos(5*f*x +
5*e) + 35*a^2*c^3*cos(3*f*x + 3*e) - 15*a^2*c^3*cos(2*f*x + 2*e) + 15*a^2*c
```

```
^3*cos(f*x + e))*sin(8*f*x + 8*e) - 78*(20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^
2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(7*f*x +
7*e) + 10*(156*a^2*c^3*cos(5*f*x + 5*e) + 75*a^2*c^3*cos(4*f*x + 4*e) + 70*
a^2*c^3*cos(3*f*x + 3*e) + 30*a^2*c^3*cos(f*x + e) + 5*a^2*c^3)*sin(6*f*x +
 6*e) - 78*(15*a^2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*
c^3)*sin(5*f*x + 5*e) + 75*(7*a^2*c^3*cos(3*f*x + 3*e) - 3*a^2*c^3*cos(2*f*
x + 2*e) + 3*a^2*c^3*cos(f*x + e))*sin(4*f*x + 4*e) - 35*(6*a^2*c^3*cos(2*f
*x + 2*e) + a^2*c^3)*sin(3*f*x + 3*e) + 15*(6*a^2*c^3*cos(f*x + e) + a^2*c^
3)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(6*cos(10*f*x + 10*e) + 15*cos(8*f
*x + 8*e) + 20*cos(6*f*x + 6*e) + 15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e)
+ 1)*cos(12*f*x + 12*e) + cos(12*f*x + 12*e)^2 + 12*(15*cos(8*f*x + 8*e) +
20*cos(6*f*x + 6*e) + 15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(10*
f*x + 10*e) + 36*cos(10*f*x + 10*e)^2 + 30*(20*cos(6*f*x + 6*e) + 15*cos(4*
f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + 225*cos(8*f*x + 8*e
)^2 + 40*(15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) +
400*cos(6*f*x + 6*e)^2 + 30*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 225
*cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 + 2*(6*sin(10*f*x + 10*e) + 15*
sin(8*f*x + 8*e) + 20*sin(6*f*x + 6*e) + 15*sin(4*f*x + 4*e) + 6*sin(2*f*x
+ 2*e))*sin(12*f*x + 12*e) + sin(12*f*x + 12*e)^2 + 12*(15*sin(8*f*x + 8*e)
 + 20*sin(6*f*x + 6*e) + 15*sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(10*f
*x + 10*e) + 36*sin(10*f*x + 10*e)^2 + 30*(20*sin(6*f*x + 6*e) + 15*sin(4*f
*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 225*sin(8*f*x + 8*e)^2 +
 120*(5*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 400*sin(6
*f*x + 6*e)^2 + 225*sin(4*f*x + 4*e)^2 + 180*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 36*sin(2*f*x + 2*e)^2 + 12*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.54, size = 163, normalized size = 1.22

$$\frac{\left(30 a^{2} c^{3} \cos\left(fx+e\right)^{5}-15 a^{2} c^{3} \cos\left(fx+e\right)^{4}-20 a^{2} c^{3} \cos\left(fx+e\right)^{3}+15 a^{2} c^{3} \cos\left(fx+e\right)^{2}+6 a^{2} c^{3} \cos\left(fx+e\right)-5 a^{2} c^{3}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{30 f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/30*(30*a^2*c^3*cos(f*x + e)^5 - 15*a^2*c^3*cos(f*x + e)^4 - 20*a^2*c^3*co
s(f*x + e)^3 + 15*a^2*c^3*cos(f*x + e)^2 + 6*a^2*c^3*cos(f*x + e) - 5*a^2*c
^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.77, size = 134, normalized size = 1.00

$$\frac{16 \left(20 \left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3 c^4+45 \left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2 c^5+36 \left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right) c^6+10\,c^7\right)\sqrt{-ac}\,a^2 c|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{15 \left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] 16/15*(20*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^4 + 45*(c*tan(1/2*f*x + 1/2*e)
^2 - c)^2*c^5 + 36*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^6 + 10*c^7)*sqrt(-a*c)*
a^2*c*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2
*f*x + 1/2*e)^2 - c)^6*f)
```

Mupad [B]
time = 6.23, size = 307, normalized size = 2.29

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,20{}\mathrm{i}}{3\,f}+\frac{a^2\,c^3\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,104{}\mathrm{i}}{5\,f}+\frac{a^2\,c^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,28{}\mathrm{i}}{3\,f}-\frac{a^2\,c^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{f}+\frac{a^2\,c^3\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{f}\right)}{{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,10{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,8{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c^3*cos(e + f*x)*exp(e*6i + f*x*6i)*(a +
a/cos(e + f*x))^(1/2)*104i)/(5*f) - (a^2*c^3*exp(e*6i + f*x*6i)*(a + a/cos(
e + f*x))^(1/2)*20i)/(3*f) + (a^2*c^3*exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*(
a + a/cos(e + f*x))^(1/2)*28i)/(3*f) - (a^2*c^3*exp(e*6i + f*x*6i)*cos(4*e
+ 4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a^2*c^3*exp(e*6i + f*x*6i)*cos
(5*e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*6i + f*x*6i)*sin(2*
e + 2*f*x)*10i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*8i + exp(e*6i + f*x*6i
)*sin(6*e + 6*f*x)*2i)
```

3.124 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=134

$$-\frac{2c^3(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{c^2(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{5f}-\frac{c(a+a\text{:}}{}$$

```
[Out] -1/5*c*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/15*c^3*
(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/5*c^2*(a+a*sec
(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.29, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4038}

$$-\frac{2c^3\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{15f\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{5f}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*c^3*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f
*x]]) - (c^2*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*
x])/(5*f) - (c*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)*Tan[e
+ f*x])/(5*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx)}{5f}$$
$$= -\frac{c^2(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+f}}{5f}$$
$$= -\frac{2c^3(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} -$$

Mathematica [A]
time = 0.76, size = 92, normalized size = 0.69

$$\frac{a^2c^2(29+20\cos(2(e+fx))+15\cos(4(e+fx)))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] (a^2*c^2*(29 + 20*Cos[2*(e + f*x)] + 15*Cos[4*(e + f*x)])*Csc[(e + f*x)/2]*
Sec[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e
 + f*x]])/(240*f)
```

Maple [A]
time = 2.70, size = 95, normalized size = 0.71

method	result	size
default	$-\frac{\left(\sin^5(fx+e)\right)\left(8\left(\cos^2(fx+e)\right)-9\cos(fx+e)+3\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}a^2}{15f(-1+\cos(fx+e))^5\cos(fx+e)^2}$	95
risch	$\frac{2ia^2c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(15\,e^{9i(fx+e)}+20\,e^{7i(fx+e)}+58\,e^{5i(fx+e)}+20\,e^{3i(fx+e)}+15\,e^{i(fx+e)}\right)}{15\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\right)^4\left(e^{i(fx+e)}-1\right)f}$	168

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/15/f*sin(f*x+e)^5*(8*cos(f*x+e)^2-9*cos(f*x+e)+3)*(a*(cos(f*x+e)+1)/cos(
f*x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/(-1+cos(f*x+e))^5/cos(f*
x+e)^2*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(125) = 250$.
time = 0.56, size = 1634, normalized size = 12.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(100*a^2*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 75*a^2*c^2*cos(f*x +
e)*sin(2*f*x + 2*e) - 75*a^2*c^2*cos(2*f*x + 2*e)*sin(f*x + e) - 15*a^2*c^2
*sin(f*x + e) - (15*a^2*c^2*sin(9*f*x + 9*e) + 20*a^2*c^2*sin(7*f*x + 7*e)
+ 58*a^2*c^2*sin(5*f*x + 5*e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*si
n(f*x + e))*cos(10*f*x + 10*e) + 75*(a^2*c^2*sin(8*f*x + 8*e) + 2*a^2*c^2*s
in(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*co
s(9*f*x + 9*e) - 5*(20*a^2*c^2*sin(7*f*x + 7*e) + 58*a^2*c^2*sin(5*f*x + 5*
e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(8*f*x + 8*e
) + 100*(2*a^2*c^2*sin(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*
sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(58*a^2*c^2*sin(5*f*x + 5*e) + 20*a
^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(6*f*x + 6*e) + 290*(
2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 5
0*(4*a^2*c^2*sin(3*f*x + 3*e) + 3*a^2*c^2*sin(f*x + e))*cos(4*f*x + 4*e) +
(15*a^2*c^2*cos(9*f*x + 9*e) + 20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*c^2*cos
(5*f*x + 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(
10*f*x + 10*e) - 15*(5*a^2*c^2*cos(8*f*x + 8*e) + 10*a^2*c^2*cos(6*f*x + 6*
e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*si
n(9*f*x + 9*e) + 5*(20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*c^2*cos(5*f*x + 5*
e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(8*f*x + 8*e
) - 20*(10*a^2*c^2*cos(6*f*x + 6*e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*sin(7*f*x + 7*e) + 10*(58*a^2*c^2*cos(5*f*x
+ 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(6*f*x +
 6*e) - 58*(10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2*e) + a^2*
c^2)*sin(5*f*x + 5*e) + 50*(4*a^2*c^2*cos(3*f*x + 3*e) + 3*a^2*c^2*cos(f*x
+ e))*sin(4*f*x + 4*e) - 20*(5*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*sin(3*f*
x + 3*e))*sqrt(a)*sqrt(c)/((2*(5*cos(8*f*x + 8*e) + 10*cos(6*f*x + 6*e) + 1
0*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) + 1)*cos(10*f*x + 10*e) + cos(10*f*
x + 10*e)^2 + 10*(10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e) + 5*cos(2*f*x +
 2*e) + 1)*cos(8*f*x + 8*e) + 25*cos(8*f*x + 8*e)^2 + 20*(10*cos(4*f*x + 4*
e) + 5*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 100*cos(6*f*x + 6*e)^2 + 20
*(5*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 100*cos(4*f*x + 4*e)^2 + 25*co
s(2*f*x + 2*e)^2 + 10*(sin(8*f*x + 8*e) + 2*sin(6*f*x + 6*e) + 2*sin(4*f*x
+ 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 50*(
2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e
) + 25*sin(8*f*x + 8*e)^2 + 100*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin
```

```
(6*f*x + 6*e) + 100*sin(6*f*x + 6*e)^2 + 100*sin(4*f*x + 4*e)^2 + 100*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + 25*sin(2*f*x + 2*e)^2 + 10*cos(2*f*x + 2*e)
+ 1)*f)
```

Fricas [A]
time = 2.05, size = 114, normalized size = 0.85

$$\frac{\left(15\,a^2c^2\cos\left(fx+e\right)^4-10\,a^2c^2\cos\left(fx+e\right)^2+3\,a^2c^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(15*a^2*c^2*cos(f*x + e)^4 - 10*a^2*c^2*cos(f*x + e)^2 + 3*a^2*c^2)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.76, size = 109, normalized size = 0.81

$$\frac{16\left(10\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^4+15\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^5+6\,c^6\right)\sqrt{-ac}\,a^2|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{15\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 16/15*(10*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^4 + 15*(c*tan(1/2*f*x + 1/2*e)
^2 - c)*c^5 + 6*c^6)*sqrt(-a*c)*a^2*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan
(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^5*f)
```

Mupad [B]
time = 5.61, size = 215, normalized size = 1.60

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,116\mathrm{i}}{15\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,16\mathrm{i}}{3\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4\mathrm{i}}{f}\right)}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,6\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c^2*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x
))^(1/2)*116i)/(15*f) + (a^2*c^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a
/cos(e + f*x))^(1/2)*16i)/(3*f) + (a^2*c^2*exp(e*5i + f*x*5i)*cos(4*e + 4*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*5i + f*x*5i)*sin(e + f*x)*4i
+ exp(e*5i + f*x*5i)*sin(3*e + 3*f*x)*6i + exp(e*5i + f*x*5i)*sin(5*e + 5*f
*x)*2i)
```

3.125 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{4f}$$

```
[Out] -1/6*c^2*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/4*c*(
a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A]
time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4038}

$$-\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*
x]]) - (c*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(4*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$
$$= -\frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{c-c\sec(e+fx)}} - \frac{c(a}{}$$

Mathematica [A]
time = 0.59, size = 96, normalized size = 1.08

$$\frac{a^2c(5\cos(e+fx)+3(\cos(2(e+fx))+\cos(3(e+fx))))\csc\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (a^2*c*(5*Cos[e + f*x] + 3*(Cos[2*(e + f*x)] + Cos[3*(e + f*x)]))*Csc[(e +
f*x)/2]*Sec[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
 c*Sec[e + f*x]])/(24*f)
```

Maple [A]
time = 2.74, size = 85, normalized size = 0.96

method	result	size
default	$-\frac{(\sin^5(fx+e))(5\cos(fx+e)-3)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}a^2}{12f(-1+\cos(fx+e))^4\cos(fx+e)^2}$	85
risch	$\frac{2ia^2c\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\left(3e^{7i(fx+e)}+3e^{6i(fx+e)}+5e^{5i(fx+e)}+5e^{3i(fx+e)}+3e^{2i(fx+e)}+3e^{i(fx+e)}\right)}{3(e^{i(fx+e)}+1)(e^{2i(fx+e)}+1)^3(e^{i(fx+e)}-1)f}$	177

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/12/f*sin(f*x+e)^5*(5*cos(f*x+e)-3)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(
c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/(-1+cos(f*x+e))^4/cos(f*x+e)^2*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. 2(83) = 166.
time = 0.58, size = 1188, normalized size = 13.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(20*a^2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a^2*c*cos(2*f*x + 2*e)
*sin(f*x + e) - 3*a^2*c*sin(f*x + e) - (3*a^2*c*sin(7*f*x + 7*e) + 3*a^2*c*
sin(6*f*x + 6*e) + 5*a^2*c*sin(5*f*x + 5*e) + 5*a^2*c*sin(3*f*x + 3*e) + 3*
a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*a^2*
c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*c
os(7*f*x + 7*e) - 2*(10*a^2*c*sin(5*f*x + 5*e) - 9*a^2*c*sin(4*f*x + 4*e) +
 10*a^2*c*sin(3*f*x + 3*e) + 6*a^2*c*sin(f*x + e))*cos(6*f*x + 6*e) + 10*(3
*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 6*(5
*a^2*c*sin(3*f*x + 3*e) + 3*a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*
cos(4*f*x + 4*e) + (3*a^2*c*cos(7*f*x + 7*e) + 3*a^2*c*cos(6*f*x + 6*e) + 5
*a^2*c*cos(5*f*x + 5*e) + 5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*
e) + 3*a^2*c*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a^2*c*cos(6*f*x + 6*e) +
 6*a^2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(7*f*x + 7
*e) + (20*a^2*c*cos(5*f*x + 5*e) - 18*a^2*c*cos(4*f*x + 4*e) + 20*a^2*c*cos
(3*f*x + 3*e) + 12*a^2*c*cos(f*x + e) - 3*a^2*c)*sin(6*f*x + 6*e) - 5*(6*a^
2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(5*f*x + 5*e) +
 6*(5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*e) + 3*a^2*c*cos(f*x +
 e))*sin(4*f*x + 4*e) - 5*(4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(3*f*x + 3*
e) + 3*(4*a^2*c*cos(f*x + e) - a^2*c)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2
*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f
*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)
 + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*s
in(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Fricas [A]
time = 2.75, size = 121, normalized size = 1.36

$$\frac{\left(12\,a^{2}c\cos\left(fx+e\right)^{3}+6\,a^{2}c\cos\left(fx+e\right)^{2}-4\,a^{2}c\cos\left(fx+e\right)-3\,a^{2}c\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{12\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(12*a^2*c*cos(f*x + e)^3 + 6*a^2*c*cos(f*x + e)^2 - 4*a^2*c*cos(f*x +
e) - 3*a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.84, size = 88, normalized size = 0.99

$$\frac{4\left(4\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{5}+3\,c^{6}\right)\sqrt{-ac}\,a^{2}|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{4}c^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 4/3*(4*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 3*c^6)*sqrt(-a*c)*a^2*abs(c)*sg
n(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2
 - c)^4*c^2*f)
```

Mupad [B]
time = 5.44, size = 195, normalized size = 2.19

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,c\,\cos(e+f\,x)\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,20\mathrm{i}}{3\,f}+\frac{a^2\,c\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4\mathrm{i}}{f}+\frac{a^2\,c\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos(3\,e+3\,f\,x)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4\mathrm{i}}{f}\right)}{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/
cos(e + f*x))^(1/2)*20i)/(3*f) + (a^2*c*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a^2*c*exp(e*4i + f*x*4i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)
*4i + exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

3.126 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}$

Optimal. Leaf size=43

$$-\frac{c(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/3*c*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4038\}$

$$-\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -1/3*(c*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]
])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 88 vs. 2(43) = 86.
time = 0.48, size = 88, normalized size = 2.05

$$\frac{a^2\cot\left(\frac{1}{2}(e+fx)\right)\left(2+4\cos(e+fx)+\cos^2(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] (a^2*Cot[(e + f*x)/2]*(2 + 4*Cos[e + f*x] + Cos[e + f*x]^2*Sec[(e + f*x)/2]
^2)*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])/(6*
f)
```

Maple [A]
time = 2.90, size = 75, normalized size = 1.74

method	result	size
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\sin^5(fx+e))\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,a^2}{3f\cos(fx+e)^2(-1+\cos(fx+e))^3}$	75
risch	$\frac{2ia^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,(3\,e^{5i(fx+e)}+6\,e^{4i(fx+e)}+10\,e^{3i(fx+e)}+6\,e^{2i(fx+e)}+3\,e^{i(fx+e)})}{3(e^{i(fx+e)}+1)(e^{2i(fx+e)}+1)^2(e^{i(fx+e)}-1)f}$	165

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*sin(f*x+e)^5*(c*(-1+cos(f*x+e))/
cos(f*x+e))^(1/2)/cos(f*x+e)^2/(-1+cos(f*x+e))^3*a^2
```

Maxima [A]
time = 0.50, size = 62, normalized size = 1.44

$$\frac{8\sqrt{-a}\,a^2\sqrt{c}}{3\,f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^3\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 8/3*sqrt(-a)*a^2*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^3*(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(40) = 80$.
time = 2.79, size = 101, normalized size = 2.35

$$\frac{\left(3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)+a^2\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^
2*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 1.75, size = 60, normalized size = 1.40

$$\frac{8\sqrt{-ac}\,a^2c^2|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 8/3*sqrt(-a*c)*a^2*c^2*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/
2*e))/((c*tan(1/2*f*x + 1/2*e)^2 - c)^3*f)
```

Mupad [B]
time = 3.61, size = 136, normalized size = 3.16

$$\frac{2\,a^2\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(10\,\sin\left(e+f\,x\right)+12\,\sin\left(2\,e+2\,f\,x\right)+13\,\sin\left(3\,e+3\,f\,x\right)+6\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(5\,e+5\,f\,x\right)\right)}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)-\cos\left(6\,e+6\,f\,x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a^2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/
cos(e + f*x))^(1/2)*(10*sin(e + f*x) + 12*sin(2*e + 2*f*x) + 13*sin(3*e + 3
*f*x) + 6*sin(4*e + 4*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*e + 2*f*x) -
2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

3.127 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}+\frac{2a^2\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{a(a+a\sec(e+fx))^{3/2}\,\mathrm{t}}{2f\sqrt{c-c\sec(e+}}$$

```
[Out] 1/2*a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+4*a^3*ln(1
-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+2*a
^2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4040, 4037}

$$\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[c - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a}{\sqrt{c-c\sec(}}$$

$$= \frac{2a^2\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{a(a+a\sec(e+fx))^{3/2}\,\mathrm{t}}{2f\sqrt{c-c\sec(e+}}$$

$$= \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\sqrt{a+a\sec(e}}{f\sqrt{c-c\,\mathrm{se}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.61, size = 328, normalized size = 2.33

$$\frac{4\sqrt{2}\,e^{\frac{1}{2}i(e+fx)}\sqrt{\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\left(2\log\left(1-e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{\sec(e+fx)}\,(a(1+\sec(e+fx)))^{5/2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,f(1+\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\sec(e+fx)\sqrt{(1+\cos(e+fx))\sec(e+fx)}\,(a(1+\sec(e+fx)))^{5/2}\left(\frac{5\sec\left(\frac{e}{2}+\frac{fx}{2}\right)}{2f}+\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sec(e+fx)}{f}\right)\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{(1+\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]
],x]
```

```
[Out] (4*Sqrt[2]*E^((I/2)*(e + f*x))*Sqrt[(1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(
e + f*x)))]*(2*Log[1 - E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))])*Sqr
t[Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*Sin[e/2 + (f*x)/2])/((1 + E^(I
*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*(1 + Sec[e +
 f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + (Sec[e + f*x]*Sqrt[(1 + Cos[e + f*
x])*Sec[e + f*x]]*(a*(1 + Sec[e + f*x]))^(5/2)*((5*Sec[e/2 + (f*x)/2])/(2*f
) + (Cos[e/2 + (f*x)/2]*Sec[e + f*x])/f)*Sin[e/2 + (f*x)/2])/((1 + Sec[e +
f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.84, size = 189, normalized size = 1.34

method	result
default	$\frac{\left(8\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-16(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+8(\cos^2(fx+e))\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e}{\sin(fx+e)}\right.}{2f\sin(fx+e)\cos(fx+e)c}$
risch	$-\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(3\,e^{2i(fx+e)}+e^{i(fx+e)}+3\right)\left(e^{2i(fx+e)}-e^{i(fx+e)}\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f\left(e^{2i(fx+e)}+1\right)^2}-\frac{8ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e}\right.}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/f*(8*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-16*cos(f*x
+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+8*cos(f*x+e)^2*ln(-(cos(f*x+e)-1+sin(
f*x+e))/sin(f*x+e))-5*cos(f*x+e)^2-6*cos(f*x+e)-1)*(a*(cos(f*x+e)+1)/cos(f*
x+e))^(1/2)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/cos(f*x+e)/c*a^
2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(137) = 274$.
time = 0.59, size = 791, normalized size = 5.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(a^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - a^2*cos(4*f*x + 4*e)*sin(2*f*x
+ 2*e) - a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x
 + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(
2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin
(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x
 + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)
*cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 3*(a^2*si
n(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 3*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a^2*cos(4*f*x + 4*e)
+ 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 3*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(4*f*
x + 4*e)^2 + 4*c*cos(2*f*x + 2*e)^2 + c*sin(4*f*x + 4*e)^2 + 4*c*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 4*c*sin(2*f*x + 2*e)^2 + 2*(2*c*cos(2*f*x + 2*e)
+ c)*cos(4*f*x + 4*e) + 4*c*cos(2*f*x + 2*e) + c)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

3.128 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\sqrt{a+a\sec(e+fx)}\ldots}{cf\sqrt{c-c\sec(e\ldots}}$$

```
[Out] -a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-4*a^3*ln(1-se
c(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-2*a^
2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4039, 4040, 4037}

$$-\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)
)) - (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]]) - (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x]
)/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(2a)\int \frac{\sec(e+fx)(a+a\sec(e+}{\sqrt{c-c\sec(e+}}}{c}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{2a^2\sqrt{a+a\sec(e+fx)}}{cf\sqrt{c-c\sec(e-}}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{4a^3\log(1-\sec(e+}{cf\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.41, size = 188, normalized size = 1.30

$$\frac{a^2\left(1-4\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(-5+8\log\left(1-e^{i(e+fx)}\right)-4\log\left(1+e^{2i(e+fx)}\right)\right)+2\log\left(1+e^{2i(e+fx)}\right)+\cos(2(e+fx))\left(-4\log\left(1-e^{i(e+fx)}\right)+2\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{cf(-1+\cos(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3
/2),x]
```

```
[Out] (a^2*(1 - 4*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(-5 + 8*Log[1 - E^(I*(e
 + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + 2*Log[1 + E^((2*I)*(e + f*x))
] + Cos[2*(e + f*x)]*(-4*Log[1 - E^(I*(e + f*x))] + 2*Log[1 + E^((2*I)*(e +
 f*x))]))*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c*f*(-
1 + Cos[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(133) = 266.
time = 2.90, size = 289, normalized size = 1.99

method	result

default	$-\frac{(-1+\cos(fx+e))\left(4(\cos^2(fx+e))\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+4\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-8(\cos^2(fx+e))\ln}{}$
risch	$\frac{2ia^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(5\,e^{3i(fx+e)}-2\,e^{2i(fx+e)}+5\,e^{i(fx+e)})}{c(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f(e^{2i(fx+e)}+1)}+\frac{8ia^2\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)\ln(e^{i(fx+e)}-1)}{c(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{4ia}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(-1+cos(f*x+e))*(4*cos(f*x+e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+
e))+4*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-8*cos(f*x+e)^
2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-3*cos(f*x+e)^2-4*cos(f*x+e)*ln(-(cos(f*x+
e)-1+sin(f*x+e))/sin(f*x+e))-4*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))+8*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-2*cos(f*x+e)+1)*(a*(
cos(f*x+e)+1)/cos(f*x+e))^(1/2)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))
^(3/2)/sin(f*x+e)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2185 vs. $2(143) = 286$.
time = 0.74, size = 2185, normalized size = 15.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2*(8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*a
^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*a^2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 2*a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x
+ 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2
*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*co
s(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*s
```

```
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a
^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) - 4*(a^2*cos(4*f*x + 4*e) + 2*a
^2*cos(2*f*x + 2*e) - 2*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*
cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*
e) - 2*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^
2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*a
rctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + (16*a^2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 5*a^2*sin(4*f*x + 4*e) + 6*a^2*sin(2*f*x + 2*e) - 8*(a^2*cos(4*f*x +
 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (5*a^2*s
in(4*f*x + 4*e) + 6*a^2*sin(2*f*x + 2*e) - 8*(a^2*cos(4*f*x + 4*e) + 2*a^2*
cos(2*f*x + 2*e) + a^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (16*a^2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 5*a^2*cos(4*f*x + 4*e) - 6*a^2*cos(2*f*x + 2*e) - 5*a^2 - 8*(
a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 (5*a^2*cos(4*f*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + 5*a^2 + 8*(a^2*sin(4*f*
x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt
(c)/((c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*
f*x + 2*e) + c^2)*cos(4*f*x + 4*e) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*
f*x + 2*e) - 2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c
^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x
 + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e) - 2*c^
2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f
*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2
*sec(f*x + e) + c^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x
)
```

3.129 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}+\frac{a^2\sqrt{a+a\sec(e+fx)}\,\tan(e+fx)}{cf(c-c\sec(e+fx))^{3/2}}+\frac{a^3\log(1-\sec(e+fx))\,\mathrm{ta}}{c^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-}}$$

```
[Out] -1/2*a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+a^2*(a+a*
sec(f*x+e))^(1/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(3/2)+a^3*ln(1-sec(f*x+e)
)*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4039, 4037}

$$\frac{a^3\tan(e+fx)\log(1-\sec(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}+\frac{a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf(c-c\sec(e+fx))^{3/2}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5
/2)) + (a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(c*f*(c - c*Sec[e + f*x]
)^(3/2)) + (a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{a\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3}}{(c-c\sec(e+fx))^{3/2}}}{c}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a^2\sqrt{a+a\sec(e+fx)}\ \mathrm{ta}}{cf(c-c\sec(e+fx}$$

$$= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a^2\sqrt{a+a\sec(e+fx)}\ \mathrm{ta}}{cf(c-c\sec(e+fx}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.32, size = 182, normalized size = 1.26

$$-\frac{a^2\left(4-6\log\left(1-e^{i(e+fx)}\right)+\cos(e+fx)\left(8\log\left(1-e^{i(e+fx)}\right)-4\log\left(1+e^{2i(e+fx)}\right)\right)+3\log\left(1+e^{2i(e+fx)}\right)+\cos(2(e+fx))\left(-2\log\left(1-e^{i(e+fx)}\right)+\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{2c^2f(-1+\cos(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5
/2),x]
```

```
[Out] -1/2*(a^2*(4 - 6*Log[1 - E^(I*(e + f*x))] + Cos[e + f*x]*(8*Log[1 - E^(I*(e
 + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + 3*Log[1 + E^((2*I)*(e + f*x))
] + Cos[2*(e + f*x)]*(-2*Log[1 - E^(I*(e + f*x))] + Log[1 + E^((2*I)*(e + f
*x))]))*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c^2*f*(-1 + Cos[e + f
*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(131) = 262.
time = 2.99, size = 366, normalized size = 2.52

method	result
risch	$\frac{8ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\ e^{2i(fx+e)}}{c^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\ f} - \frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{c^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\ f} + \frac{ia^2\sqrt{\frac{a(e^{i(fx+}}{e^{2i(fx+}}}}{c^2(e^{i(f}}$
default	$\frac{(-1+\cos(fx+e))\left(2\left(\cos^2(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)-4\left(\cos^2(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+2\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e}{\sin(fx+e)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/f*(-1+cos(f*x+e))*(2*cos(f*x+e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x
+e))-4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*ln(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-cos(f*x+e)^2-4*cos(f*x+e)*ln(-(cos(f*x+e
)-1+sin(f*x+e))/sin(f*x+e))+8*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-4*
cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)+2*ln(-(c
os(f*x+e)-1+sin(f*x+e))/sin(f*x+e))-4*ln(-(-1+cos(f*x+e))/sin(f*x+e))+2*ln(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+3)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1
/2)/cos(f*x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)*a^2
```

Maxima [**A**]
time = 0.49, size = 179, normalized size = 1.23

$$-\frac{\frac{2\sqrt{-a}\,a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{\frac{5}{2}}}+\frac{2\sqrt{-a}\,a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}}-\frac{4\sqrt{-a}\,a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}}+\frac{\left(\sqrt{-a}\,a^2\sqrt{c}+\frac{2\sqrt{-a}\,a^2\sqrt{c}\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)^4}{c^3\sin(fx+e)^4}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*(2*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(5/2) + 2*s
qrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 4*sqrt(-a)*a
^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) + (sqrt(-a)*a^2*sqrt(c) + 2
*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) +
1)^4/(c^3*sin(f*x + e)^4))/f
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^
3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: TypeError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x
)
```

3.130 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/6*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A]
time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4035\}$

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/6*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(7/2
))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

Mathematica [A]
time = 0.60, size = 76, normalized size = 1.81

$$\frac{a^2(5+3\cos(2(e+fx)))\csc^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{48c^3f\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7
/2),x]
```

```
[Out] (a^2*(5 + 3*Cos[2*(e + f*x)])*Csc[(e + f*x)/2]^5*Sec[(e + f*x)/2]*Sqrt[a*(1
 + Sec[e + f*x])])/(48*c^3*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.
time = 2.91, size = 75, normalized size = 1.79

method	result	size
default	$-\frac{\left(\sin^{5}\left(fx+e\right)\right)\sqrt{\frac{a\left(\cos\left(fx+e\right)+1\right)}{\cos\left(fx+e\right)}}\,a^{2}}{6f\left(-1+\cos\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)^{3}\left(\frac{c\left(-1+\cos\left(fx+e\right)\right)}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}$	75
risch	$\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\left(3\,e^{5i(fx+e)}+10\,e^{3i(fx+e)}+3\,e^{i(fx+e)}\right)}{3c^{3}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{5}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,f}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/6/f*sin(f*x+e)^5*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))^2/c
os(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(7/2)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1949 vs. $2(39) = 78$.
time = 0.59, size = 1949, normalized size = 46.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(208*a^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 48*a^2*cos(f*x + e)*sin(2*
f*x + 2*e) - 48*a^2*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a^2*sin(f*x + e) - (3
*a^2*sin(7*f*x + 7*e) + 13*a^2*sin(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) +
 3*a^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(8*a^2*sin(6*f*x + 6*e) + 15*a^2*
sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(13*a^2*si
n(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(6*f*x +
6*e) + 26*(15*a^2*sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*
e) - 30*(13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(4*f*x + 4*e) + (
3*a^2*cos(7*f*x + 7*e) + 13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e)
+ 3*a^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(16*a^2*cos(6*f*x + 6*e) + 30*a^
```

```
2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2)*sin(7*f*x + 7*e) + 16*(
13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f*x + e))*sin
(6*f*x + 6*e) - 13*(30*a^2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2
)*sin(5*f*x + 5*e) + 30*(13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f*x + e))*sin(
4*f*x + 4*e) - 13*(16*a^2*cos(2*f*x + 2*e) + a^2)*sin(3*f*x + 3*e))*sqrt(a)
*sqrt(c)/((c^4*cos(8*f*x + 8*e)^2 + 36*c^4*cos(7*f*x + 7*e)^2 + 256*c^4*cos
(6*f*x + 6*e)^2 + 676*c^4*cos(5*f*x + 5*e)^2 + 900*c^4*cos(4*f*x + 4*e)^2 +
 676*c^4*cos(3*f*x + 3*e)^2 + 256*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(f*x +
 e)^2 + c^4*sin(8*f*x + 8*e)^2 + 36*c^4*sin(7*f*x + 7*e)^2 + 256*c^4*sin(6*
f*x + 6*e)^2 + 676*c^4*sin(5*f*x + 5*e)^2 + 900*c^4*sin(4*f*x + 4*e)^2 + 67
6*c^4*sin(3*f*x + 3*e)^2 + 256*c^4*sin(2*f*x + 2*e)^2 - 192*c^4*sin(2*f*x +
 2*e)*sin(f*x + e) + 36*c^4*sin(f*x + e)^2 - 12*c^4*cos(f*x + e) + c^4 - 2*
(6*c^4*cos(7*f*x + 7*e) - 16*c^4*cos(6*f*x + 6*e) + 26*c^4*cos(5*f*x + 5*e)
 - 30*c^4*cos(4*f*x + 4*e) + 26*c^4*cos(3*f*x + 3*e) - 16*c^4*cos(2*f*x + 2
*e) + 6*c^4*cos(f*x + e) - c^4)*cos(8*f*x + 8*e) - 12*(16*c^4*cos(6*f*x + 6
*e) - 26*c^4*cos(5*f*x + 5*e) + 30*c^4*cos(4*f*x + 4*e) - 26*c^4*cos(3*f*x
+ 3*e) + 16*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(7*f*x + 7*
e) - 32*(26*c^4*cos(5*f*x + 5*e) - 30*c^4*cos(4*f*x + 4*e) + 26*c^4*cos(3*f
*x + 3*e) - 16*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(6*f*x +
 6*e) - 52*(30*c^4*cos(4*f*x + 4*e) - 26*c^4*cos(3*f*x + 3*e) + 16*c^4*cos(
2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(5*f*x + 5*e) - 60*(26*c^4*cos(
3*f*x + 3*e) - 16*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(4*f*
x + 4*e) - 52*(16*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(3*f*
x + 3*e) - 32*(6*c^4*cos(f*x + e) - c^4)*cos(2*f*x + 2*e) - 4*(3*c^4*sin(7*
f*x + 7*e) - 8*c^4*sin(6*f*x + 6*e) + 13*c^4*sin(5*f*x + 5*e) - 15*c^4*sin(
4*f*x + 4*e) + 13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x + 2*e) + 3*c^4*sin
(f*x + e))*sin(8*f*x + 8*e) - 24*(8*c^4*sin(6*f*x + 6*e) - 13*c^4*sin(5*f*x
 + 5*e) + 15*c^4*sin(4*f*x + 4*e) - 13*c^4*sin(3*f*x + 3*e) + 8*c^4*sin(2*f
*x + 2*e) - 3*c^4*sin(f*x + e))*sin(7*f*x + 7*e) - 64*(13*c^4*sin(5*f*x + 5
*e) - 15*c^4*sin(4*f*x + 4*e) + 13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x +
 2*e) + 3*c^4*sin(f*x + e))*sin(6*f*x + 6*e) - 104*(15*c^4*sin(4*f*x + 4*e)
 - 13*c^4*sin(3*f*x + 3*e) + 8*c^4*sin(2*f*x + 2*e) - 3*c^4*sin(f*x + e))*s
in(5*f*x + 5*e) - 120*(13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x + 2*e) + 3
*c^4*sin(f*x + e))*sin(4*f*x + 4*e) - 104*(8*c^4*sin(2*f*x + 2*e) - 3*c^4*s
in(f*x + e))*sin(3*f*x + 3*e))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(39) = 78$.
time = 2.34, size = 136, normalized size = 3.24

$$\frac{\left(3\,a^{2}\cos\left(fx+e\right)^{3}+a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{3\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-c^{4}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^3 + a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f*cos(f*x + e)^3 -
 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 1.82, size = 65, normalized size = 1.55

$$-\frac{\left(a^2-\frac{a^2}{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6}\right)a^2}{6\,\sqrt{-ac}\,c^3f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] -1/6*(a^2 - a^2/tan(1/2*f*x + 1/2*e)^6)*a^2/(sqrt(-a*c)*c^3*f*abs(a)*sgn(ta
n(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 6.04, size = 199, normalized size = 4.74

$$-\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,\cos(e+f\,x)\,{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,52\mathrm{i}}{3\,c^4\,f}+\frac{a^2\,{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos(3\,e+3\,f\,x)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4\mathrm{i}}{c^4\,f}\right)}{{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(e+f\,x\right)\,28\mathrm{i}-{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,28\mathrm{i}+{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,12\mathrm{i}-{\mathrm{e}}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,2\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] -((c - c/cos(e + f*x))^(1/2)*((a^2*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/c
os(e + f*x))^(1/2)*52i)/(3*c^4*f) + (a^2*exp(e*4i + f*x*4i)*cos(3*e + 3*f*x
)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^4*f)))/(exp(e*4i + f*x*4i)*sin(e + f*x)
*28i - exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)*28i + exp(e*4i + f*x*4i)*sin(3*e
 + 3*f*x)*12i - exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

3.131 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}}-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{48cf(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/8*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)-1/48*(a+a*s
ec(f*x+e))^(5/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A]
time = 0.20, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4036, 4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{48cf(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -1/8*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(9/2
)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(48*c*f*(c - c*Sec[e + f*x])
^(7/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}} d}{8c}$$
$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan}{48cf(c-c\sec(e+fx}$$

Mathematica [A]
time = 0.83, size = 92, normalized size = 1.05

$$-\frac{a^2(-5+17\cos(e+fx)-3\cos(2(e+fx))+3\cos(3(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{12c^4f(-1+\cos(e+fx))^4\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9
/2),x]
```

```
[Out] -1/12*(a^2*(-5 + 17*Cos[e + f*x] - 3*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])
*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(c^4*f*(-1 + Cos[e + f*x])^4*
Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.99, size = 85, normalized size = 0.97

method	result	size
default	$-\frac{(7\cos(fx+e)-1)\left(\sin^5(fx+e)\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,a^2}{48f(-1+\cos(fx+e))^2\cos(fx+e)^4\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{9}{2}}}$	85
risch	$\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(3e^{7i(fx+e)}-3e^{6i(fx+e)}+17e^{5i(fx+e)}-10e^{4i(fx+e)}+17e^{3i(fx+e)}-3e^{2i(fx+e)}+3e^{i(fx+e)}\right)}{3c^4\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^7\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$	177

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/48/f*(7*cos(f*x+e)-1)*sin(f*x+e)^5*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(
-1+cos(f*x+e))^2/cos(f*x+e)^4/(c*(-1+cos(f*x+e))/cos(f*x+e))^(9/2)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2915 vs. $2(82) = 164$.
time = 5.79, size = 2915, normalized size = 33.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(70*a^2*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 70*a^2*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 3*a^2*sin(2*f*x + 2*e) + (3*a^2*sin(6*f*x + 6*e) + 10*a^2*s
in(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) + (3*a^2*sin(8*f
*x + 8*e) + 60*a^2*sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin
(2*f*x + 2*e) - 32*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 32*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e) + 308*
a^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x + 2*e)
+ 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e) + 308*a
^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x + 2*e) +
 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a^2*sin(8*f*x + 8*e) + 60*a^2*
sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin(2*f*x + 2*e))*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*cos(6*f*x + 6*e)
+ 10*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - (70*
a^2*cos(4*f*x + 4*e) - 3*a^2)*sin(6*f*x + 6*e) + 10*(7*a^2*cos(2*f*x + 2*e)
 + a^2)*sin(4*f*x + 4*e) - (3*a^2*cos(8*f*x + 8*e) + 60*a^2*cos(6*f*x + 6*e
) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) - 32*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 3*a^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - (17*a^2*cos(8*f*x + 8*e) + 308*a^2*cos(6*f*x + 6*e) + 630
*a^2*cos(4*f*x + 4*e) + 308*a^2*cos(2*f*x + 2*e) + 32*a^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a^2)*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - (17*a^2*cos(8*f*x + 8*e) + 308*a^2*cos(6*f*x + 6*
e) + 630*a^2*cos(4*f*x + 4*e) + 308*a^2*cos(2*f*x + 2*e) + 32*a^2*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*cos(8*f*x + 8*e) + 60*a^2*cos(6*f*
x + 6*e) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) + 3*a^2)*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^5*cos
(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x + 6*e)^2 + 4900*c^5*cos(4*f*x + 4*e)^2
+ 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*cos(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 3136*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
+ 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^5*sin(8
*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*e)^2 + 4900*c^5*sin(4*f*x + 4*e)^2 +
3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*c^5*sin(2*f*x + 2*e)^2 + 6
4*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*sin(1/2*arctan2(sin(2*f
```

```
*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c^5*cos(2*f*x + 2*e) + c^5 + 2*(28*c^5
*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5
)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e)
+ c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*cos(2*f*x + 2*e) + c^5)*cos(4*f*x + 4
*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x
 + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)
*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x
+ 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f
*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8
*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28
*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) -
 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*cos(8*f*x + 8*e) + 2
8*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e)
+ c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 28*(2*c^5*sin
(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(8*f*x
+ 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(6*f*x +
6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*
x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e
) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x +
2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e),...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(82) = 164$.

time = 2.94, size = 179, normalized size = 2.03

$$\frac{\left(6\,a^{2}\cos\left(fx+e\right)^{4}-3\,a^{2}\cos\left(fx+e\right)^{3}+4\,a^{2}\cos\left(fx+e\right)^{2}-a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{6\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}+6\,c^{5}f\cos\left(fx+e\right)^{2}-4\,c^{5}f\cos\left(fx+e\right)+c^{5}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a^2*cos(f*x + e)^4 - 3*a^2*cos(f*x + e)^3 + 4*a^2*cos(f*x + e)^2 - a
^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c
^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.91, size = 91, normalized size = 1.03

$$-\frac{\left(a^2-\frac{4\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a\right)a^5+a^6}{a^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8}\right)a^2}{48\sqrt{-ac}\,c^4f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="giac")
```

```
[Out] -1/48*(a^2 - (4*(a*tan(1/2*f*x + 1/2*e)^2 - a)*a^5 + a^6)/(a^4*tan(1/2*f*x
+ 1/2*e)^8))*a^2/(sqrt(-a*c)*c^4*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(
1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 6.79, size = 350, normalized size = 3.98

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+fx\right)}}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,68{}\mathrm{i}}{3\,c^5\,f}-\frac{a^2\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,52{}\mathrm{i}}{3\,c^5\,f}+\frac{a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,80{}\mathrm{i}}{3\,c^5\,f}-\frac{a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^5\,f}+\frac{a^2\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^5\,f}\right)}{{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,84{}\mathrm{i}-{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,96{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,54{}\mathrm{i}-{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,16{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(9/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))^(
1/2)*68i)/(3*c^5*f) - (a^2*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f
*x))^(1/2)*52i)/(3*c^5*f) + (a^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a
/cos(e + f*x))^(1/2)*80i)/(3*c^5*f) - (a^2*exp(e*5i + f*x*5i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f) + (a^2*exp(e*5i + f*x*5i)*cos(4*
e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f)))/(exp(e*5i + f*x*5i)*sin
(e + f*x)*84i - exp(e*5i + f*x*5i)*sin(2*e + 2*f*x)*96i + exp(e*5i + f*x*5i
)*sin(3*e + 3*f*x)*54i - exp(e*5i + f*x*5i)*sin(4*e + 4*f*x)*16i + exp(e*5i
 + f*x*5i)*sin(5*e + 5*f*x)*2i)
```

3.132 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}}-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}}-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{240c^2f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/10*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)-1/40*(a+a
*sec(f*x+e))^(5/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(9/2)-1/240*(a+a*sec(f*x
+e))^(5/2)*tan(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A]
time = 0.31, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4036, 4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{240c^2f(c-c\sec(e+fx))^{7/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{40cf(c-c\sec(e+fx))^{9/2}}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(11/2),x
]
```

```
[Out] -1/10*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(11
/2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(40*c*f*(c - c*Sec[e + f*x
])^(9/2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(240*c^2*f*(c - c*Sec
[e + f*x])^(7/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}}{5c}$$

$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan}{40cf(c-c\sec(e+f}$$

$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan}{40cf(c-c\sec(e+f}$$

Mathematica [A]
time = 1.21, size = 102, normalized size = 0.77

$$\frac{a^2(-141+170\cos(e+fx)-140\cos(2(e+fx))+30\cos(3(e+fx))-15\cos(4(e+fx)))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{120c^5f(-1+\cos(e+fx))^5\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(1
1/2),x]
```

```
[Out] (a^2*(-141 + 170*Cos[e + f*x] - 140*Cos[2*(e + f*x)] + 30*Cos[3*(e + f*x)]
- 15*Cos[4*(e + f*x)])*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(120*c^
5*f*(-1 + Cos[e + f*x])^5*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.77, size = 95, normalized size = 0.71

method	result
default	$-\frac{\left(31\left(\cos^2(fx+e)\right)-8\cos(fx+e)+1\right)\left(\sin^5(fx+e)\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,a^2}{240f(-1+\cos(fx+e))^2\cos(fx+e)^5\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{11}{2}}}$
risch	$\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(15\,e^{9i(fx+e)}-30\,e^{8i(fx+e)}+140\,e^{7i(fx+e)}-170\,e^{6i(fx+e)}+282\,e^{5i(fx+e)}-170\,e^{4i(fx+e)}+140\,e^{3i(fx+e)}-3\right.}{15c^5\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^9\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/240/f*(31*cos(f*x+e)^2-8*cos(f*x+e)+1)*sin(f*x+e)^5*(a*(cos(f*x+e)+1)/co
s(f*x+e))^(1/2)/(-1+cos(f*x+e))^2/cos(f*x+e)^5/(c*(-1+cos(f*x+e))/cos(f*x+e
))^(11/2)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4404 vs. $2(124) = 248$.
time = 32.79, size = 4404, normalized size = 33.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="maxima")
```

```
[Out] -2/15*(1350*a^2*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 1350*a^2*cos(4*f*x + 4*
e)*sin(2*f*x + 2*e) - 30*a^2*sin(2*f*x + 2*e) - 10*(3*a^2*sin(8*f*x + 8*e)
+ 17*a^2*sin(6*f*x + 6*e) + 17*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e
))*cos(10*f*x + 10*e) - 1350*(a^2*sin(6*f*x + 6*e) + a^2*sin(4*f*x + 4*e))*
cos(8*f*x + 8*e) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*e) +
290*a^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*
e) - 80*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 192*a^2*
sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*sin(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e
) + 450*a^2*sin(6*f*x + 6*e) + 450*a^2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x
 + 2*e) - 72*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*
a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(47*a^2*sin(10*f*x + 10*e) + 855*a^
2*sin(8*f*x + 8*e) + 2730*a^2*sin(6*f*x + 6*e) + 2730*a^2*sin(4*f*x + 4*e)
+ 855*a^2*sin(2*f*x + 2*e) + 240*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 160*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10
*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e) + 450*a^2*sin(6*f*x + 6*e) + 450*a^
2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x + 2*e) + 20*a^2*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*e) + 290*a
^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*e))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*(3*a^2*cos(8*f*x +
 8*e) + 17*a^2*cos(6*f*x + 6*e) + 17*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x
 + 2*e))*sin(10*f*x + 10*e) + 30*(45*a^2*cos(6*f*x + 6*e) + 45*a^2*cos(4*f*
x + 4*e) - a^2)*sin(8*f*x + 8*e) - 10*(135*a^2*cos(2*f*x + 2*e) + 17*a^2)*s
in(6*f*x + 6*e) - 10*(135*a^2*cos(2*f*x + 2*e) + 17*a^2)*sin(4*f*x + 4*e) +
 5*(3*a^2*cos(10*f*x + 10*e) + 75*a^2*cos(8*f*x + 8*e) + 290*a^2*cos(6*f*x
+ 6*e) + 290*a^2*cos(4*f*x + 4*e) + 75*a^2*cos(2*f*x + 2*e) - 80*a^2*cos(7/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 192*a^2*cos(5/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 3*a^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 20*(7*a^2*cos(10*f*x + 10*e) + 135*a^2*cos(8*f*x + 8*e) + 450*a^
2*cos(6*f*x + 6*e) + 450*a^2*cos(4*f*x + 4*e) + 135*a^2*cos(2*f*x + 2*e) -
```

```
72*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*a^2*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7*a^2)*sin(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(47*a^2*cos(10*f*x + 10*e) + 855*a^2*
cos(8*f*x + 8*e) + 2730*a^2*cos(6*f*x + 6*e) + 2730*a^2*cos(4*f*x + 4*e) +
855*a^2*cos(2*f*x + 2*e) + 240*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 160*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 + 47*a^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*(7*a^2
*cos(10*f*x + 10*e) + 135*a^2*cos(8*f*x + 8*e) + 450*a^2*cos(6*f*x + 6*e) +
 450*a^2*cos(4*f*x + 4*e) + 135*a^2*cos(2*f*x + 2*e) + 20*a^2*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7*a^2)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 5*(3*a^2*cos(10*f*x + 10*e) + 75*a^2*cos(8*f*x
 + 8*e) + 290*a^2*cos(6*f*x + 6*e) + 290*a^2*cos(4*f*x + 4*e) + 75*a^2*cos(
2*f*x + 2*e) + 3*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))
*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 10*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 +
 44100*c^6*cos(6*f*x + 6*e)^2 + 44100*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos
(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 14400*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
63504*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^
6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 +
 2025*c^6*sin(8*f*x + 8*e)^2 + 44100*c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin
(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*si
n(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 14400*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 63504*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c
^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) +
 c^6 + 2*(45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(
4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(10*f*x + 10*e) + 90*(210*
c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) +
 c^6)*cos(8*f*x + 8*e) + 420*(210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x +
 2*e) + c^6)*cos(6*f*x + 6*e) + 420*(45*c^6*cos...
```

Fricas [A]
time = 2.87, size = 209, normalized size = 1.57

$$\frac{\left(15\,a^2\cos\left(fx+e\right)^5-15\,a^2\cos\left(fx+e\right)^4+20\,a^2\cos\left(fx+e\right)^3-10\,a^2\cos\left(fx+e\right)^2+2\,a^2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{15\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\left(fx+e\right)-c^6f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/15*(15*a^2*cos(f*x + e)^5 - 15*a^2*cos(f*x + e)^4 + 20*a^2*cos(f*x + e)^3
 - 10*a^2*cos(f*x + e)^2 + 2*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/co
```

```
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5
- 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^
2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.75, size = 115, normalized size = 0.86

$$-\frac{\left(a^{2}-\frac{10\,\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)^{2}a^{5}+5\,\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\right)a^{6}+a^{7}}{a^{5}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}\right)a^{2}}{240\,\sqrt{-ac}\,c^{5}f|a|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="giac")
```

```
[Out] -1/240*(a^2 - (10*(a*tan(1/2*f*x + 1/2*e)^2 - a)^2*a^5 + 5*(a*tan(1/2*f*x +
 1/2*e)^2 - a)*a^6 + a^7)/(a^5*tan(1/2*f*x + 1/2*e)^10))*a^2/(sqrt(-a*c)*c^
5*f*abs(a)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 7.17, size = 419, normalized size = 3.15

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,136{}\mathrm{i}}{3\,c^6\,f}-\frac{a^2\,\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,1688{}\mathrm{i}}{15\,c^6\,f}+\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,160{}\mathrm{i}}{3\,c^6\,f}-\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,124{}\mathrm{i}}{3\,c^6\,f}+\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,8{}\mathrm{i}}{c^6\,f}-\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,4{}\mathrm{i}}{c^6\,f}\right)}{{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,88{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(11/2)),x
)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(
1/2)*136i)/(3*c^6*f) - (a^2*cos(e + f*x)*exp(e*6i + f*x*6i)*(a + a/cos(e +
f*x))^(1/2)*1688i)/(15*c^6*f) + (a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a
 + a/cos(e + f*x))^(1/2)*160i)/(3*c^6*f) - (a^2*exp(e*6i + f*x*6i)*cos(3*e
+ 3*f*x)*(a + a/cos(e + f*x))^(1/2)*124i)/(3*c^6*f) + (a^2*exp(e*6i + f*x*6
i)*cos(4*e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*8i)/(c^6*f) - (a^2*exp(e*6i
```

```
+ f*x*6i)*cos(5*e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^6*f)))/(exp(e*
6i + f*x*6i)*sin(e + f*x)*264i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i +
 exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*220i - exp(e*6i + f*x*6i)*sin(4*e + 4*
f*x)*88i + exp(e*6i + f*x*6i)*sin(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin
(6*e + 6*f*x)*2i)
```

3.133 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=139

$$-\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -1/2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-4*c^3*ln(
1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-2*
c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4037}

$$-\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}-\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[a + a*Sec[e + f*x]]) - (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f
*Sqrt[a + a*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + (2c)\int \frac{\sec(e+fx)(c-c}{\sqrt{a+a\sec}}$$
$$= -\frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{c(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a+a\sec(e+}}$$
$$= -\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{2c^2\sqrt{c-c\sec(e}}{f\sqrt{a+a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.48, size = 141, normalized size = 1.01

$$\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\left(1-6\cos(e+fx)+8\log\left(1+e^{i(e+fx)}\right)+\cos(2(e+fx))\left(8\log\left(1+e^{i(e+fx)}\right)-4\log\left(1+e^{2i(e+fx)}\right)\right)-4\log\left(1+e^{2i(e+fx)}\right)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]
],x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(1 - 6*Cos[e + f*x] + 8*Log[1 + E^(I*(e + f*x))] + Co
s[2*(e + f*x)]*(8*Log[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]
) - 4*Log[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]^2*Sqrt[c - c*Sec[e + f*x]]
)/(2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 2.86, size = 165, normalized size = 1.19

method	result
default	$-\frac{\left(8\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))+8(\cos^2(fx+e))\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+7(\cos^2(fx+e))+6\cos(fx+e)-1}{2f\sin(fx+e)(-1+\cos(fx+e))^2 a}$
risch	$-\frac{2ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(3e^{2i(fx+e)}-e^{i(fx+e)}+3\right)\left(e^{2i(fx+e)}+e^{i(fx+e)}\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f\left(e^{2i(fx+e)}+1\right)^2}+\frac{8ic^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\ln(e^{i(fx+e)}}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f*(8*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+8*cos(f*x
+e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+7*cos(f*x+e)^2+6*cos(f*x+e)
```

```
-1)*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*(a*(cos(f*x+e)+1)/cos(f
*x+e))^(1/2)/sin(f*x+e)/(-1+cos(f*x+e))^2/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(135) = 270$.
time = 0.59, size = 791, normalized size = 5.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - c^2*cos(4*f*x + 4*e)*sin(2*f*x +
 2*e) - c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x
+ 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2
*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(
4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x
+ 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*
cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 3*(c^2*sin
(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 3*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(c^2*cos(4*f*x + 4*e) +
 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 3*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(4*f*x
 + 4*e)^2 + 4*a*cos(2*f*x + 2*e)^2 + a*sin(4*f*x + 4*e)^2 + 4*a*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 4*a*sin(2*f*x + 2*e)^2 + 2*(2*a*cos(2*f*x + 2*e) +
 a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 1.64, size = 160, normalized size = 1.15

$$\frac{2\,c^2\left(\frac{2\,\sqrt{-ac}\,c\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)}{a|c|}-\frac{3\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2\sqrt{-ac}\,c+4\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}\,c^2+\sqrt{-ac}\,c^3}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2a|c|}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 2*c^2*(2*sqrt(-a*c)*c*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a*abs(c)) - (3*(c*
tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*c + 4*(c*tan(1/2*f*x + 1/2*e)^2 -
c)*sqrt(-a*c)*c^2 + sqrt(-a*c)*c^3)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*a*abs
(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^{5/2}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

3.134 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=94

$$-\frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -2*c^2*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)-c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4037}

$$-\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[
a + a*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} + (2c)\int \frac{\sec(e+fx)\sqrt{c-}}{\sqrt{a+a\sec(}}$$
$$= -\frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{c\sqrt{c-c\sec(e+}}{f\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.79, size = 173, normalized size = 1.84

$$\frac{ce^{-2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^2\cos\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\left(-1+\cos(e+fx)\left(4\log\left(1+e^{i(e+fx)}\right)-2\log\left(1+e^{2i(e+fx)}\right)\right)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)}{2\left(1+e^{i(e+fx)}\right)f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]
],x]
```

```
[Out] (c*(1 + E^((2*I)*(e + f*x)))^2*Cos[(e + f*x)/2]*Cot[(e + f*x)/2]*(-1 + Cos[
e + f*x]*(4*Log[1 + E^(I*(e + f*x))] - 2*Log[1 + E^((2*I)*(e + f*x))]))*Sec
[e + f*x]^3*Sqrt[c - c*Sec[e + f*x]]*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2]
))/(2*E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])
])
```

Maple [A]
time = 3.12, size = 149, normalized size = 1.59

method	result
default	$-\frac{\left(2\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)+1\right)\cos(fx+e)\left(\frac{c(-1+\cos(f}{\cos(fx+}\right.}{f\sin(fx+e)(-1+\cos(fx+e))a}$
risch	$-\frac{2ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+e^{i(fx+e)}\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f\left(e^{2i(fx+e)}+1\right)}+\frac{4ic\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\ln\left(e^{i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f}-\frac{2ic\left(e^{i(fx+e)}+1\right.}{\sqrt{\frac{a(}{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(2*cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+2*cos(f*x+e)*l
n(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+cos(f*x+e)+1)*cos(f*x+e)*(c*(-1+c
os(f*x+e))/cos(f*x+e))^(3/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)
/(-1+cos(f*x+e))/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(93) = 186$.
time = 0.57, size = 297, normalized size = 3.16

$$-\frac{2\left(c\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\sin\left(2fx+2e\right)-\left(c\cos\left(2fx+2e\right)^{2}+c\sin\left(2fx+2e\right)^{2}+2c\cos\left(2fx+2e\right)+c\right)\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)+2\left(c\cos\left(2fx+2e\right)^{2}+c\sin\left(2fx+2e\right)^{2}+2c\cos\left(2fx+2e\right)+c\right)\arctan\left(\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right),\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)+1\right)-\left(c\cos\left(2fx+2e\right)+c\right)\sin\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)\right)\right)\right)\sqrt{a}\sqrt{c}}{\left(a\cos\left(2fx+2e\right)^{2}+a\sin\left(2fx+2e\right)^{2}+2a\cos\left(2fx+2e\right)+a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e)
 - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(c*cos(2*f*x + 2*e)^2
+ c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 1) - (c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*f)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(-c*sec(f*x + e) + c)/sqr
t(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/sqrt(a*(sec(e + f*x) +
 1)), x)
```

Giac [A]
time = 1.41, size = 129, normalized size = 1.37

$$\frac{2\left(\frac{\sqrt{-ac}\,c^{2}\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)}{a|c|}-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)\sqrt{-ac}\,c^{2}+\sqrt{-ac}\,c^{3}}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c\right)a|c|}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 2*(sqrt(-a*c)*c^2*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a*abs(c)) - ((c*tan(1/
2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*c^2 + sqrt(-a*c)*c^3)/((c*tan(1/2*f*x + 1/
2*e)^2 - c)*a*abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

3.135 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=50

$$-\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Rubi [A]
time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4037\}$

$$-\frac{c\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((c*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.39, size = 140, normalized size = 2.80

$$\frac{i\left(1+e^{i(e+fx)}\right)\sqrt{\frac{c\left(-1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)}{\left(-1+e^{i(e+fx)}\right)\sqrt{\frac{a\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],
x]
```

```
[Out] (I*(1 + E^(I*(e + f*x)))*Sqrt[(c*(-1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e
 + f*x)))]*(2*Log[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]))/((-
1 + E^(I*(e + f*x)))*Sqrt[(a*(1 + E^(I*(e + f*x)))^2)/(1 + E^((2*I)*(e + f*
x)))]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(46) = 92$.
time = 3.04, size = 115, normalized size = 2.30

method	result	size
default	$-\frac{\left(\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)\right)\cos(fx+e)\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}}{f\sin(fx+e)a}$	115
risch	$\frac{2i\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\ln\left(e^{i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}-\frac{i\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\ln\left(e^{2i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$	206

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+ln((-cos(f*x+e)+1+sin(f*x+e
))/sin(f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x
+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)/a
```

Maxima [A]
time = 0.51, size = 68, normalized size = 1.36

$$-\frac{\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}}+\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(-a) + sqrt(c)*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(-a))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)
), x)
```

Giac [A]
time = 1.24, size = 59, normalized size = 1.18

$$-\frac{c^2\log\left(\left|c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sqrt{-ac}\,f|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 - c))*sgn(tan(1/2*f*x + 1/2*e)^3 + ta
n(1/2*f*x + 1/2*e))/(sqrt(-a*c)*f*abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

3.136 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -arctanh(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4044, 3855}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -((ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx &= \frac{\tan(e+fx)\int \csc(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\ &= -\frac{\tanh^{-1}(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.81, size = 94, normalized size = 2.00

$$\frac{4i\left(-1+e^{i(e+fx)}\right)\tanh^{-1}\left(e^{i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}{\left(1+e^{i(e+fx)}\right)f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),
x]
```

```
[Out] ((4*I)*(-1 + E^(I*(e + f*x)))*ArcTanh[E^(I*(e + f*x))]*Cos[(e + f*x)/2]^2*S
ec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c
*Sec[e + f*x]])
```

Maple [A]
time = 3.11, size = 85, normalized size = 1.81

method	result	size
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\cos(fx+e)}{f\sin(fx+e)ca}$	85
risch	$-\frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$	228

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*ln(-(-1+cos(f*x+e))/sin(f*x+e))*(c
*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*cos(f*x+e)/sin(f*x+e)/c/a
```

Maxima [A]
time = 0.56, size = 48, normalized size = 1.02

$$-\frac{\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)-\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)}{\sqrt{a}\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(arctan2(sin(f*x + e), cos(f*x + e) + 1) - arctan2(sin(f*x + e), cos(f*x +
 e) - 1))/(sqrt(a)*sqrt(c)*f)
```

Fricas [A]
time = 3.17, size = 218, normalized size = 4.64

$$\left[-\frac{\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)}{2acf}, \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{ac\sin(fx+e)}\right)}{acf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x +
e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f), sqr
t(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))/(a*c*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1
))), x)
```

Giac [A]
time = 1.68, size = 70, normalized size = 1.49

$$-\frac{c^2\left(\frac{\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c}-\frac{\log(|c|)}{c}\right)}{2\sqrt{-ac}\,f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/2*c^2*(log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - log(abs(c))/c)/(sqrt(-a*c)
*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

3.137 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}-\frac{\tanh^{-1}(\cos(e+fx))\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)-1/2*arctanh
(cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4045, 4044, 3855}

$$-\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2cf\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) -
 (ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[
c - c*Sec[e + f*x]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
```

```
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{\int \frac{}{\sqrt{a}}}{}$$
$$= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} + \frac{}{2c\sqrt{a}}$$
$$= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}} - \frac{t}{2cf\sqrt{c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.88, size = 79, normalized size = 0.83

$$-\frac{\left(1+2\tanh^{-1}\left(e^{i(e+fx)}\right)(-1+\cos(e+fx))\right)\tan(e+fx)}{2cf(-1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)
),x]
```

```
[Out] -1/2*((1 + 2*ArcTanh[E^(I*(e + f*x))]*(-1 + Cos[e + f*x]))*Tan[e + f*x])/(c
*f*(-1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 3.07, size = 131, normalized size = 1.38

method	result
default	$-\frac{(-1+\cos(fx+e))\left(2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-2\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\cos(fx+e)-1\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}}{4f\cos(fx+e)\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\sin(fx+e)a}$
risch	$\frac{i\left(e^{2i(fx+e)}+e^{i(fx+e)}\right)}{c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f} - \frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{2c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)}{e^{2i(fx+e)}+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/f*(-1+cos(f*x+e))*(2*cos(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-2*ln(-
(-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e)-1)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/
2)/cos(f*x+e)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(90) = 180$.
time = 0.57, size = 442, normalized size = 4.65

$$\frac{\left(\left(2\left(2\cos\left(fx+e\right)-1\right)\cos\left(2fx+2e\right)-\cos\left(2fx+2e\right)^{2}-4\cos\left(fx+e\right)^{2}-\sin\left(2fx+2e\right)^{2}+4\sin\left(2fx+2e\right)\sin\left(fx+e\right)-4\sin\left(fx+e\right)^{2}+4\cos\left(fx+e\right)-1\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)-\left(2\left(2\cos\left(fx+e\right)-1\right)\cos\left(2fx+2e\right)-\cos\left(2fx+2e\right)^{2}-4\cos\left(fx+e\right)^{2}-\sin\left(2fx+2e\right)^{2}+4\sin\left(2fx+2e\right)\sin\left(fx+e\right)-4\sin\left(fx+e\right)^{2}+4\cos\left(fx+e\right)-1\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)+2\cos\left(fx+e\right)\sin\left(2fx+2e\right)-2\cos\left(2fx+2e\right)\sin\left(fx+e\right)-2\sin\left(fx+e\right)\right)\sqrt{a}\sqrt{c}}{2\left(ac^{2}\cos\left(2fx+2e\right)^{2}+4ac^{2}\cos\left(fx+e\right)^{2}+ac^{2}\sin\left(2fx+2e\right)^{2}-4ac^{2}\sin\left(2fx+2e\right)\sin\left(fx+e\right)+4ac^{2}\sin\left(fx+e\right)^{2}-4ac^{2}\cos\left(fx+e\right)+ac^{2}-2\left(2ac^{2}\cos\left(fx+e\right)-ac^{2}\right)\cos\left(2fx+2e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(
f*x + e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f
*x + e)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - (
2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x +
e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e
)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*cos(f
*x + e)*sin(2*f*x + 2*e) - 2*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin(f*x + e)
)*sqrt(a)*sqrt(c)/((a*c^2*cos(2*f*x + 2*e)^2 + 4*a*c^2*cos(f*x + e)^2 + a*c
^2*sin(2*f*x + 2*e)^2 - 4*a*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a*c^2*sin
(f*x + e)^2 - 4*a*c^2*cos(f*x + e) + a*c^2 - 2*(2*a*c^2*cos(f*x + e) - a*c^
2)*cos(2*f*x + 2*e))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(90) = 180$.
time = 1.98, size = 414, normalized size = 4.36

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)-1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+\left(ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{4\left(ac^{2}f\cos\left(fx+e\right)-ac^{2}f\right)\sin\left(fx+e\right)},\ \frac{\sqrt{ac}\left(\cos\left(fx+e\right)-1\right)\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{ac\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{2\left(ac^{2}f\cos\left(fx+e\right)-ac^{2}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e) - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x
 + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e) - a*c^2*
f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) - 1)*arctan(sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(
a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
```

```
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e)
 - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**
(3/2)), x)
```

Giac [A]
time = 1.90, size = 92, normalized size = 0.97

$$\frac{\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}-\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)+\log\left(|c|\right)}{4\sqrt{-ac}\,f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)/(c*tan(1/2*f*x + 1/2*e)^2) - log(abs(c)
*tan(1/2*f*x + 1/2*e)^2) + log(abs(c)))/(sqrt(-a*c)*f*abs(c)*sgn(tan(1/2*f*
x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

3.138 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{3/2}}-\frac{\tan}{4c^2f\sqrt{a-}}$$

```
[Out] -1/4*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-1/4*tan(f*x
+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e
))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4045, 4044, 3855}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4c^2f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4cf\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{3/2}}-\frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] -1/4*Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)) -
 Tan[e + f*x]/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) -
 (ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
```

```
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} + \frac{\int \frac{}{\sqrt{a}}}{}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{}{4cf\sqrt{}}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{}{4cf\sqrt{}}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))^{5/2}} - \frac{}{4cf\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.82, size = 91, normalized size = 0.65

$$-\frac{\left(-2+3\cos(e+fx)+8\tanh^{-1}\left(e^{i(e+fx)}\right)\sin^4\left(\frac{1}{2}(e+fx)\right)\right)\tan(e+fx)}{4c^2f(-1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
),x]
```

```
[Out] -1/4*((-2 + 3*Cos[e + f*x] + 8*ArcTanh[E^(I*(e + f*x))]*Sin[(e + f*x)/2]^4)
*Tan[e + f*x])/(c^2*f*(-1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt
[c - c*Sec[e + f*x]])
```

Maple [A]
time = 3.06, size = 170, normalized size = 1.21

method	result
default	$-\frac{(-1+\cos(fx+e))\left(4(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-8\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-5(\cos^2(fx+e))+4\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\right.}{16f\cos(fx+e)^2\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}\sin(fx+e)a}$

risch	$\frac{i\left(3\,\mathrm{e}^{2i(fx+e)}-4\,\mathrm{e}^{i(fx+e)}+3\right)\left(\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}\right)}{2c^2\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{2i(fx+e)}+1\right)f}+\frac{i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{i(fx+e)}+\right.}{4c^2\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{2i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/16/f*(-1+cos(f*x+e))*(4*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-8*c
os(f*x+e)*ln(-(-1+cos(f*x+e))/sin(f*x+e))-5*cos(f*x+e)^2+4*ln(-(-1+cos(f*x+
e))/sin(f*x+e))-2*cos(f*x+e)+3)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/cos(f*x
+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(132) = 264$.
time = 0.65, size = 1303, normalized size = 9.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(
4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e)
+ 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos
(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x
+ 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(4*f*x
+ 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 16*s
in(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e
) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x +
 e) + 1) - (2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1
)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x
 + e) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) -
1)*cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(
3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(
4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e)
- 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f
*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos
(f*x + e) - 1) - 2*(3*sin(3*f*x + 3*e) - 4*sin(2*f*x + 2*e) + 3*sin(f*x + e
))*cos(4*f*x + 4*e) + 2*(3*cos(3*f*x + 3*e) - 4*cos(2*f*x + 2*e) + 3*cos(f*
x + e))*sin(4*f*x + 4*e) - 2*(2*cos(2*f*x + 2*e) + 3)*sin(3*f*x + 3*e) + 4*
(cos(f*x + e) + 2)*sin(2*f*x + 2*e) + 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) -
 4*cos(2*f*x + 2*e)*sin(f*x + e) - 6*sin(f*x + e))*sqrt(a)*sqrt(c)/((a*c^3*
cos(4*f*x + 4*e)^2 + 16*a*c^3*cos(3*f*x + 3*e)^2 + 36*a*c^3*cos(2*f*x + 2*e
```

```
)^2 + 16*a*c^3*cos(f*x + e)^2 + a*c^3*sin(4*f*x + 4*e)^2 + 16*a*c^3*sin(3*f
*x + 3*e)^2 + 36*a*c^3*sin(2*f*x + 2*e)^2 - 48*a*c^3*sin(2*f*x + 2*e)*sin(f
*x + e) + 16*a*c^3*sin(f*x + e)^2 - 8*a*c^3*cos(f*x + e) + a*c^3 - 2*(4*a*c
^3*cos(3*f*x + 3*e) - 6*a*c^3*cos(2*f*x + 2*e) + 4*a*c^3*cos(f*x + e) - a*c
^3)*cos(4*f*x + 4*e) - 8*(6*a*c^3*cos(2*f*x + 2*e) - 4*a*c^3*cos(f*x + e) +
 a*c^3)*cos(3*f*x + 3*e) - 12*(4*a*c^3*cos(f*x + e) - a*c^3)*cos(2*f*x + 2*
e) - 4*(2*a*c^3*sin(3*f*x + 3*e) - 3*a*c^3*sin(2*f*x + 2*e) + 2*a*c^3*sin(f
*x + e))*sin(4*f*x + 4*e) - 16*(3*a*c^3*sin(2*f*x + 2*e) - 2*a*c^3*sin(f*x
+ e))*sin(3*f*x + 3*e))*f)
```

Fricas [A]
time = 3.50, size = 494, normalized size = 3.53

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^2+\left(ac\cos\left(fx+e\right)^2+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^2-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\left(3\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{8\left(ac^3f\cos\left(fx+e\right)^2-2ac^3f\cos\left(fx+e\right)+ac^3f\right)\sin\left(fx+e\right)}, \frac{\sqrt{ac}\left(\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)+1\right)\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{ac\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+\left(3\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{4\left(ac^3f\cos\left(fx+e\right)^2-2ac^3f\cos\left(fx+e\right)+ac^3f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x
+ e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 - 2*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin
(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*arctan(sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*cos(f*x + e)^2 - 2*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*
sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**
(5/2)), x)
```

Giac [A]
time = 1.74, size = 122, normalized size = 0.87

$$\frac{\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}-2\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)+2\log\left(|c|\right)}{16\sqrt{-ac}\,cf|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/16*((3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 2*(c*tan(1/2*f*x + 1/2*e)^2 - c
)*c)/(c^2*tan(1/2*f*x + 1/2*e)^4) - 2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) +
2*log(abs(c)))/(sqrt(-a*c)*c*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*
f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

3.139 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}}+\frac{c(c-c\sec(e+fx))^{3/2}\,\mathrm{ta}}{f(a+a\sec(e+fx}$$

```
[Out] c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)+4*c^3*ln(1+sec
(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+2*c^2
*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4039, 4040, 4037}

$$\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a\sec(e+fx)+a}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sq
rt[c - c*Sec[e + f*x]]) + (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a*
f*Sqrt[a + a*Sec[e + f*x]]) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{\sqrt{a+a\sec(e+fx}}}{a}$$

$$= \frac{2c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan}{f(a+a\sec(e+fx)}$$

$$= \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\sqrt{c-c\sec(e-}}{af\sqrt{a+a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.03, size = 183, normalized size = 1.29

$$-\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\left(-1+4\log\left(1+e^{i(e+fx)}\right)+\cos(e+fx)\left(-5+8\log\left(1+e^{i(e+fx)}\right)-4\log\left(1+e^{2i(e+fx)}\right)\right)+\cos(2(e+fx))\left(4\log\left(1+e^{i(e+fx)}\right)-2\log\left(1+e^{2i(e+fx)}\right)\right)-2\log\left(1+e^{2i(e+fx)}\right)\right)\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{af(1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((c^2*Cot[(e + f*x)/2]*(-1 + 4*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(-5
 + 8*Log[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + Cos[2*(e
+ f*x)]*(4*Log[1 + E^(I*(e + f*x))] - 2*Log[1 + E^((2*I)*(e + f*x))]) - 2*L
og[1 + E^((2*I)*(e + f*x))])*Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1
 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]))
```

Maple [A]
time = 2.86, size = 235, normalized size = 1.65

method	result

default	$-\frac{\left(4(\cos^2(fx+e))\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+4\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-(\cos^2(fx+e))+4\cos(fx+e)\ln\left(-\right.\right.}{f\sin(fx+}$
risch	$\frac{2ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,(5\,e^{3i(fx+e)}+2\,e^{2i(fx+e)}+5\,e^{i(fx+e)})}{a(e^{i(fx+e)}+1)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)f(e^{2i(fx+e)}+1)}-\frac{8ic^2(e^{i(fx+e)}+1)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\ln(e^{i(fx+e)}+1)}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,(e^{i(fx+e)}-1)f}+\frac{4ic^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(4*cos(f*x+e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+4*ln(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-cos(f*x+e)^2+4*cos(f*x+e)*ln(
-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))+4*cos(f*x+e)+1)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*co
s(f*x+e)^2*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+e))
/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2185 vs. $2(140) = 280$.
time = 0.78, size = 2185, normalized size = 15.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -2*(8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*
c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*c^2*cos(4*f*x + 4*e)*sin(2*f*x +
2*e) + 2*c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x
 + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(
2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*c
os(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*sin(1/2*arctan2(si
```

```
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*
c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e) + 4*(c^2*cos(4*f*x + 4*e) + 2*
c^2*cos(2*f*x + 2*e) + 2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + c^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2
*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2
*e) + 2*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c
^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (16*c^2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 5*c^2*sin(4*f*x + 4*e) - 6*c^2*sin(2*f*x + 2*e) + 8*(c^2*cos(4*f*x
+ 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (5*c^2*
sin(4*f*x + 4*e) + 6*c^2*sin(2*f*x + 2*e) - 8*(c^2*cos(4*f*x + 4*e) + 2*c^2
*cos(2*f*x + 2*e) + c^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (16*c^2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 5*c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + 5*c^2 + 8*
(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ (5*c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + 5*c^2 + 8*(c^2*sin(4*f
*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqr
t(c)/((a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(3/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2
*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2
*f*x + 2*e) + 2*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*
x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e) + 2*a
^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*
f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2
*sec(f*x + e) + a^2), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**A**]
time = 1.61, size = 161, normalized size = 1.13

$$-\frac{2\,c^2\left(\frac{2\,\sqrt{-ac}\,c\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)}{a^2|c|}+\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}}{a^2|c|}-\frac{2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}\,c+\sqrt{-ac}\,c^2}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)a^2|c|}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -2*c^2*(2*sqrt(-a*c)*c*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a^2*abs(c)) + (c*
tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)/(a^2*abs(c)) - (2*(c*tan(1/2*f*x + 1
/2*e)^2 - c)*sqrt(-a*c)*c + sqrt(-a*c)*c^2)/((c*tan(1/2*f*x + 1/2*e)^2 - c)
*a^2*abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)), x
)
```

3.140 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{c^2\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] c^2*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.18, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4039, 4037}

$$\frac{c^2\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a + a
*Sec[e + f*x])^(3/2))
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{c\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+f}{\sqrt{a+a\sec(e+fx)}}}{a}$$
$$= \frac{c^2\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{c\sqrt{c-c\sec(e+f}}{f(a+a\sec(e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.05, size = 132, normalized size = 1.39

$$-\frac{c\cot\left(\frac{1}{2}(e+fx)\right)\left(-2+2\log\left(1+e^{i(e+fx)}\right)+\cos(e+fx)\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)-\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{c-c\sec(e+fx)}}{af(1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((c*Cot[(e + f*x)/2]*(-2 + 2*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(2*Lo
g[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]) - Log[1 + E^((2*I)*(
e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(a*f*(1 + Cos[e + f*x])*Sqrt[a*(1 + S
ec[e + f*x])]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(87) = 174$.
time = 2.91, size = 191, normalized size = 2.01

method	result
default	$-\frac{\left(\cos(fx+e)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)\ln\left(\frac{-\cos(fx+e)+1+\sin(fx+e)}{\sin(fx+e)}\right)+\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\ln\left(\frac{-\cos(fx+}{\sin}\right.\right.}{f\sin(fx+e)^3a^2}$
risch	$\frac{4ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,e^{i(fx+e)}}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f} - \frac{2ic\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\ln\left(e^{i(fx+e)}+1\right)}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)f} + \frac{ic\left(e^{i(fx+e)}+1\right)\sqrt{}}{a\sqrt{\frac{a(e^{i(f}}{e^{2i(f}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+cos(f*x+e)*ln((-
cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))+ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+
e))+ln((-cos(f*x+e)+1+sin(f*x+e))/sin(f*x+e))-cos(f*x+e)+1)*(c*(-1+cos(f*x+
e))/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(
f*x+e)^3/a^2
```

Maxima [A]
time = 0.51, size = 105, normalized size = 1.11

$$\frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}\,a}+\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}\,a}+\frac{c^{\frac{3}{2}}\sin(fx+e)^2}{\sqrt{-a}\,a(\cos(fx+e)+1)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] (c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a) + c^(3/2)*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a) + c^(3/2)*sin(f*x + e)^
2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt
(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/(a*(sec(e + f*x) + 1))
**(3/2), x)
```

Giac [A]
time = 1.57, size = 79, normalized size = 0.83

$$\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)-c\right)c^2\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-ac}\,af|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] (c*tan(1/2*f*x + 1/2*e)^2 + c*log(c*tan(1/2*f*x + 1/2*e)^2 - c) - c)*c^2*sg
n(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a*f*abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)), x
)
```

3.141 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 1/2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4035\}$

$$\frac{\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{2f(a\sec(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A]
time = 0.18, size = 42, normalized size = 1.00

$$\frac{\csc(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2
),x]
```

```
[Out] (Csc[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 2.96, size = 73, normalized size = 1.74

method	result	size
default	$\frac{\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\cos(fx+e)(-1+\cos(fx+e))^2}{2f\sin(fx+e)^3a^2}$	73
risch	$\frac{2i\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,e^{i(fx+e)}}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1
/2)*cos(f*x+e)*(-1+cos(f*x+e))^2/sin(f*x+e)^3/a^2
```

Maxima [A]
time = 0.51, size = 58, normalized size = 1.38

$$\frac{\sqrt{c}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{2\sqrt{-a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/(sqrt(-a)*a*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(39) = 78$.
time = 4.24, size = 85, normalized size = 2.02

$$\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\,\cos(fx+e)}{(a^2f\cos(fx+e)+a^2f)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*cos(f*x + e)/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\,\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**(
3/2), x)
```

Giac [A]
time = 1.66, size = 36, normalized size = 0.86

$$\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c}{2\,\sqrt{-ac}\,af|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c/(sqrt(-a*c)*a*f*abs(c))
```

Mupad [B]
time = 2.55, size = 50, normalized size = 1.19

$$\frac{\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}{a\,f\,\sin\left(e+f\,x\right)\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)
```

```
[Out] (c - c/cos(e + f*x))^(1/2)/(a*f*sin(e + f*x)*((a*(cos(e + f*x) + 1))/cos(e
+ f*x))^(1/2))
```

3.142 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tanh^{-1}(\cos(e+fx))\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)-1/2*arctanh(
cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4045, 4044, 3855}

$$\frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(2*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
```

```
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{}{\sqrt{a\ldots}}}{}$$
$$= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{}{2a\sqrt{a\ldots}}$$
$$= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{t\ldots}{2af\sqrt{c\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.39, size = 157, normalized size = 1.65

$$-\frac{\left(1+2\tanh^{-1}\left(e^{i(e+fx)}\right)(1+\cos(e+fx))\right)\sec^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+i\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{2}\,a\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\,f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] -(((1 + 2*ArcTanh[E^(I*(e + f*x))]*(1 + Cos[e + f*x]))*Sec[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*Sin[(e + f*x)/2])/(Sqrt[2]*a*(1 + E^
(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)))]*f*Sqrt[a*(1
+ Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A]
time = 2.82, size = 123, normalized size = 1.29

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(-1+\cos(fx+e))^2\left(2\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+2\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+\cos(fx+e)-1\right)}{4f\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sin(fx+e)^3a^2}$
risch	$\frac{i\left(e^{2i(fx+e)}-e^{i(fx+e)}\right)}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)f} - \frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{2a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+\ldots\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(2*cos(f*x+e)*l
n(-(-1+cos(f*x+e))/sin(f*x+e))+2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+cos(f*x+e)
-1)/(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^3/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(90) = 180$.
time = 0.57, size = 433, normalized size = 4.56

$$-\frac{\left(\left(2\,\left(2\,\cos\left(fx+e\right)+1\right)\cos\left(2\,fx+2\,e\right)+\cos\left(2\,fx+2\,e\right)^{2}+4\,\cos\left(fx+e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+4\,\sin\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+4\,\sin\left(fx+e\right)^{2}+4\,\cos\left(fx+e\right)+1\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)+1\right)-\left(2\,\left(2\,\cos\left(fx+e\right)+1\right)\cos\left(2\,fx+2\,e\right)+\cos\left(2\,fx+2\,e\right)^{2}+4\,\cos\left(fx+e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+4\,\sin\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+4\,\sin\left(fx+e\right)^{2}+4\,\cos\left(fx+e\right)+1\right)\arctan\left(\sin\left(fx+e\right),\cos\left(fx+e\right)-1\right)-2\,\cos\left(fx+e\right)\sin\left(2\,fx+2\,e\right)+2\,\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+2\,\sin\left(fx+e\right)\right)\sqrt{a}\sqrt{c}}{2\,\left(a^{2}c\cos\left(2\,fx+2\,e\right)^{2}+4\,a^{2}c\cos\left(fx+e\right)^{2}+a^{2}c\sin\left(2\,fx+2\,e\right)^{2}+4\,a^{2}c\sin\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+4\,a^{2}c\sin\left(fx+e\right)^{2}+4\,a^{2}c\cos\left(fx+e\right)+a^{2}c+2\,\left(2\,a^{2}c\cos\left(fx+e\right)+a^{2}c\right)\cos\left(2\,fx+2\,e\right)\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos
(f*x + e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(
f*x + e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) -
(2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x +
 e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x +
e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) - 2*cos(
f*x + e)*sin(2*f*x + 2*e) + 2*cos(2*f*x + 2*e)*sin(f*x + e) + 2*sin(f*x + e
))*sqrt(a)*sqrt(c)/((a^2*c*cos(2*f*x + 2*e)^2 + 4*a^2*c*cos(f*x + e)^2 + a^
2*c*sin(2*f*x + 2*e)^2 + 4*a^2*c*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2*c*si
n(f*x + e)^2 + 4*a^2*c*cos(f*x + e) + a^2*c + 2*(2*a^2*c*cos(f*x + e) + a^2
*c)*cos(2*f*x + 2*e))*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(90) = 180$.
time = 3.19, size = 412, normalized size = 4.34

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+\left(ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{4\left(a^{2}cf\cos\left(fx+e\right)+a^{2}cf\right)\sin\left(fx+e\right)},\ \frac{\sqrt{ac}\left(\cos\left(fx+e\right)+1\right)\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{ac\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{2\left(a^{2}cf\cos\left(fx+e\right)+a^{2}cf\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x
 + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e) + a^2*c*
```

```
f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) + 1)*arctan(sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(
a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e)
 + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

Giac [A]
time = 1.89, size = 95, normalized size = 1.00

$$-\frac{c^2\left(\frac{\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c}-\frac{\log(|c|)}{c}-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{c^2}\right)}{4\sqrt{-ac}\,af|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/4*c^2*(log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - log(abs(c))/c - (c*tan(1/2
*f*x + 1/2*e)^2 - c)/c^2)/(sqrt(-a*c)*a*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3
 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

3.143 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\tanh^{-1}(\cos(e+fx))\tan(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*csc(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*arct
anh(cos(f*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Rubi [A]
time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4044, 2691, 3855}

$$\frac{\csc(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] Csc[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) -
(ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)\int \cot^2(e+fx)\csc(e+fx)\,dx}{ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{2ac\sqrt{}}$$
$$= \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{}{2acf}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.80, size = 69, normalized size = 0.66

$$\frac{\csc(e+fx) - 2\tanh^{-1}\left(e^{i(e+fx)}\right)\tan(e+fx)}{2acf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/
2)),x]
```

```
[Out] (Csc[e + f*x] - 2*ArcTanh[E^(I*(e + f*x))]*Tan[e + f*x])/(2*a*c*f*Sqrt[a*(1
 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 2.97, size = 133, normalized size = 1.28

method	result
default	$\frac{(-1+\cos(fx+e))^2\left((\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-\cos(fx+e)\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}}{2f\sin(fx+e)^3\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}\cos(fx+e)a^2}$
risch	$\frac{i e^{i(fx+e)}}{ac\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f} - \frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{2ac\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\sqrt{\frac{c\left(e^{i(fx+e)}\right.}{e^{2i(fx+e}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/f*(-1+cos(f*x+e))^2*(cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-ln(-(
-1+cos(f*x+e))/sin(f*x+e))-cos(f*x+e))*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/
sin(f*x+e)^3/(c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/cos(f*x+e)/a^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(99) = 198$.
time $= 0.57$, size $= 613$, normalized size $= 5.89$

$$\frac{\left(\left(2 \left(2 \cos \left(2 f x+2 e\right)-1\right) \cos \left(4 f x+4 e\right)-\cos \left(4 f x+4 e\right)^{2}-4 \cos \left(2 f x+2 e\right)^{2}-\sin \left(4 f x+4 e\right)^{2}+4 \sin \left(4 f x+4 e\right) \sin \left(2 f x+2 e\right)-4 \sin \left(2 f x+2 e\right)^{2}+4 \cos \left(2 f x+2 e\right)-1\right) \arctan \left(\sin \left(f x+e\right), \cos \left(f x+e\right)+1\right)-\left(2 \left(2 \cos \left(2 f x+2 e\right)-1\right) \cos \left(4 f x+4 e\right)-\cos \left(4 f x+4 e\right)^{2}-4 \cos \left(2 f x+2 e\right)^{2}-\sin \left(4 f x+4 e\right)^{2}+4 \sin \left(4 f x+4 e\right) \sin \left(2 f x+2 e\right)-4 \sin \left(2 f x+2 e\right)^{2}+4 \cos \left(2 f x+2 e\right)-1\right) \arctan \left(\sin \left(f x+e\right), \cos \left(f x+e\right)-1\right)-2 \left(\sin \left(3 f x+3 e\right)+\sin \left(f x+e\right)\right) \cos \left(4 f x+4 e\right)+2 \left(\cos \left(3 f x+3 e\right)+\cos \left(f x+e\right)\right) \sin \left(4 f x+4 e\right)+2 \left(2 \cos \left(2 f x+2 e\right)-1\right) \sin \left(3 f x+3 e\right)-4 \cos \left(3 f x+3 e\right) \sin \left(2 f x+2 e\right)-4 \cos \left(f x+e\right) \sin \left(2 f x+2 e\right)+4 \cos \left(2 f x+2 e\right) \sin \left(f x+e\right)-2 \sin \left(f x+e\right)\right) \sqrt{a} \sqrt{c}}{2 \left(a^{2} c^{2} \cos \left(4 f x+4 e\right)^{2}+4 a^{2} c^{2} \cos \left(2 f x+2 e\right)^{2}+a^{2} c^{2} \sin \left(4 f x+4 e\right)^{2}-4 a^{2} c^{2} \sin \left(4 f x+4 e\right) \sin \left(2 f x+2 e\right)+4 a^{2} c^{2} \sin \left(2 f x+2 e\right)^{2}-4 a^{2} c^{2} \cos \left(2 f x+2 e\right)+a^{2} c^{2}-2 \left(2 a^{2} c^{2} \cos \left(2 f x+2 e\right)-a^{2} c^{2}\right) \cos \left(4 f x+4 e\right)\right) f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*
cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e), c
os(f*x + e) + 1) - (2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x
 + 4*e)^2 - 4*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(s
in(f*x + e), cos(f*x + e) - 1) - 2*(sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*
f*x + 4*e) + 2*(cos(3*f*x + 3*e) + cos(f*x + e))*sin(4*f*x + 4*e) + 2*(2*co
s(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e)
- 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin
(f*x + e))*sqrt(a)*sqrt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f
*x + 2*e)^2 + a^2*c^2*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 4*a^2*c^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a
^2*c^2 - 2*(2*a^2*c^2*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

Fricas [**A**]
time $= 2.40$, size $= 434$, normalized size $= 4.17$

$$\left[-\frac{\sqrt{-a c} \left(\cos \left(f x+e\right)^{2}-1\right) \log \left(-\frac{4 \left(2 \sqrt{-a c} \sqrt{\frac{a \cos \left(f x+e\right)+a}{\cos \left(f x+e\right)}} \sqrt{\frac{c \cos \left(f x+e\right)-c}{\cos \left(f x+e\right)}} \cos \left(f x+e\right)^{2}+\left(a c \cos \left(f x+e\right)^{2}+a c\right) \sin \left(f x+e\right)\right)}{\left(\cos \left(f x+e\right)^{2}-1\right) \sin \left(f x+e\right)}\right) \sin \left(f x+e\right)-2 \sqrt{\frac{a \cos \left(f x+e\right)+a}{\cos \left(f x+e\right)}} \sqrt{\frac{c \cos \left(f x+e\right)-c}{\cos \left(f x+e\right)}} \cos \left(f x+e\right)^{2}}{4 \left(a^{2} c^{2} f \cos \left(f x+e\right)^{2}-a^{2} c^{2} f\right) \sin \left(f x+e\right)}, \frac{\sqrt{a c} \left(\cos \left(f x+e\right)^{2}-1\right) \arctan \left(\frac{\sqrt{a c} \sqrt{\frac{a \cos \left(f x+e\right)+a}{\cos \left(f x+e\right)}} \sqrt{\frac{c \cos \left(f x+e\right)-c}{\cos \left(f x+e\right)}}}{a c \sin \left(f x+e\right)}\right) \sin \left(f x+e\right)+\sqrt{\frac{a \cos \left(f x+e\right)+a}{\cos \left(f x+e\right)}} \sqrt{\frac{c \cos \left(f x+e\right)-c}{\cos \left(f x+e\right)}} \cos \left(f x+e\right)^{2}}{2 \left(a^{2} c^{2} f \cos \left(f x+e\right)^{2}-a^{2} c^{2} f\right) \sin \left(f x+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f
*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^2*f*cos(f*x + e)^2
- a^2*c^2*f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e)^2 - 1)*arctan(sqrt
(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^
2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1)
)**(3/2)), x)
```

Giac [A]
time = 3.07, size = 120, normalized size = 1.15

$$\frac{\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{c}+\frac{2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}-2\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)+2\log\left(|c|\right)-1}{8\sqrt{-ac}\,af|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/8*((c*tan(1/2*f*x + 1/2*e)^2 - c)/c + (2*c*tan(1/2*f*x + 1/2*e)^2 - c)/(c
*tan(1/2*f*x + 1/2*e)^2) - 2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) + 2*log(abs
(c)) - 1)/(sqrt(-a*c)*a*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x +
 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+fx)\left(a+\frac{a}{\cos(e+fx)}\right)^{3/2}\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

3.144 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}-\frac{3\tan}{8ac^2f\sqrt{a-}}$$

```
[Out] 3/8*csc(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2)-3/8*arctanh(cos(f*
x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4045, 4044, 2691, 3855}

$$\frac{3\csc(e+fx)}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) - Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} + \frac{3\int \cdots}{\cdots}$$
$$= -\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} - \frac{(3\,\mathrm{t}\cdots}{4ac\cdots}$$
$$= \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{\cdots}{4f(\cdots}$$
$$= \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{\cdots}{4f(\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.50, size = 122, normalized size = 0.84

$$-\frac{\left(1-2\cos(e+fx)+5\cos(2(e+fx))+24\tanh^{-1}\left(e^{i(e+fx)}\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\sin^2(e+fx)\right)\tan(e+fx)}{16ac^2f(-1+\cos(e+fx))^2(1+\cos(e+fx))\sqrt{a(1+\sec(e+fx))}\,\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)),x]
```

```
[Out] -1/16*((1 - 2*Cos[e + f*x] + 5*Cos[2*(e + f*x)] + 24*ArcTanh[E^(I*(e + f*x)
)]*Sin[(e + f*x)/2]^2*Sin[e + f*x]^2)*Tan[e + f*x])/(a*c^2*f*(-1 + Cos[e +
f*x])^2*(1 + Cos[e + f*x])*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*
x]])
```

Maple [A]
time = 2.70, size = 211, normalized size = 1.45

method	result
default	$\frac{(-1+\cos(fx+e))^2\left(12(\cos^3(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-12(\cos^2(fx+e))\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-5(\cos^3(fx+e))-12\cos(fx+e)\ln\left(-\right.\right.}{32f\cos(fx+e)^2\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}\sin(f}$
risch	$\frac{i\left(5\,e^{5i(fx+e)}-2\,e^{4i(fx+e)}+2\,e^{3i(fx+e)}-2\,e^{2i(fx+e)}+5\,e^{i(fx+e)}\right)}{4a\,c^2\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{3i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right.}{8a\,c^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right.}{e^{2i(fx+e)}+1}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/32/f*(-1+cos(f*x+e))^2*(12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))-1
2*cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-5*cos(f*x+e)^3-12*cos(f*x+e)
*ln(-(-1+cos(f*x+e))/sin(f*x+e))-15*cos(f*x+e)^2+12*ln(-(-1+cos(f*x+e))/sin
(f*x+e))+9*cos(f*x+e)+3)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/cos(f*x+e)^2/(
c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)^3/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 4.56, size = 588, normalized size = 4.03

$$\left[-\frac{3\left(\cos(fx+e)^3-\cos(fx+e)^2-\cos(fx+e)+1\right)\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)\sin(fx+e)-2\left(5\cos(fx+e)^3-\cos(fx+e)^2-2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{16\left(a^2c^3f\cos(fx+e)^3-a^2c^3f\cos(fx+e)^2-a^2c^3f\cos(fx+e)+a^2c^3f\right)\sin(fx+e)},\ \frac{3\left(\cos(fx+e)^3-\cos(fx+e)^2-\cos(fx+e)+1\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{ac\sin(fx+e)}\right)\sin(fx+e)+\left(5\cos(fx+e)^3-\cos(fx+e)^2-2\cos(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{8\left(a^2c^3f\cos(fx+e)^3-a^2c^3f\cos(fx+e)^2-a^2c^3f\cos(fx+e)+a^2c^3f\right)\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a*c)*l
og(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*
x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x +
```

```
e)^3 - cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 -
a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)
), 1/8*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a*c)*ar
ctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*x + e)^3 -
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3
*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 1.93, size = 153, normalized size = 1.05

$$\frac{\frac{2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)}{c}+\frac{9\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}+12\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c+4\,c^{2}}{c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}-6\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)+6\,\log\left(|c|\right)-4}{32\,\sqrt{-ac}\,acf|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] 1/32*(2*(c*tan(1/2*f*x + 1/2*e)^2 - c)/c + (9*(c*tan(1/2*f*x + 1/2*e)^2 - c
)^2 + 12*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 4*c^2)/(c^2*tan(1/2*f*x + 1/2*e
)^4) - 6*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) + 6*log(abs(c)) - 4)/(sqrt(-a*c
)*a*c*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

3.145 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{c^3\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}}+\frac{c(c-c\sec(e+fx))^{3/2}}{2f(a+a\sec(e+$$

```
[Out] 1/2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-c^3*ln(1+s
ec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-c
^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4039, 4037}

$$-\frac{c^3\tan(e+fx)\log(\sec(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af(a\sec(e+fx)+a)^{3/2}}+\frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c - c*Sec[e + f*x]])) - (c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a
*f*(a + a*Sec[e + f*x])^(3/2)) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(2*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}}{a}$$

$$= -\frac{c^2\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}} + \frac{c(c-c\sec(e+fx))^{3/2}\,\mathrm{ta}}{2f(a+a\sec(e+f}$$

$$= -\frac{c^3\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}} - \frac{c^2\sqrt{c-c\sec(e}}{af(a+a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.39, size = 178, normalized size = 1.23

$$\frac{c^2\cot\left(\frac{1}{2}(e+fx)\right)\left(-4+6\log\left(1+e^{i(e+fx)}\right)+\cos(e+fx)\left(8\log\left(1+e^{i(e+fx)}\right)-4\log\left(1+e^{2i(e+fx)}\right)\right)+\cos(2(e+fx))\left(2\log\left(1+e^{i(e+fx)}\right)-\log\left(1+e^{2i(e+fx)}\right)\right)-3\log\left(1+e^{2i(e+fx)}\right)\right)\sqrt{c-c\sec(e+fx)}}{2a^2f(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (c^2*Cot[(e + f*x)/2]*(-4 + 6*Log[1 + E^(I*(e + f*x))] + Cos[e + f*x]*(8*Lo
g[1 + E^(I*(e + f*x))] - 4*Log[1 + E^((2*I)*(e + f*x))]) + Cos[2*(e + f*x)]
*(2*Log[1 + E^(I*(e + f*x))] - Log[1 + E^((2*I)*(e + f*x))]) - 3*Log[1 + E^
((2*I)*(e + f*x))])*Sqrt[c - c*Sec[e + f*x]])/(2*a^2*f*(1 + Cos[e + f*x])^2
*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(131) = 262$.
time = 3.25, size = 281, normalized size = 1.94

method	result
default	$-\frac{\left(2\left(\cos^2(fx+e)\right)\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+2\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)-\left(\cos^2(fx+e)\right)+4\cos(fx+e)\ln\left(-\right.\right.}{}$
risch	$-\frac{8ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,e^{2i(fx+e)}}{a^2\left(e^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}+\frac{2ic^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\ln\left(e^{i(fx+e)}+1\right)}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}-\frac{ic^2\left(e^{i(fx+e}\right.}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

[Out] -1/2/f*(2*cos(f*x+e)^2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-cos(f*x+e)^2+4*cos(f*x+e)*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+4*cos(f*x+e)*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-2*cos(f*x+e)+2*ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(f*x+e))+2*ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))+3)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)*cos(f*x+e)^3*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3

Maxima [A]
time = 0.52, size = 141, normalized size = 0.97

$$-\frac{\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}\,a^2}+\frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}\,a^2}-\frac{\frac{2\sqrt{-a}\,c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{-a}\,c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{a^3}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] -1/2*(2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a^2) + 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a^2) - (2*sqrt(-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(-a)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/a^3)/f

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep

Giac [A]
time = 1.52, size = 114, normalized size = 0.79

$$-\frac{c^4\left(\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^2+4\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^3}{c^4}+2\,\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{2\,\sqrt{-ac}\,a^2f|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/2*c^4*(((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 + 4*(c*tan(1/2*f*x + 1/2*e)
^2 - c)*c^3)/c^4 + 2*log(c*tan(1/2*f*x + 1/2*e)^2 - c))*sgn(tan(1/2*f*x + 1
/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)), x
)
```

3.146 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$\frac{(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 1/4*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4035\}$

$$\frac{\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{4f(a\sec(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

Mathematica [A]
time = 0.29, size = 68, normalized size = 1.62

$$\frac{c\cos(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{4a^2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5
/2),x]
```

```
[Out] (c*Cos[e + f*x]*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e + f*x]
])/(4*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.
time = 3.07, size = 75, normalized size = 1.79

method	result	size
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\cos^2(fx+e))(-1+\cos(fx+e))^3\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{3}{2}}}{4f\sin(fx+e)^5a^3}$	75
risch	$\frac{2ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{a^2\left(e^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*cos(f*x+e)^2*(-1+cos(f*x+e))^3*(
c*(-1+cos(f*x+e))/cos(f*x+e))^(3/2)/sin(f*x+e)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(39) = 78$.
time = 0.51, size = 106, normalized size = 2.52

$$-\frac{\sqrt{-a}\,c^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\sin\left(fx+e\right)^4}{4\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)f(\cos\left(fx+e\right)+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*sqrt(-a)*c^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)*sin(f*x + e)^4/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2)*f*(cos(f*x + e) + 1)^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(39) = 78$.
time = 2.06, size = 103, normalized size = 2.45

$$\frac{c\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)^2}{\left(a^3f\cos\left(fx+e\right)^2+2\,a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f
)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/(a*(sec(e + f*x) + 1))
**(5/2), x)
```

Giac [A]
time = 1.61, size = 59, normalized size = 1.40

$$-\frac{\left(\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2+2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c\right)c}{4\sqrt{-ac}\,a^2f|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c
)*c/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [B]
time = 3.51, size = 119, normalized size = 2.83

$$-\frac{2\,c\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(\sin\left(e+f\,x\right)+2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{a^2\,f\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\left(4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(e+f\,x\right)+4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*c*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(sin(e + f*x) + 2*sin(2*e
 + 2*f*x) + sin(3*e + 3*f*x)))/(a^2*f*((a*(cos(e + f*x) + 1))/cos(e + f*x))
^(1/2)*(4*cos(2*e + 2*f*x) - 4*cos(e + f*x) + 4*cos(3*e + 3*f*x) + cos(4*e
+ 4*f*x) - 5))
```

3.147 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = $\{4038\}$

$$\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] (c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A]
time = 0.25, size = 71, normalized size = 1.65

$$\frac{(1+2\cos(e+fx))\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{8a^2f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2
),x]
```

```
[Out] ((1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e
+ f*x]])/(8*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 3.05, size = 75, normalized size = 1.74

method	result	size
default	$-\frac{\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,(-1+\cos(fx+e))^{2}\left(\cos^{3}(fx+e)\right)\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}}{2f\sin(fx+e)^{5}a^{3}}$	75
risch	$\frac{2i\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{3i(fx+e)}+e^{2i(fx+e)}+e^{i(fx+e)}\right)}{a^{2}\left(e^{i(fx+e)}+1\right)^{3}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{e^{2i(fx+e)}+1}}\,\left(e^{i(fx+e)}-1\right)f}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*cos(f*x+e)^3*
(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3
```

Maxima [A]
time = 0.50, size = 62, normalized size = 1.44

$$-\frac{\sqrt{c}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{2}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{2}}{8\sqrt{-a}\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/8*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)^2/(sqrt(-a)*a^2*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(40) = 80.
time = 1.71, size = 113, normalized size = 2.63

$$\frac{\left(2\cos\left(fx+e\right)^{2}+\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{2\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f
*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**(
5/2), x)
```

Giac [A]
time = 1.54, size = 37, normalized size = 0.86

$$-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}}{8\,\sqrt{-ac}\,a^{2}f|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/8*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [B]
time = 3.25, size = 120, normalized size = 2.79

$$-\frac{2\left(3\,\sin\left(e+f\,x\right)+3\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}}{a^{2}\,f\,\sqrt{\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}}\,\left(4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(e+f\,x\right)+4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*(3*sin(e + f*x) + 3*sin(2*e + 2*f*x) + sin(3*e + 3*f*x))*((c*(cos(e + f
*x) - 1))/cos(e + f*x))^(1/2))/(a^2*f*((a*(cos(e + f*x) + 1))/cos(e + f*x))
^(1/2)*(4*cos(2*e + 2*f*x) - 4*cos(e + f*x) + 4*cos(3*e + 3*f*x) + cos(4*e
+ 4*f*x) - 5))
```

3.148 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\tanh^{-}}{4a^2f\sqrt{a+a}}$$

```
[Out] 1/4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)+1/4*tan(f*x+
e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e)
)*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4045, 4044, 3855}

$$-\frac{\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{4a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + Ta
n[e + f*x]/(4*a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
```

```
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\int}{(a+a\,\mathrm{s}}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{}{4af(a}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{}{4af(a}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{}{4af(a}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.04, size = 91, normalized size = 0.65

$$-\frac{\left(2+8\tanh^{-1}\left(e^{i(e+fx)}\right)\cos^4\left(\frac{1}{2}(e+fx)\right)+3\cos(e+fx)\right)\tan(e+fx)}{4a^2f(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] -1/4*((2 + 8*ArcTanh[E^(I*(e + f*x))]*Cos[(e + f*x)/2]^4 + 3*Cos[e + f*x])*
Tan[e + f*x])/(a^2*f*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [A]
time = 3.19, size = 164, normalized size = 1.17

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(-1+\cos(fx+e))^3\left(4\left(\cos^2(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+5\left(\cos^2(fx+e)\right)+8\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right.\right.}{16f\sqrt{\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}}\,\sin(fx+e)^5a^3}$

risch	$\frac{i\left(3\,\mathrm{e}^{2i(fx+e)}+4\,\mathrm{e}^{i(fx+e)}+3\right)\left(\mathrm{e}^{2i(fx+e)}-\mathrm{e}^{i(fx+e)}\right)}{2a^2\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\,\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\,f}-\frac{i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{i(fx+e)}-1\right.}{4a^2\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+\right.}}{\mathrm{e}^{2i(fx-}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/16/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^3*(4*cos(f*x+e)
^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))+5*cos(f*x+e)^2+8*cos(f*x+e)*ln(-(-1+cos(
f*x+e))/sin(f*x+e))-2*cos(f*x+e)+4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-3)/(c*(-
1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. $2(132) = 264$.
time = 0.62, size = 1293, normalized size = 9.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos
(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e)
 + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) + 1)*co
s(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x
 + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin(4*f*x
 + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e) + 16*
sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x +
e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x
+ e) + 1) - (2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) +
1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*
x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) +
 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin
(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin
(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e)
 + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(
f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), co
s(f*x + e) - 1) + 2*(3*sin(3*f*x + 3*e) + 4*sin(2*f*x + 2*e) + 3*sin(f*x +
e))*cos(4*f*x + 4*e) - 2*(3*cos(3*f*x + 3*e) + 4*cos(2*f*x + 2*e) + 3*cos(f
*x + e))*sin(4*f*x + 4*e) + 2*(2*cos(2*f*x + 2*e) + 3)*sin(3*f*x + 3*e) - 4
*(cos(f*x + e) - 2)*sin(2*f*x + 2*e) - 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e)
+ 4*cos(2*f*x + 2*e)*sin(f*x + e) + 6*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^3*c
*cos(4*f*x + 4*e)^2 + 16*a^3*c*cos(3*f*x + 3*e)^2 + 36*a^3*c*cos(2*f*x + 2*
```

```
e)^2 + 16*a^3*c*cos(f*x + e)^2 + a^3*c*sin(4*f*x + 4*e)^2 + 16*a^3*c*sin(3*
f*x + 3*e)^2 + 36*a^3*c*sin(2*f*x + 2*e)^2 + 48*a^3*c*sin(2*f*x + 2*e)*sin(
f*x + e) + 16*a^3*c*sin(f*x + e)^2 + 8*a^3*c*cos(f*x + e) + a^3*c + 2*(4*a^
3*c*cos(3*f*x + 3*e) + 6*a^3*c*cos(2*f*x + 2*e) + 4*a^3*c*cos(f*x + e) + a^
3*c)*cos(4*f*x + 4*e) + 8*(6*a^3*c*cos(2*f*x + 2*e) + 4*a^3*c*cos(f*x + e)
+ a^3*c)*cos(3*f*x + 3*e) + 12*(4*a^3*c*cos(f*x + e) + a^3*c)*cos(2*f*x + 2
*e) + 4*(2*a^3*c*sin(3*f*x + 3*e) + 3*a^3*c*sin(2*f*x + 2*e) + 2*a^3*c*sin(
f*x + e))*sin(4*f*x + 4*e) + 16*(3*a^3*c*sin(2*f*x + 2*e) + 2*a^3*c*sin(f*x
 + e))*sin(3*f*x + 3*e))*f)
```

Fricas [A]
time = 1.95, size = 494, normalized size = 3.53

$$\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}+\left(ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\left(3\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{8\left(a^{3}cf\cos\left(fx+e\right)^{2}+2a^{3}cf\cos\left(fx+e\right)+a^{3}cf\right)\sin\left(fx+e\right)},\ \frac{\sqrt{ac}\left(\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)+1\right)\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{ac\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+\left(3\cos\left(fx+e\right)^{2}+2\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{4\left(a^{3}cf\cos\left(fx+e\right)^{2}+2a^{3}cf\cos\left(fx+e\right)+a^{3}cf\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x
+ e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 + 2*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin
(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*arctan(sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*cos(f*x + e)^2 + 2*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*
sin(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

Giac [A]
time = 1.71, size = 124, normalized size = 0.89

$$-\frac{c^2\left(\frac{2\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c}-\frac{2\log(|c|)}{c}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^3-2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^4}{c^6}\right)}{16\sqrt{-ac}\,a^2f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/16*c^2*(2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - 2*log(abs(c))/c + ((c*t
an(1/2*f*x + 1/2*e)^2 - c)^2*c^3 - 2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4)/c^
6)/(sqrt(-a*c)*a^2*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*
e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

3.149 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 3/8*csc(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+1/4*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2)-3/8*arctanh(cos(f*
x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4045, 4044, 2691, 3855}

$$\frac{3\csc(e+fx)}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) + Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx &= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} + \frac{3\int \frac{}{(a+}}{} \\
&= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} - \frac{(3\tan(}{4a^2c\sqrt{}} \\
&= \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{4f(a} \\
&= \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{}{4f(a}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.42, size = 130, normalized size = 0.89

$$-\frac{\left(1+2\cos(e+fx)+5\cos(2(e+fx))-3\tanh^{-1}\left(e^{i(e+fx)}\right)(2+\cos(e+fx)-2\cos(2(e+fx))-\cos(3(e+fx)))\right)\tan(e+fx)}{16a^2cf(-1+\cos(e+fx))(1+\cos(e+fx))^2\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)),x]
```

```
[Out] -1/16*((1 + 2*Cos[e + f*x] + 5*Cos[2*(e + f*x)] - 3*ArcTanh[E^(I*(e + f*x))
]*(2 + Cos[e + f*x] - 2*Cos[2*(e + f*x)] - Cos[3*(e + f*x)]))*Tan[e + f*x])
/(a^2*c*f*(-1 + Cos[e + f*x])*(1 + Cos[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x]
)]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A]
time = 3.55, size = 204, normalized size = 1.40

method	result
default	$-\frac{\left(12\left(\cos^3(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+12\left(\cos^2(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)+5\left(\cos^3(fx+e)\right)-12\cos(fx+e)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right.}{32f\,c^3\sin(fx+}$
risch	$\frac{i\left(5\,\mathrm{e}^{5i(fx+e)}+2\,\mathrm{e}^{4i(fx+e)}+2\,\mathrm{e}^{3i(fx+e)}+2\,\mathrm{e}^{2i(fx+e)}+5\,\mathrm{e}^{i(fx+e)}\right)}{4a^2c\left(\mathrm{e}^{2i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\left(\mathrm{e}^{i(fx+e)}-1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{\mathrm{e}^{2i(fx+e)}+1}}\,f}-\frac{3i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-\right.}{8a^2c\left(\mathrm{e}^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+\right.}{\mathrm{e}^{2i(fx+e)}+}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/32/f*(12*cos(f*x+e)^3*ln(-(-1+cos(f*x+e))/sin(f*x+e))+12*cos(f*x+e)^2*ln
(-(-1+cos(f*x+e))/sin(f*x+e))+5*cos(f*x+e)^3-12*cos(f*x+e)*ln(-(-1+cos(f*x+
e))/sin(f*x+e))-15*cos(f*x+e)^2-12*ln(-(-1+cos(f*x+e))/sin(f*x+e))-9*cos(f*
x+e)+3)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*cos(f*x+e)^2*(c*(-1+cos(f*x+e))
/cos(f*x+e))^(3/2)/c^3/sin(f*x+e)^5/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.50, size = 580, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a*c)*l
og(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*
x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x +
e)^3 + cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
```

```
e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 +
a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)
), 1/8*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(a*c)*ar
ctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*x + e)^3 +
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2
*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 1.96, size = 151, normalized size = 1.03

$$\frac{\frac{2\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^2-4\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^3}{c^4}-6\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)+6\,\log\left(|c|\right)-4}{32\,\sqrt{-ac}\,a^2f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/32*(2*(3*c*tan(1/2*f*x + 1/2*e)^2 - c)/(c*tan(1/2*f*x + 1/2*e)^2) - ((c*t
an(1/2*f*x + 1/2*e)^2 - c)^2*c^2 - 4*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3)/c^
4 - 6*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) + 6*log(abs(c)) - 4)/(sqrt(-a*c)*a
^2*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

3.150 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}} dx$

Optimal. Leaf size=160

$$\frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{}{8a^2c^2f}$$

```
[Out] 3/8*csc(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*
cot(f*x+e)^2*csc(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-3/8*arctanh(cos(f*x+e))*tan(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c
-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4044, 2691, 3855}

$$\frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{3\tan(e+fx)\tanh^{-1}(\cos(e+fx))}{8a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f
*x]]) - (Cot[e + f*x]^2*Csc[e + f*x])/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]]) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*
c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
```

```
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)\int \cot^4(e+fx)\csc(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{(3}{4a}$$
$$= \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{}{4a^2c}$$
$$= \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{}{4a^2c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.38, size = 84, normalized size = 0.52

$$\frac{(1-5\cos(2(e+fx)))\csc^3(e+fx)-12\tanh^{-1}\left(e^{i(e+fx)}\right)\tan(e+fx)}{16a^2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/
2)),x]
```

```
[Out] ((1 - 5*Cos[2*(e + f*x)])*Csc[e + f*x]^3 - 12*ArcTanh[E^(I*(e + f*x))]*Tan[
e + f*x])/(16*a^2*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]
)
```

Maple [A]
time = 3.10, size = 173, normalized size = 1.08

method	result
default	$-\frac{(-1+\cos(fx+e))^3\left(3\left(\cos^4(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-6\left(\cos^2(fx+e)\right)\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)-5\left(\cos^3(fx+e)\right)+3\ln\left(-\frac{-1+\cos(fx+}{\sin(fx+e)}\right.\right.}{8f\cos(fx+e)^2\left(\frac{c(-1+\cos(fx+e))}{\cos(fx+e)}\right)^{\frac{5}{2}}\sin(fx+e)^5a^3}$
risch	$\frac{i\left(5\,e^{7i(fx+e)}+3\,e^{5i(fx+e)}+3\,e^{3i(fx+e)}+5\,e^{i(fx+e)}\right)}{4a^2c^2\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}+\frac{3i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}\right.}{8a^2c^2\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}\right.}{e^{2i(fx+e)}}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))^3*(3*cos(f*x+e)^4*ln(-(-1+cos(f*x+e))/sin(f*x+e))-6*
cos(f*x+e)^2*ln(-(-1+cos(f*x+e))/sin(f*x+e))-5*cos(f*x+e)^3+3*ln(-(-1+cos(f
*x+e))/sin(f*x+e))+3*cos(f*x+e))*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/cos(f*
x+e)^2/(c*(-1+cos(f*x+e))/cos(f*x+e))^(5/2)/sin(f*x+e)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1791 vs. $2(153) = 306$.
time = 0.88, size = 1791, normalized size = 11.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/8*(3*(2*(4*cos(6*f*x + 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1
)*cos(8*f*x + 8*e) - cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f
*x + 2*e) + 1)*cos(6*f*x + 6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x +
 2*e) - 1)*cos(4*f*x + 4*e) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2
 + 4*(2*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f
*x + 8*e) - sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e
))*sin(6*f*x + 6*e) - 16*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*si
n(4*f*x + 4*e)*sin(2*f*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 3*(2*(4*cos(6*f*x + 6*e) -
 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f*x
+ 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)
 - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - 3
6*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*si
n(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x + 8*e)^2
+ 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*
f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e), co
s(f*x + e) - 1) - 2*(5*sin(7*f*x + 7*e) + 3*sin(5*f*x + 5*e) + 3*sin(3*f*x
+ 3*e) + 5*sin(f*x + e))*cos(8*f*x + 8*e) - 20*(2*sin(6*f*x + 6*e) - 3*sin(
4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 8*(3*sin(5*f*x + 5*e)
 + 3*sin(3*f*x + 3*e) + 5*sin(f*x + e))*cos(6*f*x + 6*e) + 12*(3*sin(4*f*x
+ 4*e) - 2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 12*(3*sin(3*f*x + 3*e) + 5*
sin(f*x + e))*cos(4*f*x + 4*e) + 2*(5*cos(7*f*x + 7*e) + 3*cos(5*f*x + 5*e)
 + 3*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(8*f*x + 8*e) + 10*(4*cos(6*f*x
+ 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*sin(7*f*x + 7*e) - 8*
(3*cos(5*f*x + 5*e) + 3*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(6*f*x + 6*e)
 - 6*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e) + 12*(3
*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(4*f*x + 4*e) + 6*(4*cos(2*f*x + 2*e
```

```
) - 1)*sin(3*f*x + 3*e) - 24*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 40*cos(f*x
 + e)*sin(2*f*x + 2*e) + 40*cos(2*f*x + 2*e)*sin(f*x + e) - 10*sin(f*x + e)
)*sqrt(a)*sqrt(c)/((a^3*c^3*cos(8*f*x + 8*e)^2 + 16*a^3*c^3*cos(6*f*x + 6*e
)^2 + 36*a^3*c^3*cos(4*f*x + 4*e)^2 + 16*a^3*c^3*cos(2*f*x + 2*e)^2 + a^3*c
^3*sin(8*f*x + 8*e)^2 + 16*a^3*c^3*sin(6*f*x + 6*e)^2 + 36*a^3*c^3*sin(4*f*
x + 4*e)^2 - 48*a^3*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*a^3*c^3*sin(
2*f*x + 2*e)^2 - 8*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3 - 2*(4*a^3*c^3*cos(6*
f*x + 6*e) - 6*a^3*c^3*cos(4*f*x + 4*e) + 4*a^3*c^3*cos(2*f*x + 2*e) - a^3*
c^3)*cos(8*f*x + 8*e) - 8*(6*a^3*c^3*cos(4*f*x + 4*e) - 4*a^3*c^3*cos(2*f*x
 + 2*e) + a^3*c^3)*cos(6*f*x + 6*e) - 12*(4*a^3*c^3*cos(2*f*x + 2*e) - a^3*
c^3)*cos(4*f*x + 4*e) - 4*(2*a^3*c^3*sin(6*f*x + 6*e) - 3*a^3*c^3*sin(4*f*x
 + 4*e) + 2*a^3*c^3*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 16*(3*a^3*c^3*sin(
4*f*x + 4*e) - 2*a^3*c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Fricas [A]
time = 1.71, size = 520, normalized size = 3.25

$$\left[-\frac{3\,\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{-ac}\,\log\left(-\frac{4\,\left(2\,\sqrt{-ac}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)^{2}+\left(ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(fx+e\right)\right)}{\left(\cos\left(fx+e\right)^{2}-1\right)\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)-2\,\left(5\,\cos\left(fx+e\right)^{4}-3\,\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{16\,\left(a^{3}c^{3}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{3}f\cos\left(fx+e\right)^{2}+a^{3}c^{3}f\right)\sin\left(fx+e\right)},\ \frac{3\,\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{ac\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+\left(5\,\cos\left(fx+e\right)^{4}-3\,\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{8\,\left(a^{3}c^{3}f\cos\left(fx+e\right)^{4}-2\,a^{3}c^{3}f\cos\left(fx+e\right)^{2}+a^{3}c^{3}f\right)\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-4*(2*sqrt
(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos
(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e)^4 - 3*cos(
f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 +
 a^3*c^3*f)*sin(f*x + e)), 1/8*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*s
qrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*
x + e)^4 - 3*cos(f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f
*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 1.92, size = 182, normalized size = 1.14

$$-\frac{\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}c^{2}-6\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c^{3}}{c^{4}}-\frac{18\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}+28\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)c+11\,c^{2}}{c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}+12\,\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)-12\,\log\left(|c|\right)+11}{64\,\sqrt{-ac}\,a^{2}cf|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/64*(((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 - 6*(c*tan(1/2*f*x + 1/2*e)^2
- c)*c^3)/c^4 - (18*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 28*(c*tan(1/2*f*x +
1/2*e)^2 - c)*c + 11*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4) + 12*log(abs(c)*tan(
1/2*f*x + 1/2*e)^2) - 12*log(abs(c)) + 11)/(sqrt(-a*c)*a^2*c*f*abs(c)*sgn(t
an(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

3.151 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal. Leaf size=101

$$-\frac{2^{\frac{1}{2}+n}c\,{}_2F_1\left(\frac{1}{2}+m,\frac{1}{2}-n;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+fx))^m(c-c\sec(e$$
$$f(1+2m)$$

```
[Out] -2^(1/2+n)*c*hypergeom([1/2+m, 1/2-n],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f*
x+e))^(1/2-n)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(1+2*
m)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4046, 72, 71}

$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\,{}_2F_1\left(m+\frac{1}{2},\frac{1}{2}-n;m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*c*Hypergeometric2F1[1/2 + m, 1/2 - n, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
```

```
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-\right.}{f\sqrt{a+a\sec(e+fx)}\ \sqrt{c-c}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+}{c}\right.\right.}{}$$

$$= -\frac{2^{\frac{1}{2}+n}c\,{}_2F_1\left(\frac{1}{2}+m,\frac{1}{2}-n;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e\right.}{}$$

Mathematica [F]
time = 1.30, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n, x]
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^m (-c(\sec(e+fx)-1))^n \sec(e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n*sec(e + f*x),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m \left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\cos(e+f\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n)/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n)/cos(e + f*x), x)
```

3.152 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$

Optimal. Leaf size=92

$$\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx))^{-1+m}(c-c\sec(e+fx$$

$$5f$$

```
[Out] 1/5*2^(1/2+m)*a*hypergeom([5/2, 1/2-m],[7/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x
+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*(c-c*sec(f*x+e))^2*tan(f*x+e)/f
```

Rubi [A]

time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4046, 72, 71}

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^2(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
```

```
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}(c-c\ldots\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec\ldots}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+\ldots}{a}\right.\right.}{\ldots}$$

$$= \frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1-\ldots}{\ldots}$$

Mathematica [A]
time = 0.25, size = 89, normalized size = 0.97

$$\frac{2^{\frac{1}{2}+m}c^2\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sec(e+fx))\right)(-1+\sec(e+fx))^2(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\tan(e+fx)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*c^2*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]
*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))
^m*Tan[e + f*x])/(5*f)
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*(a*
sec(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int (a\sec(e+fx)+a)^m\sec(e+fx)\,dx+\int \left(-2(a\sec(e+fx)+a)^m\sec^2(e+fx)\right)\,dx+\int (a\sec(e+fx)+a)^m\sec^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral((a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral(-2*(a*se
c(e + f*x) + a)**m*sec(e + f*x)**2, x) + Integral((a*sec(e + f*x) + a)**m*s
ec(e + f*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}{\cos(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^2)/cos(e + f*x), x)
```

3.153 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=90

$$\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx))^{-1+m}(c-c\sec(e+fx))}{3f}$$

```
[Out] 1/3*2^(1/2+m)*a*hypergeom([3/2, 1/2-m],[5/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x
+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*(c-c*sec(f*x+e))*tan(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4046, 72, 71}

$$\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])*Tan[e + f*x])/(3*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
```

```
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}\sqrt{c-}\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+}{a}\right.\right.}{}$$

$$= \frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1}{}$$

Mathematica [A]
time = 0.16, size = 85, normalized size = 0.94

$$-\frac{2^{\frac{1}{2}+m}c\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sec(e+fx))\right)(-1+\sec(e+fx))(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m\tan(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(2^(1/2 + m)*c*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])
/2]*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x])
)^m*Tan[e + f*x])/f
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sec(fx+e)(a+a\sec(fx+e))^m(c-c\sec(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -integrate((c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \left(-(a\sec\left(e+fx\right)+a)^{m}\sec\left(e+fx\right)\right)\,dx+\int (a\sec\left(e+fx\right)+a)^{m}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-(a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral((a*sec(e
+ f*x) + a)**m*sec(e + f*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate(-(c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{m}\left(c-\frac{c}{\cos(e+f\,x)}\right)}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x)))/cos(e + f*x), x)
```

3.154 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=90

$$-\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx))^{-1+m}\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -2^(1/2+m)*a*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e
))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A]
time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4046, 72, 71}

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{f(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((2^(1/2 + m)*a*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x])/
(f*(c - c*Sec[e + f*x])))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
```

```
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx\right.}{f\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}}}{f(c-c\sec(e+fx))}$$

Mathematica [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]), x]
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)(a+a\sec(fx+e))^m}{c-c\sec(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(a\sec(e+fx)+a)^m \sec(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x) - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{c-c\,\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] -int((a + a/cos(e + f*x))^m/(c - c*cos(e + f*x)), x)
```

3.155 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx))^{-1+m}\tan(e+fx)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] -1/3*2^(1/2+m)*a*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sec(f*x+e))*(1+sec(
f*x+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*tan(f*x+e)/f/(c-c*sec(f*x+e))^2
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4046, 72, 71}

$$-\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)}{3f(c-c\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(2^(1/2 + m)*a*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sec[e + f*x
])/2]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*
x])/(f*(c - c*Sec[e + f*x])^2)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
```

```
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+}{f\sqrt{c-c\sec(e+fx}}$$
$$= -\frac{2^{\frac{1}{2}+m}a\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx}{3f(c-c\sec(e+fx))}$$

Mathematica [F]
time = 2.47, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2, x]
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)\,(a+a\sec(fx+e))^m}{(c-c\sec(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2*sec(f*x + e)^2 - 2*c^2*se
c(f*x + e) + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(a\sec(e+fx)+a)^m\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e +
f*x) + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^2), x)
```

3.156 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=160

$$-\frac{64c^3(a+a\sec(e+fx))^m\tan(e+fx)}{f(5+2m)\,(3+8m+4m^2)\,\sqrt{c-c\sec(e+fx)}}-\frac{16c^2(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,\tan(e+}{f\,(15+16m+4m^2)}$$

```
[Out] -2*c*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(5+2*m)-64*c^3*
(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(5+2*m)/(4*m^2+8*m+3)/(c-c*sec(f*x+e))^(1/2
)-16*c^2*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(4*m^2+16*m
+15)
```

Rubi [A]
time = 0.25, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4040, 4038}

$$-\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+5)\,(4m^2+8m+3)\,\sqrt{c-c\sec(e+fx)}}-\frac{16c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}\,(a\sec(e+fx)+a)^m}{f\,(4m^2+16m+15)}-\frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m}{f(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(5 + 2*m)*(3 + 8*m + 4*m^2
)*Sqrt[c - c*Sec[e + f*x]]) - (16*c^2*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec
[e + f*x]]*Tan[e + f*x])/(f*(15 + 16*m + 4*m^2)) - (2*c*(a + a*Sec[e + f*x]
)^m*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(5 + 2*m))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/}}{f(5+2m)}$$
$$= -\frac{16c^2(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx}}{f\left(15+16m+4m^2\right)}$$
$$= -\frac{64c^3(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(15+46m+36m^2+8m^3\right)\sqrt{c-c\sec(e+}}$$

Mathematica [**F**]
time = 50.90, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2), x
]
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

Maxima [**A**]
time = 0.51, size = 240, normalized size = 1.50

$$-\frac{2\left(\frac{\sqrt{2}\,(2^{m+5}m+5\cdot 2^{m+4})(-a)^m c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{\sqrt{2}\,(2^{m+4}m^2+2^{m+6}m+15\cdot 2^{m+2})(-a)^m c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-2^{m+\frac{11}{2}}(-a)^m c^{\frac{5}{2}}\right)e^{\left(-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\right)}}{(8\,m^3+36\,m^2+46\,m+15)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] -2*(sqrt(2)*(2^(m + 5)*m + 5*2^(m + 4))*(-a)^m*c^(5/2)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - sqrt(2)*(2^(m + 4)*m^2 + 2^(m + 6)*m + 15*2^(m + 2))*(-a)
^m*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2^(m + 11/2)*(-a)^m*c^(5/2
))*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1))/((8*m^3 + 36*m^2 + 46*m + 15)*f*(sin(f*x + e)/(cos(f*x
 + e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Fricas [**A**]
time = 1.98, size = 200, normalized size = 1.25

$$\frac{2\left(4\,c^{2}m^{2}+\left(4\,c^{2}m^{2}+24\,c^{2}m+43\,c^{2}\right)\cos\left(fx+e\right)^{3}+8\,c^{2}m-\left(4\,c^{2}m^{2}+8\,c^{2}m-29\,c^{2}\right)\cos\left(fx+e\right)^{2}+3\,c^{2}-\left(4\,c^{2}m^{2}+24\,c^{2}m+11\,c^{2}\right)\cos\left(fx+e\right)\right)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^{m}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{\left(8\,fm^{3}+36\,fm^{2}+46\,fm+15\,f\right)\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2*(4*c^2*m^2 + (4*c^2*m^2 + 24*c^2*m + 43*c^2)*cos(f*x + e)^3 + 8*c^2*m - (
4*c^2*m^2 + 8*c^2*m - 29*c^2)*cos(f*x + e)^2 + 3*c^2 - (4*c^2*m^2 + 24*c^2*
m + 11*c^2)*cos(f*x + e))*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x
+ e)^2*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^(5/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x), x)
```

3.157 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=100

$$-\frac{8c^2(a+a\sec(e+fx))^m\tan(e+fx)}{f\,(3+8m+4m^2)\,\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,\tan(e+fx)}{f(3+2m)}$$

```
[Out] -8*c^2*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(4*m^2+8*m+3)/(c-c*sec(f*x+e))^(1/2)
-2*c*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(3+2*m)
```

Rubi [A]
time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4040, 4038}

$$-\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^m}{f\,(4m^2+8m+3)\,\sqrt{c-c\sec(e+fx)}}-\frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}\,(a\sec(e+fx)+a)^m}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + 8*m + 4*m^2)*Sqrt[c -
c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]]*Tan
[e + f*x])/(f*(3 + 2*m))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2c(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}}{f(3+2m)}$$
$$= -\frac{8c^2(a+a\sec(e+fx))^m\tan(e+fx)}{f\,(3+8m+4m^2)\,\sqrt{c-c\sec(e+fx)}} - \frac{2c}{}$$

Mathematica [F]
time = 68.68, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2), x
]
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

Maxima [A]
time = 0.50, size = 181, normalized size = 1.81

$$-\frac{2\left(\sqrt{2}\,2^{m+2}(-a)^m\,c^{\frac{3}{2}}-\frac{\sqrt{2}\,(2^{m+2}m+3\cdot 2^{m+1})(-a)^m c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)e^{\left(-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\right)}}{(4\,m^2+8\,m+3)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] -2*(sqrt(2)*2^(m + 2)*(-a)^m*c^(3/2) - sqrt(2)*(2^(m + 2)*m + 3*2^(m + 1))*
(-a)^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*e^(-m*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/((4*m
```

```
^2 + 8*m + 3)*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)^(3/2))
```

Fricas [A]
time = 1.31, size = 120, normalized size = 1.20

$$\frac{2\left((2\,cm+5\,c)\cos\left(fx+e\right)^{2}-2\,cm+4\,c\cos\left(fx+e\right)-c\right)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^{m}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}}{\left(4\,fm^{2}+8\,fm+3\,f\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2*((2*c*m + 5*c)*cos(f*x + e)^2 - 2*c*m + 4*c*cos(f*x + e) - c)*((a*cos(f*x
 + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((4*f*m^
2 + 8*f*m + 3*f)*cos(f*x + e)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^(3/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
x)
```

Mupad [B]
time = 3.59, size = 154, normalized size = 1.54

$$-\frac{2\,c\,\left(\frac{a\,\left(\cos(e+f\,x)+1\right)}{\cos(e+f\,x)}\right)^{m}\,\sqrt{\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}}\,\left(5\,\sin\left(e+f\,x\right)-2\,\sin\left(2\,e+2\,f\,x\right)+5\,\sin\left(3\,e+3\,f\,x\right)+2\,m\,\sin\left(e+f\,x\right)-4\,m\,\sin\left(2\,e+2\,f\,x\right)+2\,m\,\sin\left(3\,e+3\,f\,x\right)\right)}{f\,\left(4\,m^{2}+8\,m+3\right)\,\left(3\,\cos\left(e+f\,x\right)-2\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] -(2*c*((a*(cos(e + f*x) + 1))/cos(e + f*x))^m*((c*(cos(e + f*x) - 1))/cos(e
 + f*x))^(1/2)*(5*sin(e + f*x) - 2*sin(2*e + 2*f*x) + 5*sin(3*e + 3*f*x) +
2*m*sin(e + f*x) - 4*m*sin(2*e + 2*f*x) + 2*m*sin(3*e + 3*f*x)))/(f*(8*m +
4*m^2 + 3)*(3*cos(e + f*x) - 2*cos(2*e + 2*f*x) + cos(3*e + 3*f*x) - 2))
```

3.158 $\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}$

Optimal. Leaf size=46

$$-\frac{2c(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*c*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = $\{4038\}$

$$-\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.80, size = 163, normalized size = 3.54

$$\frac{\sqrt{2}\,e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\csc\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^{-m}(a(1+\sec(e+fx)))^m\sqrt{c-c\sec(e+fx)}}{(f+2fm)\sqrt{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
))]*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^m*Csc[(e + f*x)/2]*
(a*(1 + Sec[e + f*x]))^m*Sqrt[c - c*Sec[e + f*x]])/(E^((I/2)*(e + f*x))*(f
+ 2*f*m)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m)
```

Maple [**F**]
time = 0.23, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\sqrt{c-c\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(47) = 94$.
time = 0.51, size = 122, normalized size = 2.65

$$\frac{2^{m+\frac{3}{2}}(-a)^{m}\sqrt{c}\,e^{\left(-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\right)}}{f(2m+1)\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1}\,\sqrt{\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] 2^(m + 3/2)*(-a)^m*sqrt(c)*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
 m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/(f*(2*m + 1)*sqrt(sin(f*x + e)
/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Fricas [**A**]
time = 3.14, size = 76, normalized size = 1.65

$$\frac{2\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^{m}\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{\left(2\,fm+f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

[Out] 2*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*(cos(f*x + e) + 1)/((2*f*m + f)*sin(f*x + e))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^m \sqrt{-c(\sec(e+fx)-1)} \sec(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1/2),x)

[Out] Integral((a*(sec(e + f*x) + 1))**m*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m \sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\cos(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)

[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x), x)

3.159 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=69

$$-\frac{{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*tan(f*
x+e)/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4046, 70}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(1,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[
e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{c-cx}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{{}_2F_1\left(1,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [F]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]], x
]
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)(a+a\sec(fx+e))^m}{\sqrt{c-c\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*
sec(f*x + e) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec{(e+fx)}+1))^m \sec{(e+fx)}}{\sqrt{-c(\sec{(e+fx)}-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.160 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{{}_2F_1\left(2,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{2cf(1+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*ta
n(f*x+e)/c/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4046, 70}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(2,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*S
ec[e + f*x])^m*Tan[e + f*x])/(c*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{{}_2F_1\left(2,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m \text{ta}}{2cf(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),
x]
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),
 x]
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)\,(a+a\sec(fx+e))^m}{(c-c\sec(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2
*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(e+fx)+1))^m \sec(e+fx)}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(3
/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.161 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{{}_2F_1\left(3,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{4c^2f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/4*hypergeom([3, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*ta
n(f*x+e)/c^2/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4046, 70}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,{}_2F_1\left(3,m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*(Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*S
ec[e + f*x])^m*Tan[e + f*x])/(c^2*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{{}_2F_1\left(3,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m}{4c^2f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [F]
time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),
 x]
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)(a+a\sec(fx+e))^m}{(c-c\sec(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^
3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.162 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=169

$$-\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)}+\frac{2(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-3-}}{af\,(3+8m+4m^2)}$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/f/(1+2*m)+2*(a+a*sec
(f*x+e))^(1+m)*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/a/f/(4*m^2+8*m+3)-2*(a+a*
sec(f*x+e))^(2+m)*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/a^2/f/(1+2*m)/(4*m^2+1
6*m+15)
```

Rubi [A]
time = 0.27, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4036, 4035}

$$-\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+2}(c-c\sec(e+fx))^{-m-3}}{a^2f(2m+1)\,(4m^2+16m+15)}+\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-3}}{af\,(4m^2+8m+3)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-3}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + (2*(a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-3 - m)*T
an[e + f*x])/(a*f*(3 + 8*m + 4*m^2)) - (2*(a + a*Sec[e + f*x])^(2 + m)*(c -
 c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(a^2*f*(1 + 2*m)*(15 + 16*m + 4*m^2
))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

```
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx &= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3}}{f(1+2m)} \\ &= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3}}{f(1+2m)} \\ &= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3}}{f(1+2m)}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.07, size = 321, normalized size = 1.90

$$-\frac{i2^{3+m}\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(1+e^{i(e+fx)}\right)\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\left(7+12m+4m^2-4e^{i(e+fx)}(3+2m)-4e^{3i(e+fx)}(3+2m)+e^{4i(e+fx)}(7+12m+4m^2)+e^{2i(e+fx)}(22+24m+8m^2)\right)\sec^{3+m}(e+fx)(1+\sec(e+fx))^{-m}(a(1+\sec(e+fx)))^m(c-c\sec(e+fx))^{-3-m}\sin^{-2(-3-m)}\left(\frac{e}{2}+\frac{fx}{2}\right)}{\left(-1+e^{i(e+fx)}\right)^5 f(1+2m)(3+2m)(5+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)
,x]
```

```
[Out] ((-I)*2^(3 + m)*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I
)*(e + f*x))))^m*(7 + 12*m + 4*m^2 - 4*E^(I*(e + f*x))*(3 + 2*m) - 4*E^((3*
I)*(e + f*x))*(3 + 2*m) + E^((4*I)*(e + f*x))*(7 + 12*m + 4*m^2) + E^((2*I)
*(e + f*x))*(22 + 24*m + 8*m^2))*Sec[e + f*x]^(3 + m)*(a*(1 + Sec[e + f*x])
)^m*(c - c*Sec[e + f*x])^(-3 - m))/((-1 + E^(I*(e + f*x)))^5*(((-I)*(-1 + E
^(I*(e + f*x))))/E^((I/2)*(e + f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*
(e + f*x))))^m*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(1 + Sec[e + f*x])^m*Sin[e/2
 + (f*x)/2]^(2*(-3 - m)))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

Maxima [A]
time = 0.50, size = 164, normalized size = 0.97

$$\frac{\left((4\,m^2+8\,m+3)(-a)^m-\frac{2\,(4\,m^2+12\,m+5)(-a)^m\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{(4\,m^2+16\,m+15)(-a)^m\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)c^{-m-3}(\cos\left(fx+e\right)+1)^5}{4\,(8\,m^3+36\,m^2+46\,m+15)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{2\,m}\sin\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="maxima")
```

```
[Out] 1/4*((4*m^2 + 8*m + 3)*(-a)^m - 2*(4*m^2 + 12*m + 5)*(-a)^m*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + (4*m^2 + 16*m + 15)*(-a)^m*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4)*c^(-m - 3)*(cos(f*x + e) + 1)^5/((8*m^3 + 36*m^2 + 46*m + 15)*f
*(sin(f*x + e)/(cos(f*x + e) + 1))^(2*m)*sin(f*x + e)^5)
```

Fricas [A]
time = 1.51, size = 128, normalized size = 0.76

$$-\frac{\left((4\,m^2+12\,m+7)\cos\left(fx+e\right)^2-2\,(2\,m+3)\cos\left(fx+e\right)+2\right)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-3}\sin\left(fx+e\right)}{(8\,fm^3+36\,fm^2+46\,fm+15\,f)\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="fricas")
```

```
[Out] -((4*m^2 + 12*m + 7)*cos(f*x + e)^2 - 2*(2*m + 3)*cos(f*x + e) + 2)*((a*cos
(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^(-m - 3)
*sin(f*x + e)/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 3)*sec(f*x + e
), x)
```

Mupad [B]

time = 10.65, size = 290, normalized size = 1.72

$$-\frac{\left(\cos\left(3\,e+3\,f\,x\right)-\sin\left(3\,e+3\,f\,x\right)\,1{}\mathrm{i}\right)\,\left(\frac{\sin(e+f\,x)\,{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^m\,\left(\cos(3\,e+3\,f\,x)+\sin(3\,e+3\,f\,x)\,1{}\mathrm{i}\right)\,\left(4\,m^2+12\,m+15\right)\,2{}\mathrm{i}}{f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}-\frac{\sin(2\,e+2\,f\,x)\,\left(8\,m+12\right)\,{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^m\,\left(\cos(3\,e+3\,f\,x)+\sin(3\,e+3\,f\,x)\,1{}\mathrm{i}\right)\,2{}\mathrm{i}}{f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}+\frac{\sin(3\,e+3\,f\,x)\,{\left(a+\frac{a}{\cos(e+f\,x)}\right)}^m\,\left(\cos(3\,e+3\,f\,x)+\sin(3\,e+3\,f\,x)\,1{}\mathrm{i}\right)\,\left(4\,m^2+12\,m+7\right)\,2{}\mathrm{i}}{f\,\left(m^3\,8{}\mathrm{i}+m^2\,36{}\mathrm{i}+m\,46{}\mathrm{i}+15{}\mathrm{i}\right)}\right)}{8\,{\cos\left(e+f\,x\right)}^3\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^{m+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 3)),x)
```

```
[Out] -((cos(3*e + 3*f*x) - sin(3*e + 3*f*x)*1i)*((sin(e + f*x)*(a + a/cos(e + f*
x))^m*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*(12*m + 4*m^2 + 15)*2i)/(f*(
m*46i + m^2*36i + m^3*8i + 15i)) - (sin(2*e + 2*f*x)*(8*m + 12)*(a + a/cos(
e + f*x))^m*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*2i)/(f*(m*46i + m^2*36
i + m^3*8i + 15i)) + (sin(3*e + 3*f*x)*(a + a/cos(e + f*x))^m*(cos(3*e + 3*
f*x) + sin(3*e + 3*f*x)*1i)*(12*m + 4*m^2 + 7)*2i)/(f*(m*46i + m^2*36i + m^
3*8i + 15i))))/(8*cos(e + f*x)^3*(c - c/cos(e + f*x))^(m + 3))
```

3.163 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=104

$$-\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{f(1+2m)}+\frac{(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-2-m}}{af\,(3+8m+4m^2)}$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m)*tan(f*x+e)/f/(1+2*m)+(a+a*sec(f
*x+e))^(1+m)*(c-c*sec(f*x+e))^(-2-m)*tan(f*x+e)/a/f/(4*m^2+8*m+3)
```

Rubi [A]
time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4036, 4035}

$$\frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-2}}{af\,(4m^2+8m+3)}-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + ((a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-2 - m)*Tan
[e + f*x])/(a*f*(3 + 8*m + 4*m^2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2}}{f(1+2m)}$$
$$= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2}}{f(1+2m)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.08, size = 250, normalized size = 2.40

$$\frac{i2^{3+m}\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(1+e^{i(e+fx)}\right)\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\left(1-e^{i(e+fx)}+m+e^{2i(e+fx)}(1+m)\right)\sec^{2+m}(e+fx)(1+\sec(e+fx))^{-m}(a(1+\sec(e+fx)))^m(c-c\sec(e+fx))^{-2-m}\sin^{2(2+m)}\left(\frac{1}{2}(e+fx)\right)}{\left(-1+e^{i(e+fx)}\right)^3 f(1+2m)(3+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)
,x]
```

```
[Out] (I*2^(3 + m)*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(
e + f*x))))^m*(1 - E^(I*(e + f*x)) + m + E^((2*I)*(e + f*x))*(1 + m))*Sec[e
 + f*x]^(2 + m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e + f*x])^(-2 - m)*Sin[
(e + f*x)/2]^(2*(2 + m)))/((-1 + E^(I*(e + f*x)))^3*(((-I)*(-1 + E^(I*(e +
f*x))))/E^((I/2)*(e + f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
)))^m*f*(1 + 2*m)*(3 + 2*m)*(1 + Sec[e + f*x])^m)
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}\left(c-c\sec\left(fx+e\right)\right)^{-2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x)
```

Maxima [A]
time = 0.50, size = 113, normalized size = 1.09

$$-\frac{\left(\left(-a\right)^{m}\left(2\,m+1\right)-\frac{\left(-a\right)^{m}\left(2\,m+3\right)\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)c^{-m-2}\left(\cos\left(fx+e\right)+1\right)^{3}}{2\left(4\,m^{2}+8\,m+3\right)f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}\right)^{2\,m}\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="maxima")
```

```
[Out] -1/2*((-a)^m*(2*m + 1) - (-a)^m*(2*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*c^(-m - 2)*(cos(f*x + e) + 1)^3/((4*m^2 + 8*m + 3)*f*(sin(f*x + e)/(cos
(f*x + e) + 1))^(2*m)*sin(f*x + e)^3)
```

Fricas [A]
time = 2.54, size = 100, normalized size = 0.96

$$-\frac{(2(m+1)\cos(fx+e)-1)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-2}\sin(fx+e)}{(4fm^2+8fm+3f)\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="fricas")
```

```
[Out] -(2*(m + 1)*cos(f*x + e) - 1)*((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos
(f*x + e) - c)/cos(f*x + e))^(-m - 2)*sin(f*x + e)/((4*f*m^2 + 8*f*m + 3*f)
*cos(f*x + e)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-2-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 2)*sec(f*x + e
), x)
```

Mupad [B]
time = 8.00, size = 145, normalized size = 1.39

$$\frac{\sin(e+fx)\left(a+\frac{a}{\cos(e+fx)}\right)^m 1\mathrm{i}}{f\cos(e+fx)^2\left(c-\frac{c}{\cos(e+fx)}\right)^{m+2}(m^2\,4\mathrm{i}+m\,8\mathrm{i}+3\mathrm{i})}-\frac{\sin(2e+2fx)\,(2m+2)\left(a+\frac{a}{\cos(e+fx)}\right)^m 1\mathrm{i}}{2f\cos(e+fx)^2\left(c-\frac{c}{\cos(e+fx)}\right)^{m+2}(m^2\,4\mathrm{i}+m\,8\mathrm{i}+3\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 2)),x)
```

```
[Out] (sin(e + f*x)*(a + a/cos(e + f*x))^m*1i)/(f*cos(e + f*x)^2*(c - c/cos(e + f
*x))^(m + 2)*(m*8i + m^2*4i + 3i)) - (sin(2*e + 2*f*x)*(2*m + 2)*(a + a/cos
(e + f*x))^m*1i)/(2*f*cos(e + f*x)^2*(c - c/cos(e + f*x))^(m + 2)*(m*8i + m
^2*4i + 3i))
```

3.164 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=47

$$-\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\tan(e+fx)}{f(1+2m)}$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m)*tan(f*x+e)/f/(1+2*m)
```

Rubi [A]
time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4035}

$$-\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)*Tan[e + f*x])/(f*(1
 + 2*m)))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))}{f(1+2m)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.21, size = 208, normalized size = 4.43

$$-\frac{2^{1+m}e^{-\frac{1}{2}i(e+fx)}\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-1-2m}\left(1+e^{i(e+fx)}\right)\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^{m}\sec^{1+m}(e+fx)(1+\sec(e+fx))^{-m}(a(1+\sec(e+fx)))^m(c-c\sec(e+fx))^{-1-m}\sin^{2(1+m)}\left(\frac{1}{2}(e+fx)\right)}{f+2fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)
,x]
```

```
[Out] -((2^(1 + m)*(((-I)*(-1 + E^(I*(e + f*x))))/E^((I/2)*(e + f*x)))^(-1 - 2*m)
*(1 + E^(I*(e + f*x)))*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^
m*Sec[e + f*x]^(1 + m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e + f*x])^(-1 -
m)*Sin[(e + f*x)/2]^(2*(1 + m)))/(E^((I/2)*(e + f*x))*(E^(I*(e + f*x))/(1 +
 E^((2*I)*(e + f*x))))^m*(f + 2*f*m)*(1 + Sec[e + f*x])^m))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^{m}\,(c-c\sec(fx+e))^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x)
```

Maxima [A]
time = 0.50, size = 66, normalized size = 1.40

$$\frac{(-a)^{m}\,c^{-m-1}(\cos(fx+e)+1)}{f(2\,m+1)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{2\,m}\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="maxima")
```

```
[Out] (-a)^m*c^(-m - 1)*(cos(f*x + e) + 1)/(f*(2*m + 1)*(sin(f*x + e)/(cos(f*x +
e) + 1))^(2*m)*sin(f*x + e))
```

Fricas [A]
time = 1.93, size = 78, normalized size = 1.66

$$-\frac{\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^{m}\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-1}\sin(fx+e)}{(2\,fm+f)\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="fricas")
```

```
[Out] -((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^
(-m - 1)*sin(f*x + e)/((2*f*m + f)*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-1-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 1)*sec(f*x + e
), x)
```

Mupad [B]
time = 2.91, size = 105, normalized size = 2.23

$$-\frac{\left(\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)\,{\left(\frac{a\,\left(\cos\left(e+f\,x\right)+1\right)}{\cos\left(e+f\,x\right)}\right)}^m}{c\,f\,\left(2\,m+1\right)\,{\left(\frac{c\,\left(\cos\left(e+f\,x\right)-1\right)}{\cos\left(e+f\,x\right)}\right)}^m\,\left(3\,\cos\left(e+f\,x\right)-2\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 1)),x)
```

```
[Out] -((sin(e + f*x) + sin(3*e + 3*f*x))*((a*(cos(e + f*x) + 1))/cos(e + f*x))^m
)/(c*f*(2*m + 1)*((c*(cos(e + f*x) - 1))/cos(e + f*x))^m*(3*cos(e + f*x) -
2*cos(2*e + 2*f*x) + cos(3*e + 3*f*x) - 2))
```

3.165 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx$

Optimal. Leaf size=101

$$-\frac{2^{\frac{1}{2}-m}c\,{}_2F_1\left(\frac{1}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{\frac{1}{2}+m}(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\tan(e+fx)}{f(1+2m)}$$

```
[Out] -2^(1/2-m)*c*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f*
x+e))^(1/2+m)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m)*tan(f*x+e)/f/(1+2*
m)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4046, 72, 71}

$$-\frac{c2^{\frac{1}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m+\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}\,{}_2F_1\left(m+\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*c*Hypergeometric2F1[1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 + m)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 - m)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
```

```
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c \cdots\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-\cdots}}$$

$$= -\frac{\left(2^{-\frac{1}{2}-m}ac(c-c\sec(e+fx))^{-1-m}\left(\frac{c-c\sec \cdots}{c \cdots}\right.\right.}{\cdots}$$

$$= -\frac{2^{\frac{1}{2}-m}c\,{}_2F_1\left(\frac{1}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec \cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.41, size = 257, normalized size = 2.54

$$\frac{2^{-1+m}\left(-ie^{-\frac{1}{2}i(e+fx)}\left(-1+e^{i(e+fx)}\right)\right)^{-2m}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{-m}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\left(-{}_2F_1\left(1,-2m;1-2m;-\frac{i\left(-1+e^{i(e+fx)}\right)}{1+e^{i(e+fx)}}\right)+{}_2F_1\left(1,-2m;1-2m;\frac{i\left(-1+e^{i(e+fx)}\right)}{1+e^{i(e+fx)}}\right)\right)\left(\frac{\sec(e+fx)}{1+\sec(e+fx)}\right)^m(a(1+\sec(e+fx)))^m(c-c\sec(e+fx))^{-m}\sin^{2m}\left(\frac{1}{2}(e+fx)\right)}{fm}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

```
[Out] (2^(-1 + m)*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^m*(-Hyperge
ometric2F1[1, -2*m, 1 - 2*m, ((-I)*(-1 + E^(I*(e + f*x))))/(1 + E^(I*(e + f
*x)))] + Hypergeometric2F1[1, -2*m, 1 - 2*m, (I*(-1 + E^(I*(e + f*x))))/(1
+ E^(I*(e + f*x)))])*(Sec[e + f*x]/(1 + Sec[e + f*x]))^m*(a*(1 + Sec[e + f*
x]))^m*Sin[(e + f*x)/2]^(2*m))/((((-I)*(-1 + E^(I*(e + f*x))))/E^((I/2)*(e
+ f*x)))^(2*m)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^m*f*m*(c - c*Sec
[e + f*x])^m)
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/((c-c*sec(f*x+e))**m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^m),x)
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^m), x)
```

3.166 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$

Optimal. Leaf size=99

$$-\frac{2^{\frac{3}{2}-m}c\,{}_2F_1\left(-\frac{1}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}+m}(a+a\sec(e+fx))^m(c-c\,\mathrm{s}}{f(1+2m)}$$

```
[Out] -2^(3/2-m)*c*hypergeom([1/2+m, -1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f
*x+e))^(-1/2+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/((c-c*sec(f*x+e))^m
)
```

Rubi [A]
time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4046, 72, 71}

$$-\frac{c2^{\frac{3}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,{}_2F_1\left(m-\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),x]
```

```
[Out] -((2^(3/2 - m)*c*Hypergeometric2F1[-1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
 f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x]
)/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
```

```
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}(c\right.}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-}}$$

$$= -\frac{\left(2^{\frac{1}{2}-m}ac(c-c\sec(e+fx))^{-m}\left(\frac{c-c\sec(e+}{c}\right.\right.}{}$$

$$= -\frac{2^{\frac{3}{2}-m}c\,{}_2F_1\left(-\frac{1}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\right.}{}$$

Mathematica [F]
time = 2.29, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),
x]
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),
 x]
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \sec(fx+e)(a+a\sec(fx+e))^m(c-c\sec(fx+e))^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e)
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{1-m}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1 - m))/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1 - m))/cos(e + f*x), x)
```

3.167 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$

Optimal. Leaf size=101

$$-\frac{2^{\frac{5}{2}-m}c^2\,{}_2F_1\left(-\frac{3}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}+m}(a+a\sec(e+fx))^m(c-}{f(1+2m)}$$

```
[Out] -2^(5/2-m)*c^2*hypergeom([1/2+m, -3/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec
(f*x+e))^(-1/2+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/((c-c*sec(f*x+e))
^m)
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4046, 72, 71}

$$-\frac{c^2 2^{\frac{5}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,{}_2F_1\left(m-\frac{3}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),x]
```

```
[Out] -((2^(5/2 - m)*c^2*Hypergeometric2F1[-3/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e
 + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
```

```
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}(c-\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-}}$$

$$= -\frac{\left(2^{\frac{3}{2}-m}ac^2(c-c\sec(e+fx))^{-m}\left(\frac{c-c\sec(e+}{c}\right.\right.}{}$$

$$= -\frac{2^{\frac{5}{2}-m}c^2\,{}_2F_1\left(-\frac{3}{2}+m,\frac{1}{2}+m;\frac{3}{2}+m;\frac{1}{2}(1+\right.}{}$$

Mathematica [F]
time = 2.72, size = 0, normalized size = 0.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),
x]
[Out] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),
 x]
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \sec(fx+e)(a+a\sec(fx+e))^m(c-c\sec(fx+e))^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e)
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(2-m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{2-m}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(2 - m))/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(2 - m))/cos(e + f*x), x)
```

3.168 $\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=105

$$\frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f}+\frac{a^3c\sec(e+fx)\tan(e+fx)}{4f}-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f}-\frac{2a^3c\tan^3(e+fx)}{3f}-a$$

```
[Out] 1/4*a^3*c*arctanh(sin(f*x+e))/f+1/4*a^3*c*sec(f*x+e)*tan(f*x+e)/f-1/2*a^3*c
*sec(f*x+e)^3*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)^3/f-1/5*a^3*c*tan(f*x+e)^5/
f
```

Rubi [A]
time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {4047, 2687, 30, 2691, 3853, 3855, 14}

$$-\frac{a^3c\tan^5(e+fx)}{5f}-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f}-\frac{a^3c\tan(e+fx)\sec^3(e+fx)}{2f}+\frac{a^3c\tan(e+fx)\sec(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*ArcTanh[Sin[e + f*x]])/(4*f) + (a^3*c*Sec[e + f*x]*Tan[e + f*x])/(4*
f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) - (2*a^3*c*Tan[e + f*x]^3)/(
3*f) - (a^3*c*Tan[e + f*x]^5)/(5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a^2\sec^2(e+fx)\tan^2(e+fx)+2a^2\right.\right.$$
$$= -\left(\left(a^3c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right) - \left(\right.$$
$$= -\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}\left(a^3c\right)\int \sec$$
$$= \frac{a^3c\sec(e+fx)\tan(e+fx)}{4f} - \frac{a^3c\sec^3(e+fx)}{2f}$$
$$= \frac{a^3c\tanh^{-1}(\sin(e+fx))}{4f} + \frac{a^3c\sec(e+fx)\tan}{4f}$$

Mathematica [**A**]

time = 0.31, size = 68, normalized size = 0.65

$$\frac{a^3c(15\tanh^{-1}(\sin(e+fx))-\tan(e+fx)(-15\sec(e+fx)+30\sec^3(e+fx)+40\tan^2(e+fx)+12\tan^4(e+fx)))}{60f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*(15*ArcTanh[Sin[e + f*x]] - Tan[e + f*x]*(-15*Sec[e + f*x] + 30*Sec[
e + f*x]^3 + 40*Tan[e + f*x]^2 + 12*Tan[e + f*x]^4)))/(60*f)
```

Maple [A]
time = 0.27, size = 137, normalized size = 1.30

method	result
derivativedivides	$\frac{a^3c\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-2a^3c\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e}{}\right.}{f}$
default	$\frac{a^3c\left(-\frac{8}{15}-\frac{\left(\sec^4(fx+e)\right)}{5}-\frac{4\left(\sec^2(fx+e)\right)}{15}\right)\tan(fx+e)-2a^3c\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e}{}\right.}{f}$
norman	$\frac{\frac{a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}+\frac{25a^3c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{64a^3c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15f}+\frac{7a^3c\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{a^3c\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{a^3c\ln\left(\tan\left(\right.\right.}{4f}$
risch	$-\frac{ia^3c(15\,e^{9i(fx+e)}-60\,e^{8i(fx+e)}-90\,e^{7i(fx+e)}-240\,e^{6i(fx+e)}-40\,e^{4i(fx+e)}+90\,e^{3i(fx+e)}-80\,e^{2i(fx+e)}-15\,e^{i(fx+e)}-2}{30f\left(e^{2i(fx+e)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(a^3*c*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*a^3*c*(-
(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))
)+2*a^3*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^3*c*t
an(f*x+e))
```

Maxima [A]
time = 0.28, size = 186, normalized size = 1.77

$$-\frac{8\,\left(3\,\tan\left(fx+e\right)^5+10\,\tan\left(fx+e\right)^3+15\,\tan\left(fx+e\right)\right)a^3c-15\,a^3c\left(\frac{2\,\left(3\,\sin(fx+e)^3-5\,\sin(fx+e)\right)}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx+e\right)-1\right)\right)+60\,a^3c\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-120\,a^3c\tan\left(fx+e\right)}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c -
15*a^3*c*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x
```

```
 + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 60*a^3*
c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*
x + e) - 1)) - 120*a^3*c*tan(f*x + e))/f
```

Fricas [A]
time = 1.40, size = 141, normalized size = 1.34

$$\frac{15\,a^3c\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)+2\left(28\,a^3c\cos\left(fx+e\right)^4+15\,a^3c\cos\left(fx+e\right)^3-16\,a^3c\cos\left(fx+e\right)^2-30\,a^3c\cos\left(fx+e\right)-12\,a^3c\right)\sin\left(fx+e\right)}{120\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*a^3*c*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e
)^5*log(-sin(f*x + e) + 1) + 2*(28*a^3*c*cos(f*x + e)^4 + 15*a^3*c*cos(f*x
+ e)^3 - 16*a^3*c*cos(f*x + e)^2 - 30*a^3*c*cos(f*x + e) - 12*a^3*c)*sin(f*
x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^3c\left(\int\left(-\sec^2\left(e+fx\right)\right)\,dx+\int\left(-2\sec^3\left(e+fx\right)\right)\,dx+\int 2\sec^5\left(e+fx\right)dx+\int\sec^6\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-sec(e + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) +
Integral(2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Giac [A]
time = 0.50, size = 145, normalized size = 1.38

$$\frac{15\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^3c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-70\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+128\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-250\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/60*(15*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c*tan(1/2*f*x + 1/2*e)^9 - 70*a^3*c*tan(1/
2*f*x + 1/2*e)^7 + 128*a^3*c*tan(1/2*f*x + 1/2*e)^5 - 250*a^3*c*tan(1/2*f*x
 + 1/2*e)^3 - 15*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5
)/f
```

Mupad [B]
time = 6.32, size = 175, normalized size = 1.67

$$\frac{-\frac{c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{2}+\frac{7\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3}-\frac{64\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}+\frac{25\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+\frac{c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}+\frac{a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

```
[Out] ((a^3*c*tan(e/2 + (f*x)/2))/2 + (25*a^3*c*tan(e/2 + (f*x)/2)^3)/3 - (64*a^3
*c*tan(e/2 + (f*x)/2)^5)/15 + (7*a^3*c*tan(e/2 + (f*x)/2)^7)/3 - (a^3*c*tan
(e/2 + (f*x)/2)^9)/2)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4
+ 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10
- 1)) + (a^3*c*atanh(tan(e/2 + (f*x)/2)))/(2*f)
```

3.169 $\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=86

$$\frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^2c\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f}-\frac{a^2c\tan^3(e+fx)}{3f}$$

```
[Out] 1/8*a^2*c*arctanh(sin(f*x+e))/f+1/8*a^2*c*sec(f*x+e)*tan(f*x+e)/f-1/4*a^2*c
*sec(f*x+e)^3*tan(f*x+e)/f-1/3*a^2*c*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4047, 2687, 30, 2691, 3853, 3855}

$$-\frac{a^2c\tan^3(e+fx)}{3f}+\frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2c\tan(e+fx)\sec^3(e+fx)}{4f}+\frac{a^2c\tan(e+fx)\sec(e+fx)}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(8*f) + (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(8*
f) - (a^2*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (a^2*c*Tan[e + f*x]^3)/(3*
f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a\sec^2(e+fx)\tan^2(e+fx)+a\sec^3\right.\right.$$

$$= -\left(\left(a^2c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right) - (a$$

$$= -\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{1}{4}\left(a^2c\right)\int \sec^3$$

$$= \frac{a^2c\sec(e+fx)\tan(e+fx)}{8f} - \frac{a^2c\sec^3(e+fx)\,t}{4f}$$

$$= \frac{a^2c\tanh^{-1}(\sin(e+fx))}{8f} + \frac{a^2c\sec(e+fx)\tan(e}{8f}$$

Mathematica [A]
time = 0.20, size = 57, normalized size = 0.66

$$\frac{a^2c\left(3\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(3\sec(e+fx)-6\sec^3(e+fx)-8\tan^2(e+fx)\right)\right)}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*(3*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*Sec[e + f*x] - 6*Sec[e +
f*x]^3 - 8*Tan[e + f*x]^2)))/(24*f)
```

Maple [A]
time = 0.25, size = 126, normalized size = 1.47

method	result
derivativedivides	$\frac{-a^2c\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+a^2c\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e}{f}$
default	$\frac{-a^2c\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+a^2c\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e}{f}$
norman	$\frac{-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{53a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}+\frac{11a^2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}-\frac{a^2c\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f}+\frac{a^2}{}$
risch	$-\frac{ia^2c\left(3\,e^{7i(fx+e)}-24\,e^{6i(fx+e)}-21\,e^{5i(fx+e)}-24\,e^{4i(fx+e)}+21\,e^{3i(fx+e)}-8\,e^{2i(fx+e)}-3\,e^{i(fx+e)}-8\right)}{12f\left(e^{2i(fx+e)}+1\right)^4}+\frac{a^2c\ln\left(e^{i(f}\right.}{8f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-a^2*c*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+
e)+tan(f*x+e)))+a^2*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a^2*c*(1/2*sec(f*x
+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^2*c*tan(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(84) = 168.
time = 0.28, size = 173, normalized size = 2.01

$$-\frac{16\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c-3a^2c\left(\frac{2\left(3\sin\left(fx+e\right)^3-5\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^4-2\sin\left(fx+e\right)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)+12a^2c\left(\frac{2\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-48a^2c\tan\left(fx+e\right)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin
(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 12*a^2*c*(2*sin(f*x + e)/(sin(f
*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*
tan(f*x + e))/f
```

Fricas [A]
time = 1.74, size = 126, normalized size = 1.47

$$\frac{3\,a^2c\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2\left(8\,a^2c\cos\left(fx+e\right)^3+3\,a^2c\cos\left(fx+e\right)^2-8\,a^2c\cos\left(fx+e\right)-6\,a^2c\right)\sin\left(fx+e\right)}{48\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*a^2*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^4
*log(-sin(f*x + e) + 1) + 2*(8*a^2*c*cos(f*x + e)^3 + 3*a^2*c*cos(f*x + e)^
2 - 8*a^2*c*cos(f*x + e) - 6*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2c\left(\int\left(-\sec^2\left(e+fx\right)\right)\,dx+\int\left(-\sec^3\left(e+fx\right)\right)\,dx+\int\sec^4\left(e+fx\right)dx+\int\sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-sec(e + f*x)**2, x) + Integral(-sec(e + f*x)**3, x) + In
tegral(sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Giac [A]
time = 0.50, size = 128, normalized size = 1.49

$$\frac{3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-11\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+53\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^2*c*log(abs(tan(1/2*
f*x + 1/2*e) - 1)) - 2*(3*a^2*c*tan(1/2*f*x + 1/2*e)^7 - 11*a^2*c*tan(1/2*f
*x + 1/2*e)^5 + 53*a^2*c*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*tan(1/2*f*x + 1/2
*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B]
time = 4.92, size = 146, normalized size = 1.70

$$\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}-\frac{\frac{c\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7}{4}-\frac{11\,c\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{12}+\frac{53\,c\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{12}+\frac{c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

```
[Out] (a^2*c*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((a^2*c*tan(e/2 + (f*x)/2))/4 + (
53*a^2*c*tan(e/2 + (f*x)/2)^3)/12 - (11*a^2*c*tan(e/2 + (f*x)/2)^5)/12 + (a
^2*c*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x
)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.170 $\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=17

$$-\frac{ac\tan^3(e+fx)}{3f}$$

```
[Out] -1/3*a*c*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4047, 2687, 30}

$$-\frac{ac\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(a*c*Tan[e + f*x]^3)/f
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\left((ac)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)$$
$$= -\frac{(ac)\text{Subst}\left(\int x^2\,dx, x, \tan(e+fx)\right)}{f}$$
$$= -\frac{ac\tan^3(e+fx)}{3f}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$-\frac{ac\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
[Out] -1/3*(a*c*Tan[e + f*x]^3)/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.
time = 0.18, size = 36, normalized size = 2.12

method	result	size
norman	$\frac{8ac\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$	34
risch	$\frac{2iac\left(3\,e^{4i(fx+e)}+1\right)}{3f\left(e^{2i(fx+e)}+1\right)^3}$	35
derivativedivides	$\frac{ac\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+ac\tan(fx+e)}{f}$	36
default	$\frac{ac\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+ac\tan(fx+e)}{f}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*(a*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a*c*tan(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(16) = 32$.
time = 0.29, size = 39, normalized size = 2.29

$$-\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ac-3\,ac\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a*c - 3*a*c*tan(f*x + e))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.
time = 2.31, size = 38, normalized size = 2.24

$$\frac{(ac\cos(fx+e)^2 - ac)\sin(fx+e)}{3\,f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/3*(a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.
time = 1.34, size = 51, normalized size = 3.00

$$\begin{cases} \frac{-ac\left(\frac{\tan^3(e+fx)}{3}+\tan(e+fx)\right)+ac\tan(e+fx)}{f} & \text{for}\ f\neq 0 \\ x(a\sec(e)+a)(-c\sec(e)+c)\sec^2(e) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] Piecewise(((-a*c*(tan(e + f*x)**3/3 + tan(e + f*x)) + a*c*tan(e + f*x))/f,
Ne(f, 0)), (x*(a*sec(e) + a)*(-c*sec(e) + c)*sec(e)**2, True))
```

Giac [A]
time = 0.46, size = 15, normalized size = 0.88

$$-\frac{ac\tan(fx+e)^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/3*a*c*tan(f*x + e)^3/f
```

Mupad [B]
time = 1.70, size = 15, normalized size = 0.88

$$-\frac{a\,c\,\tan(e+f\,x)^3}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
[Out] -(a*c*tan(e + f*x)^3)/(3*f)
```

3.171 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{c\tan(e+fx)}{af}-\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] 2*c*arctanh(sin(f*x+e))/a/f-c*tan(f*x+e)/a/f-2*c*tan(f*x+e)/f/(a+a*sec(f*x+
e))
```

Rubi [A]
time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4093, 3872, 3855, 3852, 8}

$$-\frac{c\tan(e+fx)}{af}+\frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c*ArcTanh[Sin[e + f*x]])/(a*f) - (c*Tan[e + f*x])/(a*f) - (2*c*Tan[e + f
*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx &= -\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{\int \sec(e+fx)(-2ac+ac\sec(e+fx))\,dx}{a^2}\\ &= -\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{c\int \sec^2(e+fx)\,dx}{a}+\frac{(2c)\int \sec(e+fx)\,d}{a}\\ &= \frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{c\text{Subst}(\int 1\,dx,x,}{af}\\ &= \frac{2c\tanh^{-1}(\sin(e+fx))}{af}-\frac{c\tan(e+fx)}{af}-\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(56) = 112.
time = 0.64, size = 154, normalized size = 2.75

$$-\frac{c\left(\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{f}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*((2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/f - (2*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]])/f + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])) + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) +
 (2*Tan[(e + f*x)/2])/f))/a)
```

Maple [A]
time = 0.14, size = 78, normalized size = 1.39

method	result	size
derivativedivides	$\frac{2c\left(-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2}-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2}\right)}{fa}$	78

default	$\frac{2c\left(-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2}-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2}\right)}{fa}$	78
risch	$-\frac{2ic\left(2\,e^{2i(fx+e)}+e^{i(fx+e)}+3\right)}{fa\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\right)}+\frac{2c\ln\left(e^{i(fx+e)}+i\right)}{af}-\frac{2c\ln\left(e^{i(fx+e)}-i\right)}{af}$	104
norman	$\frac{-\frac{4c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{6c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{2c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}+\frac{2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c/a*(-tan(1/2*f*x+1/2*e)+1/2/(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*
e)-1)+ln(tan(1/2*f*x+1/2*e)+1)+1/2/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(60) = 120$.
time = 0.30, size = 210, normalized size = 3.75

$$\frac{c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] (c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*(log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) -
 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))))/f
```

Fricas [A]
time = 2.34, size = 115, normalized size = 2.05

$$\frac{(c\cos(fx+e)^2+c\cos(fx+e))\log(\sin(fx+e)+1)-(c\cos(fx+e)^2+c\cos(fx+e))\log(-\sin(fx+e)+1)-(3\,c\cos(fx+e)+c)\sin(fx+e)}{af\cos(fx+e)^2+af\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] ((c*cos(f*x + e)^2 + c*cos(f*x + e))*log(sin(f*x + e) + 1) - (c*cos(f*x + e
)^2 + c*cos(f*x + e))*log(-sin(f*x + e) + 1) - (3*c*cos(f*x + e) + c)*sin(f
*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec^2(e+fx)}{\sec(e+fx)+1}\right)dx+\int\frac{\sec^3(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x
)**3/(sec(e + f*x) + 1), x))/a
```

Giac [A]
time = 0.48, size = 87, normalized size = 1.55

$$\frac{2\left(\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}+\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 2*(c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - c*log(abs(tan(1/2*f*x + 1/2*e)
- 1))/a - c*tan(1/2*f*x + 1/2*e)/a + c*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x +
 1/2*e)^2 - 1)*a))/f
```

Mupad [B]
time = 1.74, size = 71, normalized size = 1.27

$$\frac{4\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}-\frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}-\frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))),x)
```

```
[Out] (4*c*atanh(tan(e/2 + (f*x)/2)))/(a*f) - (2*c*tan(e/2 + (f*x)/2))/(f*(a - a*
tan(e/2 + (f*x)/2)^2)) - (2*c*tan(e/2 + (f*x)/2))/(a*f)
```

3.172 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{c\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{7c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] -c*arctanh(sin(f*x+e))/a^2/f+7/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-2/3*c*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A]

time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4093, 4083, 3855, 3879}

$$-\frac{c\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{7c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a^2*f)) + (7*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec
[e + f*x])) - (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
```

```
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-4ac+3ac\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^2}$$

$$= -\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{c\int \sec(e+fx)\,dx}{a^2} + \frac{(7c)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}}{3a}$$

$$= -\frac{c\tanh^{-1}(\sin(e+fx))}{a^2 f} - \frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{7c\tan(e+}{3f\left(a^2+a^2\sec\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(70) = 140.
time = 0.46, size = 335, normalized size = 4.79

$$\frac{c\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sec^2(e+fx)\left(3\cos\left(e+\frac{3fx}{2}\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+3\cos\left(2e+\frac{3fx}{2}\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+9\cos\left(\frac{fx}{2}\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+9\cos\left(e+\frac{fx}{2}\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-3\cos\left(e+\frac{3fx}{2}\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-3\cos\left(2e+\frac{3fx}{2}\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+24\sin\left(\frac{fx}{2}\right)-6\sin\left(e+\frac{fx}{2}\right)+10\sin\left(e+\frac{3fx}{2}\right)\right)}{6a^2f(1+\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c*Cos[(e + f*x)/2]*Sec[e/2]*Sec[e + f*x]^2*(3*Cos[e + (3*f*x)/2]*Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]] + 3*Cos[2*e + (3*f*x)/2]*Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] + 9*Cos[(f*x)/2]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*Cos[e + (f*x)/2]*(Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]]) - 3*Cos[e + (3*f*x)/2]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 3
*Cos[2*e + (3*f*x)/2]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 24*Sin[(f*
x)/2] - 6*Sin[e + (f*x)/2] + 10*Sin[e + (3*f*x)/2]))/(6*a^2*f*(1 + Sec[e +
f*x])^2)
```

Maple [A]
time = 0.20, size = 60, normalized size = 0.86

method	result
derivativedivides	$\frac{c\left(\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{f\,a^2}$

default	$\frac{c\left(\frac{\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{f\,a^{2}}$
risch	$\frac{2ic\left(3\,\mathrm{e}^{2i(fx+e)}+12\,\mathrm{e}^{i(fx+e)}+5\right)}{3f\,a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}+\frac{c\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{a^{2}f}-\frac{c\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{a^{2}f}$
norman	$\frac{\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{11c\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{4c\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}+\frac{c\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}a}+\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{a^{2}f}-\frac{c\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{a^{2}f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*c/a^2*(1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+ln(tan(1/2*f*x+1/2
*e)-1)-ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(71) = 142$.
time = 0.30, size = 156, normalized size = 2.23

$$\frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}{a^{2}}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^{2}}\right)+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a^2) + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 1.30, size = 133, normalized size = 1.90

$$-\frac{3\left(c\cos\left(fx+e\right)^{2}+2\,c\cos\left(fx+e\right)+c\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c\cos\left(fx+e\right)^{2}+2\,c\cos\left(fx+e\right)+c\right)\log\left(-\sin\left(fx+e\right)+1\right)-2\left(5\,c\cos\left(fx+e\right)+7\,c\right)\sin\left(fx+e\right)}{6\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+a^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - 3
*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(-sin(f*x + e) + 1) - 2*(5*c*
```

```
cos(f*x + e) + 7*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x +
 e) + a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.47, size = 81, normalized size = 1.16

$$-\frac{\frac{3\,c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{3\,c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}-\frac{a^4c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,a^4c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 3*c*log(abs(tan(1/2*f*x
+ 1/2*e) - 1))/a^2 - (a^4*c*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c*tan(1/2*f*x +
1/2*e))/a^6)/f
```

Mupad [B]
time = 1.69, size = 44, normalized size = 0.63

$$\frac{c\left(6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (c*(6*tan(e/2 + (f*x)/2) - 6*atanh(tan(e/2 + (f*x)/2)) + tan(e/2 + (f*x)/2)
^3))/(3*a^2*f)
```

3.173 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=86

$$-\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2}-\frac{4c\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}$$

```
[Out] -2/5*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+11/15*c*tan(f*x+e)/a/f/(a+a*sec(f*x+
e))^2-4/15*c*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4093, 4085, 3879}

$$-\frac{4c\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{11c\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}-\frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (11*c*Tan[e + f*x])/(15*
a*f*(a + a*Sec[e + f*x])^2) - (4*c*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f
*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
```

```
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-6ac+5ac\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx}{5a^2}$$

$$= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{(4c)\int \frac{\sec(}{a+a\,\mathrm{se}}}{15a}$$

$$= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{4c\tan}{15f\,(a^3+a}$$

Mathematica [A]
time = 0.17, size = 43, normalized size = 0.50

$$-\frac{c(4+\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)\tan^3\left(\frac{1}{2}(e+fx)\right)}{30a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
[Out] -1/30*(c*(4 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]^3)/(a^3*f)
```

Maple [A]
time = 0.20, size = 37, normalized size = 0.43

method	result	size
derivativedivides	$\frac{c\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}\right)}{2f\,a^3}$	37
default	$\frac{c\left(-\frac{\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}\right)}{2f\,a^3}$	37
risch	$\frac{2ic\left(15\,e^{3i(fx+e)}-5\,e^{2i(fx+e)}+5\,e^{i(fx+e)}+1\right)}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}$	59
norman	$\frac{-\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}+\frac{7c\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{30af}+\frac{c\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{30af}-\frac{c\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2a^2}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f*c/a^3*(-1/5*tan(1/2*f*x+1/2*e)^5-1/3*tan(1/2*f*x+1/2*e)^3)
```

Maxima [A]
time = 0.29, size = 125, normalized size = 1.45

$$-\frac{\frac{c\left(\frac{15\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\,\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3}+\frac{3\,\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}-\frac{3\,c\left(\frac{5\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
[Out] -1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x +
e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 1.55, size = 84, normalized size = 0.98

$$\frac{\left(c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)-4\,c\right)\sin\left(fx+e\right)}{15\,\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
[Out] 1/15*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e) - 4*c)*sin(f*x + e)/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{\sec^3\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f
*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.51, size = 37, normalized size = 0.43

$$-\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{30\,a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 + 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

Mupad [B]
time = 1.70, size = 35, normalized size = 0.41

$$-\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+5\right)}{30\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))^3),x)
```

```
[Out] -(c*tan(e/2 + (f*x)/2)^3*(3*tan(e/2 + (f*x)/2)^2 + 5))/(30*a^3*f)
```

3.174 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=140

$$-\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f}-\frac{a^2c\cos^2(e+fx)^{\frac{4+p}{2}}\,{}_2F_1\left(\frac{3}{2}\right.}{}$$

```
[Out] -1/3*a^2*c*(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([3/2, 3/2+1/2*p],[5/2],sin(
f*x+e)^2)*(g*sec(f*x+e))^p*tan(f*x+e)^3/f-1/3*a^2*c*(cos(f*x+e)^2)^(2+1/2*p
)*hypergeom([3/2, 2+1/2*p],[5/2],sin(f*x+e)^2)*(g*sec(f*x+e))^(1+p)*tan(f*x
+e)^3/f/g
```

Rubi [A]
time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4047, 2697, 16}

$$-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,{}_2F_1\left(\frac{3}{2},\frac{p+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}-\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+4}{2}}(g\sec(e+fx))^{p+1}\,{}_2F_1\left(\frac{3}{2},\frac{p+4}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(a^2*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2,
5/2, Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/f - (a^2*c*(Cos[e +
 f*x]^2)^((4 + p)/2)*Hypergeometric2F1[3/2, (4 + p)/2, 5/2, Sin[e + f*x]^2]
*(g*Sec[e + f*x])^(1 + p)*Tan[e + f*x]^3)/(3*f*g)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
```

```
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\left((ac)\int \left(a(g\sec(e+fx))^p\tan^2(e+fx)+\right.\right.$$
$$= -\left((a^2c)\int (g\sec(e+fx))^p\tan^2(e+fx)\,dx\right)$$
$$= -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+\right.}{3f}$$
$$= -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+\right.}{3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 30.37, size = 7087, normalized size = 50.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
[Out] Result too large to show
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int (g\sec(fx+e))^p\,(a+a\sec(fx+e))^2\,(c-c\sec(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
[Out] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(a^2*c*sec(f*x + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e)
 - a^2*c)*(g*sec(f*x + e))^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-a^2c\left(\int \left(-(g\sec(e+fx))^p\right)dx + \int \left(-(g\sec(e+fx))^p\sec(e+fx)\right)dx + \int (g\sec(e+fx))^p\sec^2(e+fx)\,dx + \int (g\sec(e+fx))^p\sec^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral(-(g*sec(e + f*x))**p*
sec(e + f*x), x) + Integral((g*sec(e + f*x))**p*sec(e + f*x)**2, x) + Integ
ral((g*sec(e + f*x))**p*sec(e + f*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^2\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p,x)
[Out] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p, x)
```

3.175 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal. Leaf size=65

$$-\frac{ac\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f}$$

```
[Out] -1/3*a*c*(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([3/2, 3/2+1/2*p],[5/2],sin(f*
x+e)^2)*(g*sec(f*x+e))^p*tan(f*x+e)^3/f
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4047, 2697}

$$-\frac{ac\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,{}_2F_1\left(\frac{3}{2},\frac{p+3}{2};\frac{5}{2};\sin^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(a*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5/
2, Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\left((ac)\int (g\sec(e+fx))^p\tan^2(e+fx)\,dx\right)$$
$$= -\frac{ac\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{3}{2},\frac{3+p}{2};\frac{5}{2};\sin^2(e+fx\right.}{3f}$$

Mathematica [A]
time = 0.36, size = 72, normalized size = 1.11

$$-\frac{ac(g\sec(e+fx))^p\tan(e+fx)\left(p+\frac{{}_2F_1\left(\frac{1}{2},\frac{p}{2};\frac{2+p}{2};\sec^2(e+fx)\right)}{\sqrt{-\tan^2(e+fx)}}\right)}{fp(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*c*(g*Sec[e + f*x])^p*Tan[e + f*x]*(p + Hypergeometric2F1[1/2, p/2, (2
+ p)/2, Sec[e + f*x]^2]/Sqrt[-Tan[e + f*x]^2]))/(f*p*(1 + p)))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (g\sec(fx+e))^p\,(a+a\sec(fx+e))\,(c-c\sec(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(a*c*sec(f*x + e)^2 - a*c)*(g*sec(f*x + e))^p, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-ac\left(\int\left(-(g\sec\left(e+fx\right))^{p}\right)dx+\int(g\sec\left(e+fx\right))^{p}\sec^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] -a*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral((g*sec(e + f*x))**p*sec(
e + f*x)**2, x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)\,\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p,x)
```

```
[Out] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p, x)
```

3.176 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=180

$$-\frac{cg(1-2p)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)(g\sec(e+fx))^{-1+p}\sin(e+fx)}{af(1-p)\sqrt{\sin^2(e+fx)}}+\frac{2c\,{}_2F_1\left(\frac{1}{2},-\frac{p}{2};\frac{2-p}{2};\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e}$$

```
[Out] -c*g*(1-2*p)*hypergeom([1/2, 1/2-1/2*p],[3/2-1/2*p],cos(f*x+e)^2)*(g*sec(f*
x+e))^(-1+p)*sin(f*x+e)/a/f/(1-p)/(sin(f*x+e)^2)^(1/2)+2*c*hypergeom([1/2,
-1/2*p],[1-1/2*p],cos(f*x+e)^2)*(g*sec(f*x+e))^p*sin(f*x+e)/a/f/(sin(f*x+e)
^2)^(1/2)-2*c*(g*sec(f*x+e))^p*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A]
time = 0.16, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4105, 3872, 3857, 2722}

$$-\frac{cg(1-2p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)}{af(1-p)\sqrt{\sin^2(e+fx)}}+\frac{2c\sin(e+fx)(g\sec(e+fx))^p\,{}_2F_1\left(\frac{1}{2},-\frac{p}{2};\frac{2-p}{2};\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e+fx)}}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*g*(1 - 2*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*x]^
2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(a*f*(1 - p)*Sqrt[Sin[e + f*x]^2
])) + (2*c*Hypergeometric2F1[1/2, -1/2*p, (2 - p)/2, Cos[e + f*x]^2]*(g*Sec
[e + f*x])^p*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) - (2*c*(g*Sec[e + f*x
])^p*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx &= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{\int(g\sec(e+fx))^p(ac(1-2\\ &= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{(c(1-2p))\int(g\sec(e+fx)}{a}\\ &= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{\left(c(1-2p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\,s\right.}{}\\ &= -\frac{c(1-2p)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)(g\sec(e-}{af(1-p)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 16.81, size = 3396, normalized size = 18.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-6*c*Sec[e + f*x]^p*(g*Sec[e + f*x])^p*Tan[(e + f*x)/2]^3*(-((AppellF1[1/2
, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^
2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2
```

]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(a*f*(3*Sec[(e + f*x)/2]^2*S
ec[e + f*x]^p*(-((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1
[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2,
 p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2,
p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 +
 p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))
 + 6*p*Sec[e + f*x]^(1 + p)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-((AppellF1[1/2,
 p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2
)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
+ 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 6*Sec[e + f*x]^p*Tan[(e + f*
x)/2]*((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Cos[(e + f*x)/2]*Sin[(e + f*x)/2])/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p
, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (Cos
[(e + f*x)/2]^2*(-1/3*((1 - p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (p*AppellF
1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
 f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + ((p*Appel
lF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/3 + (p*AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)/(3*Ap
pellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(App
ellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + Appell
F1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
f*x)/2]^2) - (AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*(2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((p*AppellF1[3/2, p, 1 - p, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3 + (p*AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f

```
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*p*Tan[(e + f*x)/2]^2*
((-3*(1 - p)*AppellF1[5/2, p, 2 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (6*p*AppellF1[5/2, 1 + p,
1 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5 + (3*(1 + p)*AppellF1[5/2, 2 + p, -p, 7/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/(3*Appe
llF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(Appel
lF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1
[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)^2 + (AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[(e + f*x)/2]^2*(2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
 + 3*(-1/3*((1 - p)*AppellF1[3/2, p, 2 - p, 5/2...
```

Maple [**F**]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(g \sec \left(f x +e \right)\right)^{p} \left(c -c \sec \left(f x +e \right)\right)}{a +a \sec \left(f x +e \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{(g\sec(e+fx))^p}{\sec(e+fx)+1}\right)dx+\int\frac{(g\sec(e+fx))^p\sec(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x) + 1), x) + Integral((g*sec(
e + f*x))**p*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,1,2,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{2,[0,0,0,1]%%%} Er
ror: B
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)\,\left(\frac{g}{\cos(e+f\,x)}\right)^p}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x)),x)
```

```
[Out] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x)), x)
```

3.177 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=226

$$-\frac{cg(3-4p)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)(g\sec(e+fx))^{-1+p}\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{c(5-4p)\,{}_2F_1\left(\frac{1}{2},-\frac{p}{2};\frac{2-p}{2};\cos^2(\right.}{3a^2f\sqrt{}}$$

```
[Out] -1/3*c*g*(3-4*p)*hypergeom([1/2, 1/2-1/2*p],[3/2-1/2*p],cos(f*x+e)^2)*(g*se
c(f*x+e))^(-1+p)*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)+1/3*c*(5-4*p)*hyperg
eom([1/2, -1/2*p],[1-1/2*p],cos(f*x+e)^2)*(g*sec(f*x+e))^p*sin(f*x+e)/a^2/f
/(sin(f*x+e)^2)^(1/2)-1/3*c*(5-4*p)*(g*sec(f*x+e))^p*tan(f*x+e)/a^2/f/(1+se
c(f*x+e))-2/3*c*(g*sec(f*x+e))^p*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.29, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4105, 3872, 3857, 2722}

$$-\frac{cg(3-4p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}+\frac{c(5-4p)\sin(e+fx)(g\sec(e+fx))^p\,{}_2F_1\left(\frac{1}{2},-\frac{p}{2};\frac{2-p}{2};\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}-\frac{c(5-4p)\tan(e+fx)(g\sec(e+fx))^p}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(c*g*(3 - 4*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*
x]^2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(a^2*f*Sqrt[Sin[e + f*x]^2])
+ (c*(5 - 4*p)*Hypergeometric2F1[1/2, -1/2*p, (2 - p)/2, Cos[e + f*x]^2]*(g
*Sec[e + f*x])^p*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (c*(5 - 4*p
)*(g*Sec[e + f*x])^p*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - (2*c*(g*S
ec[e + f*x])^p*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{\int \frac{(g\sec(e+fx))^p(ac(3-2p)-2ac(1-p}{a+a\sec(e+fx)}}{3a^2}$$
$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c(g\sec(e+fx))^p\tan}{3f(a+a\sec(e+}$$
$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c(g\sec(e+fx))^p\tan}{3f(a+a\sec(e+}$$
$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c(g\sec(e+fx))^p\tan}{3f(a+a\sec(e+}$$
$$= -\frac{c(3-4p)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-p}{2};\frac{3-p}{2};\cos^2(e+fx)\right)(g\sec(e+}{3a^2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [F]
time = 11.16, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,
x]

[Out] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,
 x]
```

Maple [F]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(fx+e))^p (c-c\sec(fx+e))}{(a+a\sec(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a)^2,
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a^2*sec(f*x + e)^2 + 2*a
^2*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(\int\left(-\frac{(g\sec(e+fx))^p}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{(g\sec(e+fx))^p\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x
) + Integral((g*sec(e + f*x))**p*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e +
f*x) + 1), x))/a**2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,4,0]%%%}+%%%{-2,[0,1,2,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%
%{4,[0
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)\,\left(\frac{g}{\cos(e+f\,x)}\right)^p}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x))^2,x)
```

```
[Out] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x))^2, x)
```

3.178 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

Optimal. Leaf size=104

$$-\frac{2\sqrt{a}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{cf}+\frac{2g\cot(e+fx)\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e$$

```
[Out] -2*g^(3/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec
(f*x+e))^(1/2))*a^(1/2)/c/f+2*g*cot(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+a*sec(f*
x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.18, antiderivative size = 143, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4049, 49, 65, 223, 209}

$$\frac{2ag^{3/2}\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c}\,\sqrt{g\sec(e+fx)}}{\sqrt{g}\,\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\,\sqrt{c-c\sec(e+fx)}}-\frac{2ag\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x]),
x]
```

```
[Out] (-2*a*g*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c -
 c*Sec[e + f*x])) + (2*a*g^(3/2)*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqr
t[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{(acg\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{gx}}{(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(ag^2\tan(e+ \cdots}{f\sqrt{\cdots}}$$

$$(2ag\tan(e+ \cdots$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\cdots}{f\cdots}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(2ag\tan(e+ \cdots}{f\sqrt{\cdots}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{2ag^{3/2}\tan^{-1} \cdots}{\sqrt{c}\,f\sqrt{a \cdots}}$$

Mathematica [A]
time = 2.27, size = 162, normalized size = 1.56

$$\frac{2\cot\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^{3/2}\sqrt{a(1+\sec(e+fx))}\left(\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}+\left(\log(1+\sec(e+fx))-\log\left(\sqrt{\sec(e+fx)}+\sec^{\frac{3}{2}}(e+fx)+\sqrt{1+\sec(e+fx)}\sqrt{\tan^2(e+fx)}\right)\right)\sqrt{\tan^2(e+fx)}\right)}{cf\sec^{\frac{3}{2}}(e+fx)(1+\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e +
f*x]),x]
```

```
[Out] (2*Cot[(e + f*x)/2]*(g*Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt
[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (Log[1 + Sec[e + f*x]] - Log[Sqrt[S
ec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x
]^2]])*Sqrt[Tan[e + f*x]^2]))/(c*f*Sec[e + f*x]^(3/2)*(1 + Sec[e + f*x])^(3
/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(88) = 176.
time = 4.12, size = 236, normalized size = 2.27

method	result

default	$-\frac{\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{\cos(fx+e)+1}}\,(1+\cos(fx+e)+\sin(fx+e))}{2}\right)\cos(fx+e)+\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{\cos(fx+e)+1}}\,(-1-\cos(fx+e)+\sin(fx+e))}{2}\right)\cos}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/c/f*(arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)+sin(f*x+e)))*cos
(f*x+e)+arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(-1-cos(f*x+e)+sin(f*x+e)))*co
s(f*x+e)+2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)-arctanh(1/2*(1/(cos(f*x+e)+1
))^(1/2)*(1+cos(f*x+e)+sin(f*x+e)))-arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(-
1-cos(f*x+e)+sin(f*x+e))))*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-1+cos(f*x+
e))*cos(f*x+e)^2*(g/cos(f*x+e))^(3/2)/(1/(cos(f*x+e)+1))^(3/2)/sin(f*x+e)^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(94) = 188$.
time = 0.64, size = 1051, normalized size = 10.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sqrt(2)*g*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*sqrt(2)*g*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) +
(g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(
2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (g*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f*x
```

```
 + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)
*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) + (g*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin
(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 2))*sqrt(a)*sqrt(g)/((c*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 - 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + c)*f)
```

Fricas [**A**]
time = 1.62, size = 369, normalized size = 3.55

$$\left[\frac{\sqrt{ag}\,g\log\left(\frac{ag\cos(fx+e)^3-7\,ag\cos(fx+e)^2+4\,\sqrt{ag}\,\left(\cos(fx+e)^2-2\,\cos(fx+e)\right)\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{g}{\cos\left(fx+e\right)}}\,\sin(fx+e)+8\,ag}{\cos(fx+e)^3+\cos(fx+e)^2}\right)\sin\left(fx+e\right)+4\,g\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{g}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{2\,cf\sin\left(fx+e\right)}, -\frac{\sqrt{-ag}\,g\arctan\left(\frac{2\,\sqrt{-ag}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{g}{\cos\left(fx+e\right)}}\,\cos(fx+e)\sin(fx+e)}{ag\cos(fx+e)^2-ag\cos(fx+e)-2\,ag}\right)\sin\left(fx+e\right)-2\,g\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{g}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{cf\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2 + 4*sqrt(a
*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 + cos(f*x + e
)^2))*sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos
(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), -(sqrt(-a*g)*g*arctan(2*sqrt(-
a*g)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x +
 e)*sin(f*x + e)/(a*g*cos(f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g))*sin(f*x +
 e) - 2*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(
f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}\sqrt{a\sec\left(e+fx\right)+a}}{\sec\left(e+fx\right)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral((g*sec(e + f*x))**(3/2)*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) -
1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(-sqrt(a*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e)
- c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c-\frac{c}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c - c/cos(e + f*x)
),x)
```

```
[Out] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c - c/cos(e + f*x)
), x)
```

3.179 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=81

$$-\frac{\operatorname{ArcTan}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,cf}+\frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

```
[Out] -1/2*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1
/2)/a^(1/2)+cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A]
time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4049, 79, 65, 214}

$$-\frac{\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}-\frac{\tan(e+fx)\tanh^{-1}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -(Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]))) - (ArcTan
h[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c
]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{x}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(a\tan(e+fx}{2f\sqrt{}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} - \frac{(a\tan(e+fx}{cf\sqrt{}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} - \frac{\tanh^{-1}\Big(}{\sqrt{2}\,\sqrt{c}\,f\sqrt{a}}$$

Mathematica [A]
time = 0.47, size = 73, normalized size = 0.90

$$-\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(-2+\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{2cf\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -1/2*(Cot[(e + f*x)/2]*(-2 + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]
]*Sqrt[-1 + Sec[e + f*x]]))/(c*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(70) = 140$.
time = 3.11, size = 204, normalized size = 2.52

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^2(fx+e))\ln\left(\frac{\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cos(fx+e)+1}{\sin(fx+e)}\right)-\sqrt{-\frac{2\cos(fx+}{\cos(fx+e)}}\right.}{2cf(\cos^2(fx+e)-1)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2/c/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*((-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*cos(f*x+e)^2*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-co
s(f*x+e)+1)/sin(f*x+e))-(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*ln((sin(f*x+e)
*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/sin(f*x+e))+2*cos(f*x+e
)*sin(f*x+e))/(cos(f*x+e)^2-1)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] -integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)),
x)
```

Fricas [A]
time = 1.74, size = 282, normalized size = 3.48

$$\left[\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+4\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{4\,acf\sin(fx+e)}, \frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{a}\sin(fx+e)}\right)\sin(fx+e)+2\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)}{2\,acf\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
```

```
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqr
t(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(sec(e + f*x)**2/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*s
ec(e + f*x) + a)), x)/c
```

Giac [**A**]
time = 0.99, size = 132, normalized size = 1.63

$$\frac{\frac{\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\right)}{\sqrt{-a}\,\mathrm{csgn}(\cos(fx+e))}-\frac{4\sqrt{2}\sqrt{-a}}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)\mathrm{csgn}(\cos(fx+e))}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/4*(sqrt(2)*log((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^2)/(sqrt(-a)*c*sgn(cos(f*x + e))) - 4*sqrt(2)*sqrt(-a)/(((sqrt(
-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*c*sg
n(cos(f*x + e))))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c\,\cos(e+f\,x)-c\,\left(\frac{\cos(2\,e+2\,f\,x)}{2}+\frac{1}{2}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)
```

```
[Out] -int(1/((a + a/cos(e + f*x))^(1/2)*(c*cos(e + f*x) - c*(cos(2*e + 2*f*x)/2
+ 1/2))), x)
```

3.180 $\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=140

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,cf}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(e+fx)}\sin(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,cf}+\frac{\csc(e+fx)\sqrt{a+a\sec(e+f}{acf\sqrt{\sec(e+fx)}}$$

```
[Out] -2*arcsinh(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)+1/2*arcta
nh(1/2*sin(f*x+e)*a^(1/2)*sec(f*x+e)^(1/2)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/
c/f*2^(1/2)/a^(1/2)+csc(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f/sec(f*x+e)^(1/2
)
```

Rubi [A]
time = 0.18, antiderivative size = 213, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4049, 100, 163, 65, 223, 209, 95, 211}

$$\frac{2\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{\sin(e+fx)\sec^{\frac{3}{2}}(e+fx)}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -((Sec[e + f*x]^(3/2)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x]))) + (2*ArcTan[(Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x
]]]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*
x]]) - (ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]
]]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{x^{3/2}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\tan(e+fx)}{cf}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\tan(e+fx)}{f\sqrt{a}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(2\tan(e+f}{f\sqrt{}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} - \frac{\tan^{-1}\left(\right.}{\sqrt{2}\,\sqrt{c}\,f\sqrt{}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{2\tan^{-1}\left(\right.}{\sqrt{c}\,f\sqrt{a+}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 724 vs. 2(140) = 280.
time = 8.05, size = 724, normalized size = 5.17

$$\frac{\sec^{\frac{3}{2}}(e+fx)\sqrt{(1+\cos(e+fx))\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}\left(-\frac{2\cot(e)}{f}+\frac{\csc\left(\frac{e}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{f}+\frac{\sec\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{f}\right)\sin^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{\sqrt{a(1+\sec(e+fx))}\,(c-c\sec(e+fx))} + \frac{\cos(e+fx)\left(\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)-2\sqrt{2}\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}\sqrt{-1+\sec^2(e+fx)}\right)-\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)+2\sqrt{2}\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}\sqrt{-1+\sec^2(e+fx)}\right)\right)(1+\sec(e+fx))^{3/2}\sqrt{-1+\sec^2(e+fx)}\sin^2\left(\frac{e}{2}+\frac{fx}{2}\right)\sin(e+fx)}{2f(1+\cos(e+fx))\sqrt{2-2\cos^2(e+fx)}\sqrt{1-\cos^2(e+fx)}\sqrt{a(1+\sec(e+fx))}\,(c-c\sec(e+fx))} + \text{[illegible]}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*Sqrt[(1 + Cos[e + f*x])*Sec[e + f*x]]*Sqrt[1 + Sec[e +
f*x]]*((-2*Cot[e])/f + (Csc[e/2]*Csc[e/2 + (f*x)/2]*Sin[(f*x)/2])/f + (Sec[
e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/f)*Sin[e/2 + (f*x)/2]^2)/(Sqrt[a*(1 +
 Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e + f*x]*(Log[1 - 2*Sec[e + f*
x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]
*Sqrt[-1 + Sec[e + f*x]^2]] - Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2
*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2
]])*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e/2 + (f*x)/2]^2
```

```
*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*Sqrt[2 - 2*Cos[e + f*x]^2]*Sqrt[1 -
Cos[e + f*x]^2]*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e +
 f*x]*(-8*Log[1 + Sec[e + f*x]] + 8*Log[Sqrt[Sec[e + f*x]] + Sec[e + f*x]^(
3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Sqrt[2]*(-Log[1
- 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 +
 Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Log[1 - 2*Sec[e + f*x] - 3*Sec[
e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 +
Sec[e + f*x]^2]]))*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e
/2 + (f*x)/2]^2*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*(1 - Cos[e + f*x]^2)*
Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(120) = 240$.
time = 4.94, size = 317, normalized size = 2.26

method	result
default	$\frac{\left(\frac{1}{\cos(fx+e)}\right)^{\frac{5}{2}} \left(\cos^{3}(fx+e)\right) \sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}} \left(\sqrt{2}\, \arctan\left(\frac{\sqrt{-\frac{2}{\cos(fx+e)+1}}\,(1+\cos(fx+e)+\sin(fx+e))\sqrt{2}}{4}\right)\cos(fx+e)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/c/f*(1/cos(f*x+e))^(5/2)*cos(f*x+e)^3*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)
*(2^(1/2)*arctan(1/4*(-2/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)+sin(f*x+e))*2^
(1/2))*cos(f*x+e)-2^(1/2)*arctan(1/4*(-2/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e
)-sin(f*x+e))*2^(1/2))*cos(f*x+e)-2^(1/2)*arctan(1/4*(-2/(cos(f*x+e)+1))^(1
/2)*(1+cos(f*x+e)+sin(f*x+e))*2^(1/2))+2^(1/2)*arctan(1/4*(-2/(cos(f*x+e)+1
))^(1/2)*(1+cos(f*x+e)-sin(f*x+e))*2^(1/2))-arctan(1/2*sin(f*x+e)*(-2/(cos(
f*x+e)+1))^(1/2))*cos(f*x+e)+sin(f*x+e)*(-2/(cos(f*x+e)+1))^(1/2)+arctan(1/
2*sin(f*x+e)*(-2/(cos(f*x+e)+1))^(1/2)))/sin(f*x+e)^2/(-2/(cos(f*x+e)+1))^(
1/2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1406 vs. $2(128) = 256$.
time = 0.62, size = 1406, normalized size = 10.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqr
t(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*log(2*cos(1/4*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*lo
g(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 2) + (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*
sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt
(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/
4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 1) + (cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*log(cos(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 1) - 4*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 4*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))/((sqrt(2)*
c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

Fricas [A]

time = 2.35, size = 502, normalized size = 3.59

$$\left[\frac{\sqrt{2}\,\sqrt{a}\,\log\left(-\frac{\cos\left(fx+e\right)^{2}-\frac{2\,\sqrt{2}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\cos\left(fx+e\right)}\,\sin\left(fx+e\right)}{\sqrt{a}}-2\,\cos\left(fx+e\right)-3}{\cos\left(fx+e\right)^{2}+2\,\cos\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+2\,\sqrt{a}\,\log\left(\frac{a\cos\left(fx+e\right)^{3}-7\,a\cos\left(fx+e\right)^{2}+\frac{4\,\left(\cos\left(fx+e\right)^{2}-2\,\cos\left(fx+e\right)\right)\sqrt{a}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{\sqrt{\cos\left(fx+e\right)}}+8\,a}{\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}}\right)\sin\left(fx+e\right)+4\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\cos\left(fx+e\right)}}{4\,acf\sin\left(fx+e\right)},-\frac{\sqrt{2}\,a\sqrt{-\frac{1}{a}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{-\frac{1}{a}}\,\sqrt{\cos\left(fx+e\right)}}{\sin\left(fx+e\right)}\right)\sin\left(fx+e\right)+2\,\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\cos\left(fx+e\right)}\,\sin\left(fx+e\right)}{a\cos\left(fx+e\right)^{2}-a\cos\left(fx+e\right)-2\,a}\right)\sin\left(fx+e\right)-2\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\cos\left(fx+e\right)}}{2\,acf\sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(a)*log(-(cos(f*x + e)^2 - 2*sqrt(2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(cos(f*x + e))*sin(f*x + e)/sqrt(a) - 2*cos(f*x + e
) - 3)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 2*sqrt(a)*log(
(a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2*cos(f*x + e)
)*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)/sqrt(cos(f*x
 + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))*sin(f*x + e) + 4*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x + e)), -1
/2*(sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*sqrt(cos(f*x + e))/sin(f*x + e))*sin(f*x + e) + 2*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)
)*sin(f*x + e)/(a*cos(f*x + e)^2 - a*cos(f*x + e) - 2*a))*sin(f*x + e) - 2*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x
+ e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-sec(f*x + e)^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c
)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
[Out] int((1/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

3.181 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=116

$$-\frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\,cf}+\frac{g\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+f}{acf}$$

```
[Out] -1/2*g^(3/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2^(1/2)/(g*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2)+g*cot(f*x+e)*(g*sec(f*x+e
))^(1/2)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A]
time = 0.19, antiderivative size = 150, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4049, 96, 95, 211}

$$\frac{g^{3/2}\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),
x]
```

```
[Out] -((g*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*
Sec[e + f*x]))) + (g^(3/2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(S
qrt[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c]*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{(acg\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{gx}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+f\ldots\right.}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(ag^2\tan(e+\ldots}{\ldots}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(ag^2\tan(e+\ldots}{f\ldots}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{g^{3/2}\tan^{-1}\left(\ldots\right.}{\sqrt{2}\,\sqrt{c}\,f\sqrt{\ldots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 236 vs. 2(116) = 232.
time = 2.83, size = 236, normalized size = 2.03

$$-\frac{a\cos\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^{5/2}\sin^3\left(\frac{1}{2}(e+fx)\right)\left(-4-4\sec(e+fx)+\frac{\left(\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)-2\sqrt{2}\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}\sqrt{\tan^2(e+fx)}\right)-\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)+2\sqrt{2}\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}\sqrt{\tan^2(e+fx)}\right)\right)\sqrt{\tan^2(e+fx)}}{\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}}\right)}{cfg(-1+\sec(e+fx))^2(a(1+\sec(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f
*x])),x]
```

```
[Out] -((a*Cos[(e + f*x)/2]*(g*Sec[e + f*x])^(5/2)*Sin[(e + f*x)/2]^3*(-4 - 4*Sec
[e + f*x] + ((Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Se
c[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 - 2*Sec[e
+ f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f
*x]]*Sqrt[Tan[e + f*x]^2]])*Sqrt[Tan[e + f*x]^2])/Sqrt[Sec[(e + f*x)/2]^2])
)/(c*f*g*(-1 + Sec[e + f*x])^2*(a*(1 + Sec[e + f*x]))^(3/2)))
```

Maple [A]
time = 3.98, size = 152, normalized size = 1.31

method	result
default	$\frac{\left(\operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\sqrt{2}\,\cos(fx+e)-\operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\sqrt{2}-2\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\right)\left(\frac{g}{\cos(fx+e)}\right)^{\frac{3}{2}}(-1+\cos(\ldots}{2cf\left(\frac{1}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\sin(fx+e)^{4}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/2/c/f*(arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(1/2)*cos(f*x+e)-arcsinh((-1
+cos(f*x+e))/sin(f*x+e))*2^(1/2)-2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2))*(g/
cos(f*x+e))^(3/2)*(-1+cos(f*x+e))*cos(f*x+e)^2*(a*(cos(f*x+e)+1)/cos(f*x+e)
)^(1/2)/(1/(cos(f*x+e)+1))^(3/2)/sin(f*x+e)^4/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(103) = 206$.
time = 0.60, size = 576, normalized size = 4.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (g*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f
```

```
*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*g*sin(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(g)/((sqrt(2)*c*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

Fricas [A]
time = 1.86, size = 356, normalized size = 3.07

$$\left[\frac{\sqrt{2}\, a g \sqrt{\frac{g}{a}} \log\left(-\frac{2\sqrt{2}\sqrt{\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+g\cos(fx+e)^2-2g\cos(fx+e)-3g}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+4g\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)}{4acf\sin(fx+e)}, \frac{\sqrt{2}\, a g \sqrt{-\frac{g}{a}} \arctan\left(\frac{\sqrt{2}\sqrt{-\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)}{g\sin(fx+e)}\right)\sin(fx+e)+2g\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)}{2acf\sin(fx+e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*g*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos(f
*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*
sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x +
 e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*a*g*sqrt(-g/a)*arctan
(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x
+ e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*g*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) -
sqrt(a*sec(e + f*x) + a)), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(-(g*sec(f*x + e))^(3/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e)
 - c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

3.182 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx$

Optimal. Leaf size=179

$$-\frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,cf}+\frac{g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e-}\right.}{\sqrt{2}\sqrt{a}\,cf}$$

```
[Out] -2*g^(5/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec
(f*x+e))^(1/2))/c/f/a^(1/2)+1/2*g^(5/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x
+e)*2^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2
)+g^2*cot(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A]
time = 0.22, antiderivative size = 242, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4049, 100, 163, 65, 223, 209, 95, 211}

$$\frac{2g^{5/2}\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g^{5/2}\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}\,f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g^2\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}\,(c-c\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),
x]
```

```
[Out] -((g^2*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c -
c*Sec[e + f*x]))) + (2*g^(5/2)*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqrt[
g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]]) - (g^(5/2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Se
c[e + f*x]])/(Sqrt[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
```

```
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))}\,dx = -\frac{(acg\tan(e+fx))\text{Subst}\left(\int \frac{(gx)^{3/2}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+f\ldots\right)}{f\sqrt{a+a\sec(e+fx)}\,\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(g\tan(e+\ldots}{c\ldots}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{(g^3\tan(e+\ldots}{f\ldots}$$

$$(2g^2\tan(e+\ldots$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{\ldots}{f\ldots}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} - \frac{g^{5/2}\tan^{-1}\left(\ldots\right.}{\sqrt{2}\,\sqrt{c}\,f\sqrt{\ldots}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\,\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\,(c-c\sec(e+fx))} + \frac{2g^{5/2}\tan^{-1}\ldots}{\sqrt{c}\,f\sqrt{a\ldots}}$$

Mathematica [A]
time = 2.19, size = 328, normalized size = 1.83

$$-\frac{(g\sec(e+fx))^{5/2}\sqrt{1+\sec(e+fx)}\,\sin^3(e+fx)\left(8\sqrt{\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}+\left(16\log(1+\sec(e+fx))-16\log\left(\sqrt{\sec(e+fx)}+\sec^{\frac{3}{2}}(e+fx)+\sqrt{1+\sec(e+fx)}\,\sqrt{\tan^2(e+fx)}\right)+\sqrt{2}\left(\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)-2\sqrt{2}\,\sqrt{\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}\,\sqrt{\tan^2(e+fx)}\right)-\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)+2\sqrt{2}\,\sqrt{\sec(e+fx)}\,\sqrt{1+\sec(e+fx)}\,\sqrt{\tan^2(e+fx)}\right)\right)\right)\sqrt{\tan^2(e+fx)}\right)}{8cf(-1+\cos(e+fx))(1+\cos(e+fx))^2(-1+\sec(e+fx))\sec^{\frac{5}{2}}(e+fx)\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f
*x])),x]
```

```
[Out] -1/8*((g*Sec[e + f*x])^(5/2)*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]^3*(8*Sqrt[
Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (16*Log[1 + Sec[e + f*x]] - 16*Log[S
qrt[Sec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e
+ f*x]^2]] + Sqrt[2]*(Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]
*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 -
2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + S
```

```
ec[e + f*x]]*Sqrt[Tan[e + f*x]^2]]))*Sqrt[Tan[e + f*x]^2]))/(c*f*(-1 + Cos[
e + f*x])*(1 + Cos[e + f*x])^2*(-1 + Sec[e + f*x])*Sec[e + f*x]^(5/2)*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [A]
time = 4.50, size = 294, normalized size = 1.64

method	result
default	$\frac{\left(\operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\sqrt{2}\,\cos(fx+e)-\operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\sqrt{2}+2\sin(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}+2\operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{\cos(fx+e}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/2/c/f*(arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(1/2)*cos(f*x+e)-arcsinh((-1
+cos(f*x+e))/sin(f*x+e))*2^(1/2)+2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)+2*ar
ctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(-1-cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)+2
*arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)+sin(f*x+e)))*cos(f*x+e)
-2*arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(-1-cos(f*x+e)+sin(f*x+e)))-2*arcta
nh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)+sin(f*x+e))))*(g/cos(f*x+e))^
(5/2)*(-1+cos(f*x+e))^2*cos(f*x+e)^3*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(1
/(cos(f*x+e)+1))^(5/2)/sin(f*x+e)^6/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1496 vs. 2(158) = 316.
time = 0.63, size = 1496, normalized size = 8.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*g^2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g^2*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 4*g^2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (sqrt(
2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(
1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x +
```

```
2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 2) + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*
cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*g^2*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2)
 + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sq
rt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2
)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*l
og(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 2) + (g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 - 2*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g^2)*log(c
os(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 1) - (g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*
g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g^2)*log(cos(1/4
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1))*sqrt(g)/((sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + sqrt(2)*c)*sqrt(a)*f)
```

Fricas [A]
time = 2.57, size = 614, normalized size = 3.43

$$\left[\frac{\sqrt{2}\, a g^{2} \sqrt{\frac{g}{a}} \log\left(\frac{2 \sqrt{2} \sqrt{\frac{g}{a}} \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{g}{\cos\left(fx+e\right)}} \cos\left(fx+e\right) \sin\left(fx+e\right) - g \cos\left(fx+e\right)^{2} + 2 g \cos\left(fx+e\right) + 3 g}{\cos\left(fx+e\right)^{2} + 2 \cos\left(fx+e\right) + 1}\right) \sin\left(fx+e\right) + 2 a g^{2} \sqrt{\frac{g}{a}} \log\left(\frac{g \cos\left(fx+e\right)^{3} + 4 \left(\cos\left(fx+e\right)^{2} - 2 \cos\left(fx+e\right)\right) \sqrt{\frac{g}{a}} \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{g}{\cos\left(fx+e\right)}} \sin\left(fx+e\right) - 7 g \cos\left(fx+e\right)^{2} + 8 g}{\cos\left(fx+e\right)^{3} + \cos\left(fx+e\right)^{2}}\right) \sin\left(fx+e\right) + 4 g^{2} \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{g}{\cos\left(fx+e\right)}} \cos\left(fx+e\right)}{4 a c f \sin\left(fx+e\right)}, -\frac{\sqrt{2}\, a g^{2} \sqrt{-\frac{g}{a}} \arctan\left(\frac{\sqrt{2} \sqrt{-\frac{g}{a}} \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{g}{\cos\left(fx+e\right)}} \cos\left(fx+e\right)}{g \sin\left(fx+e\right)}\right) \sin\left(fx+e\right) + 2 a g^{2} \sqrt{-\frac{g}{a}} \arctan\left(\frac{2 \sqrt{-\frac{g}{a}} \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{g}{\cos\left(fx+e\right)}} \cos\left(fx+e\right) \sin\left(fx+e\right)}{g \cos\left(fx+e\right)^{2} - g \cos\left(fx+e\right) - 2 g}\right) \sin\left(fx+e\right) - 2 g^{2} \sqrt{\frac{a \cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}} \sqrt{\frac{g}{\cos\left(fx+e\right)}} \cos\left(fx+e\right)}{2 a c f \sin\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*g^2*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(
```

```
f*x + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
*sin(f*x + e) + 2*a*g^2*sqrt(g/a)*log((g*cos(f*x + e)^3 + 4*(cos(f*x + e)^2
 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g
/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g)/(cos(f*x + e)^3 + c
os(f*x + e)^2))*sin(f*x + e) + 4*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), -1/2*(sqrt(2)*a*
g^2*sqrt(-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*
a*g^2*sqrt(-g/a)*arctan(2*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(
f*x + e) - 2*g))*sin(f*x + e) - 2*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
[Out] integrate(-(g*sec(f*x + e))^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e)
 - c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
[Out] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

3.183 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\log(\tan(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(tan(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {4048, 2700, 29}

$$\frac{\tan(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Tan[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4048

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[(
(-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e
+ f*x]])), Int[(g*Csc[e + f*x])^p*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b
, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && Intege
rQ[m + 1/2]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)\int \csc(e+fx)\sec(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\log(\tan(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.81, size = 94, normalized size = 2.04

$$\frac{4i\left(-1+e^{i(e+fx)}\right)\tanh^{-1}\left(e^{2i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}{\left(1+e^{i(e+fx)}\right)f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] ((4*I)*(-1 + E^(I*(e + f*x)))*ArcTanh[E^((2*I)*(e + f*x))]*Cos[(e + f*x)/2]
^2*Sec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(42) = 84$.
time = 3.20, size = 139, normalized size = 3.02

method	result
default	$\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\ln\left(-\frac{\cos(fx+e)-1+\sin(fx+e)}{\sin(fx+e)}\right)+\ln\left(-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\right)\cos(fx+e)\sqrt{\frac{c(-1+\mathrm{co}}{\cos(f}}}{f\sin(fx+e)ca}$
risch	$\frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{2i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}-\frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{2i(fx+e)}-1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{e^{2i(fx+e)}+1}}\left(e^{2i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{e^{2i(fx+e)}+1}}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(ln(-(cos(f*x+e)-1+sin(f*x+e))/sin(
f*x+e))+ln(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))-ln(-(-1+cos(f*x+e))/sin(
f*x+e)))*cos(f*x+e)*(c*(-1+cos(f*x+e))/cos(f*x+e))^(1/2)/sin(f*x+e)/c/a
```

Maxima [A]
time = 0.61, size = 60, normalized size = 1.30

$$-\frac{\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)-\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)-1\right)}{\sqrt{a}\,\sqrt{c}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -(arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt(c)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(46) = 92$.
time = 2.44, size = 274, normalized size = 5.96

$$\left[-\frac{\sqrt{-ac}\,\log\left(-\frac{8\left(2\,\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)\right)\sqrt{-ac}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}+\left(2\,ac\cos\left(fx+e\right)^{4}-2\,ac\cos\left(fx+e\right)^{2}+ac\right)\sin\left(fx+e\right)}{\left(\cos\left(fx+e\right)^{4}-\cos\left(fx+e\right)^{2}\right)\sin\left(fx+e\right)}\right)}{2\,acf},\ \frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{\frac{a\cos\left(fx+e\right)+a}{\cos\left(fx+e\right)}}\,\sqrt{\frac{c\cos\left(fx+e\right)-c}{\cos\left(fx+e\right)}}\,\cos\left(fx+e\right)}{\left(2\,ac\cos\left(fx+e\right)^{2}-ac\right)\sin\left(fx+e\right)}\right)}{acf}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-8*((2*cos(f*x + e)^3 - cos(f*x + e))*sqrt(-a*c)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))
+ (2*a*c*cos(f*x + e)^4 - 2*a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f
*x + e)^4 - cos(f*x + e)^2)*sin(f*x + e)))/(a*c*f), sqrt(a*c)*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/((2*a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)))/(a*c*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(e+fx)^2\sqrt{a+\frac{a}{\cos(e+fx)}}\,\sqrt{c-\frac{c}{\cos(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)
), x)
```

3.184 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx$

Optimal. Leaf size=65

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{c-d}\sqrt{d}f}$$

```
[Out] 2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(c-d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^
(1/2)/f/(c-d)^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4052, 214}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c - d]*Sqrt[a + a*S
ec[e + f*x]])])/(Sqrt[c - d]*Sqrt[d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = -\frac{(2a)\mathrm{Subst}\left(\int \frac{1}{ac-ad-dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{c-d}\sqrt{d}\,f}$$

Mathematica [**A**]
time = 0.27, size = 98, normalized size = 1.51

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c-d}\sqrt{\cos(e+fx)}}\right)\sqrt{\cos(e+fx)}\,\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{c-d}\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c - d]*Sqrt[Cos[e
 + f*x]])])*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/
(Sqrt[c - d]*Sqrt[d]*f)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(51) = 102$.
time = 8.21, size = 412, normalized size = 6.34

method	result
default	$-\frac{\left(\ln\left(\frac{2\sqrt{-\frac{2d}{c+d}}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,c\sin(fx+e)+2\sqrt{-\frac{2d}{c+d}}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,d\sin(fx+e)+2\sqrt{(c+d)(c-d)}\,\cos(f}{\sqrt{(c+d)(c-d)}\,\sin(fx+e)-c\cos(fx+e)-d\cos(fx+e)+c+d}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/f*(ln(2*((-2*d/(c+d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f
*x+e)+(-2*d/(c+d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+
((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2
))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)-d*cos(f*x+e)+c+d))-ln(2*(-(
-2*d/(c+d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-(-2*d/(
```

```
c+d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))
^(1/2)*cos(f*x+e)+c*sin(f*x+e)+d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c
-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)+d*cos(f*x+e)-c-d))*(-2*cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(-2*d/(c+d))^(1/2)/((c
+d)*(c-d))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) - c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(53) = 106$.
time = 2.77, size = 374, normalized size = 5.75

$$\left[\frac{\sqrt{\frac{a}{cd-d^2}}\log\left(-\frac{(ac^2-8acd+8ad^2)\cos(fx+e)^3+ad^2+(ac^2-2acd)\cos(fx+e)^2+4\left((c^2d-3cd^2+2d^3)\cos(fx+e)^2+(cd^2-d^3)\cos(fx+e)\right)\sqrt{\frac{a}{cd-d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)+(6acd-7ad^2)\cos(fx+e)}{c^2\cos(fx+e)^3+(c^2-2cd)\cos(fx+e)^2+d^2-(2cd-d^2)\cos(fx+e)}\right)}{2f}, -\frac{\sqrt{-\frac{a}{cd-d^2}}\arctan\left(\frac{2(cd-d^2)\sqrt{-\frac{a}{cd-d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{(ac-2ad)\cos(fx+e)^2+ad+(ac-ad)\cos(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*sqrt(a/(c*d - d^2))*log(-((a*c^2 - 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 +
 a*d^2 + (a*c^2 - 2*a*c*d)*cos(f*x + e)^2 + 4*((c^2*d - 3*c*d^2 + 2*d^3)*co
s(f*x + e)^2 + (c*d^2 - d^3)*cos(f*x + e))*sqrt(a/(c*d - d^2))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (6*a*c*d - 7*a*d^2)*cos(f*x + e)
)/(c^2*cos(f*x + e)^3 + (c^2 - 2*c*d)*cos(f*x + e)^2 + d^2 - (2*c*d - d^2)*
cos(f*x + e)))/f, -sqrt(-a/(c*d - d^2))*arctan(2*(c*d - d^2)*sqrt(-a/(c*d -
 d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((
a*c - 2*a*d)*cos(f*x + e)^2 + a*d + (a*c - a*d)*cos(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a(\sec(e+fx)+1)}\sec(e+fx)}{c-d\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c - d*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(51) = 102$.
time = 0.91, size = 136, normalized size = 2.09

$$\frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c+\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2d+ac-3ad\right)}{4\sqrt{cd-d^2}a}\right)\mathrm{sgn}(\cos(fx+e))}{\sqrt{cd-d^2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] 2*sqrt(-a)*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^2*c + (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c - 3*a*d)/(sqrt(c*d - d^2)*a))*sgn(cos(
f*x + e))/(sqrt(c*d - d^2)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{d-c\,\cos(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - d/cos(e + f*x))),x)
```

```
[Out] -int((a + a/cos(e + f*x))^(1/2)/(d - c*cos(e + f*x)), x)
```

3.185 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=236

$$\frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\tan(e+fx)}{30f}$$

```
[Out] 1/8*a*(8*c^4+16*c^3*d+24*c^2*d^2+12*c*d^3+3*d^4)*arctanh(sin(f*x+e))/f+1/30
*a*(12*c^4+95*c^3*d+112*c^2*d^2+80*c*d^3+16*d^4)*tan(f*x+e)/f+1/120*a*d*(24
*c^3+130*c^2*d+116*c*d^2+45*d^3)*sec(f*x+e)*tan(f*x+e)/f+1/60*a*(12*c^2+35*
c*d+16*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/20*a*(4*c+5*d)*(c+d*sec(f*x+e
))^3*tan(f*x+e)/f+1/5*a*(c+d*sec(f*x+e))^4*tan(f*x+e)/f
```

Rubi [A]
time = 0.30, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{a(12c^2+35cd+16d^2)\tan(e+fx)(c+d\sec(e+fx))^2}{60f}+\frac{ad(24c^3+130c^2d+116cd^2+45d^3)\tan(e+fx)\sec(e+fx)}{120f}+\frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\tan(e+fx)}{30f}+\frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}+\frac{a(4c+5d)\tan(e+fx)(c+d\sec(e+fx))^3}{20f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f*x]]
)/(8*f) + (a*(12*c^4 + 95*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Tan[e +
f*x])/(30*f) + (a*d*(24*c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Sec[e + f*x]*
Tan[e + f*x])/(120*f) + (a*(12*c^2 + 35*c*d + 16*d^2)*(c + d*Sec[e + f*x])^
2*Tan[e + f*x])/(60*f) + (a*(4*c + 5*d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(20*f) + (a*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx &= \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f}+\frac{1}{5}\int \sec(e+ \\
&= \frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{a(\cdots}{} \\
&= \frac{a(12c^2+35cd+16d^2)\,(c+d\sec(e+fx))^2\tan(e}{60f} \\
&= \frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(e+fx)\,t}{120f} \\
&= \frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(e+fx)\,t}{120f} \\
&= \frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\tanh^{-1}(si}{8f} \\
&= \frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\tanh^{-1}(si}{8f}
\end{aligned}$$

Mathematica [A]

time = 1.82, size = 153, normalized size = 0.65

$$\frac{a\left(15\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(120(c+d)^4+15d\left(16c^3+24c^2d+12cd^2+3d^3\right)\sec(e+fx)+30d^3(4c+d)\sec^3(e+fx)+80d^2\left(3c^2+2cd+d^2\right)\tan^2(e+fx)+24d^4\tan^4(e+fx)\right)\right)}{120f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(15*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f
*x]] + Tan[e + f*x]*(120*(c + d)^4 + 15*d*(16*c^3 + 24*c^2*d + 12*c*d^2 + 3
*d^3)*Sec[e + f*x] + 30*d^3*(4*c + d)*Sec[e + f*x]^3 + 80*d^2*(3*c^2 + 2*c*
d + d^2)*Tan[e + f*x]^2 + 24*d^4*Tan[e + f*x]^4)))/(120*f)
```

Maple [A]
time = 0.40, size = 313, normalized size = 1.33 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^4*tan(f*x+e)+4*a*c^3*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e
)+tan(f*x+e)))-6*a*c^2*d^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+4*a*c*d^3*(-(
-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
-a*d^4*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+a*c^4*ln(sec(f
*x+e)+tan(f*x+e))+4*a*c^3*d*tan(f*x+e)+6*a*c^2*d^2*(1/2*sec(f*x+e)*tan(f*x+
e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-4*a*c*d^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x
+e)+a*d^4*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)
+tan(f*x+e))))
```

Maxima [A]
time = 0.29, size = 410, normalized size = 1.74

$$\frac{480\left(\tan(fx+e)^3+3\tan(fx+e)\right)ac^2d^2+320\left(\tan(fx+e)^3+3\tan(fx+e)\right)acd^3+16\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)ad^4-60\,acd^3\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log(\sin(fx+e)-1)\right)-15\,ad^4\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log(\sin(fx+e)-1)\right)-240\,ac^3d\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-360\,ac^2d^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+240\,ac^4\log(\sec(fx+e)+\tan(fx+e))+240\,ac^4\tan(fx+e)+960\,ac^3d\tan(fx+e)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2*d^2 + 320*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15
*tan(f*x + e))*a*d^4 - 60*a*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) - 240*a*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
 e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*lo
```

```
g(sec(f*x + e) + tan(f*x + e)) + 240*a*c^4*tan(f*x + e) + 960*a*c^3*d*tan(f
*x + e))/f
```

Fricas [A]
time = 2.35, size = 291, normalized size = 1.23

$$\frac{15\left(8ac^4+16ac^3d+24ac^2d^2+12acd^3+3ad^4\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(8ac^4+16ac^3d+24ac^2d^2+12acd^3+3ad^4\right)\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)+2\left(24ad^4+8\left(15ac^4+60ac^3d+60ac^2d^2+40acd^3+8ad^4\right)\cos\left(fx+e\right)^4+15\left(16ac^3d+24ac^2d^2+12acd^3+3ad^4\right)\cos\left(fx+e\right)^3+16\left(15ac^2d^2+10acd^3+2ad^4\right)\cos\left(fx+e\right)^2+30\left(4acd^3+ad^4\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{240f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(
f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2
+ 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*a*d^4
 + 8*(15*a*c^4 + 60*a*c^3*d + 60*a*c^2*d^2 + 40*a*c*d^3 + 8*a*d^4)*cos(f*x
+ e)^4 + 15*(16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)
^3 + 16*(15*a*c^2*d^2 + 10*a*c*d^3 + 2*a*d^4)*cos(f*x + e)^2 + 30*(4*a*c*d^
3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^{4}\sec{\left(e+fx\right)}\,dx+\int c^{4}\sec^{2}{\left(e+fx\right)}\,dx+\int d^{4}\sec^{5}{\left(e+fx\right)}\,dx+\int d^{4}\sec^{6}{\left(e+fx\right)}\,dx+\int 4cd^{3}\sec^{4}{\left(e+fx\right)}\,dx+\int 4cd^{3}\sec^{5}{\left(e+fx\right)}\,dx+\int 6c^{2}d^{2}\sec^{3}{\left(e+fx\right)}\,dx+\int 6c^{2}d^{2}\sec^{4}{\left(e+fx\right)}\,dx+\int 4c^{3}d\sec^{2}{\left(e+fx\right)}\,dx+\int 4c^{3}d\sec^{3}{\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(c**4*sec(e + f*x), x) + Integral(c**4*sec(e + f*x)**2, x) + Int
egral(d**4*sec(e + f*x)**5, x) + Integral(d**4*sec(e + f*x)**6, x) + Integr
al(4*c*d**3*sec(e + f*x)**4, x) + Integral(4*c*d**3*sec(e + f*x)**5, x) + I
ntegral(6*c**2*d**2*sec(e + f*x)**3, x) + Integral(6*c**2*d**2*sec(e + f*x)
**4, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(4*c**3*d*sec(e +
 f*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(224) = 448.
time = 0.52, size = 566, normalized size = 2.40

$$\frac{15\left(8ac^4+16ac^3d+24ac^2d^2+12acd^3+3ad^4\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15\left(8ac^4+16ac^3d+24ac^2d^2+12acd^3+3ad^4\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(120ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+240ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+360ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+180acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+45ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9-480ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-1440ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-1200ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-1160acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-130ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+720ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+2880ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+2400ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+1600acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+464ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-480ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2400ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2640ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-1400acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-190ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+120ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+720ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1080ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+780acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+195ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^5}}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*log(
abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 +
```

```
12*a*c*d^3 + 3*a*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(120*a*c^4*tan
(1/2*f*x + 1/2*e)^9 + 240*a*c^3*d*tan(1/2*f*x + 1/2*e)^9 + 360*a*c^2*d^2*ta
n(1/2*f*x + 1/2*e)^9 + 180*a*c*d^3*tan(1/2*f*x + 1/2*e)^9 + 45*a*d^4*tan(1/
2*f*x + 1/2*e)^9 - 480*a*c^4*tan(1/2*f*x + 1/2*e)^7 - 1440*a*c^3*d*tan(1/2*
f*x + 1/2*e)^7 - 1200*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 1160*a*c*d^3*tan(1
/2*f*x + 1/2*e)^7 - 130*a*d^4*tan(1/2*f*x + 1/2*e)^7 + 720*a*c^4*tan(1/2*f*
x + 1/2*e)^5 + 2880*a*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 2400*a*c^2*d^2*tan(1/2
*f*x + 1/2*e)^5 + 1600*a*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 464*a*d^4*tan(1/2*f
*x + 1/2*e)^5 - 480*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 2400*a*c^3*d*tan(1/2*f*x
 + 1/2*e)^3 - 2640*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 1400*a*c*d^3*tan(1/2*
f*x + 1/2*e)^3 - 190*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 120*a*c^4*tan(1/2*f*x +
 1/2*e) + 720*a*c^3*d*tan(1/2*f*x + 1/2*e) + 1080*a*c^2*d^2*tan(1/2*f*x + 1
/2*e) + 780*a*c*d^3*tan(1/2*f*x + 1/2*e) + 195*a*d^4*tan(1/2*f*x + 1/2*e))/
(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

Mupad [B]
time = 5.51, size = 361, normalized size = 1.53

$$\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{2\,\left(4\,c^4+8\,c^3\,d+12\,c^2\,d^2+6\,c\,d^3+\frac{3\,d^4}{2}\right)}\right)\,\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{4\,f}-\frac{\left(2\,a\,c^4+4\,a\,c^3\,d+6\,a\,c^2\,d^2+3\,a\,c\,d^3+\frac{3\,a\,d^4}{4}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9+\left(-8\,a\,c^4-24\,a\,c^3\,d-20\,a\,c^2\,d^2-\frac{58\,a\,c\,d^3}{3}-\frac{13\,a\,d^4}{6}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+\left(12\,a\,c^4+48\,a\,c^3\,d+40\,a\,c^2\,d^2+\frac{80\,a\,c\,d^3}{3}+\frac{116\,a\,d^4}{15}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(-8\,a\,c^4-40\,a\,c^3\,d-44\,a\,c^2\,d^2-\frac{70\,a\,c\,d^3}{3}-\frac{19\,a\,d^4}{6}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(2\,a\,c^4+12\,a\,c^3\,d+18\,a\,c^2\,d^2+13\,a\,c\,d^3+\frac{13\,a\,d^4}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (a*atanh((tan(e/2 + (f*x)/2)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*
d^2))/(2*(6*c*d^3 + 8*c^3*d + 4*c^4 + (3*d^4)/2 + 12*c^2*d^2)))*(12*c*d^3 +
 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2))/(4*f) - (tan(e/2 + (f*x)/2)^9*(2*a
*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*c*d^3 + 4*a*c^3*d) - tan(e/2 + (f*x)
/2)^7*(8*a*c^4 + (13*a*d^4)/6 + 20*a*c^2*d^2 + (58*a*c*d^3)/3 + 24*a*c^3*d)
 - tan(e/2 + (f*x)/2)^3*(8*a*c^4 + (19*a*d^4)/6 + 44*a*c^2*d^2 + (70*a*c*d^
3)/3 + 40*a*c^3*d) + tan(e/2 + (f*x)/2)^5*(12*a*c^4 + (116*a*d^4)/15 + 40*a
*c^2*d^2 + (80*a*c*d^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*(2*a*c^4 + (13
*a*d^4)/4 + 18*a*c^2*d^2 + 13*a*c*d^3 + 12*a*c^3*d))/(f*(5*tan(e/2 + (f*x)/
2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x
)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.186 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=171

$$\frac{a(8c^3+12c^2d+12cd^2+3d^3)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a(3c^3+16c^2d+12cd^2+4d^3)\tan(e+fx)}{6f}+\frac{ad(6c^2+20c}{}$$

```
[Out] 1/8*a*(8*c^3+12*c^2*d+12*c*d^2+3*d^3)*arctanh(sin(f*x+e))/f+1/6*a*(3*c^3+16
*c^2*d+12*c*d^2+4*d^3)*tan(f*x+e)/f+1/24*a*d*(6*c^2+20*c*d+9*d^2)*sec(f*x+e
)*tan(f*x+e)/f+1/12*a*(3*c+4*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/4*a*(c+d*
sec(f*x+e))^3*tan(f*x+e)/f
```

Rubi [A]
time = 0.20, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{ad(6c^2+20cd+9d^2)\tan(e+fx)\sec(e+fx)}{24f}+\frac{a(3c^3+16c^2d+12cd^2+4d^3)\tan(e+fx)}{6f}+\frac{a(8c^3+12c^2d+12cd^2+3d^3)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}+\frac{a(3c+4d)\tan(e+fx)(c+d\sec(e+fx))^2}{12f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]])/(8*f) + (a*
(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(6*f) + (a*d*(6*c^2 + 2
0*c*d + 9*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + (a*(3*c + 4*d)*(c + d*Se
c[e + f*x])^2*Tan[e + f*x])/(12*f) + (a*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx &= \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}\int \sec(e\\ &= \frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{a}{}\\ &= \frac{ad(6c^2+20cd+9d^2)\sec(e+fx)\tan(e+fx)}{24f}+\\ &= \frac{ad(6c^2+20cd+9d^2)\sec(e+fx)\tan(e+fx)}{24f}+\\ &= \frac{a(8c^3+12c^2d+12cd^2+3d^3)\tanh^{-1}(\sin(e+f}{8f}\\ &= \frac{a(8c^3+12c^2d+12cd^2+3d^3)\tanh^{-1}(\sin(e+f}{8f}\end{aligned}$$

Mathematica [A]

time = 0.72, size = 103, normalized size = 0.60

$$\frac{a(3(8c^3+12c^2d+12cd^2+3d^3)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\,(24(c+d)^3+9d(2c+d)^2\sec(e+fx)+6d^3\sec^3(e+fx)+8d^2(3c+d)\tan^2(e+fx)))}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(3*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]] + Tan[e +
 f*x]*(24*(c + d)^3 + 9*d*(2*c + d)^2*Sec[e + f*x] + 6*d^3*Sec[e + f*x]^3 +
 8*d^2*(3*c + d)*Tan[e + f*x]^2)))/(24*f)
```

Maple [A]
time = 0.30, size = 223, normalized size = 1.30

method	result
derivativedivides	$\frac{a\,c^3\tan(fx+e)+3a\,c^2d\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-3ac\,d^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+a\,d^3\left(\right.}{}$
default	$\frac{a\,c^3\tan(fx+e)+3a\,c^2d\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-3ac\,d^2\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+a\,d^3\left(\right.}{}$
norman	$\frac{-\frac{a\left(8c^3+12c^2d+12c\,d^2+3d^3\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{a\left(8c^3+36c^2d+36c\,d^2+13d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{a\left(72c^3+180c^2d+84c\,d^2+49d^3\right)\left(\tan^5\left(\right.\right.}{12f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}$
risch	$\frac{ia\left(16d^3+24c^3-36c\,d^2e^{5i(fx+e)}+216c^2d\,e^{4i(fx+e)}+36c\,d^2e^{3i(fx+e)}+216c^2d\,e^{2i(fx+e)}+144c\,d^2e^{4i(fx+e)}+36c^2d\,e^{3i(fx+e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^3*tan(f*x+e)+3*a*c^2*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e
)+tan(f*x+e)))-3*a*c*d^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a*d^3*(-(-1/4*s
ec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+a*c^3
*ln(sec(f*x+e)+tan(f*x+e))+3*a*c^2*d*tan(f*x+e)+3*a*c*d^2*(1/2*sec(f*x+e)*t
an(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a*d^3*(-2/3-1/3*sec(f*x+e)^2)*tan(
f*x+e))
```

Maxima [A]
time = 0.30, size = 288, normalized size = 1.68

$$\frac{48\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)acd^2+16\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ad^3-3\,ad^3\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-36\,ac^2d\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-36\,acd^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+48\,ac^3\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)+48\,ac^3\tan\left(fx+e\right)+144\,ac^2d\tan\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c*d^2 + 16*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a*d^3 - 3*a*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(
f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 36*a*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
```

```
x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c^3*log
(sec(f*x + e) + tan(f*x + e)) + 48*a*c^3*tan(f*x + e) + 144*a*c^2*d*tan(f*x
 + e))/f
```

Fricas [A]
time = 2.00, size = 220, normalized size = 1.29

$$\frac{3\,(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3)\cos(fx+e)^4\log(\sin(fx+e)+1)-3\,(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3)\cos(fx+e)^4\log(-\sin(fx+e)+1)+2\,(6\,ad^3+8\,(3\,ac^3+9\,ac^2d+6\,acd^2+2\,ad^3)\cos(fx+e)^3+9\,(4\,ac^2d+4\,acd^2+ad^3)\cos(fx+e)^2+8\,(3\,acd^2+ad^3)\cos(fx+e))\sin(fx+e)}{48\,f\cos(fx+e)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^4*log(si
n(f*x + e) + 1) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) + 2*(6*a*d^3 + 8*(3*a*c^3 + 9*a*c^2*d + 6*a*c*
d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*a*c^2*d + 4*a*c*d^2 + a*d^3)*cos(f*x +
 e)^2 + 8*(3*a*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^3\sec(e+fx)\,dx+\int c^3\sec^2(e+fx)\,dx+\int d^3\sec^4(e+fx)\,dx+\int d^3\sec^5(e+fx)\,dx+\int 3cd^2\sec^3(e+fx)\,dx+\int 3cd^2\sec^4(e+fx)\,dx+\int 3c^2d\sec^2(e+fx)\,dx+\int 3c^2d\sec^3(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a*(Integral(c**3*sec(e + f*x), x) + Integral(c**3*sec(e + f*x)**2, x) + Int
egral(d**3*sec(e + f*x)**4, x) + Integral(d**3*sec(e + f*x)**5, x) + Integr
al(3*c*d**2*sec(e + f*x)**3, x) + Integral(3*c*d**2*sec(e + f*x)**4, x) + I
ntegral(3*c**2*d*sec(e + f*x)**2, x) + Integral(3*c**2*d*sec(e + f*x)**3, x
))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(161) = 322.
time = 0.54, size = 380, normalized size = 2.22

$$\frac{3\,(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(24\,ac^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7+36\,ac^2d\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7+36\,acd^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7+9\,ad^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7-72\,ac^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-180\,ac^2d\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-84\,acd^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-49\,ad^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5+72\,ac^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3+252\,ac^2d\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3+156\,acd^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3+31\,ad^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3-24\,ac^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-108\,ac^2d\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-108\,acd^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-39\,ad^3\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)\right)}{\left(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*log(abs(tan(1/2*f*x +
 1/2*e) + 1)) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*log(abs(tan
```

```
(1/2*f*x + 1/2*e) - 1)) - 2*(24*a*c^3*tan(1/2*f*x + 1/2*e)^7 + 36*a*c^2*d*t
an(1/2*f*x + 1/2*e)^7 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 9*a*d^3*tan(1/2
*f*x + 1/2*e)^7 - 72*a*c^3*tan(1/2*f*x + 1/2*e)^5 - 180*a*c^2*d*tan(1/2*f*x
 + 1/2*e)^5 - 84*a*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 49*a*d^3*tan(1/2*f*x + 1/
2*e)^5 + 72*a*c^3*tan(1/2*f*x + 1/2*e)^3 + 252*a*c^2*d*tan(1/2*f*x + 1/2*e)
^3 + 156*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 31*a*d^3*tan(1/2*f*x + 1/2*e)^3 -
 24*a*c^3*tan(1/2*f*x + 1/2*e) - 108*a*c^2*d*tan(1/2*f*x + 1/2*e) - 108*a*c
*d^2*tan(1/2*f*x + 1/2*e) - 39*a*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1
/2*e)^2 - 1)^4)/f
```

Mupad [B]
time = 5.34, size = 255, normalized size = 1.49

$$\frac{\left(-2\,a\,c^3-3\,a\,c^2\,d-3\,a\,c\,d^2-\frac{3\,a\,d^3}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(6\,a\,c^3+15\,a\,c^2\,d+7\,a\,c\,d^2+\frac{49\,a\,d^3}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-6\,a\,c^3-21\,a\,c^2\,d-13\,a\,c\,d^2-\frac{31\,a\,d^3}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(2\,a\,c^3+9\,a\,c^2\,d+9\,a\,c\,d^2+\frac{13\,a\,d^3}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{2\,\left(4\,c^3+6\,c^2\,d+6\,c\,d^2+\frac{3\,d^3}{2}\right)}\right)\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(2*a*c^3 + (13*a*d^3)/4 + 9*a*c*d^2 + 9*a*c^2*d) - tan(
e/2 + (f*x)/2)^7*(2*a*c^3 + (3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d) - tan(e/2
+ (f*x)/2)^3*(6*a*c^3 + (31*a*d^3)/12 + 13*a*c*d^2 + 21*a*c^2*d) + tan(e/2
+ (f*x)/2)^5*(6*a*c^3 + (49*a*d^3)/12 + 7*a*c*d^2 + 15*a*c^2*d))/(f*(6*tan(
e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/
2 + (f*x)/2)^8 + 1)) + (a*atanh((tan(e/2 + (f*x)/2)*(12*c*d^2 + 12*c^2*d +
8*c^3 + 3*d^3))/(2*(6*c*d^2 + 6*c^2*d + 4*c^3 + (3*d^3)/2)))*(12*c*d^2 + 12
*c^2*d + 8*c^3 + 3*d^3))/(4*f)
```

3.187 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=108

$$\frac{a(2c^2+2cd+d^2)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2a(c^2+3cd+d^2)\tan(e+fx)}{3f}+\frac{ad(2c+3d)\sec(e+fx)\tan(e+f$$

```
[Out] 1/2*a*(2*c^2+2*c*d+d^2)*arctanh(sin(f*x+e))/f+2/3*a*(c^2+3*c*d+d^2)*tan(f*x
+e)/f+1/6*a*d*(2*c+3*d)*sec(f*x+e)*tan(f*x+e)/f+1/3*a*(c+d*sec(f*x+e))^2*ta
n(f*x+e)/f
```

Rubi [A]
time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{2a(c^2+3cd+d^2)\tan(e+fx)}{3f}+\frac{a(2c^2+2cd+d^2)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}+\frac{ad(2c+3d)\tan(e+fx)\sec(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a*(c^2 + 3*c*d +
 d^2)*Tan[e + f*x])/(3*f) + (a*d*(2*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(6*
f) + (a*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx &= \frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}\int \sec(e+\\
&= \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{a(c+d\,\mathrm{se}}{}\\
&= \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{a(c+d\,\mathrm{se}}{}\\
&= \frac{a(2c^2+2cd+d^2)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad(2c+}{}\\
&= \frac{a(2c^2+2cd+d^2)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2a(c^2+}{}
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 75, normalized size = 0.69

$$\frac{a\left(3(2c^2+2cd+d^2)\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(3d(2c+d)\sec(e+fx)+2(3(c+d)^2+d^2\tan^2(e+fx))\right)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*(3*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*d*(2*c
+ d)*Sec[e + f*x] + 2*(3*(c + d)^2 + d^2*Tan[e + f*x]^2))))/(6*f)
```

Maple [A]
time = 0.24, size = 143, normalized size = 1.32

method	result
derivativedivides	$\frac{a\,c^{2} \tan(fx+e)+2acd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,d^{2}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(fx+e)\right)}{3}\right)\tan(fx+e)+a\,c^{2}\ln(\text{s}}{f}$
default	$\frac{a\,c^{2} \tan(fx+e)+2acd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,d^{2}\left(-\frac{2}{3}-\frac{\left(\sec^{2}(fx+e)\right)}{3}\right)\tan(fx+e)+a\,c^{2}\ln(\text{s}}{f}$
norman	$\frac{-\frac{a\left(2c^{2}+2cd+d^{2}\right)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a\left(2c^{2}+6cd+3d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{4a\left(3c^{2}+6cd+d^{2}\right)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}}{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{a(2c^{2}+2cd+d^{2})\ln}{2}$
risch	$-\frac{ia\left(6cd\,e^{5i(fx+e)}+3d^{2}e^{5i(fx+e)}-6c^{2}e^{4i(fx+e)}-12cd\,e^{4i(fx+e)}-12c^{2}e^{2i(fx+e)}-24cd\,e^{2i(fx+e)}-12d^{2}e^{2i(fx+e)}-6d\,e^{i}\right.}{3f\left(e^{2i(fx+e)}+1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^2*tan(f*x+e)+2*a*c*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+
tan(f*x+e)))-a*d^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a*c^2*ln(sec(f*x+e)+t
an(f*x+e))+2*a*c*d*tan(f*x+e)+a*d^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f
*x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.28, size = 179, normalized size = 1.66

$$\frac{4\left(\tan(fx+e)^{3}+3\tan(fx+e)\right)ad^{2}-6\,acd\left(\frac{2\sin(fx+e)}{\sin(fx+e)^{2}-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-3\,ad^{2}\left(\frac{2\sin(fx+e)}{\sin(fx+e)^{2}-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+12\,ac^{2}\log(\sec(fx+e)+\tan(fx+e))+12\,ac^{2}\tan(fx+e)+24\,acd\tan(fx+e)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*d^2 - 6*a*c*d*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*a*
d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*a*c^2*tan(f
*x + e) + 24*a*c*d*tan(f*x + e))/f
```

Fricas [A]
time = 1.85, size = 158, normalized size = 1.46

$$\frac{3\left(2\,ac^{2}+2\,acd+ad^{2}\right)\cos(fx+e)^{3}\log(\sin(fx+e)+1)-3\left(2\,ac^{2}+2\,acd+ad^{2}\right)\cos(fx+e)^{3}\log(-\sin(fx+e)+1)+2\left(2\,ad^{2}+2\left(3\,ac^{2}+6\,acd+2\,ad^{2}\right)\cos(fx+e)^{2}+3\left(2\,acd+ad^{2}\right)\cos(fx+e)\right)\sin(fx+e)}{12\,f\cos(fx+e)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*
a*d^2 + 2*(3*a*c^2 + 6*a*c*d + 2*a*d^2)*cos(f*x + e)^2 + 3*(2*a*c*d + a*d^2
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c^{2}\sec\left(e+fx\right)dx+\int c^{2}\sec^{2}\left(e+fx\right)dx+\int d^{2}\sec^{3}\left(e+fx\right)dx+\int d^{2}\sec^{4}\left(e+fx\right)dx+\int 2cd\sec^{2}\left(e+fx\right)dx+\int 2cd\sec^{3}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(c**2*sec(e + f*x), x) + Integral(c**2*sec(e + f*x)**2, x) + Int
egral(d**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**4, x) + Integr
al(2*c*d*sec(e + f*x)**2, x) + Integral(2*c*d*sec(e + f*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(100) = 200.
time = 0.51, size = 232, normalized size = 2.15

$$\frac{3\left(2ac^{2}+2acd+ad^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3\left(2ac^{2}+2acd+ad^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(6ac^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+6acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+3ad^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-12ac^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-24acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-4ad^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+6ac^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+18acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+9ad^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{3}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(
2*a*c^2 + 2*a*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(6*a*c^2*
tan(1/2*f*x + 1/2*e)^5 + 6*a*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*a*d^2*tan(1/2*f
*x + 1/2*e)^5 - 12*a*c^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*c*d*tan(1/2*f*x + 1/
2*e)^3 - 4*a*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*c^2*tan(1/2*f*x + 1/2*e) + 18
*a*c*d*tan(1/2*f*x + 1/2*e) + 9*a*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x +
1/2*e)^2 - 1)^3)/f
```

Mupad [B]
time = 4.76, size = 196, normalized size = 1.81

$$\frac{a\,\mathrm{atanh}\left(\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)}{4\,c^2+4\,c\,d+2\,d^2}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)}{f}-\frac{\left(2\,a\,c^2+2\,a\,c\,d+a\,d^2\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(-4\,a\,c^2-8\,a\,c\,d-\frac{4\,a\,d^2}{3}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(2\,a\,c^2+6\,a\,c\,d+3\,a\,d^2\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a*atanh((2*tan(e/2 + (f*x)/2)*(2*c*d + 2*c^2 + d^2))/(4*c*d + 4*c^2 + 2*d^
2))*(2*c*d + 2*c^2 + d^2))/f - (tan(e/2 + (f*x)/2)*(2*a*c^2 + 3*a*d^2 + 6*a
*c*d) + tan(e/2 + (f*x)/2)^5*(2*a*c^2 + a*d^2 + 2*a*c*d) - tan(e/2 + (f*x)/
2)^3*(4*a*c^2 + (4*a*d^2)/3 + 8*a*c*d))/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(
e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

3.188 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=56

$$\frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a(c+d)\tan(e+fx)}{f}+\frac{ad\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*a*(2*c+d)*arctanh(sin(f*x+e))/f+a*(c+d)*tan(f*x+e)/f+1/2*a*d*sec(f*x+e)
*tan(f*x+e)/f
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4082, 3872, 3855, 3852, 8}

$$\frac{a(c+d)\tan(e+fx)}{f}+\frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*(2*c + d)*ArcTanh[Sin[e + f*x]])/(2*f) + (a*(c + d)*Tan[e + f*x])/f + (a
*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{ad\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}\int \sec(e+fx)(a$$

$$= \frac{ad\sec(e+fx)\tan(e+fx)}{2f} + (a(c+d))\int \sec^2($$

$$= \frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{ad\sec(e+fx)}{2f}$$

$$= \frac{a(2c+d)\tanh^{-1}(\sin(e+fx))}{2f} + \frac{a(c+d)\tan(e}{f}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.34

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{f} + \frac{ad\tanh^{-1}(\sin(e+fx))}{2f} + \frac{ac\tan(e+fx)}{f} + \frac{ad\tan(e+fx)}{f} + \frac{ad\sec(e+fx)\tan(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/f + (a*d*ArcTanh[Sin[e + f*x]])/(2*f) + (a*c*Ta
n[e + f*x])/f + (a*d*Tan[e + f*x])/f + (a*d*Sec[e + f*x]*Tan[e + f*x])/(2*f
)
```

Maple [A]
time = 0.20, size = 75, normalized size = 1.34

method	result
derivativedivides	$\frac{ac\tan(fx+e)+ad\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)}{f}$
default	$\frac{ac\tan(fx+e)+ad\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)}{f}$

norman	$\frac{\frac{a(2c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{a(2c+d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{a(2c+d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a(2c+d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$
risch	$-\frac{ia\left(d\,\mathrm{e}^{3i(fx+e)}-2\,\mathrm{e}^{2i(fx+e)}c-2d\,\mathrm{e}^{2i(fx+e)}-d\,\mathrm{e}^{i(fx+e)}-2c-2d\right)}{f\left(\mathrm{e}^{2i(fx+e)}+1\right)^2}+\frac{ac\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{f}+\frac{a\ln\left(\mathrm{e}^{i(fx+e)}+i\right)d}{2f}-\frac{ac\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c*tan(f*x+e)+a*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*
x+e)))+a*c*ln(sec(f*x+e)+tan(f*x+e))+a*d*tan(f*x+e))
```

Maxima [A]
time = 0.28, size = 96, normalized size = 1.71

$$-\frac{ad\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-4\,ac\tan\left(fx+e\right)-4\,ad\tan\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -1/4*(a*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*a*c*tan(f
*x + e) - 4*a*d*tan(f*x + e))/f
```

Fricas [A]
time = 1.62, size = 103, normalized size = 1.84

$$\frac{(2\,ac+ad)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,ac+ad)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(ad+2\left(ac+ad\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*a*c + a*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + a*d)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a*d + 2*(a*c + a*d)*cos(f*x + e))*s
in(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int c\sec\left(e+fx\right)dx+\int c\sec^2\left(e+fx\right)dx+\int d\sec^2\left(e+fx\right)dx+\int d\sec^3\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] a*(Integral(c*sec(e + f*x), x) + Integral(c*sec(e + f*x)**2, x) + Integral(
d*sec(e + f*x)**2, x) + Integral(d*sec(e + f*x)**3, x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(52) = 104$.
time = 0.46, size = 124, normalized size = 2.21

$$\frac{(2\,ac + ad)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) - (2\,ac + ad)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right) - \frac{2\left(2\,ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a*c + a*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - (2*a*c + a*d)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(2*a*c*tan(1/2*f*x + 1/2*e)^3 + a*d*tan(1
/2*f*x + 1/2*e)^3 - 2*a*c*tan(1/2*f*x + 1/2*e) - 3*a*d*tan(1/2*f*x + 1/2*e)
)/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

Mupad [**B**]
time = 2.57, size = 111, normalized size = 1.98

$$\frac{a\,\mathrm{atanh}\left(\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c+d)}{4\,c+2\,d}\right)\,(2\,c+d)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}\,(2\,a\,c+a\,d) - \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,a\,c+3\,a\,d)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4} - 2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (a*atanh((2*tan(e/2 + (f*x)/2)*(2*c + d))/(4*c + 2*d))*(2*c + d))/f - (tan(
e/2 + (f*x)/2)^3*(2*a*c + a*d) - tan(e/2 + (f*x)/2)*(2*a*c + 3*a*d))/(f*(ta
n(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

3.189 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=69

$$\frac{a\tanh^{-1}(\sin(e+fx))}{df}-\frac{2a\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d\sqrt{c+d}\,f}$$

```
[Out] a*arctanh(sin(f*x+e))/d/f-2*a*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^
(1/2))*(c-d)^(1/2)/d/f/(c+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4083, 3855, 3916, 2738, 214}

$$\frac{a\tanh^{-1}(\sin(e+fx))}{df}-\frac{2a\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*ArcTanh[Sin[e + f*x]])/(d*f) - (2*a*Sqrt[c - d]*ArcTanh[(Sqrt[c - d]*Tan
[(e + f*x)/2])/Sqrt[c + d]])/(d*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\int \sec(e+fx)\,dx}{d} + \frac{(-ac+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{d}$$

$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{(a(c-d))\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d^2}$$

$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{(2a(c-d))\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\right.}{d^2f}$$

$$= \frac{a\tanh^{-1}(\sin(e+fx))}{df} - \frac{2a\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d\sqrt{c+d}\,f}$$

Mathematica [A]
time = 0.20, size = 107, normalized size = 1.55

$$\frac{a\left(\frac{2(c-d)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right) + \log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*((2*(c - d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c
^2 - d^2] - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]]))/(d*f)
```

Maple [A]
time = 0.23, size = 90, normalized size = 1.30

method	result

derivativedivides	$\frac{4a\left(-\frac{(c-d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2d\sqrt{(c+d)(c-d)}}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4d}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4d}\right)}{f}$
default	$\frac{4a\left(-\frac{(c-d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2d\sqrt{(c+d)(c-d)}}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4d}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4d}\right)}{f}$
risch	$\frac{\sqrt{(c+d)(c-d)}\,a\ln\left(\mathrm{e}^{i(fx+e)}-\frac{i\sqrt{(c+d)(c-d)}-d}{c}\right)}{(c+d)fd}-\frac{\sqrt{(c+d)(c-d)}\,a\ln\left(\mathrm{e}^{i(fx+e)}+\frac{i\sqrt{}}{}\right.}{(c+d)fd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a*(-1/2*(c-d)/d/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((
c+d)*(c-d))^(1/2))+1/4/d*ln(tan(1/2*f*x+1/2*e)+1)-1/4/d*ln(tan(1/2*f*x+1/2*
e)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 1.84, size = 267, normalized size = 3.87

$$\left[\frac{a\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,(c^2+cd+(cd+d^2)\cos(fx+e))\sqrt{\frac{c-d}{c+d}}\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)+a\log\left(\sin\left(fx+e\right)+1\right)-a\log\left(-\sin\left(fx+e\right)+1\right)}{2\,df},\ -\frac{2\,a\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right)-a\log\left(\sin\left(fx+e\right)+1\right)+a\log\left(-\sin\left(fx+e\right)+1\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [1/2*(a*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f
*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*
sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)
) + a*log(sin(f*x + e) + 1) - a*log(-sin(f*x + e) + 1))/(d*f), -1/2*(2*a*sq
rt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((
c - d)*sin(f*x + e))) - a*log(sin(f*x + e) + 1) + a*log(-sin(f*x + e) + 1))
/(d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec (e+fx)}{c+d\sec (e+fx)}\,dx+\int \frac{\sec^2 (e+fx)}{c+d\sec (e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**
2/(c + d*sec(e + f*x)), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(60) = 120.
time = 0.51, size = 127, normalized size = 1.84

$$\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d}+\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(ac-ad)}{\sqrt{-c^2+d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d - a*log(abs(tan(1/2*f*x + 1/2*e) -
1))/d + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - a*d)/(
sqrt(-c^2 + d^2)*d))/f
```

Mupad [B]
time = 2.16, size = 195, normalized size = 2.83

$$\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c+d\right)}+\frac{2\,a\,\left(\mathrm{atanh}\left(\frac{d^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,d^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c^2\,d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c^2-d^2\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}\,\left(d^2+c\,d\right)}\right)\,\sqrt{c^2-d^2}+c\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{d\,f\,\left(c+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (2*a*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*a*(atan
h((d^3*sin(e/2 + (f*x)/2) - c^3*sin(e/2 + (f*x)/2) + c*d^2*sin(e/2 + (f*x)/
2) - c^2*d*sin(e/2 + (f*x)/2) + c*sin(e/2 + (f*x)/2)*(c^2 - d^2))/(cos(e/2
+ (f*x)/2)*(c^2 - d^2)^(1/2)*(c*d + d^2)))*(c^2 - d^2)^(1/2) + c*atanh(sin(
e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/(d*f*(c + d))
```

3.190 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2} dx$

Optimal. Leaf size=79

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,(c+d)^{3/2}f}+\frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))}$$

```
[Out] 2*a*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c+d)^(3/2)/f/(c-d)
^(1/2)+a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\,(c+d)^{3/2}}+\frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*(c +
 d)^(3/2)*f) + (a*Tan[e + f*x])/((c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx &= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} - \frac{\int \frac{a(c-d)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{-c^2+d^2} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{a\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c+d} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{a\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d(c+d)} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{(2a)\mathrm{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\right.\right.}{d(c+d)f} \\
&= \frac{2a\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,(c+d)^{3/2}f} + \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 75, normalized size = 0.95

$$\frac{a\left(-\frac{2\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{\sin(e+fx)}{d+c\cos(e+fx)}\right)}{(c+d)f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*((-2*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^
2] + Sin[e + f*x]/(d + c*Cos[e + f*x])))/((c + d)*f)
```

Maple [A]
time = 0.23, size = 105, normalized size = 1.33

method	result
derivativedivides	$\frac{4a\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}+\frac{\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{f}$
default	$\frac{4a\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}+\frac{\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{f}$
risch	$\frac{2ia\left(d\,\mathrm{e}^{i(fx+e)}+c\right)}{cf(c+d)\left(\mathrm{e}^{2i(fx+e)}c+2d\,\mathrm{e}^{i(fx+e)}+c\right)}+\frac{a\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c+d)f}-\frac{a\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2}}{\sqrt{c^2-}}\right.}{\sqrt{c^2-d^2}\,(c+d)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+
1/2*e)^2-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e
)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(73) = 146.
time = 1.66, size = 371, normalized size = 4.70

$$\left[\frac{(ac\cos(fx+e)+ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}\,(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)+2\,(ac^2-ad^2)\sin(fx+e)}{2\,((c^4+c^3d-c^2d^2-cd^3)f\cos(fx+e)+(c^3d+c^2d^2-cd^3-d^4)f)},\ \frac{(ac\cos(fx+e)+ad)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}\,(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)+(ac^2-ad^2)\sin(fx+e)}{(c^4+c^3d-c^2d^2-cd^3)f\cos(fx+e)+(c^3d+c^2d^2-cd^3-d^4)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*c*cos(f*x + e) + a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c
^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x
 + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(
a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*cos(f*x + e
) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f), ((a*c*cos(f*x + e) + a*d)*sqrt(-c^2
 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x
+ e))) + (a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*c
os(f*x + e) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx+\int \frac{\sec^2(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**2)
, x) + Integral(sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f
*x)**2), x))
```

Giac [A]
time = 0.48, size = 137, normalized size = 1.73

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a}{\sqrt{-c^2+d^2}\,(c+d)}+\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(c+d)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a/(sqrt(-c^2 + d^2)
*(c + d)) + a*tan(1/2*f*x + 1/2*e)/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f
*x + 1/2*e)^2 - c - d)*(c + d)))/f
```

Mupad [B]

time = 1.91, size = 85, normalized size = 1.08

$$\frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(c+d\right)\,\left(\left(d-c\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+c+d\right)}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)}{f\,\left(c+d\right)^{3/2}\,\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*a*tan(e/2 + (f*x)/2))/(f*(c + d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
 + (2*a*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)
^(3/2)*(c - d)^(1/2))
```

3.191 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{5/2}f}+\frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2}+\frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))}$$

```
[Out] a*(2*c-d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/2)/(
c+d)^(5/2)/f+1/2*a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^2+1/2*a*(c-2*d)*tan(
f*x+e)/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$\frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{5/2}}+\frac{a(c-2d)\tan(e+fx)}{2f(c-d)(c+d)^2(c+d\sec(e+fx))}+\frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(2*c - d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^
(3/2)*(c + d)^(5/2)*f) + (a*Tan[e + f*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])
^2) + (a*(c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sec[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx &= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-2a(c-d)-a(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^2}}{2\left(c^2-d^2\right)} \\
&= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+f} \\
&= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+f} \\
&= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+f} \\
&= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+f} \\
&= \frac{a(2c-d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{5/2}f} + \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+}
\end{aligned}$$

Mathematica [A]
time = 1.28, size = 167, normalized size = 1.27

$$\frac{a(1+\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)\left(-2(2c-d)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^2+\sqrt{c^2-d^2}\left((c-2d)d+\left(2c^2-2cd-d^2\right)\cos(e+fx)\right)\sin(e+fx)\right)}{4(c-d)(c+d)^2\sqrt{c^2-d^2}\,f(d+c\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(-2*(2*c - d)*ArcTanh[((-c + d)*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2 + Sqrt[c^2 - d^2]*(
(c - 2*d)*d + (2*c^2 - 2*c*d - d^2)*Cos[e + f*x])*Sin[e + f*x]))/(4*(c - d)
*(c + d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^2)
```

Maple [A]
time = 0.36, size = 178, normalized size = 1.36

method	result
derivativedivides	$\frac{4a\left(\frac{-\frac{(2c-d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4\left(c^2+2cd+d^2\right)}+\frac{(2c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)(c-d)}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}+\frac{(2c-d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{4\left(c^3+c^2d-c\,d^2-d^3\right)\sqrt{(c+d)(c-d)}}\right)}{f}$
default	$\frac{4a\left(\frac{-\frac{(2c-d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4\left(c^2+2cd+d^2\right)}+\frac{(2c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)(c-d)}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}+\frac{(2c-d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{4\left(c^3+c^2d-c\,d^2-d^3\right)\sqrt{(c+d)(c-d)}}\right)}{f}$
risch	$\frac{ia\left(-3c^3d\,e^{3i(fx+e)}+2c^2d^2e^{3i(fx+e)}+2c\,d^3e^{3i(fx+e)}-2c^4e^{2i(fx+e)}+2c^3d\,e^{2i(fx+e)}-3c^2d^2e^{2i(fx+e)}+4c\,d^3e^{2i(fx+e)}+2d\right.}{c^2\left(-c^2+d^2\right)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)^2(c+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a*((-1/4*(2*c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/4*(2*c-3*d)/(c+
d)/(c-d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2
-c-d)^2+1/4*(2*c-d)/(c^3+c^2*d-c*d^2-d^3)/((c+d)*(c-d))^(1/2)*arctanh((c-d)
*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(123) = 246$.
time = 2.41, size = 756, normalized size = 5.77

$$\left[\frac{\left(2\,acd^{2}-ad^{3}+\left(2\,ac^{3}-ac^{2}d\right)\cos\left(fx+e\right)^{2}+2\left(2\,ac^{2}d-acd^{2}\right)\cos\left(fx+e\right)\right)\sqrt{c^{2}-d^{2}}\log\left(\frac{2\,cd\cos(fx+e)-(c^{2}-2\,d^{2})\cos(fx+e)^{2}+2\,\sqrt{c^{2}-d^{2}}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^{2}-d^{2}}{c^{2}\cos(fx+e)^{2}+2\,cd\cos(fx+e)+d^{2}}\right)+2\left(ac^{3}d-2\,ac^{2}d^{2}-acd^{3}+2\,ad^{4}+\left(2\,ac^{4}-2\,ac^{3}d-3\,ac^{2}d^{2}+2\,acd^{3}+ad^{4}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4\left(\left(c^{7}+c^{6}d-2\,c^{5}d^{2}-2\,c^{4}d^{3}+c^{3}d^{4}+c^{2}d^{5}\right)f\cos\left(fx+e\right)^{2}+2\left(c^{6}d+c^{5}d^{2}-2\,c^{4}d^{3}-2\,c^{3}d^{4}+c^{2}d^{5}+cd^{6}\right)f\cos\left(fx+e\right)+\left(c^{5}d^{2}+c^{4}d^{3}-2\,c^{3}d^{4}-2\,c^{2}d^{5}+cd^{6}+d^{7}\right)f\right)},\ \frac{\left(2\,acd^{2}-ad^{3}+\left(2\,ac^{3}-ac^{2}d\right)\cos\left(fx+e\right)^{2}+2\left(2\,ac^{2}d-acd^{2}\right)\cos\left(fx+e\right)\right)\sqrt{-c^{2}+d^{2}}\arctan\left(-\frac{\sqrt{-c^{2}+d^{2}}(d\cos(fx+e)+c)}{(c^{2}-d^{2})\sin(fx+e)}\right)+\left(ac^{3}d-2\,ac^{2}d^{2}-acd^{3}+2\,ad^{4}+\left(2\,ac^{4}-2\,ac^{3}d-3\,ac^{2}d^{2}+2\,acd^{3}+ad^{4}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{2\left(\left(c^{7}+c^{6}d-2\,c^{5}d^{2}-2\,c^{4}d^{3}+c^{3}d^{4}+c^{2}d^{5}\right)f\cos\left(fx+e\right)^{2}+2\left(c^{6}d+c^{5}d^{2}-2\,c^{4}d^{3}-2\,c^{3}d^{4}+c^{2}d^{5}+cd^{6}\right)f\cos\left(fx+e\right)+\left(c^{5}d^{2}+c^{4}d^{3}-2\,c^{3}d^{4}-2\,c^{2}d^{5}+cd^{6}+d^{7}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*a*c*d^2 - a*d^3 + (2*a*c^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*
d - a*c*d^2)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e
) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a*c^
3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2
+ 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c^6*d - 2*c^5*d^2
- 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6*d + c^5*d^2 - 2*
c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^5*d^2 + c^4*d^3
- 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f), 1/2*((2*a*c*d^2 - a*d^3 + (2*a*c
^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*cos(f*x + e))*sqrt(-
c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3
*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c
^6*d - 2*c^5*d^2 - 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6
*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c
^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\sec\left(e+fx\right)}{c^{3}+3c^{2}d\sec\left(e+fx\right)+3cd^{2}\sec^{2}\left(e+fx\right)+d^{3}\sec^{3}\left(e+fx\right)}\,dx+\int\frac{\sec^{2}\left(e+fx\right)}{c^{3}+3c^{2}d\sec\left(e+fx\right)+3cd^{2}\sec^{2}\left(e+fx\right)+d^{3}\sec^{3}\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f
*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**2/(c**3 + 3*c**
2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(118) = 236$.
time = 0.58, size = 263, normalized size = 2.01

$$\frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^{2}+d^{2}}}\right)\right)(2ac-ad)}{(c^{3}+c^{2}d-cd^{2}-d^{3})\sqrt{-c^{2}+d^{2}}}-\frac{2ac^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-3acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+ad^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-2ac^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+3ad^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(c^{3}+c^{2}d-cd^{2}-d^{3})\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-c-d\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x
 + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*a*c - a*d)/((c^3
+ c^2*d - c*d^2 - d^3)*sqrt(-c^2 + d^2)) - (2*a*c^2*tan(1/2*f*x + 1/2*e)^3
- 3*a*c*d*tan(1/2*f*x + 1/2*e)^3 + a*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*c^2*t
an(1/2*f*x + 1/2*e) + a*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2*f*x + 1/
2*e))/((c^3 + c^2*d - c*d^2 - d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*
x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B]
time = 3.78, size = 171, normalized size = 1.31

$$\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)\,\left(2\,c-d\right)}{f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{3/2}}-\frac{\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,a\,c-a\,d\right)}{{\left(c+d\right)}^2}-\frac{a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-3\,d\right)}{\left(c+d\right)\,\left(c-d\right)}}{f\,\left(2\,c\,d-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2-2\,d^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(c^2-2\,c\,d+d^2\right)+c^2+d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] (a*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(2*c - d))/(f*(c
 + d)^(5/2)*(c - d)^(3/2)) - ((tan(e/2 + (f*x)/2)^3*(2*a*c - a*d))/(c + d)^
2 - (a*tan(e/2 + (f*x)/2)*(2*c - 3*d))/((c + d)*(c - d)))/(f*(2*c*d - tan(e
/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2)
+ c^2 + d^2))
```

3.192 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=189

$$\frac{a(2c^2-2cd+d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{7/2}f}+\frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3}+\frac{a(2c-3d)\tan(e}{6(c-d)(c+d)^2f(c+d\,s}$$

```
[Out] a*(2*c^2-2*c*d+d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-
d)^(5/2)/(c+d)^(7/2)/f+1/3*a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^3+1/6*a*(2
*c-3*d)*tan(f*x+e)/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))^2+1/6*a*(c-4*d)*(2*c-d)
*tan(f*x+e)/(c-d)^2/(c+d)^3/f/(c+d*sec(f*x+e))
```

Rubi [A]

time = 0.32, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$\frac{a(2c^2-2cd+d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{7/2}}+\frac{a(c-4d)(2c-d)\tan(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sec(e+fx))}+\frac{a(2c-3d)\tan(e+fx)}{6f(c-d)(c+d)^2(c+d\sec(e+fx))^2}+\frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]
])/((c - d)^(5/2)*(c + d)^(7/2)*f) + (a*Tan[e + f*x])/(3*(c + d)*f*(c + d*S
ec[e + f*x])^3) + (a*(2*c - 3*d)*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c +
d*Sec[e + f*x])^2) + (a*(c - 4*d)*(2*c - d)*Tan[e + f*x])/(6*(c - d)^2*(c +
 d)^3*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-3a(c-d)-2a(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^3}\,d\cdots}{3\left(c^2-d^2\right)}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2 f(c+d\sec(e+fx)\cdots}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2 f(c+d\sec(e+fx)\cdots}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2 f(c+d\sec(e+fx)\cdots}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2 f(c+d\sec(e+fx)\cdots}$$

$$= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2 f(c+d\sec(e+fx)\cdots}$$

$$= \frac{a(2c^2-2cd+d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{7/2}f} + \frac{a\tan(e\cdots}{3(c+d)f(c+d\,\mathrm{s}\cdots}$$

Mathematica [A]
time = 3.34, size = 247, normalized size = 1.31

$$-\frac{a(1+\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)\left(6(2c^2-2cd+d^2)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^3-\frac{1}{2}\sqrt{c^2-d^2}\,(6c^4-12c^3d+2c^2d^2-15cd^3+10d^4+6d(2c^3-7c^2d+2cd^2+d^3)\cos(e+fx)+(6c^4-12c^3d-2c^2d^2+3cd^3+2d^4)\cos(2(e+fx)))\sin(e+fx)\right)}{12(c-d)^2(c+d)^3\sqrt{c^2-d^2}\,f(d+c\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -1/12*(a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(6*(2*c^2 - 2*c*d + d^2)*Arc
Tanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3 -
(Sqrt[c^2 - d^2]*(6*c^4 - 12*c^3*d + 2*c^2*d^2 - 15*c*d^3 + 10*d^4 + 6*d*(2
*c^3 - 7*c^2*d + 2*c*d^2 + d^3)*Cos[e + f*x] + (6*c^4 - 12*c^3*d - 2*c^2*d^
2 + 3*c*d^3 + 2*d^4)*Cos[2*(e + f*x)])*Sin[e + f*x])/2))/((c - d)^2*(c + d)
^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

Maple [A]
time = 0.49, size = 271, normalized size = 1.43 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a*((-1/4*(2*c^2-2*c*d+d^2)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)
^5+1/3*(3*c^2-6*c*d+d^2)/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/4*(2*
c^2-6*c*d+3*d^2)/(c+d)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1
/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^3+1/4*(2*c^2-2*c*d+d^2)/(c^5+c^4*d-2*c^
3*d^2-2*c^2*d^3+c*d^4+d^5)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/
2*e)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(181) = 362$.
time = 2.51, size = 1304, normalized size = 6.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d + a*c^3*d^
2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*x + e)^2
+ 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^
5 - 4*a*d^6 + (6*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4*a*c^2*
d^4 - 3*a*c*d^5 - 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*
a*c^2*d^4 - 2*a*c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10 + c^9*d -
 3*c^8*d^2 - 3*c^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^7)*f*cos(f
*x + e)^3 + 3*(c^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d^5 + 3*c^4*
d^6 - c^3*d^7 - c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^
4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f*x + e) + (
c^7*d^3 + c^6*d^4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 -
 d^10)*f), 1/6*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d +
 a*c^3*d^2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*
x + e)^2 + 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(-c^2
+ d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +
 e))) + (2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6 + (6
*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4*a*c^2*d^4 - 3*a*c*d^5
- 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*
c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10 + c^9*d - 3*c^8*d^2 - 3*c
^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^7)*f*cos(f*x + e)^3 + 3*(c
^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d^5 + 3*c^4*d^6 - c^3*d^7 -
c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^4 - 3*c^5*d^5 +
3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f*x + e) + (c^7*d^3 + c^6*d^
4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{\sec(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx+\int \frac{\sec^2(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e
+ f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral
(sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**
2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(174) = 348.
time = 0.53, size = 449, normalized size = 2.38

$$-\frac{\frac{3\left(2ac^2-2acd+ad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^5+c^4d-2c^3d^2-2c^2d^3+cd^4+d^5\right)\sqrt{-c^2+d^2}}+\frac{6ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+21ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+3ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+24ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+8ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-24acd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+4ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6ac^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-21ac^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+9ad^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c^5+c^4d-2c^3d^2-2c^2d^3+cd^4+d^5\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*a*c^2 - 2*a*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2
*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-
c^2 + d^2)))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*sqrt(-c^2
 + d^2)) + (6*a*c^4*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^3*d*tan(1/2*f*x + 1/2*e
)^5 + 21*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a*c*d^3*tan(1/2*f*x + 1/2*e)
^5 + 3*a*d^4*tan(1/2*f*x + 1/2*e)^5 - 12*a*c^4*tan(1/2*f*x + 1/2*e)^3 + 24*
a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 8*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*
c*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 6*a*c^4*tan
(1/2*f*x + 1/2*e) - 6*a*c^3*d*tan(1/2*f*x + 1/2*e) - 21*a*c^2*d^2*tan(1/2*f
*x + 1/2*e) + 9*a*d^4*tan(1/2*f*x + 1/2*e))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c
^2*d^3 + c*d^4 + d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2
- c - d)^3))/f
```

Mupad [B]
time = 5.28, size = 321, normalized size = 1.70

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(2\,a\,c^2-2\,a\,c\,d+a\,d^2\right)}{{\left(c+d\right)}^3}+\frac{a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2-6\,c\,d+3\,d^2\right)}{\left(c+d\right)\,\left(c^2-2\,c\,d+d^2\right)}-\frac{4\,a\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\,c^2-6\,c\,d+d^2\right)}{3\,{\left(c+d\right)}^2\,\left(c-d\right)}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3\right)+3\,c\,d^2+3\,c^2\,d+c^3+d^3-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6\,\left(c^3-3\,c^2\,d+3\,c\,d^2-d^3\right)\right)}+\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-2\,d\right)\,\left(c^2-2\,c\,d+d^2\right)}{2\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{f\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(2*a*c^2 + a*d^2 - 2*a*c*d))/(c + d)^3 + (a*tan(e/2
+ (f*x)/2)*(2*c^2 - 6*c*d + 3*d^2))/((c + d)*(c^2 - 2*c*d + d^2)) - (4*a*ta
n(e/2 + (f*x)/2)^3*(3*c^2 - 6*c*d + d^2))/(3*(c + d)^2*(c - d)))/(f*(tan(e/
2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(
3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/
2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (a*atanh((tan(e/2 + (f*x
)/2)*(2*c - 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*(c - d)^(5/2)))*(2*c
^2 - 2*c*d + d^2))/(f*(c + d)^(7/2)*(c - d)^(5/2))
```

3.193 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=327

$$\frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tanh^{-1}(\sin(e+fx))}{16f}-\frac{a^2(4c^5-48c^4d-311c^3d^2-448c^2d^3-288c}{60df}$$

```
[Out] 1/16*a^2*(24*c^4+64*c^3*d+84*c^2*d^2+48*c*d^3+11*d^4)*arctanh(sin(f*x+e))/f
-1/60*a^2*(4*c^5-48*c^4*d-311*c^3*d^2-448*c^2*d^3-288*c*d^4-64*d^5)*tan(f*x
+e)/d/f-1/240*a^2*(8*c^4-96*c^3*d-438*c^2*d^2-464*c*d^3-165*d^4)*sec(f*x+e)
*tan(f*x+e)/f-1/120*a^2*(4*c^3-48*c^2*d-123*c*d^2-64*d^3)*(c+d*sec(f*x+e))^
2*tan(f*x+e)/d/f-1/120*a^2*(4*c^2-48*c*d-55*d^2)*(c+d*sec(f*x+e))^3*tan(f*x
+e)/d/f-1/30*a^2*(c-12*d)*(c+d*sec(f*x+e))^4*tan(f*x+e)/d/f+1/6*a^2*(c+d*se
c(f*x+e))^5*tan(f*x+e)/d/f
```

Rubi [A]
time = 0.29, antiderivative size = 371, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4072, 102, 158, 152, 52, 65, 223, 209}

$$\frac{a^3(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)}{16f}+\frac{(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{48f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2\left(d(48c^2+32cd+19d^2)\sec(e+fx)+2(52c^3+56c^2d+48cd^2+9d^3)\right)}{120f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^3}{6f}+\frac{d(9c+2d)\tan(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^2}{30f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Tan[e + f*x])/(16
*f) + (a^3*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*ArcTan[Sqrt
[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(8*f*Sqrt[a
 - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((24*c^4 + 64*c^3*d + 84*c^2*
d^2 + 48*c*d^3 + 11*d^4)*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(48*f) + (d
*(9*c + 2*d)*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3
0*f) + (d*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(6*f)
 + (d*(a + a*Sec[e + f*x])^2*(2*(52*c^3 + 56*c^2*d + 48*c*d^2 + 9*d^3) + d*
(48*c^2 + 32*c*d + 19*d^2)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^4}{\sqrt{a-ax}}\,dx,\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(}{6f}$$

$$= \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+f}{30f}$$

$$= \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+f}{30f}$$

$$= \frac{(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\,(a^2+}{48f}$$

$$= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(}{16f}$$

$$= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(}{16f}$$

$$= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(}{16f}$$

$$= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(}{16f}$$

Mathematica [A]
time = 1.98, size = 460, normalized size = 1.41

$$-\frac{a^2(1+\cos(e+fx))^2\sec^4\left(\frac{1}{2}(e+fx)\right)\sec^6(e+fx)\left(240(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\cos^6(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-2(360c^4+2880c^3d+5220c^2d^2+4080cd^3+1255d^4+32(75c^4+310c^3d+480c^2d^2+336cd^3+88d^4)\cos(e+fx)+20(24c^4+192c^3d+324c^2d^2+240cd^3+55d^4)\cos(2(e+fx))+1200c^4\cos(3(e+fx))+4640c^3d\cos(3(e+fx))+6720c^2d^2\cos(3(e+fx))+4032cd^3\cos(3(e+fx))+896d^4\cos(3(e+fx))+120c^4\cos(4(e+fx))+960c^3d\cos(4(e+fx))+1260c^2d^2\cos(4(e+fx))+720cd^3\cos(4(e+fx))+165d^4\cos(4(e+fx))+240c^4\cos(5(e+fx))+800c^3d\cos(5(e+fx))+960c^2d^2\cos(5(e+fx))+576cd^3\cos(5(e+fx))+128d^4\cos(5(e+fx)))\sin(e+fx)\right)}{15360f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -1/15360*(a^2*(1 + Cos[e + f*x])^2*Sec[(e + f*x)/2]^4*Sec[e + f*x]^6*(240*(
24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Cos[e + f*x]^6*(Log[Cos
```

```
[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
]) - 2*(360*c^4 + 2880*c^3*d + 5220*c^2*d^2 + 4080*c*d^3 + 1255*d^4 + 32*(7
5*c^4 + 310*c^3*d + 480*c^2*d^2 + 336*c*d^3 + 88*d^4)*Cos[e + f*x] + 20*(24
*c^4 + 192*c^3*d + 324*c^2*d^2 + 240*c*d^3 + 55*d^4)*Cos[2*(e + f*x)] + 120
0*c^4*Cos[3*(e + f*x)] + 4640*c^3*d*Cos[3*(e + f*x)] + 6720*c^2*d^2*Cos[3*(
e + f*x)] + 4032*c*d^3*Cos[3*(e + f*x)] + 896*d^4*Cos[3*(e + f*x)] + 120*c^
4*Cos[4*(e + f*x)] + 960*c^3*d*Cos[4*(e + f*x)] + 1260*c^2*d^2*Cos[4*(e + f
*x)] + 720*c*d^3*Cos[4*(e + f*x)] + 165*d^4*Cos[4*(e + f*x)] + 240*c^4*Cos[
5*(e + f*x)] + 800*c^3*d*Cos[5*(e + f*x)] + 960*c^2*d^2*Cos[5*(e + f*x)] +
576*c*d^3*Cos[5*(e + f*x)] + 128*d^4*Cos[5*(e + f*x)])*Sin[e + f*x]))/f
```

Maple [A]
time = 0.47, size = 552, normalized size = 1.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(c^4*a^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-4*a^
2*c^3*d*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+6*a^2*c^2*d^2*(-(-1/4*sec(f*x+e)
^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-4*a^2*c*d^3*(-
8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+a^2*d^4*(-(-1/6*sec(f*x
+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(
f*x+e)))+2*c^4*a^2*tan(f*x+e)+8*a^2*c^3*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln
(sec(f*x+e)+tan(f*x+e)))-12*a^2*c^2*d^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+
8*a^2*c*d^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+
e)+tan(f*x+e)))-2*a^2*d^4*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*
x+e)+c^4*a^2*ln(sec(f*x+e)+tan(f*x+e))+4*a^2*c^3*d*tan(f*x+e)+6*a^2*c^2*d^2
*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-4*a^2*c*d^3*(-2/
3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a^2*d^4*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))
*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(325) = 650$.
time = 0.30, size = 737, normalized size = 2.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/480*(640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3*d + 1920*(tan(f*x + e)
^3 + 3*tan(f*x + e))*a^2*c^2*d^2 + 128*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^
3 + 15*tan(f*x + e))*a^2*c*d^3 + 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*
c*d^3 + 64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*d^4
```

```
 - 5*a^2*d^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(
sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
+ e) + 1) + 15*log(sin(f*x + e) - 1)) - 180*a^2*c^2*d^2*(2*(3*sin(f*x + e)^
3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
 + e) + 1) + 3*log(sin(f*x + e) - 1)) - 240*a^2*c*d^3*(2*(3*sin(f*x + e)^3
- 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
 e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a^2*d^4*(2*(3*sin(f*x + e)^3 - 5*s
in(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) +
 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^
2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 960*a^2*c^3*d*(2*
sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e
) - 1)) - 720*a^2*c^2*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^2*c^4*log(sec(f*x + e) + tan(f
*x + e)) + 960*a^2*c^4*tan(f*x + e) + 1920*a^2*c^3*d*tan(f*x + e))/f
```

Fricas [**A**]
time = 2.13, size = 398, normalized size = 1.22

$$\frac{15\,(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4)\cos(fx+e)^6\log(\sin(fx+e)+1)-15\,(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4)\cos(fx+e)^6\log(-\sin(fx+e)+1)+2\,(40\,a^2d^4+32\,(15\,a^2c^4+50\,a^2c^3d+60\,a^2c^2d^2+36\,a^2cd^3+8\,a^2d^4)\cos(fx+e)^5+15\,(8\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4)\cos(fx+e)^4+64\,(5\,a^2c^3d+15\,a^2c^2d^2+9\,a^2cd^3+2\,a^2d^4)\cos(fx+e)^3+10\,(36\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4)\cos(fx+e)^2+96\,(2\,a^2cd^3+a^2d^4)\cos(fx+e))\sin(fx+e)}{480\,f\cos(fx+e)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a
^2*d^4)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*(24*a^2*c^4 + 64*a^2*c^3*
d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^6*log(-sin(f*x
 + e) + 1) + 2*(40*a^2*d^4 + 32*(15*a^2*c^4 + 50*a^2*c^3*d + 60*a^2*c^2*d^2
 + 36*a^2*c*d^3 + 8*a^2*d^4)*cos(f*x + e)^5 + 15*(8*a^2*c^4 + 64*a^2*c^3*d
+ 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^4 + 64*(5*a^2*c^
3*d + 15*a^2*c^2*d^2 + 9*a^2*c*d^3 + 2*a^2*d^4)*cos(f*x + e)^3 + 10*(36*a^2
*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^2 + 96*(2*a^2*c*d^3 + a^
2*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c^4\sec(e+fx)\,dx+\int 2c^4\sec^2(e+fx)\,dx+\int c^4\sec^3(e+fx)\,dx+\int d^4\sec^5(e+fx)\,dx+\int 2d^4\sec^6(e+fx)\,dx+\int d^4\sec^7(e+fx)\,dx+\int 4cd^3\sec^4(e+fx)\,dx+\int 8cd^3\sec^5(e+fx)\,dx+\int 4cd^3\sec^6(e+fx)\,dx+\int 6c^2d^2\sec^3(e+fx)\,dx+\int 12c^2d^2\sec^4(e+fx)\,dx+\int 6c^2d^2\sec^5(e+fx)\,dx+\int 4c^3d\sec^2(e+fx)\,dx+\int 8c^3d\sec^3(e+fx)\,dx+\int 4c^3d\sec^4(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(c**4*sec(e + f*x), x) + Integral(2*c**4*sec(e + f*x)**2, x)
+ Integral(c**4*sec(e + f*x)**3, x) + Integral(d**4*sec(e + f*x)**5, x) + I
ntegral(2*d**4*sec(e + f*x)**6, x) + Integral(d**4*sec(e + f*x)**7, x) + In
tegral(4*c*d**3*sec(e + f*x)**4, x) + Integral(8*c*d**3*sec(e + f*x)**5, x)
```

+ Integral(4*c*d**3*sec(e + f*x)**6, x) + Integral(6*c**2*d**2*sec(e + f*x)**3, x) + Integral(12*c**2*d**2*sec(e + f*x)**4, x) + Integral(6*c**2*d**2*sec(e + f*x)**5, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(8*c**3*d*sec(e + f*x)**3, x) + Integral(4*c**3*d*sec(e + f*x)**4, x))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(313) = 626.
time = 0.63, size = 736, normalized size = 2.25

$$\frac{15\,(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15\,(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\,\left(360\,a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+960\,a^2c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+1260\,a^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+720\,a^2cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+165\,a^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}-2040\,a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}-5440\,a^2c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}-7140\,a^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}-4080\,a^2cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}-935\,a^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}+4560\,a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+13440\,a^2c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+15480\,a^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+10272\,a^2cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+1986\,a^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}-5040\,a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-17280\,a^2c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-19080\,a^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-11232\,a^2cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-3006\,a^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+2760\,a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+11200\,a^2c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+13980\,a^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+7440\,a^2cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+1305\,a^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-600\,a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2880\,a^2c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-4500\,a^2c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3120\,a^2cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-795\,a^2d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^6}}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a
^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(24*a^2*c^4 + 64*a^2*c^3*d
+ 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e)
- 1)) - 2*(360*a^2*c^4*tan(1/2*f*x + 1/2*e)^11 + 960*a^2*c^3*d*tan(1/2*f*x
+ 1/2*e)^11 + 1260*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^11 + 720*a^2*c*d^3*tan(
1/2*f*x + 1/2*e)^11 + 165*a^2*d^4*tan(1/2*f*x + 1/2*e)^11 - 2040*a^2*c^4*ta
n(1/2*f*x + 1/2*e)^9 - 5440*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^9 - 7140*a^2*c^2
*d^2*tan(1/2*f*x + 1/2*e)^9 - 4080*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^9 - 935*a
^2*d^4*tan(1/2*f*x + 1/2*e)^9 + 4560*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 13440
*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^7 + 15480*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^
7 + 10272*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^7 + 1986*a^2*d^4*tan(1/2*f*x + 1/2
*e)^7 - 5040*a^2*c^4*tan(1/2*f*x + 1/2*e)^5 - 17280*a^2*c^3*d*tan(1/2*f*x +
 1/2*e)^5 - 19080*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 11232*a^2*c*d^3*tan(
1/2*f*x + 1/2*e)^5 - 3006*a^2*d^4*tan(1/2*f*x + 1/2*e)^5 + 2760*a^2*c^4*tan
(1/2*f*x + 1/2*e)^3 + 11200*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 13980*a^2*c^
2*d^2*tan(1/2*f*x + 1/2*e)^3 + 7440*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 1305
*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 600*a^2*c^4*tan(1/2*f*x + 1/2*e) - 2880*a
^2*c^3*d*tan(1/2*f*x + 1/2*e) - 4500*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e) - 312
0*a^2*c*d^3*tan(1/2*f*x + 1/2*e) - 795*a^2*d^4*tan(1/2*f*x + 1/2*e))/(tan(1
/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B]
time = 5.35, size = 484, normalized size = 1.48

$$\frac{\left(-3\,a^2c^4-8\,a^2c^3d-\frac{21\,a^2c^2d^2}{2}-6\,a^2cd^3-\frac{11\,a^2d^4}{8}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}+\left(17\,a^2c^4+\frac{136\,a^2c^3d}{3}+\frac{119\,a^2c^2d^2}{2}+34\,a^2cd^3+\frac{187\,a^2d^4}{24}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}+\left(-38\,a^2c^4-112\,a^2c^3d-129\,a^2c^2d^2-\frac{428\,a^2cd^3}{5}-\frac{331\,a^2d^4}{20}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}+\left(42\,a^2c^4+144\,a^2c^3d+159\,a^2c^2d^2+\frac{468\,a^2cd^3}{5}+\frac{501\,a^2d^4}{20}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}+\left(-23\,a^2c^4-\frac{280\,a^2c^3d}{3}-\frac{233\,a^2c^2d^2}{2}-62\,a^2cd^3-\frac{87\,a^2d^4}{8}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}+\left(5\,a^2c^4+24\,a^2c^3d+\frac{75\,a^2c^2d^2}{2}+26\,a^2cd^3+\frac{53\,a^2d^4}{8}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}-6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+1\right)}+\frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(24\,c^4+64\,c^3d+84\,c^2d^2+48\,cd^3+11\,d^4\right)}{4\left(6\,c^4+16\,c^3d+21\,c^2d^2+12\,cd^3+\frac{11\,d^4}{4}\right)}\right)\left(24\,c^4+64\,c^3d+84\,c^2d^2+48\,cd^3+11\,d^4\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(5*a^2*c^4 + (53*a^2*d^4)/8 + 26*a^2*c*d^3 + 24*a^2*c^3
*d + (75*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^11*(3*a^2*c^4 + (11*a^2*d^4)/
```

```
8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (21*a^2*c^2*d^2)/2) + tan(e/2 + (f*x)/2)^9*
(17*a^2*c^4 + (187*a^2*d^4)/24 + 34*a^2*c*d^3 + (136*a^2*c^3*d)/3 + (119*a^
2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^3*(23*a^2*c^4 + (87*a^2*d^4)/8 + 62*a^2*
c*d^3 + (280*a^2*c^3*d)/3 + (233*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^7*(38
*a^2*c^4 + (331*a^2*d^4)/20 + (428*a^2*c*d^3)/5 + 112*a^2*c^3*d + 129*a^2*c
^2*d^2) + tan(e/2 + (f*x)/2)^5*(42*a^2*c^4 + (501*a^2*d^4)/20 + (468*a^2*c*
d^3)/5 + 144*a^2*c^3*d + 159*a^2*c^2*d^2))/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*
tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 -
6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (a^2*atanh((tan(e/2
 + (f*x)/2)*(48*c*d^3 + 64*c^3*d + 24*c^4 + 11*d^4 + 84*c^2*d^2))/(4*(12*c*
d^3 + 16*c^3*d + 6*c^4 + (11*d^4)/4 + 21*c^2*d^2)))*(48*c*d^3 + 64*c^3*d +
24*c^4 + 11*d^4 + 84*c^2*d^2))/(8*f)
```

3.194 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=242

$$\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2\left(c^4-10c^3d-44c^2d^2-40cd^3-12d^4\right)\tan(e+fx)}{10df}-\frac{a}{}$$

```
[Out] 3/8*a^2*(2*c+d)*(2*c^2+3*c*d+2*d^2)*arctanh(sin(f*x+e))/f-1/10*a^2*(c^4-10*
c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)*tan(f*x+e)/d/f-1/40*a^2*(2*c^3-20*c^2*d-5
7*c*d^2-30*d^3)*sec(f*x+e)*tan(f*x+e)/f-1/20*a^2*(c^2-10*c*d-12*d^2)*(c+d*s
ec(f*x+e))^2*tan(f*x+e)/d/f-1/20*a^2*(c-10*d)*(c+d*sec(f*x+e))^3*tan(f*x+e)
/d/f+1/5*a^2*(c+d*sec(f*x+e))^4*tan(f*x+e)/d/f
```

Rubi [A]
time = 0.22, antiderivative size = 277, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 102, 152, 52, 65, 223, 209}

$$\frac{3a^3(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f}+\frac{(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{8f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2\left(2\left(8c^2+5cd+2d^2\right)+d(7c+2d)\sec(e+fx)\right)}{20f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^2}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*Tan[e + f*x])/(8*f) + (3*a^3*(2*c
+ d)*(2*c^2 + 3*c*d + 2*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Se
c[e + f*x])]]*Tan[e + f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) + ((2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*(a^2 + a^2*Sec[e + f*x])*Tan[
e + f*x])/(8*f) + (d*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e +
f*x])/(5*f) + (d*(a + a*Sec[e + f*x])^2*(2*(8*c^2 + 5*c*d + 2*d^2) + d*(7*c
 + 2*d)*Sec[e + f*x])*Tan[e + f*x])/(20*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^3}{\sqrt{a-ax}}\,dx, x\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e}{5f}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e}{5f}$$

$$= \frac{(2c+d)\left(2c^2+3cd+2d^2\right)\left(a^2+a^2\sec(e+fx)\right)}{8f}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c}{}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{3a}{}$$

Mathematica [A]
time = 1.37, size = 326, normalized size = 1.35

$$-\frac{a^2(1+\cos(e+fx))^2\sec^4\left(\frac{1}{2}(e+fx)\right)\sec^5(e+fx)\left(120\left(4c^3+8c^2d+7cd^2+2d^3\right)\cos^5(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-2\left(120c^3+380c^2d+400cd^2+152d^3+5\left(12c^3+72c^2d+87cd^2+34d^3\right)\cos(e+fx)+16\left(10c^3+30c^2d+30cd^2+9d^3\right)\cos(2(e+fx))+20c^3\cos(3(e+fx))+120c^2d\cos(3(e+fx))+105cd^2\cos(3(e+fx))+30d^3\cos(3(e+fx))+40c^3\cos(4(e+fx))+100c^2d\cos(4(e+fx))+80cd^2\cos(4(e+fx))+24d^3\cos(4(e+fx))\right)\sin(e+fx)\right)}{1280f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -1/1280*(a^2*(1 + Cos[e + f*x])^2*Sec[(e + f*x)/2]^4*Sec[e + f*x]^5*(120*(4
*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3)*Cos[e + f*x]^5*(Log[Cos[(e + f*x)/2] - Si
n[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(120*c^3 +
```

```
380*c^2*d + 400*c*d^2 + 152*d^3 + 5*(12*c^3 + 72*c^2*d + 87*c*d^2 + 34*d^3)
*Cos[e + f*x] + 16*(10*c^3 + 30*c^2*d + 30*c*d^2 + 9*d^3)*Cos[2*(e + f*x)]
+ 20*c^3*Cos[3*(e + f*x)] + 120*c^2*d*Cos[3*(e + f*x)] + 105*c*d^2*Cos[3*(e
 + f*x)] + 30*d^3*Cos[3*(e + f*x)] + 40*c^3*Cos[4*(e + f*x)] + 100*c^2*d*Co
s[4*(e + f*x)] + 80*c*d^2*Cos[4*(e + f*x)] + 24*d^3*Cos[4*(e + f*x)])*Sin[e
 + f*x]))/f
```

Maple [A]
time = 0.39, size = 395, normalized size = 1.63 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(c^3*a^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-3*a^
2*c^2*d*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+3*a^2*c*d^2*(-(-1/4*sec(f*x+e)^3
-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-a^2*d^3*(-8/15-1
/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+2*c^3*a^2*tan(f*x+e)+6*a^2*c^
2*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-6*a^2*c*d^2*(
-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a^2*d^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*
x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+c^3*a^2*ln(sec(f*x+e)+tan(f
*x+e))+3*a^2*c^2*d*tan(f*x+e)+3*a^2*c*d^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln
(sec(f*x+e)+tan(f*x+e)))-a^2*d^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(240) = 480$.
time = 0.29, size = 506, normalized size = 2.09

$$\frac{240\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2c^2d+480\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2cd^2+16\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^2d^3+80\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2d^3-45a^2cd^2\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin(fx+e)+1\right)+3\log\left(\sin(fx+e)-1\right)\right)-30a^2d^3\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log\left(\sin(fx+e)+1\right)+3\log\left(\sin(fx+e)-1\right)\right)-60a^2c^3\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin(fx+e)+1\right)+\log\left(\sin(fx+e)-1\right)\right)-360a^2c^2d\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin(fx+e)+1\right)+\log\left(\sin(fx+e)-1\right)\right)-180a^2cd^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin(fx+e)+1\right)+\log\left(\sin(fx+e)-1\right)\right)+240a^2c^3\log\left(\sec(fx+e)+\tan(fx+e)\right)+480a^2c^3\tan(fx+e)+720a^2c^2d\tan(fx+e)}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2*d + 480*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a^2*c*d^2 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 +
15*tan(f*x + e))*a^2*d^3 + 80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d^3 - 4
5*a^2*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(
f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a
^2*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
 e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 60*a^2*c^
3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*
x + e) - 1)) - 360*a^2*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin
(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 180*a^2*c*d^2*(2*sin(f*x + e)/(si
n(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a^
2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 480*a^2*c^3*tan(f*x + e) + 720*a^2
*c^2*d*tan(f*x + e))/f
```

Fricas [A]
time = 2.16, size = 304, normalized size = 1.26

$$\frac{15 \, {\left(4 \, a^{2} c^{3} + 8 \, a^{2} c^{2} d + 7 \, a^{2} c d^{2} + 2 \, a^{2} d^{3}\right)} \cos\left(f x + e\right)^{5} \log\left(\sin\left(f x + e\right) + 1\right) - 15 \, {\left(4 \, a^{2} c^{3} + 8 \, a^{2} c^{2} d + 7 \, a^{2} c d^{2} + 2 \, a^{2} d^{3}\right)} \cos\left(f x + e\right)^{5} \log\left(-\sin\left(f x + e\right) + 1\right) + 2 \, {\left(8 \, a^{2} d^{3} + 8 \, {\left(10 \, a^{2} c^{3} + 25 \, a^{2} c^{2} d + 20 \, a^{2} c d^{2} + 6 \, a^{2} d^{3}\right)} \cos\left(f x + e\right)^{4} + 5 \, {\left(4 \, a^{2} c^{3} + 24 \, a^{2} c^{2} d + 21 \, a^{2} c d^{2} + 6 \, a^{2} d^{3}\right)} \cos\left(f x + e\right)^{3} + 8 \, {\left(5 \, a^{2} c^{2} d + 10 \, a^{2} c d^{2} + 3 \, a^{2} d^{3}\right)} \cos\left(f x + e\right)^{2} + 10 \, {\left(3 \, a^{2} c d^{2} + 2 \, a^{2} d^{3}\right)} \cos\left(f x + e\right)\right)} \sin\left(f x + e\right)}{80 \, f \cos\left(f x + e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^5
*log(sin(f*x + e) + 1) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*
d^3)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^2*d^3 + 8*(10*a^2*c^3 +
 25*a^2*c^2*d + 20*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^4 + 5*(4*a^2*c^3 + 2
4*a^2*c^2*d + 21*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^3 + 8*(5*a^2*c^2*d + 1
0*a^2*c*d^2 + 3*a^2*d^3)*cos(f*x + e)^2 + 10*(3*a^2*c*d^2 + 2*a^2*d^3)*cos(
f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int c^{3} \sec{\left(e + f x \right)}\, dx + \int 2 c^{3} \sec^{2}{\left(e + f x \right)}\, dx + \int c^{3} \sec^{3}{\left(e + f x \right)}\, dx + \int d^{3} \sec^{4}{\left(e + f x \right)}\, dx + \int 2 d^{3} \sec^{5}{\left(e + f x \right)}\, dx + \int d^{3} \sec^{6}{\left(e + f x \right)}\, dx + \int 3 c d^{2} \sec^{3}{\left(e + f x \right)}\, dx + \int 6 c d^{2} \sec^{4}{\left(e + f x \right)}\, dx + \int 3 c d^{2} \sec^{5}{\left(e + f x \right)}\, dx + \int 3 c^{2} d \sec^{2}{\left(e + f x \right)}\, dx + \int 6 c^{2} d \sec^{3}{\left(e + f x \right)}\, dx + \int 3 c^{2} d \sec^{4}{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**2*(Integral(c**3*sec(e + f*x), x) + Integral(2*c**3*sec(e + f*x)**2, x)
+ Integral(c**3*sec(e + f*x)**3, x) + Integral(d**3*sec(e + f*x)**4, x) + I
ntegral(2*d**3*sec(e + f*x)**5, x) + Integral(d**3*sec(e + f*x)**6, x) + In
tegral(3*c*d**2*sec(e + f*x)**3, x) + Integral(6*c*d**2*sec(e + f*x)**4, x)
 + Integral(3*c*d**2*sec(e + f*x)**5, x) + Integral(3*c**2*d*sec(e + f*x)**
2, x) + Integral(6*c**2*d*sec(e + f*x)**3, x) + Integral(3*c**2*d*sec(e + f
*x)**4, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(230) = 460.
time = 0.65, size = 506, normalized size = 2.09

$$\frac{15 \, {\left(4 \, a^{2} c^{3} + 8 \, a^{2} c^{2} d + 7 \, a^{2} c d^{2} + 2 \, a^{2} d^{3}\right)} \log\left({\left| \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 1 \right|}\right) - 15 \, {\left(4 \, a^{2} c^{3} + 8 \, a^{2} c^{2} d + 7 \, a^{2} c d^{2} + 2 \, a^{2} d^{3}\right)} \log\left({\left| \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) - 1 \right|}\right) - \frac{2 \, {\left(60 \, a^{2} c^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{9} + 120 \, a^{2} c^{2} d \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{9} + 105 \, a^{2} c d^{2} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{9} + 30 \, a^{2} d^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{9} - 280 \, a^{2} c^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{7} - 560 \, a^{2} c^{2} d \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{7} - 490 \, a^{2} c d^{2} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{7} - 140 \, a^{2} d^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{7} + 480 \, a^{2} c^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{5} + 1120 \, a^{2} c^{2} d \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{5} + 800 \, a^{2} c d^{2} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{5} + 288 \, a^{2} d^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{5} - 360 \, a^{2} c^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} - 1040 \, a^{2} c^{2} d \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} - 790 \, a^{2} c d^{2} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} - 180 \, a^{2} d^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{3} + 100 \, a^{2} c^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 360 \, a^{2} c^{2} d \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 375 \, a^{2} c d^{2} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right) + 130 \, a^{2} d^{3} \tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right)}}{{\left(\tan\left(\frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{2} - 1\right)}^{5}}}{40 \, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/40*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*log(abs(tan(1/
2*f*x + 1/2*e) + 1)) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^
```

```
3)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(60*a^2*c^3*tan(1/2*f*x + 1/2*e)^
9 + 120*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^9 + 105*a^2*c*d^2*tan(1/2*f*x + 1/2*
e)^9 + 30*a^2*d^3*tan(1/2*f*x + 1/2*e)^9 - 280*a^2*c^3*tan(1/2*f*x + 1/2*e)
^7 - 560*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^7 - 490*a^2*c*d^2*tan(1/2*f*x + 1/2
*e)^7 - 140*a^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 480*a^2*c^3*tan(1/2*f*x + 1/2*
e)^5 + 1120*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 800*a^2*c*d^2*tan(1/2*f*x +
1/2*e)^5 + 288*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 360*a^2*c^3*tan(1/2*f*x + 1
/2*e)^3 - 1040*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 790*a^2*c*d^2*tan(1/2*f*x
 + 1/2*e)^3 - 180*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 100*a^2*c^3*tan(1/2*f*x
+ 1/2*e) + 360*a^2*c^2*d*tan(1/2*f*x + 1/2*e) + 375*a^2*c*d^2*tan(1/2*f*x +
 1/2*e) + 130*a^2*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)
/f
```

Mupad [B]
time = 5.49, size = 394, normalized size = 1.63

$$\frac{3\,a^2\,\mathrm{atanh}\left(\frac{3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c+d\right)\,\left(2\,c^2+3\,c\,d+2\,d^2\right)}{2\,\left(6\,c^3+12\,c^2\,d+\frac{21\,c\,d^2}{2}+3\,d^3\right)}\right)\,\left(2\,c+d\right)\,\left(2\,c^2+3\,c\,d+2\,d^2\right)}{4\,f}-\frac{\left(3\,a^2\,c^3+6\,a^2\,c^2\,d+\frac{21\,a^2\,c\,d^2}{4}+\frac{3\,a^2\,d^3}{2}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9+\left(-14\,a^2\,c^3-28\,a^2\,c^2\,d-\frac{49\,a^2\,c\,d^2}{2}-7\,a^2\,d^3\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+\left(24\,a^2\,c^3+56\,a^2\,c^2\,d+40\,a^2\,c\,d^2+\frac{72\,a^2\,d^3}{5}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(-18\,a^2\,c^3-52\,a^2\,c^2\,d-\frac{79\,a^2\,c\,d^2}{2}-9\,a^2\,d^3\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(5\,a^2\,c^3+18\,a^2\,c^2\,d+\frac{75\,a^2\,c\,d^2}{4}+\frac{13\,a^2\,d^3}{2}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (3*a^2*atanh((3*tan(e/2 + (f*x)/2)*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2))/(2*((
21*c*d^2)/2 + 12*c^2*d + 6*c^3 + 3*d^3)))*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2)
)/(4*f) - (tan(e/2 + (f*x)/2)^9*(3*a^2*c^3 + (3*a^2*d^3)/2 + (21*a^2*c*d^2)
/4 + 6*a^2*c^2*d) - tan(e/2 + (f*x)/2)^7*(14*a^2*c^3 + 7*a^2*d^3 + (49*a^2*
c*d^2)/2 + 28*a^2*c^2*d) - tan(e/2 + (f*x)/2)^3*(18*a^2*c^3 + 9*a^2*d^3 + (
79*a^2*c*d^2)/2 + 52*a^2*c^2*d) + tan(e/2 + (f*x)/2)^5*(24*a^2*c^3 + (72*a^
2*d^3)/5 + 40*a^2*c*d^2 + 56*a^2*c^2*d) + tan(e/2 + (f*x)/2)*(5*a^2*c^3 + (
13*a^2*d^3)/2 + (75*a^2*c*d^2)/4 + 18*a^2*c^2*d))/(f*(5*tan(e/2 + (f*x)/2)^
2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2
)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.195 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=176

$$\frac{a^2(12c^2+16cd+7d^2)\tanh^{-1}(\sin(e+fx))}{8f}-\frac{a^2(c^3-8c^2d-20cd^2-8d^3)\tan(e+fx)}{6df}-\frac{a^2(2c(c-8d)-21d^2$$

```
[Out] 1/8*a^2*(12*c^2+16*c*d+7*d^2)*arctanh(sin(f*x+e))/f-1/6*a^2*(c^3-8*c^2*d-20
*c*d^2-8*d^3)*tan(f*x+e)/d/f-1/24*a^2*(2*c*(c-8*d)-21*d^2)*sec(f*x+e)*tan(f
*x+e)/f-1/12*a^2*(c-8*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/d/f+1/4*a^2*(c+d*sec
(f*x+e))^3*tan(f*x+e)/d/f
```

Rubi [A]
time = 0.17, antiderivative size = 234, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 92, 81, 52, 65, 223, 209}

$$\frac{a^3(12c^2+16cd+7d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f}+\frac{(12c^2+16cd+7d^2)\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{24f}+\frac{d(5c+2d)\tan(e+fx)(a\sec(e+fx)+a)^2}{12f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*Tan[e + f*x])/(8*f) + (a^3*(12*c^2 + 16*c*d
+ 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e
+ f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d*(5*c +
 2*d)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + ((12*c^2 + 16*c*d + 7*d
^2)*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(24*f) + (d*(a + a*Sec[e + f*x])
^2*(c + d*Sec[e + f*x])*Tan[e + f*x])/(4*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^2}{\sqrt{a-ax}}\,dx, x \ldots\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+\ldots}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+\ldots}{4f}$$

$$= \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{d\ldots}{}$$

$$= \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{(\ldots}{}$$

$$= \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{d(5c+2d\ldots}{}$$

$$= \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{d(5c+2d\ldots}{}$$

$$= \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{d(5c+2d\ldots}{}$$

$$= \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{a^3(12c^2+\ldots}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 479 vs. $2(176) = 352$.
time = 1.00, size = 479, normalized size = 2.72

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] -1/192*(a^2*Sec[e + f*x]^4*(108*c^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 144*c*d*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 63*d^2*Log[Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]] + 12*(12*c^2 + 16*c*d + 7*d^2)*Cos[2*(e + f*x)]
```

```
*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2]]) + 3*(12*c^2 + 16*c*d + 7*d^2)*Cos[4*(e + f*x)]*(Log[Cos[(e + f*
x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 108
*c^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 144*c*d*Log[Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]] - 63*d^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 2
4*c^2*Sin[e + f*x] - 96*c*d*Sin[e + f*x] - 90*d^2*Sin[e + f*x] - 96*c^2*Sin
[2*(e + f*x)] - 224*c*d*Sin[2*(e + f*x)] - 128*d^2*Sin[2*(e + f*x)] - 24*c^
2*Sin[3*(e + f*x)] - 96*c*d*Sin[3*(e + f*x)] - 42*d^2*Sin[3*(e + f*x)] - 48
*c^2*Sin[4*(e + f*x)] - 80*c*d*Sin[4*(e + f*x)] - 32*d^2*Sin[4*(e + f*x)]))
/f
```

Maple [A]
time = 0.31, size = 270, normalized size = 1.53

method	result
norman	$\frac{\frac{11a^2\left(12c^2+16cd+7d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}-\frac{a^2\left(12c^2+16cd+7d^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}+\frac{a^2\left(20c^2+48cd+25d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{a^2\left(156c^2\right.}{}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}$
derivativedivides	$\frac{a^2c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-2a^2cd\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+a^2d^2\left(-\left(-\frac{\left(\sec^3(fx+e\right.}{4}\right.\right.}{}$
default	$\frac{a^2c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-2a^2cd\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+a^2d^2\left(-\left(-\frac{\left(\sec^3(fx+e\right.}{4}\right.\right.}{}$
risch	$-\frac{ia^2\left(12c^2e^{7i(fx+e)}+48cd\,e^{7i(fx+e)}+21d^2e^{7i(fx+e)}-48c^2e^{6i(fx+e)}-48cd\,e^{6i(fx+e)}+12c^2e^{5i(fx+e)}+48cd\,e^{5i(fx+e)}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(a^2*c^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-2*a^
2*c*d*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a^2*d^2*(-(-1/4*sec(f*x+e)^3-3/8*s
ec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+2*a^2*c^2*tan(f*x+e)+4
*a^2*c*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-2*a^2*d^
2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a^2*c^2*ln(sec(f*x+e)+tan(f*x+e))+2*a^
2*c*d*tan(f*x+e)+a^2*d^2*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f
*x+e))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(174) = 348.
time = 0.28, size = 350, normalized size = 1.99

$$\frac{32\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2cd+32\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2d^2-3a^2d^2\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log(\sin(fx+e)-1)\right)-12a^2c^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-48a^2cd\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-12a^2d^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+48a^2c^2\log(\sec(fx+e)+\tan(fx+e))+96a^2c^2\tan(fx+e)+96a^2cd\tan(fx+e)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c*d + 32*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a^2*d^2 - 3*a^2*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(
sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(si
n(f*x + e) - 1)) - 12*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(si
n(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*d*(2*sin(f*x + e)/(sin(
f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*a^2*d
^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f
*x + e) - 1)) + 48*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)) + 96*a^2*c^2*ta
n(f*x + e) + 96*a^2*c*d*tan(f*x + e))/f
```

Fricas [A]
time = 2.31, size = 218, normalized size = 1.24

$$\frac{3\,(12\,a^2c^2+16\,a^2cd+7\,a^2d^2)\cos{(fx+e)}^4\log{(\sin{(fx+e)}+1)}-3\,(12\,a^2c^2+16\,a^2cd+7\,a^2d^2)\cos{(fx+e)}^4\log{(-\sin{(fx+e)}+1)}+2\,(6\,a^2d^2+16\,(3\,a^2c^2+5\,a^2cd+2\,a^2d^2)\cos{(fx+e)}^3+3\,(4\,a^2c^2+16\,a^2cd+7\,a^2d^2)\cos{(fx+e)}^2+16\,(a^2cd+a^2d^2)\cos{(fx+e)})\sin{(fx+e)}}{48\,f\cos{(fx+e)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(sin(f*x +
e) + 1) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(-sin(f
*x + e) + 1) + 2*(6*a^2*d^2 + 16*(3*a^2*c^2 + 5*a^2*c*d + 2*a^2*d^2)*cos(f*
x + e)^3 + 3*(4*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^2 + 16*(a^2*
c*d + a^2*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c^2\sec{(e+fx)}\,dx+\int 2c^2\sec^2{(e+fx)}\,dx+\int c^2\sec^3{(e+fx)}\,dx+\int d^2\sec^3{(e+fx)}\,dx+\int 2d^2\sec^4{(e+fx)}\,dx+\int d^2\sec^5{(e+fx)}\,dx+\int 2cd\sec^2{(e+fx)}\,dx+\int 4cd\sec^3{(e+fx)}\,dx+\int 2cd\sec^4{(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(c**2*sec(e + f*x), x) + Integral(2*c**2*sec(e + f*x)**2, x)
+ Integral(c**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**3, x) + I
ntegral(2*d**2*sec(e + f*x)**4, x) + Integral(d**2*sec(e + f*x)**5, x) + In
tegral(2*c*d*sec(e + f*x)**2, x) + Integral(4*c*d*sec(e + f*x)**3, x) + Int
egral(2*c*d*sec(e + f*x)**4, x))
```

Giac [A]
time = 0.54, size = 320, normalized size = 1.82

$$\frac{3\,(12\,a^2c^2+16\,a^2cd+7\,a^2d^2)\log{(|\tan{(\frac{1}{2}\,fx+\frac{1}{2}\,e)}+1|)}-3\,(12\,a^2c^2+16\,a^2cd+7\,a^2d^2)\log{(|\tan{(\frac{1}{2}\,fx+\frac{1}{2}\,e)}-1|)}-\frac{2\left(36\,a^2c^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7+48\,a^2cd\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7+21\,a^2d^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7-132\,a^2c^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-176\,a^2cd\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5-77\,a^2d^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^5+156\,a^2c^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3+272\,a^2cd\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3+83\,a^2d^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^3-60\,a^2c^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-144\,a^2cd\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-75\,a^2d^2\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)\right)}{\left(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*log(abs(tan(1/2*f*x + 1/2*e)
+ 1)) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*log(abs(tan(1/2*f*x + 1/2*e
) - 1)) - 2*(36*a^2*c^2*tan(1/2*f*x + 1/2*e)^7 + 48*a^2*c*d*tan(1/2*f*x + 1
/2*e)^7 + 21*a^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^2*tan(1/2*f*x + 1/2
*e)^5 - 176*a^2*c*d*tan(1/2*f*x + 1/2*e)^5 - 77*a^2*d^2*tan(1/2*f*x + 1/2*e
)^5 + 156*a^2*c^2*tan(1/2*f*x + 1/2*e)^3 + 272*a^2*c*d*tan(1/2*f*x + 1/2*e)
^3 + 83*a^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 60*a^2*c^2*tan(1/2*f*x + 1/2*e) -
144*a^2*c*d*tan(1/2*f*x + 1/2*e) - 75*a^2*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/
2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B]
time = 5.46, size = 237, normalized size = 1.35

$$\frac{\left(-3\,a^2\,c^2-4\,a^2\,c\,d-\frac{7\,a^2\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(11\,a^2\,c^2+\frac{44\,a^2\,c\,d}{3}+\frac{77\,a^2\,d^2}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-13\,a^2\,c^2-\frac{68\,a^2\,c\,d}{3}-\frac{83\,a^2\,d^2}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(5\,a^2\,c^2+12\,a^2\,c\,d+\frac{25\,a^2\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)}{2\,\left(6\,c^2+8\,c\,d+\frac{7\,d^2}{2}\right)}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(5*a^2*c^2 + (25*a^2*d^2)/4 + 12*a^2*c*d) - tan(e/2 + (
f*x)/2)^7*(3*a^2*c^2 + (7*a^2*d^2)/4 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^5*(1
1*a^2*c^2 + (77*a^2*d^2)/12 + (44*a^2*c*d)/3) - tan(e/2 + (f*x)/2)^3*(13*a^
2*c^2 + (83*a^2*d^2)/12 + (68*a^2*c*d)/3))/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*t
an(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) +
 (a^2*atanh((tan(e/2 + (f*x)/2)*(16*c*d + 12*c^2 + 7*d^2))/(2*(8*c*d + 6*c^
2 + (7*d^2)/2)))*(16*c*d + 12*c^2 + 7*d^2))/(4*f)
```

3.196 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=103

$$\frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2a^2(3c+2d)\tan(e+fx)}{3f}+\frac{a^2(3c+2d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{d(a+a\,\text{s}}{}$$

[Out] 1/2*a^2*(3*c+2*d)*arctanh(sin(f*x+e))/f+2/3*a^2*(3*c+2*d)*tan(f*x+e)/f+1/6*a^2*(3*c+2*d)*sec(f*x+e)*tan(f*x+e)/f+1/3*d*(a+a*sec(f*x+e))^2*tan(f*x+e)/f

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4086, 3873, 3852, 8, 4131, 3855}

$$\frac{2a^2(3c+2d)\tan(e+fx)}{3f}+\frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{a^2(3c+2d)\tan(e+fx)\sec(e+fx)}{6f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]

[Out] (a^2*(3*c + 2*d)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a^2*(3*c + 2*d)*Tan[e + f*x])/(3*f) + (a^2*(3*c + 2*d)*Sec[e + f*x]*Tan[e + f*x])/(6*f) + (d*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
```

```
e, f, n}, x]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx &= \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}(3c+2d\\ &= \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}(3c+2d\\ &= \frac{a^2(3c+2d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{d(a+a\\ &= \frac{a^2(3c+2d)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2a^2(3c+2d}{3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 481 vs. 2(103) = 206.
time = 6.30, size = 481, normalized size = 4.67

$$\frac{a^2\cos^3(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^2(c+d\sec(e+fx))\left(-6(3c+2d)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+6(3c+2d)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{2d\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{(3c+7d)\cos\left(\frac{e}{2}\right)-(3c+5d)\sin\left(\frac{e}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{4(6c+5d)\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{2d\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{(3c+7d)\cos\left(\frac{e}{2}\right)+(3c+5d)\sin\left(\frac{e}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{4(6c+5d)\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{48f(d+c\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]^3*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^2*(c + d*Sec[e +
f*x])*(-6*(3*c + 2*d)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 6*(3*c + 2
*d)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (2*d*Sin[(f*x)/2])/((Cos[e/2
```

```
] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + ((3*c + 7*d)*Cos[e
/2] - (3*c + 5*d)*Sin[e/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^2) + (4*(6*c + 5*d)*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])) + (2*d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/
2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - ((3*c + 7*d)*Cos[e/2] + (3*c
 + 5*d)*Sin[e/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^2) + (4*(6*c + 5*d)*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))))/(48*f*(d + c*Cos[e + f*x]))
```

Maple [A]
time = 0.24, size = 145, normalized size = 1.41

method	result
derivativedivides	$\frac{a^2 c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2 d\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+2a^2 c\tan(fx+e)+2a^2 d\left(\frac{\sec(f}{}\right.}{f}$
default	$\frac{a^2 c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2 d\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+2a^2 c\tan(fx+e)+2a^2 d\left(\frac{\sec(f}{}\right.}{f}$
norman	$\frac{\frac{8a^2(3c+2d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{a^2(3c+2d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{a^2(5c+6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{a^2(3c+2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a^2(}{}$
risch	$-\frac{ia^2\left(3c\,e^{5i(fx+e)}+6d\,e^{5i(fx+e)}-12c\,e^{4i(fx+e)}-6d\,e^{4i(fx+e)}-24\,e^{2i(fx+e)}c-24d\,e^{2i(fx+e)}-3\,e^{i(fx+e)}c-6d\,e^{i(fx+e)}-1\right.}{3f\left(e^{2i(fx+e)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a^2*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a^2*d*
(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a^2*c*tan(f*x+e)+2*a^2*d*(1/2*sec(f*x+
e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^2*c*ln(sec(f*x+e)+tan(f*x+e)
)+a^2*d*tan(f*x+e))
```

Maxima [A]
time = 0.29, size = 181, normalized size = 1.76

$$\frac{4\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2d-3a^2c\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-6a^2d\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+12a^2c\log(\sec(fx+e)+\tan(fx+e))+24a^2c\tan(fx+e)+12a^2d\tan(fx+e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d - 3*a^2*c*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 6*a^
2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a^2*c*log(sec(f*x + e) + tan(f*x + e)) + 24*a^2*c*tan(f
*x + e) + 12*a^2*d*tan(f*x + e))/f
```

Fricas [A]
time = 1.17, size = 146, normalized size = 1.42

$$\frac{3\,(3\,a^2c+2\,a^2d)\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,(3\,a^2c+2\,a^2d)\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2\,a^2d+2\,(6\,a^2c+5\,a^2d)\cos\left(fx+e\right)^2+3\,(a^2c+2\,a^2d)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(3*a^2*c + 2*a^2*d)*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*(3*a^2
*c + 2*a^2*d)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*a^2*d + 2*(6*a^2
*c + 5*a^2*d)*cos(f*x + e)^2 + 3*(a^2*c + 2*a^2*d)*cos(f*x + e))*sin(f*x +
e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int c\sec\left(e+fx\right)dx+\int 2c\sec^2\left(e+fx\right)dx+\int c\sec^3\left(e+fx\right)dx+\int d\sec^2\left(e+fx\right)dx+\int 2d\sec^3\left(e+fx\right)dx+\int d\sec^4\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e)),x)
```

```
[Out] a**2*(Integral(c*sec(e + f*x), x) + Integral(2*c*sec(e + f*x)**2, x) + Inte
gral(c*sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**2, x) + Integral(2*d*
sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**4, x))
```

Giac [A]
time = 0.51, size = 178, normalized size = 1.73

$$\frac{3\,(3\,a^2c+2\,a^2d)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,(3\,a^2c+2\,a^2d)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(9\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+6\,a^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-16\,a^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+18\,a^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(3*a^2*c + 2*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(3*a^2*c
+ 2*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(9*a^2*c*tan(1/2*f*x + 1/
2*e)^5 + 6*a^2*d*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c*tan(1/2*f*x + 1/2*e)^3 -
 16*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c*tan(1/2*f*x + 1/2*e) + 18*a^2*d
*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B]
time = 4.52, size = 161, normalized size = 1.56

$$\frac{2\,a^2\,\mathrm{atanh}\left(\frac{4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,c}{2}+d\right)}{6\,c+4\,d}\right)\,\left(\frac{3\,c}{2}+d\right)}{f}-\frac{\left(3\,a^2\,c+2\,a^2\,d\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(-8\,a^2\,c-\frac{16\,a^2\,d}{3}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(5\,a^2\,c+6\,a^2\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (2*a^2*atanh((4*tan(e/2 + (f*x)/2)*((3*c)/2 + d))/(6*c + 4*d))*((3*c)/2 + d
))/f - (tan(e/2 + (f*x)/2)*(5*a^2*c + 6*a^2*d) + tan(e/2 + (f*x)/2)^5*(3*a^
2*c + 2*a^2*d) - tan(e/2 + (f*x)/2)^3*(8*a^2*c + (16*a^2*d)/3))/(f*(3*tan(e
/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

3.197 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^2(c-2d)\tanh^{-1}(\sin(e+fx))}{d^2f}+\frac{2a^2(c-d)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2\sqrt{c+d}\,f}+\frac{a^2\tan(e+fx)}{df}$$

```
[Out] -a^2*(c-2*d)*arctanh(sin(f*x+e))/d^2/f+2*a^2*(c-d)^(3/2)*arctanh((c-d)^(1/2
)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/d^2/f/(c+d)^(1/2)+a^2*tan(f*x+e)/d/f
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 208 vs. $2(95) = 190$.
time = 0.17, antiderivative size = 208, normalized size of antiderivative = 2.19, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4072, 104, 163, 65, 223, 209, 95, 211}

$$-\frac{2a^3(c-2d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2a^3(c-d)^{3/2}\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^2f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2\tan(e+fx)}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Tan[e + f*x])/(d*f) - (2*a^3*(c - 2*d)*ArcTan[Sqrt[a - a*Sec[e + f*x]]
/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^2*f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]) - (2*a^3*(c - d)^(3/2)*ArcTan[(Sqrt[c + d]*Sqrt[a
 + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(
d^2*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2\tan(e+fx)}{df} + \frac{(a\tan(e+fx))\mathrm{Subst}\left(\int \frac{-a^3d+a^3(c-2d)x}{\sqrt{a-ax}\,\sqrt{a+ax}\,(c+a}\right.}{df\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(}$$

$$= \frac{a^2\tan(e+fx)}{df} + \frac{(a^4(c-2d)\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,\sqrt{a}}\right.}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{(2a^3(c-2d)\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,d\right.}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a}}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{2a^3(c-d)^{3/2}\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+f}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+f}}\right.}{d^2\sqrt{c+d}\,f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{2a^3(c-2d)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+f}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.09, size = 329, normalized size = 3.46

$$\frac{a^2\cos(e+fx)(d+c\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^2\left((c-2d)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-(c-2d)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-\frac{2i(c-d)^2\mathrm{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{d\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{d\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{4d^2f(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x]
)^2*((c - 2*d)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - (c - 2*d)*Log[Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]] - ((2*I)*(c - d)^2*ArcTan[((I*Cos[e] + Si
n[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos
[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I
*Sin[e])^2]) + (d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])) + (d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))))/(4*d^2*f*(c + d*Sec[e + f*x]))
```

Maple [A]
time = 0.31, size = 150, normalized size = 1.58

method	result
derivativedivides	$\frac{8a^2\left(-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{(c-2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8d^2}-\frac{\left(-c^2+2cd-d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{4d^2\sqrt{(c+d)(c-d)}}-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{f}$
default	$\frac{8a^2\left(-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{(c-2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8d^2}-\frac{\left(-c^2+2cd-d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{4d^2\sqrt{(c+d)(c-d)}}-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{f}$
risch	$\frac{2ia^2}{fd\left(e^{2i(fx+e)}+1\right)}+\frac{\sqrt{(c+d)(c-d)}\,a^2\ln\left(e^{i(fx+e)}+\frac{i\sqrt{(c+d)(c-d)}+d}{c}\right)c}{(c+d)fd^2}-\frac{\sqrt{(c+d)(c-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*a^2*(-1/8/d/(tan(1/2*f*x+1/2*e)-1)+1/8*(c-2*d)/d^2*ln(tan(1/2*f*x+1/2*e
)-1)-1/4/d^2*(-c^2+2*c*d-d^2)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x
+1/2*e)/((c+d)*(c-d))^(1/2))-1/8/d/(tan(1/2*f*x+1/2*e)+1)+1/8/d^2*(-c+2*d)*
ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.09, size = 420, normalized size = 4.42

$$\left[\frac{2\,a^2d\sin\left(fx+e\right)-\left(a^2c-a^2d\right)\sqrt{\frac{c-d}{c+d}}\cos\left(fx+e\right)\log\left(\frac{2\,cd\cos(fx+e)-\left(c^2-2\,d^2\right)\cos(fx+e)^2-2\left(c^2+cd+\left(cd+d^2\right)\cos(fx+e)\right)\sqrt{\frac{c-d}{c+d}}\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)-\left(a^2c-2\,a^2d\right)\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)+\left(a^2c-2\,a^2d\right)\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)}{2\,d^2f\cos\left(fx+e\right)},\ \frac{2\,a^2d\sin\left(fx+e\right)+2\left(a^2c-a^2d\right)\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{\left(d\cos(fx+e)+c\right)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right)\cos\left(fx+e\right)-\left(a^2c-2\,a^2d\right)\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)+\left(a^2c-2\,a^2d\right)\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)}{2\,d^2f\cos\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*a^2*d*sin(f*x + e) - (a^2*c - a^2*d)*sqrt((c - d)/(c + d))*cos(f*x
+ e)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d
+ (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^
2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (a^2*c - 2*a^2*d)*cos
(f*x + e)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(f
*x + e) + 1))/(d^2*f*cos(f*x + e)), 1/2*(2*a^2*d*sin(f*x + e) + 2*(a^2*c -
a^2*d)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c
 + d))/((c - d)*sin(f*x + e)))*cos(f*x + e) - (a^2*c - 2*a^2*d)*cos(f*x + e
)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(f*x + e)
+ 1))/(d^2*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{\sec{\left(e+fx \right)}}{c+d\sec{\left(e+fx \right)}}\, dx+\int \frac{2\sec^{2}{\left(e+fx \right)}}{c+d\sec{\left(e+fx \right)}}\, dx+\int \frac{\sec^{3}{\left(e+fx \right)}}{c+d\sec{\left(e+fx \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(2*sec(e + f
*x)**2/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**3/(c + d*sec(e + f
*x)), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(86) = 172.
time = 0.52, size = 196, normalized size = 2.06

$$-\frac{\frac{2\,a^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)d}+\frac{\left(a^{2}c-2\,a^{2}d\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d^{2}}-\frac{\left(a^{2}c-2\,a^{2}d\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d^{2}}+\frac{2\left(a^{2}c^{2}-2\,a^{2}cd+a^{2}d^{2}\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,c-2\,d\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^{2}+d^{2}}}\right)\right)}{\sqrt{-c^{2}+d^{2}}\,d^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(2*a^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*d) + (a^2*c - 2*
a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^2 - (a^2*c - 2*a^2*d)*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/d^2 + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) -
d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/(sqrt(-c^2 + d^2)*d^2))/f
```

Mupad [B]
time = 2.59, size = 529, normalized size = 5.57

$$\frac{2\,a^2\,\left(\frac{\sin\left(e+f\,x\right)}{2}+2\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}+\frac{2\,a^2\,\left(\frac{c\,\sin\left(e+f\,x\right)}{2}+c\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{d\,f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}-\frac{2\,a^2\,\left(c^2\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)+\cos\left(e+f\,x\right)\,\mathrm{atan}\left(\frac{\left(2\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(c^4-2\,c^3\,d+2\,c\,d^3-d^4\right)}^{3/2}-2\,c^5\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^4-2\,c^3\,d+2\,c\,d^3-d^4}+5\,d^5\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^4-2\,c^3\,d+2\,c\,d^3-d^4}-c\,d^4\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^4-2\,c^3\,d+2\,c\,d^3-d^4}+4\,c^4\,d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^4-2\,c^3\,d+2\,c\,d^3-d^4}-9\,c^2\,d^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^4-2\,c^3\,d+2\,c\,d^3-d^4}+3\,c^3\,d^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^4-2\,c^3\,d+2\,c\,d^3-d^4}\right)\,1{}\mathrm{i}}{d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c+d\right)\,\left(3\,c^4\,d-8\,c^3\,d^2+2\,c^2\,d^3+8\,c\,d^4-5\,d^5\right)}\right)\,\sqrt{\left(c+d\right)\,{\left(c-d\right)}^3}\,1{}\mathrm{i}\right)}{d^2\,f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (2*a^2*(sin(e + f*x)/2 + 2*cos(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 +
(f*x)/2))))/(f*cos(e + f*x)*(c + d)) + (2*a^2*((c*sin(e + f*x))/2 + c*cos(e
 + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/(d*f*cos(e + f*x)*(c
 + d)) - (2*a^2*(c^2*cos(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/
2)) + cos(e + f*x)*atan(((2*c*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 -
 d^4)^(3/2) - 2*c^5*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2
) + 5*d^5*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) - c*d^4*
sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) + 4*c^4*d*sin(e/2
+ (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) - 9*c^2*d^3*sin(e/2 + (f*x
)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) + 3*c^3*d^2*sin(e/2 + (f*x)/2)*(
2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2))*1i)/(d*cos(e/2 + (f*x)/2)*(c + d)*(8*
c*d^4 + 3*c^4*d - 5*d^5 + 2*c^2*d^3 - 8*c^3*d^2)))*((c + d)*(c - d)^3)^(1/2
)*1i))/(d^2*f*cos(e + f*x)*(c + d))
```

3.198 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=117

$$\frac{a^2\tanh^{-1}(\sin(e+fx))}{d^2f}-\frac{2a^2\sqrt{c-d}\,(c+2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2(c+d)^{3/2}f}-\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))}$$

```
[Out] a^2*arctanh(sin(f*x+e))/d^2/f-2*a^2*(c+2*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x
+1/2*e)/(c+d)^(1/2))*(c-d)^(1/2)/d^2/(c+d)^(3/2)/f-a^2*(c-d)*tan(f*x+e)/d/(
c+d)/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.17, antiderivative size = 231, normalized size of antiderivative = 1.97, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4072, 100, 163, 65, 223, 209, 95, 211}

$$\frac{2a^3\sqrt{c-d}\,(c+2d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{d^2f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2a^3\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{a^2(c-d)\tan(e+fx)}{df(c+d)(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(d^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*Sq
rt[c - d]*(c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^2*(c + d)^(3/2)*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^2*(c - d)*Tan[e + f*x])/(d*(
c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{-}{\sqrt{a-}}\right.}{d(c+d)f\sqrt{a-a\sec(e}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{(a^4\tan(e+fx))\,\text{Subst}\left(\int \frac{}{\sqrt{a-}}\right.}{d^2f\sqrt{a-a\sec(e+f}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(2a^3\tan(e+fx))\,\text{Subst}\left(\int \frac{}{\sqrt{2a}}\right.}{d^2f\sqrt{a-a\sec(e+f}}$$

$$= \frac{2a^3\sqrt{c-d}\,(c+2d)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e}{d^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{2a^3\sqrt{c-d}\,(c+}{d^2(c+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.55, size = 312, normalized size = 2.67

$$\frac{a^2(d+c\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^2\left(-\left((d+c\cos(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+(d+c\cos(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{2(c^2+cd-2d^2)\text{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}\right)(d+c\cos(e+fx))(i\cos(e)+\sin(e))}{(c+d)\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{(c-d)d(d\sin(e)-c\sin(fx))}{c(c+d)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)}\right)}{4d^2f(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^2*(-((d + c
*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (d + c*Cos[e + f
*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (2*(c^2 + c*d - 2*d^2)*ArcT
an[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^
2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x])*(I*Cos[e] + Sin
[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((c - d)*d*(d
*Sin[e] - c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2
]))))/(4*d^2*f*(c + d*Sec[e + f*x])^2)
```

Maple [A]
time = 0.38, size = 157, normalized size = 1.34

method	result
derivativedivides	$\frac{8a^2\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^2}+\frac{(c-d)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{(c+2d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right.}{(c+d)\sqrt{(c+d)(c-d)}}\right.}{4d^2}\right.}{f}$
default	$\frac{8a^2\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^2}+\frac{(c-d)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{(c+2d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right.}{(c+d)\sqrt{(c+d)(c-d)}}\right.}{4d^2}\right.}{f}$
risch	$-\frac{2ia^2(c-d)\left(d\,e^{i(fx+e)}+c\right)}{df(c+d)c\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{\sqrt{(c+d)(c-d)}\,a^2\ln\left(e^{i(fx+e)}-\frac{i\sqrt{(c+d)(c-d)}-d}{c}\right)c}{(c+d)^2f\,d^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*a^2*(1/8/d^2*ln(tan(1/2*f*x+1/2*e)+1)+1/4*(c-d)/d^2*(d/(c+d)*tan(1/2*f*
x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)-(c+2*d)/(c+d)/
((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))-
1/8/d^2*ln(tan(1/2*f*x+1/2*e)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see 'assume?' for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(112) = 224$.
time = 1.69, size = 589, normalized size = 5.03

$$\left[\frac{(a^2cd+2a^2d^2+(a^2c^2+2a^2cd)\cos(fx+e))\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2cd\cos(fx+e)-(c^2-2d^2)\cos(fx+e)^2-2(c^2+cd+(cd+d^2)\cos(fx+e))\sqrt{\frac{c-d}{c+d}}\sin(fx+e)+2c^2-d^2}{c^2\cos(fx+e)^2+2cd\cos(fx+e)+d^2}\right)+(a^2cd+a^2d^2+(a^2c^2+a^2cd)\cos(fx+e))\log(\sin(fx+e)+1)-(a^2cd+a^2d^2+(a^2c^2+a^2cd)\cos(fx+e))\log(-\sin(fx+e)+1)-2(a^2cd-a^2d^2)\sin(fx+e)}{2((c^2d^2+cd^3)f\cos(fx+e)+(cd^3+d^4)f)}, -\frac{2(a^2cd+2a^2d^2+(a^2c^2+2a^2cd)\cos(fx+e))\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right)-(a^2cd+a^2d^2+(a^2c^2+a^2cd)\cos(fx+e))\log(\sin(fx+e)+1)+(a^2cd+a^2d^2+(a^2c^2+a^2cd)\cos(fx+e))\log(-\sin(fx+e)+1)+2(a^2cd-a^2d^2)\sin(fx+e)}{2((c^2d^2+cd^3)f\cos(fx+e)+(cd^3+d^4)f)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e))*sqrt((c -
d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2
 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c
^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (a^2*c*d + a^2
*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(sin(f*x + e) + 1) - (a^2*c*d +
 a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(a^
2*c*d - a^2*d^2)*sin(f*x + e))/((c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 +
 d^4)*f), -1/2*(2*(a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e)
)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d)
)/((c - d)*sin(f*x + e))) - (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*
x + e))*log(sin(f*x + e) + 1) + (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*co
s(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^2*c*d - a^2*d^2)*sin(f*x + e))/((
c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 + d^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx+\int \frac{2\sec^2(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx+\int \frac{\sec^3(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)*
*2), x) + Integral(2*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(
e + f*x)**2), x) + Integral(sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) + d*
*2*sec(e + f*x)**2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(108) = 216$.
time = 0.53, size = 230, normalized size = 1.97

$$\frac{\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d^2}-\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d^2}+\frac{2(a^2c^2+a^2cd-2a^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(cd^2+d^3)\sqrt{-c^2+d^2}}+\frac{2(a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-a^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right))}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(cd+d^2)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^2 - a^2*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/d^2 + 2*(a^2*c^2 + a^2*c*d - 2*a^2*d^2)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
1/2*e))/sqrt(-c^2 + d^2)))/((c*d^2 + d^3)*sqrt(-c^2 + d^2)) + 2*(a^2*c*tan(
1/2*f*x + 1/2*e) - a^2*d*tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 -
 d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c*d + d^2)))/f
```

Mupad [B]
time = 4.79, size = 2563, normalized size = 21.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (a^2*atan(((a^2*((a^2*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2
*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*a^2*tan(e/2 + (f*x
)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d
^2 + (32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*
d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2))*1i)/d^2 - (a^2*((a^2*((32*
(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 +
 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 - (32*tan(e/2 + (f*x)/2)
*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*
d^3 + d^4 + c^2*d^2))*1i)/d^2)/((64*(2*a^6*d^4 - a^6*c^4 - 5*a^6*c*d^3 + a^
6*c^3*d + 3*a^6*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((a^2*((32*(3*a^
2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^
4 + d^5 + c^2*d^3) - (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^
5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 + (32*tan(e/2 + (f*x)/2)*(2*a
^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 +
 d^4 + c^2*d^2)))/d^2 + (a^2*((a^2*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^
2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (32*a^2*t
an(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 +
c^2*d^2))))/d^2 - (32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d
^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2)))/d^2))*2i)/(d
^2*f) + (a^2*atan(((a^2*((32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*
a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) + (a^2*
((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c
^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*a^2*
tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6
 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*
```

```
d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))*((c + d)^3*(c - d))^(1/2)*(c
 + 2*d)*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) + (a^2*((32*tan(e/2 + (f*
x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/
(2*c*d^3 + d^4 + c^2*d^2) - (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(
3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*
c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/
2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*
(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d
^2))*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c
^3*d^2))/((64*(2*a^6*d^4 - a^6*c^4 - 5*a^6*c*d^3 + a^6*c^3*d + 3*a^6*c^2*d^
2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5
*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2
*d^2) + (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(3*a^2*d^8 - 2*a^2*c*
d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^
3) - (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*(2*c*d^
8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*(3*c*d^4 + d^5 + 3*c
^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))*((c + d)^3*(c -
 d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) - (a^2*((32*tan
(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*
c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2
*d)*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4
*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c
 - d))^(1/2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 +
 c^2*d^2)*(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d
^3 + c^3*d^2))*((c + d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*
d^3 + c^3*d^2)))*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*2i)/(f*(3*c*d^4 + d^5
+ 3*c^2*d^3 + c^3*d^2)) - (2*a^2*tan(e/2 + (f*x)/2)*(c - d))/(d*f*(c + d)*(
c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
```

3.199 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=130

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,(c+d)^{5/2}f}+\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2}+\frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx))}$$

```
[Out] 3*a^2*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c+d)^(5/2)/f/(c-
d)^(1/2)+1/2*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^2+3/2
*a^2*tan(f*x+e)/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4072, 96, 95, 211}

$$-\frac{3a^3\tan(e+fx)\,\mathrm{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f\sqrt{c-d}\,(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{3a^2\tan(e+fx)}{2f(c+d)^2(c+d\sec(e+fx))}+\frac{\tan(e+fx)\,(a^2\sec(e+fx)+a^2)}{2f(c+d)(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (-3*a^3*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
 a*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c - d]*(c + d)^(5/2)*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])^2) + (3*a^2*Tan[e + f*x])/(2*(c + d)
^2*f*(c + d*Sec[e + f*x]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} - \frac{\left(3a^3\tan(e+fx)\right)\text{Subst}\left(\int - \cdots\right.}{2(c+d)f\sqrt{a-a\sec(e \cdots}}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx) \cdots}$$

$$= \frac{\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx) \cdots}$$

$$= -\frac{3a^3\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c-d}\,(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.25, size = 249, normalized size = 1.92

$$\frac{a^2(d+c\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(1+\sec(e+fx))^2\left(-\frac{6i\text{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(d+c\cos(e+fx))^2(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{(c-d)(c+d)\sec(e)(-d\sin(e)+c\sin(fx))}{c^2}+\frac{(d+c\cos(e+fx))\sec(e)\left(\left(c^2-4cd-2d^2\right)\sin(e)+c(4c+d)\sin(fx)\right)}{c^2}\right)}{8(c+d)^2f(c+d\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Sec[e + f*x]*(1 + Sec[e + f*x]
)^2*(((-6*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f
*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x]
)^2*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((
c - d)*(c + d)*Sec[e]*(-(d*Sin[e]) + c*Sin[f*x]))/c^2 + ((d + c*Cos[e + f*x
])*Sec[e]*((c^2 - 4*c*d - 2*d^2)*Sin[e] + c*(4*c + d)*Sin[f*x]))/c^2))/(8*(
c + d)^2*f*(c + d*Sec[e + f*x])^3)
```

Maple [A]
time = 0.39, size = 167, normalized size = 1.28

method	result
derivativedivides	$\frac{8a^2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}+\frac{-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{}{\sqrt{}}\right.}{8(c+d)\sqrt{}}}{c+d}\right.}{f}$
default	$\frac{8a^2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}+\frac{-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{}{\sqrt{}}\right.}{8(c+d)\sqrt{}}}{c+d}\right.}{f}$
risch	$\frac{ia^2\left(-c^3e^{3i(fx+e)}+4c^2d\,e^{3i(fx+e)}+2c\,d^2e^{3i(fx+e)}+4c^3e^{2i(fx+e)}+c^2d\,e^{2i(fx+e)}+8c\,d^2e^{2i(fx+e)}+2d^3e^{2i(fx+e)}+c^3e^{i(fx+}\right.}{c^2(c+d)^2f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*a^2*(1/4*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x
+1/2*e)^2-c-d)^2+3/4/(c+d)*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+1/
2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c
-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(123) = 246$.
time = 2.11, size = 642, normalized size = 4.94

$$\left[\frac{3\left(a^{2}c^{2}\cos\left(fx+e\right)^{2}+2\,a^{2}cd\cos\left(fx+e\right)+a^{2}d^{2}\right)\sqrt{c^{2}-d^{2}}\log\left(\frac{2\,cd\cos(fx+e)-(c^{2}-2\,d^{2})\cos(fx+e)^{2}+2\,\sqrt{c^{2}-d^{2}}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^{2}-d^{2}}{c^{2}\cos(fx+e)^{2}+2\,cd\cos(fx+e)+d^{2}}\right)+2\left(a^{2}c^{3}+4\,a^{2}c^{2}d-a^{2}cd^{2}-4\,a^{2}d^{3}+\left(4\,a^{2}c^{3}+a^{2}c^{2}d-4\,a^{2}cd^{2}-a^{2}d^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4\left(\left(c^{6}+2\,c^{5}d-2\,c^{3}d^{3}-c^{2}d^{4}\right)f\cos\left(fx+e\right)^{2}+2\left(c^{5}d+2\,c^{4}d^{2}-2\,c^{2}d^{4}-cd^{5}\right)f\cos\left(fx+e\right)+\left(c^{4}d^{2}+2\,c^{3}d^{3}-2\,cd^{5}-d^{6}\right)f\right)},\frac{3\left(a^{2}c^{2}\cos\left(fx+e\right)^{2}+2\,a^{2}cd\cos\left(fx+e\right)+a^{2}d^{2}\right)\sqrt{-c^{2}+d^{2}}\arctan\left(-\frac{\sqrt{-c^{2}+d^{2}}(d\cos(fx+e)+c)}{(c^{2}-d^{2})\sin(fx+e)}\right)+\left(a^{2}c^{3}+4\,a^{2}c^{2}d-a^{2}cd^{2}-4\,a^{2}d^{3}+\left(4\,a^{2}c^{3}+a^{2}c^{2}d-4\,a^{2}cd^{2}-a^{2}d^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{2\left(\left(c^{6}+2\,c^{5}d-2\,c^{3}d^{3}-c^{2}d^{4}\right)f\cos\left(fx+e\right)^{2}+2\left(c^{5}d+2\,c^{4}d^{2}-2\,c^{2}d^{4}-cd^{5}\right)f\cos\left(fx+e\right)+\left(c^{4}d^{2}+2\,c^{3}d^{3}-2\,cd^{5}-d^{6}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e) + a^2*d^2)*sqrt(c^
2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^
2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)
^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*
a^2*d^3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))*sin
(f*x + e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 + 2*(c^5
*d + 2*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^3*d^3 -
 2*c*d^5 - d^6)*f), 1/2*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e)
 + a^2*d^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/
((c^2 - d^2)*sin(f*x + e))) + (a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^
3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))*sin(f*x +
 e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 + 2*(c^5*d + 2
*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^3*d^3 - 2*c*d
^5 - d^6)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{\sec\left(e+fx\right)}{c^{3}+3c^{2}d\sec\left(e+fx\right)+3cd^{2}\sec^{2}\left(e+fx\right)+d^{3}\sec^{3}\left(e+fx\right)}\,dx+\int \frac{2\sec^{2}\left(e+fx\right)}{c^{3}+3c^{2}d\sec\left(e+fx\right)+3cd^{2}\sec^{2}\left(e+fx\right)+d^{3}\sec^{3}\left(e+fx\right)}\,dx+\int \frac{\sec^{3}\left(e+fx\right)}{c^{3}+3c^{2}d\sec\left(e+fx\right)+3cd^{2}\sec^{2}\left(e+fx\right)+d^{3}\sec^{3}\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(2*sec(e + f*x)**2/(c**3 +
3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x
```

```
) + Integral(sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

Giac [A]
time = 0.56, size = 211, normalized size = 1.62

$$-\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^2}{(c^2+2cd+d^2)\sqrt{-c^2+d^2}}+\frac{3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-5a^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^2(c^2+2cd+d^2)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -(3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^2/((c^2 + 2*c*d +
 d^2)*sqrt(-c^2 + d^2)) + (3*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a^2*d*tan(1/2
*f*x + 1/2*e)^3 - 5*a^2*c*tan(1/2*f*x + 1/2*e) - 5*a^2*d*tan(1/2*f*x + 1/2*
e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2*(c^2 +
 2*c*d + d^2)))/f
```

Mupad [B]
time = 3.57, size = 158, normalized size = 1.22

$$\frac{\frac{5a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+d}-\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(a^2c-a^2d)}{(c+d)^2}}{f\left(2cd-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(2c^2-2d^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(c^2-2cd+d^2)+c^2+d^2\right)}+\frac{3a^2\operatorname{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)}{f(c+d)^{5/2}\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] ((5*a^2*tan(e/2 + (f*x)/2))/(c + d) - (3*tan(e/2 + (f*x)/2)^3*(a^2*c - a^2*
d))/(c + d)^2)/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 +
 (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) + (3*a^2*atanh((tan(e/2 + (f*
x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)^(5/2)*(c - d)^(1/2))
```

3.200 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=213

$$\frac{a^2(3c-2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{7/2}f}-\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3}+\frac{(3c-2d)\left(a^2+a^2\sec(e+\right.}{6(c-d)(c+d)^2f(c+d}$$

```
[Out] a^2*(3*c-2*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/
2)/(c+d)^(7/2)/f-1/3*d*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(f
*x+e))^3+1/6*(3*c-2*d)*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/(c-d)/(c+d)^2/f/(c+d
*sec(f*x+e))^2+1/2*a^2*(3*c-2*d)*tan(f*x+e)/(c-d)/(c+d)^3/f/(c+d*sec(f*x+e)
)
```

Rubi [A]
time = 0.19, antiderivative size = 268, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4072, 98, 96, 95, 211}

$$-\frac{a^3(3c-2d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{3/2}(c+d)^{7/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{a^2(3c-2d)\tan(e+fx)}{2f(c-d)(c+d)^3(c+d\sec(e+fx))}+\frac{(3c-2d)\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{6f(c-d)(c+d)^2(c+d\sec(e+fx))^2}-\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -((a^3*(3*c - 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(3/2)*(c + d)^(7/2)*f*
Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])) - (d*(a + a*Sec[e + f*x
])^2*Tan[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((3*c - 2*d)*
(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sec[e
+ f*x])^2) + (a^2*(3*c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^3*f*(c + d*S
ec[e + f*x]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
```

```
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = -\frac{(a^2\tan(e+fx))\operatorname{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)^4}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} - \frac{(a^2(3c-2d)\tan(e+fx))\,\mathrm{S}}{3(c^2-d^2)f\sqrt{a-}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)(a^2+a^2\sec(e+}{6(c-d)(c+d)^2f(c+d}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)(a^2+a^2\sec(e+}{6(c-d)(c+d)^2f(c+d}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)(a^2+a^2\sec(e+}{6(c-d)(c+d)^2f(c+d}$$

$$= -\frac{a^3(3c-2d)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{(c-d)^{3/2}(c+d)^{7/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 4.86, size = 211, normalized size = 0.99

$$\frac{a^2(c-d)^2\left(24(3c-2d)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^3-2\sqrt{c^2-d^2}\,(12c^3-5c^2d+6cd^2-22d^3+6(c^3+6c^2d-7cd^2-2d^3)\cos(e+fx)+(12c^3-7c^2d-6cd^2-2d^3)\cos(2(e+fx)))\sin(e+fx)\right)}{24(-c+d)^3(c+d)^3\sqrt{c^2-d^2}\,f(d+c\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(c - d)^2*(24*(3*c - 2*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2
 - d^2]]*(d + c*Cos[e + f*x])^3 - 2*Sqrt[c^2 - d^2]*(12*c^3 - 5*c^2*d + 6*c
*d^2 - 22*d^3 + 6*(c^3 + 6*c^2*d - 7*c*d^2 - 2*d^3)*Cos[e + f*x] + (12*c^3
- 7*c^2*d - 6*c*d^2 - 2*d^3)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(24*(-c + d)^
3*(c + d)^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

Maple [A]
time = 0.56, size = 228, normalized size = 1.07

method	result

derivativedivides	$\frac{8a^2\left(-\frac{-\frac{(3c-2d)(c-d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8c^3+24c^2d+24c\,d^2+8d^3}-\frac{(3c-2d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3\left(c^2+2cd+d^2\right)}+\frac{(5c-6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)(c-d)}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^3}+\frac{(3c-2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-}}\right.}{8\left(c^4+2c^3d-2c\,d^3-d^4\right)\sqrt{(c+d)(c}}\right.}{f}$
default	$\frac{8a^2\left(-\frac{-\frac{(3c-2d)(c-d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{8c^3+24c^2d+24c\,d^2+8d^3}-\frac{(3c-2d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3\left(c^2+2cd+d^2\right)}+\frac{(5c-6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)(c-d)}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^3}+\frac{(3c-2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-}}\right.}{8\left(c^4+2c^3d-2c\,d^3-d^4\right)\sqrt{(c+d)(c}}\right.}{f}$
risch	$\frac{ia^2\left(7c^5d+2c^3d^3+6c^4d^2-12c^6-24c^6e^{2i(fx+e)}+3c^6e^{5i(fx+e)}-12c^6e^{4i(fx+e)}+8d^6e^{3i(fx+e)}-3c^6e^{i(fx+e)}-54c^5d\,e^{i(fx+e)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*a^2*(-(1/8*(3*c-2*d)*(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)
^5-1/3*(3*c-2*d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/8*(5*c-6*d)/(c+d)/(
c-d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d
)^3+1/8*(3*c-2*d)/(c^4+2*c^3*d-2*c*d^3-d^4)/((c+d)*(c-d))^(1/2)*arctanh((c-
d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(207) = 414$.
time = 3.95, size = 1260, normalized size = 5.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^4 - 2*a^2*c^3*d)*cos(f*x + e)^
3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x + e)^2 + 3*(3*a^2*c^2*d^2 - 2*a
^2*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*
d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^4
*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*a^2*c^
5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a^2*d^5)
*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^2*d^3
+ 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c^8*d - c^
7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)^3 + 3*(c^
8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^2*d^7)*f*co
s(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c
^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^3*d^6
 - c^2*d^7 + 2*c*d^8 + d^9)*f), 1/6*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^
4 - 2*a^2*c^3*d)*cos(f*x + e)^3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x +
 e)^2 + 3*(3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arct
an(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (a^
2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*a
^2*c^5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a^2
*d^5)*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^2
*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c^8*d
 - c^7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)^3 +
3*(c^8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^2*d^7)
*f*cos(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6
+ 2*c^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^
3*d^6 - c^2*d^7 + 2*c*d^8 + d^9)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{\sec{\left(e+fx \right)}}{c^{4}+4c^{3}d\sec{\left(e+fx \right)}+6c^{2}d^{2}\sec^{2}{\left(e+fx \right)}+4cd^{3}\sec^{3}{\left(e+fx \right)}+d^{4}\sec^{4}{\left(e+fx \right)}}\,dx+\int \frac{2\sec^{2}{\left(e+fx \right)}}{c^{4}+4c^{3}d\sec{\left(e+fx \right)}+6c^{2}d^{2}\sec^{2}{\left(e+fx \right)}+4cd^{3}\sec^{3}{\left(e+fx \right)}+d^{4}\sec^{4}{\left(e+fx \right)}}\,dx+\int \frac{\sec^{3}{\left(e+fx \right)}}{c^{4}+4c^{3}d\sec{\left(e+fx \right)}+6c^{2}d^{2}\sec^{2}{\left(e+fx \right)}+4cd^{3}\sec^{3}{\left(e+fx \right)}+d^{4}\sec^{4}{\left(e+fx \right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec
(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integ
ral(2*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f
*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(se
c(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2 +
 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(198) = 396.

time = 0.55, size = 403, normalized size = 1.89

$$-\frac{\frac{3\,(3\,a^2c-2\,a^2d)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4+2\,c^3d-2\,cd^3-d^4)\sqrt{-c^2+d^2}}+\frac{9\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,a^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+21\,a^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-6\,a^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+16\,a^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+24\,a^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-16\,a^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+12\,a^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-21\,a^2cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-18\,a^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{(c^4+2\,c^3d-2\,cd^3-d^4)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)^3}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(3*a^2*c - 2*a^2*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d
) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^
2)))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*sqrt(-c^2 + d^2)) + (9*a^2*c^3*tan(1/
2*f*x + 1/2*e)^5 - 24*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 21*a^2*c*d^2*tan(1
/2*f*x + 1/2*e)^5 - 6*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c^3*tan(1/2*f
*x + 1/2*e)^3 + 16*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 24*a^2*c*d^2*tan(1/2*
f*x + 1/2*e)^3 - 16*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^3*tan(1/2*f*x
 + 1/2*e) + 12*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - 21*a^2*c*d^2*tan(1/2*f*x +
1/2*e) - 18*a^2*d^3*tan(1/2*f*x + 1/2*e))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*
(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

Mupad [B]
time = 5.09, size = 286, normalized size = 1.34

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(3\,a^2\,c^2-5\,a^2\,c\,d+2\,a^2\,d^2)}{(c+d)^3}-\frac{8\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(3\,a^2\,c-2\,a^2\,d)}{3\,(c+d)^2}+\frac{a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(5\,c-6\,d)}{(c+d)\,(c-d)}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3)+3\,c\,d^2+3\,c^2\,d+c^3+d^3-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6(c^3-3\,c^2\,d+3\,c\,d^2-d^3)\right)}+\frac{2\,a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)\left(\frac{3\,c}{2}-d\right)}{f\,(c+d)^{7/2}\,(c-d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(3*a^2*c^2 + 2*a^2*d^2 - 5*a^2*c*d))/(c + d)^3 - (8*
tan(e/2 + (f*x)/2)^3*(3*a^2*c - 2*a^2*d))/(3*(c + d)^2) + (a^2*tan(e/2 + (f
*x)/2)*(5*c - 6*d))/((c + d)*(c - d)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 -
3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3
- 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 -
3*c^2*d + c^3 - d^3))) + (2*a^2*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c
 + d)^(1/2))*((3*c)/2 - d))/(f*(c + d)^(7/2)*(c - d)^(3/2))
```

3.201 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$

Optimal. Leaf size=276

$$\frac{a^2(12c^2-16cd+7d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4(c-d)^{5/2}(c+d)^{9/2}f}-\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4}+\frac{a^2(c+8d)\tan}{12d(c+d)^2f(c+d}$$

```
[Out] 1/4*a^2*(12*c^2-16*c*d+7*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^
(1/2))/(c-d)^(5/2)/(c+d)^(9/2)/f-1/4*a^2*(c-d)*tan(f*x+e)/d/(c+d)/f/(c+d*se
c(f*x+e))^4+1/12*a^2*(c+8*d)*tan(f*x+e)/d/(c+d)^2/f/(c+d*sec(f*x+e))^3+1/24
*a^2*(2*c^2+16*c*d-21*d^2)*tan(f*x+e)/(c-d)/d/(c+d)^3/f/(c+d*sec(f*x+e))^2+
1/24*a^2*(2*c^3+16*c^2*d-59*c*d^2+32*d^3)*tan(f*x+e)/(c-d)^2/d/(c+d)^4/f/(c
+d*sec(f*x+e))
```

Rubi [A]

time = 0.38, antiderivative size = 330, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 100, 156, 12, 95, 211}

$$-\frac{a^3(12c^2-16cd+7d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{4f(c-d)^{5/2}(c+d)^{9/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24df(c-d)(c+d)^3(c+d\sec(e+fx))^2}+\frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\tan(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sec(e+fx))}+\frac{a^2(c+8d)\tan(e+fx)}{12df(c+d)^2(c+d\sec(e+fx))^3}-\frac{a^2(c-d)\tan(e+fx)}{4df(c+d)(c+d\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] -1/4*(a^3*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(5/2)
*(c + d)^(9/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^2*
(c - d)*Tan[e + f*x])/(4*d*(c + d)*f*(c + d*Sec[e + f*x])^4) + (a^2*(c + 8*
d)*Tan[e + f*x])/(12*d*(c + d)^2*f*(c + d*Sec[e + f*x])^3) + (a^2*(2*c^2 +
16*c*d - 21*d^2)*Tan[e + f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sec[e + f*x
])^2) + (a^2*(2*c^3 + 16*c^2*d - 59*c*d^2 + 32*d^3)*Tan[e + f*x])/(24*(c -
d)^2*d*(c + d)^4*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}\,(c+dx)^5}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{(a\tan(e+fx))\mathrm{Subst}\left(\int \frac{}{\sqrt{a-}}\right.}{4d(c+d)f\sqrt{a-a\,\mathrm{se}}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx}$$

$$= -\frac{a^3(12c^2-16cd+7d^2)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\mathrm{ta}}{4(c-d)^{5/2}(c+d)^{9/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+}}$$

Mathematica [A]
time = 9.59, size = 322, normalized size = 1.17

$$\frac{a^2\left(-\frac{24(12c^2-16cd+7d^2)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{(24c^5+192c^4d-446c^3d^2+128c^2d^3-148cd^4+160d^5+(144c^5-172c^4d+208c^3d^2-785c^2d^3+368cd^4+102d^5)\cos(e+fx)+2(12c^5+96c^4d-227c^3d^2+32c^2d^3+44cd^4+16d^5)\cos(2(e+fx))+48c^5\cos(3(e+fx))-68c^4d\cos(3(e+fx))-16c^3d^2\cos(3(e+fx))+5c^2d^3\cos(3(e+fx))+16cd^4\cos(3(e+fx))+6d^5\cos(3(e+fx)))\sin(e+fx)}{(d+c\cos(e+fx))^4}\right)}{96(c-d)^2(c+d)^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*((-24*(12*c^2 - 16*c*d + 7*d^2)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sq
rt[c^2 - d^2]])/Sqrt[c^2 - d^2] + ((24*c^5 + 192*c^4*d - 446*c^3*d^2 + 128*
c^2*d^3 - 148*c*d^4 + 160*d^5 + (144*c^5 - 172*c^4*d + 208*c^3*d^2 - 785*c^
2*d^3 + 368*c*d^4 + 102*d^5)*Cos[e + f*x] + 2*(12*c^5 + 96*c^4*d - 227*c^3*
```

```
d^2 + 32*c^2*d^3 + 44*c*d^4 + 16*d^5)*Cos[2*(e + f*x)] + 48*c^5*Cos[3*(e +
f*x)] - 68*c^4*d*Cos[3*(e + f*x)] - 16*c^3*d^2*Cos[3*(e + f*x)] + 5*c^2*d^3
*Cos[3*(e + f*x)] + 16*c*d^4*Cos[3*(e + f*x)] + 6*d^5*Cos[3*(e + f*x)])*Sin
[e + f*x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)^2*(c + d)^4*f)
```

Maple [A]
time = 0.79, size = 352, normalized size = 1.28

method	result
derivativedivides	$\frac{8a^2\left(-\frac{\frac{\left(12c^2-16cd+7d^2\right)(c-d)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32c^4+128c^3d+192c^2d^2+128c\,d^3+32d^4}-\frac{11\left(12c^2-16cd+7d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{\left(156c^2-272cd+83d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96(c-d)\left(c^2+2cd+d^2\right)}-\cdots}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^4}\cdots\right)}{f}$
default	$\frac{8a^2\left(-\frac{\frac{\left(12c^2-16cd+7d^2\right)(c-d)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{32c^4+128c^3d+192c^2d^2+128c\,d^3+32d^4}-\frac{11\left(12c^2-16cd+7d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{\left(156c^2-272cd+83d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{96(c-d)\left(c^2+2cd+d^2\right)}-\cdots}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^4}\cdots\right)}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*a^2*(-(1/32*(12*c^2-16*c*d+7*d^2)*(c-d)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+
d^4)*tan(1/2*f*x+1/2*e)^7-11/96*(12*c^2-16*c*d+7*d^2)/(c^3+3*c^2*d+3*c*d^2+
d^3)*tan(1/2*f*x+1/2*e)^5+1/96*(156*c^2-272*c*d+83*d^2)/(c-d)/(c^2+2*c*d+d^
2)*tan(1/2*f*x+1/2*e)^3-1/32*(20*c^2-48*c*d+25*d^2)/(c+d)/(c^2-2*c*d+d^2)*t
an(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^4+1/
32*(12*c^2-16*c*d+7*d^2)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6
)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see 'assume?' for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(266) = 532$.
time = 3.55, size = 1940, normalized size = 7.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] [1/48*(3*(12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2*
c^5*d + 7*a^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 +
7*a^2*c^3*d^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a^2*
c^2*d^4)*cos(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)
*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(
f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 -
d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^2*c^5*d^2 +
16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^
7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*a^2*c^5*d^2 + 73*a^2*c^4*d^3 + 32*a^2*c
^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6 - 6*a^2*d^7)*cos(f*x + e)^3 + (12*a^2*c
^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2 - 64*a^2*c^4*d^3 + 271*a^2*c^3*d^4 - 16
*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x + e)^2 + (8*a^2*c^6*d + 6
4*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2
*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*sin(f*x + e))/((c^12 + 2*c^11*d - 2*c^10
*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6*d^6 - 2*c^5*d^7 - c^4*d^8)*f*cos(f*x +
 e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c^9*d^3 - 6*c^8*d^4 + 6*c^6*d^6 + 2*c^5*
d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x + e)^3 + 6*(c^10*d^2 + 2*c^9*d^3 - 2*c
^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c^4*d^8 - 2*c^3*d^9 - c^2*d^10)*f*cos(f*
x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c
^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e) + (c^8*d^4 + 2*c^7*d^5 - 2*c^6
*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f), 1/24*(3*(1
2*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2*c^5*d + 7*a
^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 7*a^2*c^3*d
^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a^2*c^2*d^4)*co
s(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)*cos(f*x +
e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
d^2)*sin(f*x + e))) + (2*a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16
*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*
a^2*c^5*d^2 + 73*a^2*c^4*d^3 + 32*a^2*c^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6
- 6*a^2*d^7)*cos(f*x + e)^3 + (12*a^2*c^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2
- 64*a^2*c^4*d^3 + 271*a^2*c^3*d^4 - 16*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2
*d^7)*cos(f*x + e)^2 + (8*a^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16
```

```
*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*s
in(f*x + e))/((c^12 + 2*c^11*d - 2*c^10*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6
*d^6 - 2*c^5*d^7 - c^4*d^8)*f*cos(f*x + e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c
^9*d^3 - 6*c^8*d^4 + 6*c^6*d^6 + 2*c^5*d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x
 + e)^3 + 6*(c^10*d^2 + 2*c^9*d^3 - 2*c^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c
^4*d^8 - 2*c^3*d^9 - c^2*d^10)*f*cos(f*x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 -
2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(
f*x + e) + (c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2
*d^10 - 2*c*d^11 - d^12)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{\sec(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e+fx)}\,dx+\int \frac{2\sec^2(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e+fx)}\,dx+\int \frac{\sec^3(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**5,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*se
c(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d
**5*sec(e + f*x)**5), x) + Integral(2*sec(e + f*x)**2/(c**5 + 5*c**4*d*sec(
e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*
c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(sec(e + f*x)*
*3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d
**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(257) = 514.
time = 0.69, size = 710, normalized size = 2.57

$$\frac{\frac{3\left(12a^2c^2-16a^2cd+7a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^6+2c^5d-c^4d^2-4c^3d^3-c^2d^4+2cd^5+d^6\right)\sqrt{-c^2+d^2}}-\frac{36a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-156a^2c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+273a^2c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-243a^2c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+111a^2cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-21a^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-132a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+308a^2c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-121a^2c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-231a^2c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+253a^2cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-77a^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+156a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-116a^2c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-345a^2c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+199a^2c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+189a^2cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-83a^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-60a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-36a^2c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+177a^2c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+147a^2c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-81a^2cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-75a^2d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c^6+2c^5d-c^4d^2-4c^3d^3-c^2d^4+2cd^5+d^6\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^4}}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/12*(3*(12*a^2*c^2 - 16*a^2*c*d + 7*a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/
2*e))/sqrt(-c^2 + d^2)))/((c^6 + 2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4 +
2*c*d^5 + d^6)*sqrt(-c^2 + d^2)) - (36*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 - 156
*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^7 + 273*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^7
- 243*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 111*a^2*c*d^4*tan(1/2*f*x + 1/2*
e)^7 - 21*a^2*d^5*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^5*tan(1/2*f*x + 1/2*e)
^5 + 308*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 121*a^2*c^3*d^2*tan(1/2*f*x + 1
/2*e)^5 - 231*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 253*a^2*c*d^4*tan(1/2*f*
x + 1/2*e)^5 - 77*a^2*d^5*tan(1/2*f*x + 1/2*e)^5 + 156*a^2*c^5*tan(1/2*f*x
```

```
+ 1/2*e)^3 - 116*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 345*a^2*c^3*d^2*tan(1/2
*f*x + 1/2*e)^3 + 199*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 189*a^2*c*d^4*ta
n(1/2*f*x + 1/2*e)^3 - 83*a^2*d^5*tan(1/2*f*x + 1/2*e)^3 - 60*a^2*c^5*tan(1
/2*f*x + 1/2*e) - 36*a^2*c^4*d*tan(1/2*f*x + 1/2*e) + 177*a^2*c^3*d^2*tan(1
/2*f*x + 1/2*e) + 147*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 81*a^2*c*d^4*tan(1
/2*f*x + 1/2*e) - 75*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 + 2*c^5*d - c^4*d^
2 - 4*c^3*d^3 - c^2*d^4 + 2*c*d^5 + d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(
1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

Mupad [B]

time = 5.24, size = 438, normalized size = 1.59

$$\frac{\frac{11\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(12a^2c^2-16a^2cd+7a^2d^2)}{12(c+d)^3}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7(12a^2c^3-28a^2c^2d+23a^2cd^2-7a^2d^3)}{4(c+d)^4}-\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(156c^2-272cd+83d^2)}{12(c+d)^2(c-d)}+\frac{a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(20c^2-48cd+25d^2)}{4(c+d)(c^2-2cd+d^2)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(6c^4-12c^2d^2+6d^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(-4c^4-8c^3d+8cd^3+4d^4)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(4c^4-8c^3d+8cd^3-4d^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(c^4-4c^3d+6c^2d^2-4cd^3+d^4)+4cd^3+4c^3d+c^4+d^4+6c^2d^2\right)}+\frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2c-2d)(c^2-2cd+d^2)}{2\sqrt{c+d}\,(c-d)^{5/2}}\right)(12c^2-16cd+7d^2)}{4f(c+d)^{9/2}(c-d)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^5),x)
```

```
[Out] ((11*tan(e/2 + (f*x)/2)^5*(12*a^2*c^2 + 7*a^2*d^2 - 16*a^2*c*d))/(12*(c + d
)^3) - (tan(e/2 + (f*x)/2)^7*(12*a^2*c^3 - 7*a^2*d^3 + 23*a^2*c*d^2 - 28*a^
2*c^2*d))/(4*(c + d)^4) - (a^2*tan(e/2 + (f*x)/2)^3*(156*c^2 - 272*c*d + 83
*d^2))/(12*(c + d)^2*(c - d)) + (a^2*tan(e/2 + (f*x)/2)*(20*c^2 - 48*c*d +
25*d^2))/(4*(c + d)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f*x)/2)^4*(6*c^4 +
 6*d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/2)^2*(8*c*d^3 - 8*c^3*d - 4*c^4 + 4*
d^4) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 - 8*c^3*d + 4*c^4 - 4*d^4) + tan(e/2 +
 (f*x)/2)^8*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2) + 4*c*d^3 + 4*c^3*d
 + c^4 + d^4 + 6*c^2*d^2)) + (a^2*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(c^
2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*(c - d)^(5/2)))*(12*c^2 - 16*c*d + 7*d^2
))/(4*f*(c + d)^(9/2)*(c - d)^(5/2))
```

3.202 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=288

$$\frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tanh^{-1}(\sin(e+fx))}{16f}+\frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f}+\frac{(40c^3$$

```
[Out] 1/16*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)*arctanh(sin(f*x+e))/f+1/16*a^3*(
40*c^3+90*c^2*d+78*c*d^2+23*d^3)*tan(f*x+e)/f+1/48*(40*c^3+90*c^2*d+78*c*d^
2+23*d^3)*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f+1/30*a*(3*c+8*d)*(a+a*sec(f*x+e
))^2*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/6*a*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+
e))^3*tan(f*x+e)/f+1/120*a*(a+a*sec(f*x+e))^2*(8*c^3+148*c^2*d+132*c*d^2+42
*d^3+d*(6*c^2+62*c*d+31*d^2)*sec(f*x+e))*tan(f*x+e)/f
```

Rubi [A]
time = 0.28, antiderivative size = 333, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 102, 152, 52, 65, 223, 209}

$$\frac{a^4(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f}+\frac{(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{48f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3\left(70c^2+4d(8c+3d)\sec(e+fx)+54cd+19d^2\right)}{120f}+\frac{a(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)(a\sec(e+fx)+a)^2}{120f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))^2}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Tan[e + f*x])/(16*f) + (a^4*(4
0*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(8*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) + (a*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*(a + a*Sec[e
 + f*x])^2*Tan[e + f*x])/(120*f) + ((40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)
*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(48*f) + (d*(a + a*Sec[e + f*x])^3*
(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(6*f) + (d*(a + a*Sec[e + f*x])^3*(70*
c^2 + 54*c*d + 19*d^2 + 4*d*(8*c + 3*d)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
```

```
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(a+ax)^{5/2}(c+dx)^3}{\sqrt{a-ax}}\,dx, x\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+}}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e}{6f}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e}{6f}$$

$$= \frac{a(40c^3+90c^2d+78cd^2+23d^3)\,(a+a\sec(e+f}{120f}$$

$$= \frac{a(40c^3+90c^2d+78cd^2+23d^3)\,(a+a\sec(e+f}{120f}$$

$$= \frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f} +$$

$$= \frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f} +$$

$$= \frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f} +$$

$$= \frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f} +$$

Mathematica [A]
time = 2.65, size = 380, normalized size = 1.32

$$-\frac{a^3(1+\cos(e+fx))^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^6(e+fx)\left(240(40c^3+90c^2d+78cd^2+23d^3)\cos^6(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-2(1080c^3+4770c^2d+5670cd^2+2275d^3+16(305c^3+945c^2d+984cd^2+344d^3)\cos(e+fx)+20(72c^3+306c^2d+342cd^2+115d^3)\cos(2(e+fx))+2360c^3\cos(3(e+fx))+6840c^2d\cos(3(e+fx))+6384cd^2\cos(3(e+fx))+1904d^3\cos(3(e+fx))+360c^3\cos(4(e+fx))+1350c^2d\cos(4(e+fx))+1170cd^2\cos(4(e+fx))+345d^3\cos(4(e+fx))+440c^3\cos(5(e+fx))+1080c^2d\cos(5(e+fx))+912cd^2\cos(5(e+fx))+272d^3\cos(5(e+fx)))\sin(e+fx)\right)}{30720f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -1/30720*(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^6*(240*(
40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Cos[e + f*x]^6*(Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(1080*
c^3 + 4770*c^2*d + 5670*c*d^2 + 2275*d^3 + 16*(305*c^3 + 945*c^2*d + 984*c*
d^2 + 344*d^3)*Cos[e + f*x] + 20*(72*c^3 + 306*c^2*d + 342*c*d^2 + 115*d^3)
*Cos[2*(e + f*x)] + 2360*c^3*Cos[3*(e + f*x)] + 6840*c^2*d*Cos[3*(e + f*x)]
 + 6384*c*d^2*Cos[3*(e + f*x)] + 1904*d^3*Cos[3*(e + f*x)] + 360*c^3*Cos[4*
(e + f*x)] + 1350*c^2*d*Cos[4*(e + f*x)] + 1170*c*d^2*Cos[4*(e + f*x)] + 34
5*d^3*Cos[4*(e + f*x)] + 440*c^3*Cos[5*(e + f*x)] + 1080*c^2*d*Cos[5*(e + f
*x)] + 912*c*d^2*Cos[5*(e + f*x)] + 272*d^3*Cos[5*(e + f*x)])*Sin[e + f*x])
)/f
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(273) = 546$.
time = 0.46, size = 573, normalized size = 1.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-c^3*a^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+3*a^3*c^2*d*(-(-1/4*sec(f*
x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-3*a^3*c*d^
2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+a^3*d^3*(-(-1/6*sec
(f*x+e)^5-5/24*sec(f*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)+5/16*ln(sec(f*x+e)+
tan(f*x+e)))+3*c^3*a^3*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x
+e)))-9*a^3*c^2*d*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+9*a^3*c*d^2*(-(-1/4*se
c(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-3*a^3*
d^3*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+3*c^3*a^3*tan(f*x
+e)+9*a^3*c^2*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-9
*a^3*c*d^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+3*a^3*d^3*(-(-1/4*sec(f*x+e)^
3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+c^3*a^3*ln(sec(
f*x+e)+tan(f*x+e))+3*a^3*c^2*d*tan(f*x+e)+3*a^3*c*d^2*(1/2*sec(f*x+e)*tan(f
*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a^3*d^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*
x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(286) = 572$.
time = 0.30, size = 757, normalized size = 2.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/480*(160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 + 1440*(tan(f*x + e)^3
 + 3*tan(f*x + e))*a^3*c^2*d + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 1
5*tan(f*x + e))*a^3*c*d^2 + 1440*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c*d^
2 + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*d^3 + 1
60*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^3 - 5*a^3*d^3*(2*(15*sin(f*x + e
)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)
^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e)
 - 1)) - 90*a^3*c^2*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) -
1)) - 270*a^3*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4
- 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)
) - 90*a^3*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*s
in(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 3
60*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + l
og(sin(f*x + e) - 1)) - 1080*a^3*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1)
 - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 360*a^3*c*d^2*(2*sin(f*
x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)
) + 480*a^3*c^3*log(sec(f*x + e) + tan(f*x + e)) + 1440*a^3*c^3*tan(f*x + e
) + 1440*a^3*c^2*d*tan(f*x + e))/f
```

Fricas [**A**]
time = 3.15, size = 348, normalized size = 1.21

$$\frac{15\left(40\,a^{3}c^{3}+90\,a^{3}c^{2}d+78\,a^{3}cd^{2}+23\,a^{3}d^{3}\right)\cos\left(fx+e\right)^{6}\log\left(\sin\left(fx+e\right)+1\right)-15\left(40\,a^{3}c^{3}+90\,a^{3}c^{2}d+78\,a^{3}cd^{2}+23\,a^{3}d^{3}\right)\cos\left(fx+e\right)^{6}\log\left(-\sin\left(fx+e\right)+1\right)+2\left(40\,a^{3}d^{3}+16\left(55\,a^{3}c^{3}+135\,a^{3}c^{2}d+114\,a^{3}cd^{2}+34\,a^{3}d^{3}\right)\cos\left(fx+e\right)^{5}+15\left(24\,a^{3}c^{3}+90\,a^{3}c^{2}d+78\,a^{3}cd^{2}+23\,a^{3}d^{3}\right)\cos\left(fx+e\right)^{4}+16\left(5\,a^{3}c^{3}+45\,a^{3}c^{2}d+57\,a^{3}cd^{2}+17\,a^{3}d^{3}\right)\cos\left(fx+e\right)^{3}+10\left(18\,a^{3}c^{2}d+54\,a^{3}cd^{2}+23\,a^{3}d^{3}\right)\cos\left(fx+e\right)^{2}+144\left(a^{3}cd^{2}+a^{3}d^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{480\,f\cos\left(fx+e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x +
 e)^6*log(sin(f*x + e) + 1) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2
+ 23*a^3*d^3)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(40*a^3*d^3 + 16*(5
5*a^3*c^3 + 135*a^3*c^2*d + 114*a^3*c*d^2 + 34*a^3*d^3)*cos(f*x + e)^5 + 15
*(24*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^4 + 1
6*(5*a^3*c^3 + 45*a^3*c^2*d + 57*a^3*c*d^2 + 17*a^3*d^3)*cos(f*x + e)^3 + 1
0*(18*a^3*c^2*d + 54*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^2 + 144*(a^3*c*d^
2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int c^{3}\sec\left(e+fx\right)dx+\int 3c^{3}\sec^{2}\left(e+fx\right)dx+\int 3c^{3}\sec^{3}\left(e+fx\right)dx+\int c^{3}\sec^{4}\left(e+fx\right)dx+\int d^{3}\sec^{4}\left(e+fx\right)dx+\int 3d^{3}\sec^{5}\left(e+fx\right)dx+\int 3d^{3}\sec^{6}\left(e+fx\right)dx+\int d^{3}\sec^{7}\left(e+fx\right)dx+\int 3cd^{2}\sec^{3}\left(e+fx\right)dx+\int 9cd^{2}\sec^{4}\left(e+fx\right)dx+\int 9cd^{2}\sec^{5}\left(e+fx\right)dx+\int 3cd^{2}\sec^{6}\left(e+fx\right)dx+\int 3c^{2}d\sec^{2}\left(e+fx\right)dx+\int 9c^{2}d\sec^{3}\left(e+fx\right)dx+\int 9c^{2}d\sec^{4}\left(e+fx\right)dx+\int 3c^{2}d\sec^{5}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(c**3*sec(e + f*x), x) + Integral(3*c**3*sec(e + f*x)**2, x)
+ Integral(3*c**3*sec(e + f*x)**3, x) + Integral(c**3*sec(e + f*x)**4, x) +
 Integral(d**3*sec(e + f*x)**4, x) + Integral(3*d**3*sec(e + f*x)**5, x) +
Integral(3*d**3*sec(e + f*x)**6, x) + Integral(d**3*sec(e + f*x)**7, x) + I
ntegral(3*c*d**2*sec(e + f*x)**3, x) + Integral(9*c*d**2*sec(e + f*x)**4, x
) + Integral(9*c*d**2*sec(e + f*x)**5, x) + Integral(3*c*d**2*sec(e + f*x)*
*6, x) + Integral(3*c**2*d*sec(e + f*x)**2, x) + Integral(9*c**2*d*sec(e +
f*x)**3, x) + Integral(9*c**2*d*sec(e + f*x)**4, x) + Integral(3*c**2*d*sec
(e + f*x)**5, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(273) = 546$.
time = 0.60, size = 584, normalized size = 2.03

$$\frac{15\left(40a^3c^3+90a^3c^2d+78a^3cd^2+23a^3d^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15\left(40a^3c^3+90a^3c^2d+78a^3cd^2+23a^3d^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(600a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+1350a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+1170a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}+345a^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}-3400a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}-7650a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}-6630a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}-1955a^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{9}+7920a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+17820a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+15444a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+4554a^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}-9360a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-22500a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-17964a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}-5814a^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+5560a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+15390a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+12570a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+3165a^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-1320a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-4410a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-4590a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1575a^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^6}}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*log(abs(t
an(1/2*f*x + 1/2*e) + 1)) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 +
23*a^3*d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(600*a^3*c^3*tan(1/2*f*x
 + 1/2*e)^11 + 1350*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^11 + 1170*a^3*c*d^2*tan(
1/2*f*x + 1/2*e)^11 + 345*a^3*d^3*tan(1/2*f*x + 1/2*e)^11 - 3400*a^3*c^3*ta
n(1/2*f*x + 1/2*e)^9 - 7650*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^9 - 6630*a^3*c*d
^2*tan(1/2*f*x + 1/2*e)^9 - 1955*a^3*d^3*tan(1/2*f*x + 1/2*e)^9 + 7920*a^3*
c^3*tan(1/2*f*x + 1/2*e)^7 + 17820*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^7 + 15444
*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 4554*a^3*d^3*tan(1/2*f*x + 1/2*e)^7 - 9
360*a^3*c^3*tan(1/2*f*x + 1/2*e)^5 - 22500*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^5
 - 17964*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 5814*a^3*d^3*tan(1/2*f*x + 1/2*
e)^5 + 5560*a^3*c^3*tan(1/2*f*x + 1/2*e)^3 + 15390*a^3*c^2*d*tan(1/2*f*x +
1/2*e)^3 + 12570*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 3165*a^3*d^3*tan(1/2*f*
x + 1/2*e)^3 - 1320*a^3*c^3*tan(1/2*f*x + 1/2*e) - 4410*a^3*c^2*d*tan(1/2*f
*x + 1/2*e) - 4590*a^3*c*d^2*tan(1/2*f*x + 1/2*e) - 1575*a^3*d^3*tan(1/2*f*
x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B]
time = 5.24, size = 411, normalized size = 1.43

$$\frac{a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(40\,c^3+90\,c^2\,d+78\,c\,d^2+23\,d^3\right)}{4\,\left(10\,c^3+\frac{45\,c^2\,d}{2}+\frac{39\,c\,d^2}{2}+\frac{23\,d^3}{4}\right)}\right)\,\left(40\,c^3+90\,c^2\,d+78\,c\,d^2+23\,d^3\right)}{8\,f}-\frac{\left(5\,a^3\,c^3+\frac{45\,a^3\,c^2\,d}{4}+\frac{39\,a^3\,c\,d^2}{4}+\frac{23\,a^3\,d^3}{8}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{11}+\left(-\frac{85\,a^3\,c^3}{3}-\frac{255\,a^3\,c^2\,d}{4}-\frac{221\,a^3\,c\,d^2}{4}-\frac{391\,a^3\,d^3}{24}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^9+\left(66\,a^3\,c^3+\frac{297\,a^3\,c^2\,d}{2}+\frac{1287\,a^3\,c\,d^2}{10}+\frac{759\,a^3\,d^3}{20}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^7+\left(-78\,a^3\,c^3-\frac{375\,a^3\,c^2\,d}{2}-\frac{1497\,a^3\,c\,d^2}{10}-\frac{969\,a^3\,d^3}{20}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(\frac{139\,a^3\,c^3}{3}+\frac{513\,a^3\,c^2\,d}{4}+\frac{419\,a^3\,c\,d^2}{4}+\frac{211\,a^3\,d^3}{8}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(-11\,a^3\,c^3-\frac{147\,a^3\,c^2\,d}{4}-\frac{153\,a^3\,c\,d^2}{4}-\frac{105\,a^3\,d^3}{8}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-20\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+15\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (a^3*atanh((tan(e/2 + (f*x)/2)*(78*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/(4*
((39*c*d^2)/2 + (45*c^2*d)/2 + 10*c^3 + (23*d^3)/4)))*(78*c*d^2 + 90*c^2*d
+ 40*c^3 + 23*d^3))/(8*f) - (tan(e/2 + (f*x)/2)^11*(5*a^3*c^3 + (23*a^3*d^3
)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4) - tan(e/2 + (f*x)/2)^9*((85*a^3*
c^3)/3 + (391*a^3*d^3)/24 + (221*a^3*c*d^2)/4 + (255*a^3*c^2*d)/4) + tan(e/
2 + (f*x)/2)^3*((139*a^3*c^3)/3 + (211*a^3*d^3)/8 + (419*a^3*c*d^2)/4 + (51
3*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^7*(66*a^3*c^3 + (759*a^3*d^3)/20 + (12
87*a^3*c*d^2)/10 + (297*a^3*c^2*d)/2) - tan(e/2 + (f*x)/2)^5*(78*a^3*c^3 +
(969*a^3*d^3)/20 + (1497*a^3*c*d^2)/10 + (375*a^3*c^2*d)/2) - tan(e/2 + (f*
x)/2)*(11*a^3*c^3 + (105*a^3*d^3)/8 + (153*a^3*c*d^2)/4 + (147*a^3*c^2*d)/4
))/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x
)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x
)/2)^12 + 1))
```

3.203 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=257

$$\frac{a^3(20c^2+30cd+13d^2)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)\tan(e+fx)}{30d^2f}+\frac{a^3(4$$

```
[Out] 1/8*a^3*(20*c^2+30*c*d+13*d^2)*arctanh(sin(f*x+e))/f+1/30*a^3*(2*c^4-15*c^3
*d+72*c^2*d^2+180*c*d^3+76*d^4)*tan(f*x+e)/d^2/f+1/120*a^3*(4*c^3-30*c^2*d+
146*c*d^2+195*d^3)*sec(f*x+e)*tan(f*x+e)/d/f+1/60*a^3*(2*c^2-15*c*d+76*d^2)
*(c+d*sec(f*x+e))^2*tan(f*x+e)/d^2/f-1/20*a^3*(2*c-11*d)*(c+d*sec(f*x+e))^3
*tan(f*x+e)/d^2/f+1/5*(a^3+a^3*sec(f*x+e))*(c+d*sec(f*x+e))^3*tan(f*x+e)/d/
f
```

Rubi [A]

time = 0.20, antiderivative size = 273, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 92, 81, 52, 65, 223, 209}

$$\frac{a^4(20c^2+30cd+13d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f}+\frac{(20c^2+30cd+13d^2)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{24f}+\frac{a(20c^2+30cd+13d^2)\tan(e+fx)(a\sec(e+fx)+a)^2}{60f}+\frac{3d(2c+d)\tan(e+fx)(a\sec(e+fx)+a)^3}{20f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*Tan[e + f*x])/(8*f) + (a^4*(20*c^2 + 30*c*d
 + 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[
e + f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a*(20*
c^2 + 30*c*d + 13*d^2)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + (3*d*(
2*c + d)*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + ((20*c^2 + 30*c*d +
13*d^2)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(24*f) + (d*(a + a*Sec[e + f
*x])^3*(c + d*Sec[e + f*x])*Tan[e + f*x])/(5*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c+dx)^2}{\sqrt{a-ax}}\,dx,\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e}{5f}$$

$$= \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} +$$

$$= \frac{a(20c^2+30cd+13d^2)\,(a+a\sec(e+fx))^2\tan}{60f}$$

$$= \frac{a(20c^2+30cd+13d^2)\,(a+a\sec(e+fx))^2\tan}{60f}$$

$$= \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{a(20c^2}{}$$

$$= \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{a(20c^2}{}$$

$$= \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{a(20c^2}{}$$

$$= \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{a^4(20c^2}{}$$

Mathematica [A]
time = 2.51, size = 433, normalized size = 1.68

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] -1/15360*(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^5*(240*(
20*c^2 + 30*c*d + 13*d^2)*Cos[e + f*x]^5*(Log[Cos[(e + f*x)/2] - Sin[(e + f
```

```
*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - Sec[e]*(80*(34*c^2 +
60*c*d + 29*d^2)*Sin[f*x] - 240*(7*c^2 + 10*c*d + 3*d^2)*Sin[2*e + f*x] + 3
60*c^2*Sin[e + 2*f*x] + 1140*c*d*Sin[e + 2*f*x] + 750*d^2*Sin[e + 2*f*x] +
360*c^2*Sin[3*e + 2*f*x] + 1140*c*d*Sin[3*e + 2*f*x] + 750*d^2*Sin[3*e + 2*
f*x] + 1840*c^2*Sin[2*e + 3*f*x] + 3360*c*d*Sin[2*e + 3*f*x] + 1520*d^2*Sin
[2*e + 3*f*x] - 360*c^2*Sin[4*e + 3*f*x] - 240*c*d*Sin[4*e + 3*f*x] + 180*c
^2*Sin[3*e + 4*f*x] + 450*c*d*Sin[3*e + 4*f*x] + 195*d^2*Sin[3*e + 4*f*x] +
 180*c^2*Sin[5*e + 4*f*x] + 450*c*d*Sin[5*e + 4*f*x] + 195*d^2*Sin[5*e + 4*
f*x] + 440*c^2*Sin[4*e + 5*f*x] + 720*c*d*Sin[4*e + 5*f*x] + 304*d^2*Sin[4*
e + 5*f*x])))/f
```

Maple [A]
time = 0.38, size = 385, normalized size = 1.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-c^2*a^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a^3*c*d*(-(-1/4*sec(f*x+
e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-a^3*d^2*(-8/
15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+3*c^2*a^3*(1/2*sec(f*x+e)
*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-6*a^3*c*d*(-2/3-1/3*sec(f*x+e)^2
)*tan(f*x+e)+3*a^3*d^2*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*
ln(sec(f*x+e)+tan(f*x+e)))+3*c^2*a^3*tan(f*x+e)+6*a^3*c*d*(1/2*sec(f*x+e)*t
an(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-3*a^3*d^2*(-2/3-1/3*sec(f*x+e)^2)*
tan(f*x+e)+c^2*a^3*ln(sec(f*x+e)+tan(f*x+e))+2*a^3*c*d*tan(f*x+e)+a^3*d^2*(
1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.29, size = 496, normalized size = 1.93

$$\frac{80\,\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}c^{2}+480\,\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}cd+16\,\left(3\,\tan\left(fx+e\right)^{5}+10\,\tan\left(fx+e\right)^{3}+15\,\tan\left(fx+e\right)\right)a^{3}d^{2}+240\,\left(\tan\left(fx+e\right)^{3}+3\,\tan\left(fx+e\right)\right)a^{3}d^{2}-30\,a^{3}cd\left(\frac{2\,\left(3\,\sin\left(fx+e\right)^{3}-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx+e\right)-1\right)\right)-45\,a^{3}d^{2}\left(\frac{2\,\left(3\,\sin\left(fx+e\right)^{3}-5\,\sin\left(fx+e\right)\right)}{\sin\left(fx+e\right)^{4}-2\,\sin\left(fx+e\right)^{2}+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx+e\right)-1\right)\right)-180\,a^{3}c^{2}\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-360\,a^{3}cd\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-60\,a^{3}d^{2}\left(\frac{2\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+240\,a^{3}c^{2}\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)+720\,a^{3}c^{2}\tan\left(fx+e\right)+480\,a^{3}cd\tan\left(fx+e\right)}{240\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 480*(tan(f*x + e)^3 +
 3*tan(f*x + e))*a^3*c*d + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*ta
n(f*x + e))*a^3*d^2 + 240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^2 - 30*a^
3*c*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 45*a^3*d^2
*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2
+ 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 180*a^3*c^2*(2*
sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e
) - 1)) - 360*a^3*c*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
```

```
e) + 1) + log(sin(f*x + e) - 1)) - 60*a^3*d^2*(2*sin(f*x + e)/(sin(f*x + e)
^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a^3*c^2*log(
sec(f*x + e) + tan(f*x + e)) + 720*a^3*c^2*tan(f*x + e) + 480*a^3*c*d*tan(f
*x + e))/f
```

Fricas [A]
time = 3.84, size = 255, normalized size = 0.99

$$\frac{15\,(20\,a^3c^2+30\,a^3cd+13\,a^3d^2)\cos(fx+e)^5\log(\sin(fx+e)+1)-15\,(20\,a^3c^2+30\,a^3cd+13\,a^3d^2)\cos(fx+e)^5\log(-\sin(fx+e)+1)+2\left(24\,a^3d^2+8\,(55\,a^3c^2+90\,a^3cd+38\,a^3d^2)\cos(fx+e)^4+15\,(12\,a^3c^2+30\,a^3cd+13\,a^3d^2)\cos(fx+e)^3+8\,(5\,a^3c^2+30\,a^3cd+19\,a^3d^2)\cos(fx+e)^2+30\,(2\,a^3cd+3\,a^3d^2)\cos(fx+e)\right)\sin(fx+e)}{240\,f\cos(fx+e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(sin(f*x
 + e) + 1) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(-
sin(f*x + e) + 1) + 2*(24*a^3*d^2 + 8*(55*a^3*c^2 + 90*a^3*c*d + 38*a^3*d^2
)*cos(f*x + e)^4 + 15*(12*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^3
 + 8*(5*a^3*c^2 + 30*a^3*c*d + 19*a^3*d^2)*cos(f*x + e)^2 + 30*(2*a^3*c*d +
 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c^2\sec(e+fx)\,dx+\int 3c^2\sec^2(e+fx)\,dx+\int 3c^2\sec^3(e+fx)\,dx+\int c^2\sec^4(e+fx)\,dx+\int d^2\sec^3(e+fx)\,dx+\int 3d^2\sec^4(e+fx)\,dx+\int 3d^2\sec^5(e+fx)\,dx+\int d^2\sec^6(e+fx)\,dx+\int 2cd\sec^2(e+fx)\,dx+\int 6cd\sec^3(e+fx)\,dx+\int 6cd\sec^4(e+fx)\,dx+\int 2cd\sec^5(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(c**2*sec(e + f*x), x) + Integral(3*c**2*sec(e + f*x)**2, x)
+ Integral(3*c**2*sec(e + f*x)**3, x) + Integral(c**2*sec(e + f*x)**4, x) +
 Integral(d**2*sec(e + f*x)**3, x) + Integral(3*d**2*sec(e + f*x)**4, x) +
Integral(3*d**2*sec(e + f*x)**5, x) + Integral(d**2*sec(e + f*x)**6, x) + I
ntegral(2*c*d*sec(e + f*x)**2, x) + Integral(6*c*d*sec(e + f*x)**3, x) + In
tegral(6*c*d*sec(e + f*x)**4, x) + Integral(2*c*d*sec(e + f*x)**5, x))
```

Giac [A]
time = 0.62, size = 376, normalized size = 1.46

$$\frac{15\,(20\,a^3c^2+30\,a^3cd+13\,a^3d^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,(20\,a^3c^2+30\,a^3cd+13\,a^3d^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(300\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+450\,a^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+195\,a^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-1400\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-2100\,a^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-910\,a^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+2560\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3840\,a^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+1664\,a^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-2120\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3660\,a^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-1330\,a^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+660\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1470\,a^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+765\,a^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^5}}{120\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*
e) + 1)) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*log(abs(tan(1/2*f*x +
```

```
1/2*e) - 1)) - 2*(300*a^3*c^2*tan(1/2*f*x + 1/2*e)^9 + 450*a^3*c*d*tan(1/2*
f*x + 1/2*e)^9 + 195*a^3*d^2*tan(1/2*f*x + 1/2*e)^9 - 1400*a^3*c^2*tan(1/2*
f*x + 1/2*e)^7 - 2100*a^3*c*d*tan(1/2*f*x + 1/2*e)^7 - 910*a^3*d^2*tan(1/2*
f*x + 1/2*e)^7 + 2560*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 3840*a^3*c*d*tan(1/2
*f*x + 1/2*e)^5 + 1664*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 2120*a^3*c^2*tan(1/
2*f*x + 1/2*e)^3 - 3660*a^3*c*d*tan(1/2*f*x + 1/2*e)^3 - 1330*a^3*d^2*tan(1
/2*f*x + 1/2*e)^3 + 660*a^3*c^2*tan(1/2*f*x + 1/2*e) + 1470*a^3*c*d*tan(1/2
*f*x + 1/2*e) + 765*a^3*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 -
 1)^5)/f
```

Mupad [B]

time = 5.52, size = 287, normalized size = 1.12

$$\frac{a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(20\,c^2+30\,c\,d+13\,d^2\right)}{2\,\left(10\,c^2+15\,c\,d+\frac{13\,d^2}{2}\right)}\right)\,\left(20\,c^2+30\,c\,d+13\,d^2\right)}{4\,f}-\frac{\left(5\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+\left(-\frac{70\,a^3\,c^2}{3}-35\,a^3\,c\,d-\frac{91\,a^3\,d^2}{6}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(\frac{128\,a^3\,c^2}{3}+64\,a^3\,c\,d+\frac{416\,a^3\,d^2}{15}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-\frac{106\,a^3\,c^2}{3}-61\,a^3\,c\,d-\frac{133\,a^3\,d^2}{6}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(11\,a^3\,c^2+\frac{49\,a^3\,c\,d}{2}+\frac{51\,a^3\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8+10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-10\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+5\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a^3*atanh((tan(e/2 + (f*x)/2)*(30*c*d + 20*c^2 + 13*d^2))/(2*(15*c*d + 10*
c^2 + (13*d^2)/2)))*(30*c*d + 20*c^2 + 13*d^2))/(4*f) - (tan(e/2 + (f*x)/2)
*(11*a^3*c^2 + (51*a^3*d^2)/4 + (49*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^9*(5*a
^3*c^2 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^7*((70*a^3*c
^2)/3 + (91*a^3*d^2)/6 + 35*a^3*c*d) - tan(e/2 + (f*x)/2)^3*((106*a^3*c^2)/
3 + (133*a^3*d^2)/6 + 61*a^3*c*d) + tan(e/2 + (f*x)/2)^5*((128*a^3*c^2)/3 +
 (416*a^3*d^2)/15 + 64*a^3*c*d))/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 +
(f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (
f*x)/2)^10 - 1))
```

3.204 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=125

$$\frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}+\frac{3a^3(4c+3d)\sec(e+fx)\tan(e+fx)}{8f}+\frac{d(a+$$

[Out] 5/8*a^3*(4*c+3*d)*arctanh(sin(f*x+e))/f+a^3*(4*c+3*d)*tan(f*x+e)/f+3/8*a^3*(4*c+3*d)*sec(f*x+e)*tan(f*x+e)/f+1/4*d*(a+a*sec(f*x+e))^3*tan(f*x+e)/f+1/12*a^3*(4*c+3*d)*tan(f*x+e)^3/f

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4086, 3876, 3855, 3852, 8, 3853}

$$\frac{a^3(4c+3d)\tan^3(e+fx)}{12f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}+\frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{3a^3(4c+3d)\tan(e+fx)\sec(e+fx)}{8f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3}{4f}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]

[Out] (5*a^3*(4*c + 3*d)*ArcTanh[Sin[e + f*x]])/(8*f) + (a^3*(4*c + 3*d)*Tan[e + f*x])/f + (3*a^3*(4*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (d*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(4*f) + (a^3*(4*c + 3*d)*Tan[e + f*x]^3)/(12*f)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx &= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}(4c+3d)\\ &= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}(4c+3d)\\ &= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}\left(a^3(4c+3\right.\\ &= \frac{a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{4f}+\frac{3a^3(4c+3d)}{}\\ &= \frac{5a^3(4c+3d)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{a^3(4c+3d)}{f}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 273 vs. 2(125) = 250.
time = 1.31, size = 273, normalized size = 2.18

$$-\frac{a^3(1+\cos(e+fx))^3\sec^6\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\left(120(4c+3d)\cos^4(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-\sec(e)(-24(11c+9d)\sin(e)+(36c+69d)\sin(fx)+36c\sin(2e+fx)+69d\sin(2e+fx)+280c\sin(e+2fx)+264d\sin(e+2fx)-72c\sin(3e+2fx)-24d\sin(3e+2fx)+36c\sin(2e+3fx)+45d\sin(2e+3fx)+36c\sin(4e+3fx)+45d\sin(4e+3fx)+88c\sin(3e+4fx)+72d\sin(3e+4fx))\right)}{1536f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]
```

```
[Out] -1/1536*(a^3*(1 + Cos[e + f*x])^3*Sec[(e + f*x)/2]^6*Sec[e + f*x]^4*(120*(4
*c + 3*d)*Cos[e + f*x]^4*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]]) - Sec[e]*(-24*(11*c + 9*d)*Sin[e] + (36
*c + 69*d)*Sin[f*x] + 36*c*Sin[2*e + f*x] + 69*d*Sin[2*e + f*x] + 280*c*Sin
[e + 2*f*x] + 264*d*Sin[e + 2*f*x] - 72*c*Sin[3*e + 2*f*x] - 24*d*Sin[3*e +
 2*f*x] + 36*c*Sin[2*e + 3*f*x] + 45*d*Sin[2*e + 3*f*x] + 36*c*Sin[4*e + 3*
f*x] + 45*d*Sin[4*e + 3*f*x] + 88*c*Sin[3*e + 4*f*x] + 72*d*Sin[3*e + 4*f*x
]))/f
```

Maple [A]
time = 0.31, size = 219, normalized size = 1.75

method	result
norman	$\frac{\frac{a^3(49d+44c)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{73(4c+3d)a^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}+\frac{55(4c+3d)a^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12f}-\frac{5(4c+3d)a^3\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{5(4c$
derivativedivides	$-a^3c\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+a^3d\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right.$
default	$-a^3c\left(-\frac{2}{3}-\frac{\left(\sec^2(fx+e)\right)}{3}\right)\tan(fx+e)+a^3d\left(-\left(-\frac{\left(\sec^3(fx+e)\right)}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right.$
risch	$-\frac{ia^3\left(36c\,e^{7i(fx+e)}+45d\,e^{7i(fx+e)}-72c\,e^{6i(fx+e)}-24d\,e^{6i(fx+e)}+36c\,e^{5i(fx+e)}+69d\,e^{5i(fx+e)}-264c\,e^{4i(fx+e)}-216d\right.}{12f\left(e^{2i(fx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^3*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a^3*d*(-(-1/4*sec(f*x+e)^3-3
/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+3*a^3*c*(1/2*sec(f
*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-3*a^3*d*(-2/3-1/3*sec(f*x+e
)^2)*tan(f*x+e)+3*a^3*c*tan(f*x+e)+3*a^3*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*l
n(sec(f*x+e)+tan(f*x+e)))+a^3*c*ln(sec(f*x+e)+tan(f*x+e))+a^3*d*tan(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(124) = 248.
time = 0.29, size = 284, normalized size = 2.27

$$\frac{16\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c+48\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3d-3a^3d\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log(\sin(fx+e)-1)\right)-36a^3c\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-36a^3d\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+48a^3c\log(\sec(fx+e)+\tan(fx+e))+144a^3c\tan(fx+e)+48a^3d\tan(fx+e)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c + 48*(tan(f*x + e)^3 + 3*t
an(f*x + e))*a^3*d - 3*a^3*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*
```

```
x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x
+ e) - 1)) - 36*a^3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
e) + 1) + log(sin(f*x + e) - 1)) - 36*a^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2
 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a^3*c*log(sec(f
*x + e) + tan(f*x + e)) + 144*a^3*c*tan(f*x + e) + 48*a^3*d*tan(f*x + e))/f
```

Fricas [A]
time = 2.15, size = 170, normalized size = 1.36

$$\frac{15\,(4\,a^3c+3\,a^3d)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-15\,(4\,a^3c+3\,a^3d)\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)+2\left(6\,a^3d+8\,(11\,a^3c+9\,a^3d)\cos\left(fx+e\right)^3+9\,(4\,a^3c+5\,a^3d)\cos\left(fx+e\right)^2+8\,(a^3c+3\,a^3d)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*(4*a^3*c + 3*a^3*d)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*(4*a
^3*c + 3*a^3*d)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*(6*a^3*d + 8*(11*
a^3*c + 9*a^3*d)*cos(f*x + e)^3 + 9*(4*a^3*c + 5*a^3*d)*cos(f*x + e)^2 + 8*
(a^3*c + 3*a^3*d)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int c\sec\left(e+fx\right)dx+\int 3c\sec^2\left(e+fx\right)dx+\int 3c\sec^3\left(e+fx\right)dx+\int c\sec^4\left(e+fx\right)dx+\int d\sec^2\left(e+fx\right)dx+\int 3d\sec^3\left(e+fx\right)dx+\int 3d\sec^4\left(e+fx\right)dx+\int d\sec^5\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e)),x)
```

```
[Out] a**3*(Integral(c*sec(e + f*x), x) + Integral(3*c*sec(e + f*x)**2, x) + Inte
gral(3*c*sec(e + f*x)**3, x) + Integral(c*sec(e + f*x)**4, x) + Integral(d*
sec(e + f*x)**2, x) + Integral(3*d*sec(e + f*x)**3, x) + Integral(3*d*sec(e
 + f*x)**4, x) + Integral(d*sec(e + f*x)**5, x))
```

Giac [A]
time = 0.51, size = 212, normalized size = 1.70

$$\frac{15\,(4\,a^3c+3\,a^3d)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,(4\,a^3c+3\,a^3d)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(60\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+45\,a^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-220\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-165\,a^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+292\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+219\,a^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-132\,a^3c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-147\,a^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^4}}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/24*(15*(4*a^3*c + 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(4*a^3
*c + 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(60*a^3*c*tan(1/2*f*x
+ 1/2*e)^7 + 45*a^3*d*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c*tan(1/2*f*x + 1/2*
e)^5 - 165*a^3*d*tan(1/2*f*x + 1/2*e)^5 + 292*a^3*c*tan(1/2*f*x + 1/2*e)^3
```

```
+ 219*a^3*d*tan(1/2*f*x + 1/2*e)^3 - 132*a^3*c*tan(1/2*f*x + 1/2*e) - 147*a
^3*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B]
time = 5.31, size = 203, normalized size = 1.62

$$\frac{\left(-5\,a^3\,c-\frac{15\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(\frac{55\,a^3\,c}{3}+\frac{55\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-\frac{73\,a^3\,c}{3}-\frac{73\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(11\,a^3\,c+\frac{49\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^8-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6+6\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-4\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}+\frac{5\,a^3\,\mathrm{atanh}\left(\frac{5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(4\,c+3\,d\right)}{2\,\left(10\,c+\frac{15\,d}{2}\right)}\right)\,\left(4\,c+3\,d\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(11*a^3*c + (49*a^3*d)/4) - tan(e/2 + (f*x)/2)^7*(5*a^3
*c + (15*a^3*d)/4) + tan(e/2 + (f*x)/2)^5*((55*a^3*c)/3 + (55*a^3*d)/4) - t
an(e/2 + (f*x)/2)^3*((73*a^3*c)/3 + (73*a^3*d)/4))/(f*(6*tan(e/2 + (f*x)/2)
^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8
 + 1)) + (5*a^3*atanh((5*tan(e/2 + (f*x)/2)*(4*c + 3*d))/(2*(10*c + (15*d)/
2)))*(4*c + 3*d))/(4*f)
```

3.205 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=153

$$\frac{a^3\tanh^{-1}(\sin(e+fx))}{2df}+\frac{a^3(c^2-3cd+3d^2)\tanh^{-1}(\sin(e+fx))}{d^3f}-\frac{2a^3(c-d)^{5/2}\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3\sqrt{c+d}\,f}$$

```
[Out] 1/2*a^3*arctanh(sin(f*x+e))/d/f+a^3*(c^2-3*c*d+3*d^2)*arctanh(sin(f*x+e))/d
^3/f-2*a^3*(c-d)^(5/2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/
d^3/f/(c+d)^(1/2)-a^3*(c-3*d)*tan(f*x+e)/d^2/f+1/2*a^3*sec(f*x+e)*tan(f*x+e
)/d/f
```

Rubi [A]

time = 0.22, antiderivative size = 257, normalized size of antiderivative = 1.68, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4072, 104, 159, 163, 65, 223, 209, 95, 211}

$$\frac{a^4(2c^2-6cd+7d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2a^4(c-d)^{5/2}\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{d^3f\sqrt{c+d}\,\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f}+\frac{\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{2df}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

```
[Out] -1/2*(a^3*(2*c - 5*d)*Tan[e + f*x])/(d^2*f) + (a^4*(2*c^2 - 6*c*d + 7*d^2)*
ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(
d^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*(c - d)^(
5/2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*
Sec[e + f*x]])]*Tan[e + f*x])/(d^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]]) + ((a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(2*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(a^3+a^3\sec(e+fx))\tan(e+fx)}{2df} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a+\cdots}}{\cdots}\right.}{2df\sqrt{a-a\sec(\cdots}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{(a^3+a^3\sec(e+fx))\tan(e+fx)}{2df} - \cdots$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{(a^3+a^3\sec(e+fx))\tan(e+fx)}{2df} + \cdots$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{(a^3+a^3\sec(e+fx))\tan(e+fx)}{2df} + \cdots$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{2a^4(c-d)^{5/2}\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\cdots}}{\sqrt{c-d}\,\sqrt{a-a\cdots}}\right.}{d^3\sqrt{c+d}\,f\sqrt{a-a\sec(e+fx)}\cdots}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{a^4(2c^2-6cd+7d^2)\tan^{-1}\left(\frac{\sqrt{a-a\,\text{se}\cdots}}{\sqrt{a+a\,\text{se}\cdots}}\right.}{d^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.43, size = 419, normalized size = 2.74

$$\frac{a^3\cos^2(e+fx)(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left(-2(2c^2-6cd+7d^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+2(2c^2-6cd+7d^2)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{8(c-d)^3\text{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(i\cos(e)+\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{d^2}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{4(c-3d)d\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}-\frac{d^2}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{4(c-3d)d\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{32d^3f(c+d\sec(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*
x])^3*(-2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
+ 2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (8*(
c - d)^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/
2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(I*Cos[e] + Sin[e]))/(S
qrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + d^2/(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])) - d^2/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]))))/(32*d^3*f*(c + d*Sec[e + f*x]))
```

Maple [A]
time = 0.38, size = 224, normalized size = 1.46

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{8d^3\sqrt{(c+d)(c-d)}}-\frac{1}{32d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-2c+5d}{32d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{(2c^2-}{}$ ... $}{f}$
default	$\frac{16a^3\left(-\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{8d^3\sqrt{(c+d)(c-d)}}-\frac{1}{32d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-2c+5d}{32d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{(2c^2-}{}$ ... $}{f}$
risch	$-\frac{ia^3\left(d\,e^{3i(fx+e)}+2\,e^{2i(fx+e)}c-6d\,e^{2i(fx+e)}-d\,e^{i(fx+e)}+2c-6d\right)}{f\,d^2\left(e^{2i(fx+e)}+1\right)^2}+\frac{a^3\ln\left(e^{i(fx+e)}+i\right)c^2}{d^3f}-\frac{3a^3\ln\left(e^{i(fx+e)}+i\right)c}{d^2f}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/f*a^3*(-1/8*(c^3-3*c^2*d+3*c*d^2-d^3)/d^3/((c+d)*(c-d))^(1/2)*arctanh((c
-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))-1/32/d/(tan(1/2*f*x+1/2*e)+1)^2
-1/32*(-2*c+5*d)/d^2/(tan(1/2*f*x+1/2*e)+1)+1/32*(2*c^2-6*c*d+7*d^2)/d^3*ln
(tan(1/2*f*x+1/2*e)+1)+1/32/d/(tan(1/2*f*x+1/2*e)-1)^2-1/32*(-2*c+5*d)/d^2/
(tan(1/2*f*x+1/2*e)-1)+1/32/d^3*(-2*c^2+6*c*d-7*d^2)*ln(tan(1/2*f*x+1/2*e)-
1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.45, size = 556, normalized size = 3.63

$$\left[\frac{2\left(a^3c^2-2a^3cd+a^3d^2\right)\sqrt{\frac{c-d}{c+d}}\cos\left(fx+e\right)^2\log\left(\frac{2cd\cos(fx+e)-(c^2-2d^2)\cos(fx+e)^2-2\left(c^2+cd+(cd+d^2)\cos(fx+e)\right)\sqrt{\frac{c-d}{c+d}}\sin(fx+e)+2c^2-d^2}{c^2\cos(fx+e)^2+2cd\cos(fx+e)+d^2}\right)+\left(2a^3c^2-6a^3cd+7a^3d^2\right)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-\left(2a^3c^2-6a^3cd+7a^3d^2\right)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(a^3d^2-2\left(a^3cd-3a^3d^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4d^3f\cos\left(fx+e\right)^2},-\frac{4\left(a^3c^2-2a^3cd+a^3d^2\right)\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right)\cos\left(fx+e\right)^2-\left(2a^3c^2-6a^3cd+7a^3d^2\right)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)+\left(2a^3c^2-6a^3cd+7a^3d^2\right)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)-2\left(a^3d^2-2\left(a^3cd-3a^3d^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4d^3f\cos\left(fx+e\right)^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/4*(2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt((c - d)/(c + d))*cos(f*x + e)^
2*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (
c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/
(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (2*a^3*c^2 - 6*a^3*c*d +
 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a^3*c^2 - 6*a^3*c*d +
 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a^3*d^2 - 2*(a^3*c*d
 - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f*x + e)^2), -1/4*(4*(
a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x +
e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e)))*cos(f*x + e)^2 - (2*
a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*
a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*
(a^3*d^2 - 2*(a^3*c*d - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f
*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int\frac{3\sec^2\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int\frac{3\sec^3\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx+\int\frac{\sec^4\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f
*x)**2/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f*x)**3/(c + d*sec(e +
 f*x)), x) + Integral(sec(e + f*x)**4/(c + d*sec(e + f*x)), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(140) = 280$.

time = 0.57, size = 285, normalized size = 1.86

$$\frac{\frac{\left(2\,a^{3}c^{2}-6\,a^{3}cd+7\,a^{3}d^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d^{3}}-\frac{\left(2\,a^{3}c^{2}-6\,a^{3}cd+7\,a^{3}d^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d^{3}}+\frac{4\left(a^{3}c^{3}-3\,a^{3}c^{2}d+3\,a^{3}cd^{2}-a^{3}d^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,c-2\,d\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^{2}+d^{2}}}\right)\right)}{\sqrt{-c^{2}+d^{2}}\,d^{3}}+\frac{2\left(2\,a^{3}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-5\,a^{3}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,a^{3}c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,a^{3}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}d^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))
/d^3 - (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1
))/d^3 + 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f
*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1
/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/(sqrt(-c^2 + d^2)*d^3) + 2*(2*a^3*c*tan
(1/2*f*x + 1/2*e)^3 - 5*a^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*c*tan(1/2*f*x
+ 1/2*e) + 7*a^3*d*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*d^
2))/f
```

Mupad [B]

time = 2.89, size = 1902, normalized size = 12.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (atanh((18824*a^9*c^2*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (
16680*a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/
d^4 - 11560*a^9*c*d) - (16680*a^9*c^3*tan(e/2 + (f*x)/2))/(2968*a^9*d^3 - 1
6680*a^9*c^3 - 11560*a^9*c*d^2 + 18824*a^9*c^2*d + (8608*a^9*c^4)/d - (2480
*a^9*c^5)/d^2 + (320*a^9*c^6)/d^3) + (8608*a^9*c^4*tan(e/2 + (f*x)/2))/(860
8*a^9*c^4 + 2968*a^9*d^4 - 11560*a^9*c*d^3 - 16680*a^9*c^3*d + 18824*a^9*c^
2*d^2 - (2480*a^9*c^5)/d + (320*a^9*c^6)/d^2) - (2480*a^9*c^5*tan(e/2 + (f*
x)/2))/(2968*a^9*d^5 - 2480*a^9*c^5 - 11560*a^9*c*d^4 + 8608*a^9*c^4*d + 18
824*a^9*c^2*d^3 - 16680*a^9*c^3*d^2 + (320*a^9*c^6)/d) + (320*a^9*c^6*tan(e
/2 + (f*x)/2))/(320*a^9*c^6 + 2968*a^9*d^6 - 11560*a^9*c*d^5 - 2480*a^9*c^5
*d + 18824*a^9*c^2*d^4 - 16680*a^9*c^3*d^3 + 8608*a^9*c^4*d^2) + (2968*a^9*
d^2*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (16680*a^9*c^3)/d +
 (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/d^4 - 11560*a^9*c*
d) - (11560*a^9*c*d*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (16
680*a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/d^
4 - 11560*a^9*c*d))*(2*a^3*c^2 + 7*a^3*d^2 - 6*a^3*c*d))/(d^3*f) - ((tan(e/
2 + (f*x)/2)*(2*a^3*c - 7*a^3*d))/d^2 - (a^3*tan(e/2 + (f*x)/2)^3*(2*c - 5*
d))/d^2)/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1)) - (a^3*ata
```

```
n(((a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^
6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 -
492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 + (a^3*((c + d)*(c - d)^5)^(1/2)*((
8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^4
*d^6))/d^6 - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8 -
 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d)))*1i)/(d^3*(c + d))
+ (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6
*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 - 4
92*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 - (a^3*((c + d)*(c - d)^5)^(1/2)*((8
*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^4*
d^6))/d^6 + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8 -
16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d)))*1i)/(d^3*(c + d)))/
((16*(4*a^9*c^8 + 35*a^9*d^8 - 219*a^9*c*d^7 - 42*a^9*c^7*d + 592*a^9*c^2*d
^6 - 904*a^9*c^3*d^5 + 855*a^9*c^4*d^4 - 515*a^9*c^5*d^3 + 194*a^9*c^6*d^2)
)/d^6 - (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 -
53*a^6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d
^4 - 492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 + (a^3*((c + d)*(c - d)^5)^(1/
2)*((8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^
3*c^4*d^6))/d^6 - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*
d^8 - 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d))))/(d^3*(c + d)
) + (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a
^6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 -
 492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 - (a^3*((c + d)*(c - d)^5)^(1/2)*(
(8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^
4*d^6))/d^6 + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8
- 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d))))/(d^3*(c + d))))*
((c + d)*(c - d)^5)^(1/2)*2i)/(d^3*f*(c + d))
```

3.206 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=161

$$-\frac{a^3(2c-3d)\tanh^{-1}(\sin(e+fx))}{d^3f}+\frac{2a^3(c-d)^{3/2}(2c+3d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3(c+d)^{3/2}f}+\frac{2a^3c\tan(e+fx)}{d^2(c+d)f}$$

[Out] -a^3*(2*c-3*d)*arctanh(sin(f*x+e))/d^3/f+2*a^3*(c-d)^(3/2)*(2*c+3*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/d^3/(c+d)^(3/2)/f+2*a^3*c*tan(f*x+e)/d^2/(c+d)/f-(c-d)*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/d/(c+d)/f/(c+d*sec(f*x+e))

Rubi [A]
time = 0.24, antiderivative size = 274, normalized size of antiderivative = 1.70, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4072, 100, 159, 163, 65, 223, 209, 95, 211}

$$-\frac{2a^4(2c-3d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{2a^4(c-d)^{3/2}(2c+3d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^3f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{2a^3c\tan(e+fx)}{d^2f(c+d)}-\frac{(c-d)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{df(c+d)(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]

[Out] (2*a^3*c*Tan[e + f*x])/(d^2*(c + d)*f) - (2*a^4*(2*c - 3*d)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^4*(c - d)^(3/2)*(2*c + 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^3*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(d*(c + d)*f*(c + d*Sec[e + f*x]))

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(a\tan(e+fx))\text{Subs}}{d(c+d)f\sqrt{}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\text{ta}}{}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(c}{}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{(2}{}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{2a^4(c-d)^{3/2}(2c+3d)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+}}{\sqrt{c-d}\sqrt{a-}}\right.}{d^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{2a^4(2c-3d)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\text{ta}}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.17, size = 455, normalized size = 2.83

$$\frac{a^3\cos(e+fx)(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left((2c-3d)(d+c\cos(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+(-2c+3d)(d+c\cos(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-\frac{2i(c-d)^2(2c+3d)\text{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(d+c\cos(e+fx))(\cos(e)-i\sin(e))}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{(c-d)^2d(-d\sin(e)+c\sin(fx))}{c(c+d)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)}+\frac{d(d+c\cos(e+fx))\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{d(d+c\cos(e+fx))\sin\left(\frac{fx}{2}\right)}{\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{8d^3f(c+d\sec(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x]
)^3*((2*c - 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]] + (-2*c + 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x
)/2]] - ((2*I)*(c - d)^2*(2*c + 3*d)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e]
+ (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^
2])]*(d + c*Cos[e + f*x])*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqr
t[(Cos[e] - I*Sin[e])^2]) + ((c - d)^2*d*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c
+ d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])) + (d*(d + c*Cos[e + f*x])
*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
) + (d*(d + c*Cos[e + f*x])*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]))))/(8*d^3*f*(c + d*Sec[e + f*x])^2)
```

Maple [A]
time = 0.44, size = 216, normalized size = 1.34

method	result
derivativedivides	$\frac{16a^3\left(-\frac{1}{16d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{(-2c+3d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16d^3}-\frac{1}{16d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{(2c-3d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16d^3}-\frac{\left(c^2-2cd\cdots\right.}{\cdots}\cdots\right.}{f}$
default	$\frac{16a^3\left(-\frac{1}{16d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{(-2c+3d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16d^3}-\frac{1}{16d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{(2c-3d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16d^3}-\frac{\left(c^2-2cd\cdots\right.}{\cdots}\cdots\right.}{f}$
risch	$\frac{2ia^3\left(c^2d\,e^{3i(fx+e)}-2c\,d^2e^{3i(fx+e)}+d^3e^{3i(fx+e)}+2c^3e^{2i(fx+e)}-c^2d\,e^{2i(fx+e)}+c\,d^2e^{2i(fx+e)}+3c^2d\,e^{i(fx+e)}+d^3e^{i(fx+e)}\cdots\right.}{f\,d^2\left(e^{2i(fx+e)}+1\right)(c+d)c\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/f*a^3*(-1/16/d^2/(tan(1/2*f*x+1/2*e)+1)+1/16/d^3*(-2*c+3*d)*ln(tan(1/2*f
*x+1/2*e)+1)-1/16/d^2/(tan(1/2*f*x+1/2*e)-1)+1/16*(2*c-3*d)/d^3*ln(tan(1/2*
f*x+1/2*e)-1)-1/4*(c^2-2*c*d+d^2)/d^3*(1/2*d/(c+d)*tan(1/2*f*x+1/2*e)/(c*ta
n(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)-1/2*(2*c+3*d)/(c+d)/((c+d)*(
c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(158) = 316$.
time = 2.72, size = 891, normalized size = 5.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d
 + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*co
s(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(
f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
e)^2 + 2*c*d*cos(f*x + e) + d^2)) + ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*
cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(si
n(f*x + e) + 1) - ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (
2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) -
 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))
*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*
x + e)), 1/2*(2*((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*
a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c + d))*arc
tan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e))) -
((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*
c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) + ((2*a^3*c^3 - a^3*
c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)
```

```
*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2
*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*
cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{\sec\left(e+fx\right)}{c^{2}+2cd\sec\left(e+fx\right)+d^{2}\sec^{2}\left(e+fx\right)}\,dx+\int \frac{3\sec^{2}\left(e+fx\right)}{c^{2}+2cd\sec\left(e+fx\right)+d^{2}\sec^{2}\left(e+fx\right)}\,dx+\int \frac{3\sec^{3}\left(e+fx\right)}{c^{2}+2cd\sec\left(e+fx\right)+d^{2}\sec^{2}\left(e+fx\right)}\,dx+\int \frac{\sec^{4}\left(e+fx\right)}{c^{2}+2cd\sec\left(e+fx\right)+d^{2}\sec^{2}\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)*
*2), x) + Integral(3*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(
e + f*x)**2), x) + Integral(3*sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) +
d**2*sec(e + f*x)**2), x) + Integral(sec(e + f*x)**4/(c**2 + 2*c*d*sec(e +
f*x) + d**2*sec(e + f*x)**2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(152) = 304.
time = 0.51, size = 317, normalized size = 1.97

$$-\frac{\frac{2\left(2a^{3}c^{3}-a^{3}c^{2}d-4a^{3}cd^{2}+3a^{3}d^{3}\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2c-2d\right)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^{2}+d^{2}}}\right)\right)}{\left(cd^{3}+d^{4}\right)\sqrt{-c^{2}+d^{2}}}+\frac{4\left(a^{3}c^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-a^{3}cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}-a^{3}c^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-a^{3}d^{2}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}-2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+c+d\right)\left(cd^{2}+d^{3}\right)}+\frac{\left(2a^{3}c-3a^{3}d\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d^{3}}-\frac{\left(2a^{3}c-3a^{3}d\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d^{3}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(2*a^3*c^3 - a^3*c^2*d - 4*a^3*c*d^2 + 3*a^3*d^3)*(pi*floor(1/2*(f*x +
e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*
x + 1/2*e))/sqrt(-c^2 + d^2)))/((c*d^3 + d^4)*sqrt(-c^2 + d^2)) + 4*(a^3*c^
2*tan(1/2*f*x + 1/2*e)^3 - a^3*c*d*tan(1/2*f*x + 1/2*e)^3 - a^3*c^2*tan(1/2
*f*x + 1/2*e) - a^3*d^2*tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^4 -
d*tan(1/2*f*x + 1/2*e)^4 - 2*c*tan(1/2*f*x + 1/2*e)^2 + c + d)*(c*d^2 + d^3
)) + (2*a^3*c - 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^3 - (2*a^3*c
- 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/d^3)/f
```

Mupad [B]
time = 5.27, size = 3135, normalized size = 19.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

[Out] (a^3*atan(((a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d
^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*
c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10
- 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^
8 + c^2*d^6) - (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 +
c^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d)*1
i)/d^3 + (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6
- 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^
5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 -
4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8
+ c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c
^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d)*1i)
/d^3)/((128*(4*a^9*c^7 - 9*a^9*c*d^6 - 16*a^9*c^6*d + 36*a^9*c^2*d^5 - 50*a
^9*c^3*d^4 + 20*a^9*c^4*d^3 + 15*a^9*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6) +
(a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6
*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(
2*c*d^5 + d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2
*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6
) - (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/
(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d))/d^3 - (a^3*
((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*
d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d
^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9
+ 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (
64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/(d^3*
(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d))/d^3))*(2*c - 3*d
)*2i)/(d^3*f) - ((4*tan(e/2 + (f*x)/2)^3*(a^3*c^2 - a^3*c*d))/(d^2*(c + d))
- (4*a^3*tan(e/2 + (f*x)/2)*(c^2 + d^2))/(d^2*(c + d)))/(f*(c + d + tan(e/
2 + (f*x)/2)^4*(c - d) - 2*c*tan(e/2 + (f*x)/2)^2)) + (a^3*atan(((a^3*((64*
tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*d - 1
6*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 +
d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a
^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (64*a^
3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^
3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c
^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d
^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*1i)/(3*c*d^5 + d^6 +
3*c^2*d^4 + c^3*d^3) + (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7
+ 27*a^6*c*d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c
^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 -
3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/
(2*c*d^7 + d^8 + c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3
)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d
^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*
c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2

```
)*(2*c + 3*d)*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))/((128*(4*a^9*c^7 -
 9*a^9*c*d^6 - 16*a^9*c^6*d + 36*a^9*c^2*d^5 - 50*a^9*c^3*d^4 + 20*a^9*c^4*
d^3 + 15*a^9*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((64*tan(e/2 + (f*x
)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5
- 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4)
 + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a
^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (64*a^3*tan(e/2 + (f
*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^
6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c
+ d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))
*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*
d^3) - (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 -
 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*
d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*
a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 +
c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*
d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^
6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^
5 + d^6 + 3*c^2*d^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3
*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*
d)*2i)/(f*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))
```

3.207 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=188

$$\frac{a^3\tanh^{-1}(\sin(e+fx))}{d^3f}-\frac{a^3\sqrt{c-d}\,(2c^2+6cd+7d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3(c+d)^{5/2}f}-\frac{(c-d)\,(a^3+a^3\sec(e}{2d(c+d)f(c+}$$

```
[Out] a^3*arctanh(sin(f*x+e))/d^3/f-a^3*(2*c^2+6*c*d+7*d^2)*arctanh((c-d)^(1/2)*t
an(1/2*f*x+1/2*e)/(c+d)^(1/2))*(c-d)^(1/2)/d^3/(c+d)^(5/2)/f-1/2*(c-d)*(a^3
+a^3*sec(f*x+e))*tan(f*x+e)/d/(c+d)/f/(c+d*sec(f*x+e))^2-1/2*a^3*(c-d)*(2*c
+5*d)*tan(f*x+e)/d^2/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A]

time = 0.27, antiderivative size = 301, normalized size of antiderivative = 1.60, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4072, 100, 154, 163, 65, 223, 209, 95, 211}

$$\frac{a^4\sqrt{c-d}\,(2c^2+6cd+7d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{d^3f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{2a^4\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2f(c+d)^2(c+d\sec(e+fx))}-\frac{(c-d)\tan(e+fx)\,(a^3\sec(e+fx)+a^3)}{2df(c+d)(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^4*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(d^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^4*Sqrt
[c - d]*(2*c^2 + 6*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]
])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^3*(c + d)^(5/2)
*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)*(a^3 + a^3
*Sec[e + f*x])*Tan[e + f*x])/(2*d*(c + d)*f*(c + d*Sec[e + f*x])^2) - (a^3*
(c - d)*(2*c + 5*d)*Tan[e + f*x])/(2*d^2*(c + d)^2*f*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} + \frac{(a\tan(e+fx))\mathrm{Subs}}{2d(c+d)}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)}{2d^2(c+d)^2f(c+d\,\mathrm{s}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)}{2d^2(c+d)^2f(c+d\,\mathrm{s}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)}{2d^2(c+d)^2f(c+d\,\mathrm{s}}$$

$$= \frac{a^4\sqrt{c-d}\left(2c^2+6cd+7d^2\right)\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right.}{d^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx}}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^3f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{a^4\sqrt{c-d}\,(2c^2}{d^3(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.48, size = 393, normalized size = 2.09

$$\frac{a^3(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left(-4(d+c\cos(e+fx))^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+4(d+c\cos(e+fx))^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{4\left(2c^3+4c^2d+cd^2-7d^3\right)\operatorname{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}\right)(d+c\cos(e+fx))^2(i\cos(e)+\sin(e))}{(c+d)^2\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{(c-d)d\sec(e)\left(\left(2c^4+6c^3d+5c^2d^2+12cd^3+2d^4\right)\sin(e)-c\left(d\left(7c^2+18cd+2d^2\right)\sin(fx)-d\left(c^2+6cd+2d^2\right)\sin(2e+fx)+c\left(2c^2+6cd+d^2\right)\sin(e+2fx)\right)\right)}{c^2(c+d)^2}\right)}{32d^3f(c+d\sec(e+fx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x])^3*(-4*(d +
c*Cos[e + f*x])^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*(d + c*Cos[e
 + f*x])^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (4*(2*c^3 + 4*c^2*d +
 c*d^2 - 7*d^3)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan
[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f
*x])^2*(I*Cos[e] + Sin[e]))/((c + d)^2*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin
[e])^2]) + ((c - d)*d*Sec[e]*((2*c^4 + 6*c^3*d + 5*c^2*d^2 + 12*c*d^3 + 2*d
^4)*Sin[e] - c*(d*(7*c^2 + 18*c*d + 2*d^2)*Sin[f*x] - d*(c^2 + 6*c*d + 2*d^
2)*Sin[2*e + f*x] + c*(2*c^2 + 6*c*d + d^2)*Sin[e + 2*f*x])))/(c^2*(c + d)^
2)))/(32*d^3*f*(c + d*Sec[e + f*x])^3)
```

Maple [A]

time = 0.61, size = 227, normalized size = 1.21

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16d^3}+\frac{(c-d)\left(\frac{\frac{d(2c^2+3cd-5d^2)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c^2+4cd+2d^2}-\frac{d(2c+7d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}-\frac{(2c^2+6cd+7d^2)\operatorname{arctanh}\left(\cdots\right)}{2(c^2+2cd+d^2)\sqrt{(c+\cdots}}\right)}{8d^3}\cdots\right)}{f}$
default	$\frac{16a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16d^3}+\frac{(c-d)\left(\frac{\frac{d(2c^2+3cd-5d^2)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2c^2+4cd+2d^2}-\frac{d(2c+7d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}-\frac{(2c^2+6cd+7d^2)\operatorname{arctanh}\left(\cdots\right)}{2(c^2+2cd+d^2)\sqrt{(c+\cdots}}\right)}{8d^3}\cdots\right)}{f}$
risch	$\frac{ia^3\left(-c^4d\,e^{3i(fx+e)}-5c^3d^2e^{3i(fx+e)}+4c^2d^3e^{3i(fx+e)}+2c\,d^4e^{3i(fx+e)}-2c^5e^{2i(fx+e)}-4c^4d\,e^{2i(fx+e)}+c^3d^2e^{2i(fx+e)}-7c^2\cdots\right.}{c^2f(c+d)^2d^2(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/f*a^3*(-1/16/d^3*ln(tan(1/2*f*x+1/2*e)-1)+1/8*(c-d)/d^3*((1/2*d*(2*c^2+3
*c*d-5*d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/2*d*(2*c+7*d)/(c+d)*tan(
1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^2-1/2*(
2*c^2+6*c*d+7*d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/
2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))+1/16/d^3*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

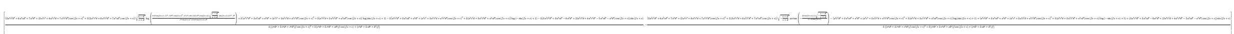
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(182) = 364$.
time = 4.27, size = 1208, normalized size = 6.43



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (2*a^3*c^4 + 6*a^3*c^3*d +
 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2*d^2 + 7*a^3*c*d
^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*
d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)
/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x +
 e) + d^2)) + 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3
*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3
)*cos(f*x + e))*log(sin(f*x + e) + 1) - 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*
d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d +
 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(3*a^3
*c^2*d^2 + 3*a^3*c*d^3 - 6*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^2 - 5*a^3*c
*d^3 - a^3*d^4)*cos(f*x + e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5
)*f*cos(f*x + e)^2 + 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^2*
d^5 + 2*c*d^6 + d^7)*f), -1/2*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (
2*a^3*c^4 + 6*a^3*c^3*d + 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d +
6*a^3*c^2*d^2 + 7*a^3*c*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c + d))*arctan(-(
```

```
d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e))) - (a^3*c
^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(
f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(si
n(f*x + e) + 1) + (a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c
^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d
^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + (3*a^3*c^2*d^2 + 3*a^3*c*d^3 - 6
*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^2 - 5*a^3*c*d^3 - a^3*d^4)*cos(f*x +
e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^
3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^2*d^5 + 2*c*d^6 + d^7)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \frac{\sec(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx+\int \frac{3\sec^2(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx+\int \frac{3\sec^3(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx+\int \frac{\sec^4(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(3*sec(e + f*x)**2/(c**3 +
3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x
) + Integral(3*sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec
(e + f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**4/(c**3 +
 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3),
x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(175) = 350$.
time = 0.64, size = 376, normalized size = 2.00

$$\frac{\frac{a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d^3}-\frac{a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d^3}+\frac{(2a^3c^3+4a^3c^2d+a^3cd^2-7a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^2d^3+2cd^4+d^5)\sqrt{-c^2+d^2}}+\frac{2a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-8a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+5a^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^3c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-7a^3c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+2a^3cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+7a^3d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(c^2d^2+2cd^3+d^4)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] (a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^3 - a^3*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/d^3 + (2*a^3*c^3 + 4*a^3*c^2*d + a^3*c*d^2 - 7*a^3*d^3)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) -
d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(
-c^2 + d^2)) + (2*a^3*c^3*tan(1/2*f*x + 1/2*e)^3 + a^3*c^2*d*tan(1/2*f*x +
1/2*e)^3 - 8*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a^3*d^3*tan(1/2*f*x + 1/2
*e)^3 - 2*a^3*c^3*tan(1/2*f*x + 1/2*e) - 7*a^3*c^2*d*tan(1/2*f*x + 1/2*e) +
 2*a^3*c*d^2*tan(1/2*f*x + 1/2*e) + 7*a^3*d^3*tan(1/2*f*x + 1/2*e))/((c^2*d
^2 + 2*c*d^3 + d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 -
c - d)^2))/f
```

Mupad [B]
time = 8.50, size = 2500, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
[Out] - ((a^3*tan(e/2 + (f*x)/2)*(5*c*d + 2*c^2 - 7*d^2))/(d^2*(c + d)) - (a^3*ta
n(e/2 + (f*x)/2)^3*(c^2*d - 8*c*d^2 + 2*c^3 + 5*d^3))/(d^2*(c + d)^2))/(f*(
2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 -
2*c*d + d^2) + c^2 + d^2)) - (a^3*atan(((a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*
c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^
3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3
*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 -
 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^
9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*a^3*tan(e/2 + (f*x)/2)*(8*
c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8
*c^7*d^6))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3)*1
i)/d^3 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6
 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*
c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) - (a^3*((8*(18*
a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^
8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^
7 + c^4*d^6) + (8*a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^
10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/(d^3*(4*c*d^7 + d^8
+ 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3)*1i)/d^3)/((16*(4*a^9*c^6 - 35*a^
9*d^6 + 61*a^9*c*d^5 + 10*a^9*c^5*d + 5*a^9*c^2*d^4 - 35*a^9*c^3*d^3 - 10*a
^9*c^4*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (a^3*((8*
tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d +
53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7
 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*
c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7
 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*
a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 -
8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^
3*d^5 + c^4*d^4))))/d^3))/d^3 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53
*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 -
52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c
^4*d^4) - (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*
c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10
 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (8*a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 +
 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6
))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3))/d^3))*2i
)/(d^3*f) - (a^3*atan(((a^3*((c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2
```

)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/((4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3))*(3*c*d + c^2 + (7*d^2)/2)*1i)/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3) + (a^3*((c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) - (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/((4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3))*(3*c*d + c^2 + (7*d^2)/2)*1i)/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3))/((16*(4*a^9*c^6 - 35*a^9*d^6 + 61*a^9*c*d^5 + 10*a^9*c^5*d + 5*a^9*c^2*d^4 - 35*a^9*c^3*d^3 - 10*a^9*c^4*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (a^3*((c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9...

3.208 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=178

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,(c+d)^{7/2}f}+\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3}-\frac{5a^3(c-d)\tan(e+fx)}{6d(c+d)^2f(c+d\sec(e+fx))^2}+$$

```
[Out] 5*a^3*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c+d)^(7/2)/f/(c-
d)^(1/2)+1/3*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^3-5/6
*a^3*(c-d)*tan(f*x+e)/d/(c+d)^2/f/(c+d*sec(f*x+e))^2+5/6*a^3*(c+4*d)*tan(f*
x+e)/d/(c+d)^3/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4072, 96, 95, 211}

$$-\frac{5a^4\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f\sqrt{c-d}\,(c+d)^{7/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{5a^3\tan(e+fx)}{2f(c+d)^3(c+d\sec(e+fx))}+\frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{6f(c+d)^2(c+d\sec(e+fx))^2}+\frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c+d)(c+d\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (-5*a^4*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
 a*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c - d]*(c + d)^(7/2)*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^2*Tan[e + f
*x])/(3*(c + d)*f*(c + d*Sec[e + f*x])^3) + (5*(a^3 + a^3*Sec[e + f*x])*Tan
[e + f*x])/(6*(c + d)^2*f*(c + d*Sec[e + f*x])^2) + (5*a^3*Tan[e + f*x])/(2
*(c + d)^3*f*(c + d*Sec[e + f*x]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)^4}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{(5a^3\tan(e+fx))\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{3(c+d)f\sqrt{a-a\sec(e}}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5(a^3+a^3\sec(e+fx))\tan(e+}{6(c+d)^2f(c+d\sec(e+fx}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5(a^3+a^3\sec(e+fx))\tan(e+}{6(c+d)^2f(c+d\sec(e+fx}$$

$$= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{5(a^3+a^3\sec(e+fx))\tan(e+}{6(c+d)^2f(c+d\sec(e+fx}$$

$$= -\frac{5a^4\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c-d}\,(c+d)^{7/2}f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{a}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 3.52, size = 398, normalized size = 2.24

$$\frac{a^3(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(1+\sec(e+fx))^3\left(-\frac{120i\,\mathrm{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(d+c\cos(e+fx))^3(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\frac{c\sec(e)\left(6\left(8c^4+6c^3d+30c^2d^2+9cd^3+2d^4\right)\sin(fx)-3\left(6c^4-3c^3d+30c^2d^2+18cd^3+4d^4\right)\sin(2e+fx)+c\left(3\left(3c^3+38c^2d+12cd^2+2d^3\right)\sin(e+2fx)+3\left(3c^3-6c^2d-6cd^2-2d^3\right)\sin(3e+2fx)+c\left(22c^2+9cd+2d^2\right)\sin(2e+3fx)\right)\right)-2d\left(66c^4+27c^3d+50c^2d^2+18cd^3+4d^4\right)\tan(e)}{c^3}\right)}{192(c+d)^3f(c+d\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*Sec[e + f*x]*(1 + Sec[e + f*x]
)^3*(((-120*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[
(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*
x])^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) +
(c*Sec[e]*(6*(8*c^4 + 6*c^3*d + 30*c^2*d^2 + 9*c*d^3 + 2*d^4)*Sin[f*x] - 3*
(6*c^4 - 3*c^3*d + 30*c^2*d^2 + 18*c*d^3 + 4*d^4)*Sin[2*e + f*x] + c*(3*(3*
c^3 + 38*c^2*d + 12*c*d^2 + 2*d^3)*Sin[e + 2*f*x] + 3*(3*c^3 - 6*c^2*d - 6*
c*d^2 - 2*d^3)*Sin[3*e + 2*f*x] + c*(22*c^2 + 9*c*d + 2*d^2)*Sin[2*e + 3*f*
x])) - 2*d*(66*c^4 + 27*c^3*d + 50*c^2*d^2 + 18*c*d^3 + 4*d^4)*Tan[e])/c^3)
)/(192*(c + d)^3*f*(c + d*Sec[e + f*x])^4)
```

Maple [A]

time = 0.57, size = 227, normalized size = 1.28

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{6(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^3}-\frac{5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}-\frac{3\left(-\right.}{}\right.}{}\right.}{f}$

default	$\frac{16a^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{6(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^3}-\frac{5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}-\frac{3\left(-\frac{}{2c}\right.}{}\right)}{}\right)}{f}$
risch	$\frac{ia^3(22c^5+9c^4d+2c^3d^2+132c^4d\,e^{3i(fx+e)}+54c^3d^2e^{3i(fx+e)}+100c^2d^3e^{3i(fx+e)}+36c\,d^4e^{3i(fx+e)}+36c^4d\,e^{2i(fx+e)}+180c^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/f*a^3*(-1/6*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f
*x+1/2*e)^2-c-d)^3-5/6/(c+d)*(-1/4*tan(1/2*f*x+1/2*e)/(c+d)/(c*tan(1/2*f*x+
1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^2-3/4/(c+d)*(-1/2*tan(1/2*f*x+1/2*e)/(
c+d)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)+1/2/(c+d)/((c+d)*(
c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(171) = 342$.
time = 2.65, size = 1038, normalized size = 5.83

$$\left[\frac{15\left(a^3c^3\cos\left(fx+e\right)^3+3a^3c^2d\cos\left(fx+e\right)^2+3a^3cd^2\cos\left(fx+e\right)+a^3d^3\right)\sqrt{c^2-d^2}\log\left(\frac{2cd\cos(fx+e)-(c^2-2d^2)\cos(fx+e)^2+2\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2c^2-d^2}{c^2\cos(fx+e)^2+2cd\cos(fx+e)+d^2}\right)+2\left(2a^3c^4+9a^3c^3d+20a^3c^2d^2-9a^3cd^3-22a^3d^4+\left(22a^3c^4+9a^3c^3d-20a^3c^2d^2-9a^3cd^3-2a^3d^4\right)\cos\left(fx+e\right)^2+3\left(3a^3c^4+16a^3c^3d-16a^3cd^3-3a^3d^4\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{12\left(\left(c^8+3c^7d+2c^6d^2-2c^5d^3-3c^4d^4-c^3d^5\right)f\cos\left(fx+e\right)^3+3\left(c^7d+3c^6d^2+2c^5d^3-2c^4d^4-3c^3d^5-c^2d^6\right)f\cos\left(fx+e\right)^2+3\left(c^6d^2+3c^5d^3+2c^4d^4-2c^3d^5-3c^2d^6-cd^7\right)f\cos\left(fx+e\right)+\left(c^5d^3+3c^4d^4+2c^3d^5-2c^2d^6-3cd^7-d^8\right)f\right)},\ \frac{15\left(a^3c^3\cos\left(fx+e\right)^3+3a^3c^2d\cos\left(fx+e\right)^2+3a^3cd^2\cos\left(fx+e\right)+a^3d^3\right)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)+\left(2a^3c^4+9a^3c^3d+20a^3c^2d^2-9a^3cd^3-22a^3d^4+\left(22a^3c^4+9a^3c^3d-20a^3c^2d^2-9a^3cd^3-2a^3d^4\right)\cos\left(fx+e\right)^2+3\left(3a^3c^4+16a^3c^3d-16a^3cd^3-3a^3d^4\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{6\left(\left(c^8+3c^7d+2c^6d^2-2c^5d^3-3c^4d^4-c^3d^5\right)f\cos\left(fx+e\right)^3+3\left(c^7d+3c^6d^2+2c^5d^3-2c^4d^4-3c^3d^5-c^2d^6\right)f\cos\left(fx+e\right)^2+3\left(c^6d^2+3c^5d^3+2c^4d^4-2c^3d^5-3c^2d^6-cd^7\right)f\cos\left(fx+e\right)+\left(c^5d^3+3c^4d^4+2c^3d^5-2c^2d^6-3cd^7-d^8\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*d*cos(f*x + e)^2 + 3*a^3*c*d^
2*cos(f*x + e) + a^3*d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3
*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a^3*c^
4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e)^2
+ 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e))*sin
(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^3*d^5)*f
*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3*c^3*d^5
- c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 - 2*c^3*d^
5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^5 -
2*c^2*d^6 - 3*c*d^7 - d^8)*f), 1/6*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*
d*cos(f*x + e)^2 + 3*a^3*c*d^2*cos(f*x + e) + a^3*d^3)*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2
*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a^
3*c^4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e
)^2 + 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e))
*sin(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^3*d^
5)*f*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3*c^3*
d^5 - c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 - 2*c^
3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^
5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int\frac{\sec\left(e+fx\right)}{c^{4}+4c^{3}d\sec\left(e+fx\right)+6c^{2}d^{2}\sec^{2}\left(e+fx\right)+4cd^{3}\sec^{3}\left(e+fx\right)+d^{4}\sec^{4}\left(e+fx\right)}\,dx+\int\frac{3\sec^{2}\left(e+fx\right)}{c^{4}+4c^{3}d\sec\left(e+fx\right)+6c^{2}d^{2}\sec^{2}\left(e+fx\right)+4cd^{3}\sec^{3}\left(e+fx\right)+d^{4}\sec^{4}\left(e+fx\right)}\,dx+\int\frac{3\sec^{3}\left(e+fx\right)}{c^{4}+4c^{3}d\sec\left(e+fx\right)+6c^{2}d^{2}\sec^{2}\left(e+fx\right)+4cd^{3}\sec^{3}\left(e+fx\right)+d^{4}\sec^{4}\left(e+fx\right)}\,dx+\int\frac{\sec^{4}\left(e+fx\right)}{c^{4}+4c^{3}d\sec\left(e+fx\right)+6c^{2}d^{2}\sec^{2}\left(e+fx\right)+4cd^{3}\sec^{3}\left(e+fx\right)+d^{4}\sec^{4}\left(e+fx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec
(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integ
ral(3*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f
*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(3*
```

```
sec(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2
 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(sec(e +
f*x)**4/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2 + 4*c*d
**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Giac [A]
time = 0.60, size = 307, normalized size = 1.72

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^3}{(c^3+3c^2d+3cd^2+d^3)\sqrt{-c^2+d^2}}+\frac{15a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-30a^3cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+15a^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-40a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+40a^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+33a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+66a^3cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+33a^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(c^3+3c^2d+3cd^2+d^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1
/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^3/((c^3 + 3*
c^2*d + 3*c*d^2 + d^3)*sqrt(-c^2 + d^2)) + (15*a^3*c^2*tan(1/2*f*x + 1/2*e)
^5 - 30*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 + 15*a^3*d^2*tan(1/2*f*x + 1/2*e)^5
- 40*a^3*c^2*tan(1/2*f*x + 1/2*e)^3 + 40*a^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3
3*a^3*c^2*tan(1/2*f*x + 1/2*e) + 66*a^3*c*d*tan(1/2*f*x + 1/2*e) + 33*a^3*d
^2*tan(1/2*f*x + 1/2*e))/((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*(c*tan(1/2*f*x +
1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

Mupad [B]
time = 4.98, size = 264, normalized size = 1.48

$$\frac{\frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(a^3c^2-2a^3cd+a^3d^2\right)}{(c+d)^3}+\frac{11a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+d}-\frac{40\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(a^3c-a^3d\right)}{3(c+d)^2}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(-3c^3-3c^2d+3cd^2+3d^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(-3c^3+3c^2d+3cd^2-3d^3\right)+3cd^2+3c^2d+c^3+d^3-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(c^3-3c^2d+3cd^2-d^3\right)\right)}+\frac{5a^3\operatorname{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)}{f(c+d)^{7/2}\sqrt{c-d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((5*tan(e/2 + (f*x)/2)^5*(a^3*c^2 + a^3*d^2 - 2*a^3*c*d))/(c + d)^3 + (11*a
^3*tan(e/2 + (f*x)/2))/(c + d) - (40*tan(e/2 + (f*x)/2)^3*(a^3*c - a^3*d))/
(3*(c + d)^2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3)
 - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c
^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) +
 (5*a^3*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)
^(7/2)*(c - d)^(1/2))
```

3.209 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal. Leaf size=266

$$\frac{5a^3(4c-3d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4(c-d)^{3/2}(c+d)^{9/2}f}-\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4}+\frac{a(4c-3d)(a+a\sec(e+\ldots}{12(c-d)(c+d)^2f(c+\ldots}$$

```
[Out] 5/4*a^3*(4*c-3*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)
^(3/2)/(c+d)^(9/2)/f-1/4*d*(a+a*sec(f*x+e))^3*tan(f*x+e)/(c^2-d^2)/f/(c+d*s
ec(f*x+e))^4+1/12*a*(4*c-3*d)*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c-d)/(c+d)^2/f
/(c+d*sec(f*x+e))^3-5/24*a^3*(4*c-3*d)*tan(f*x+e)/d/(c+d)^3/f/(c+d*sec(f*x+
e))^2+5/24*a^3*(4*c-3*d)*(c+4*d)*tan(f*x+e)/(c-d)/d/(c+d)^4/f/(c+d*sec(f*x+
e))
```

Rubi [A]
time = 0.23, antiderivative size = 327, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4072, 98, 96, 95, 211}

$$-\frac{5a^4(4c-3d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{4f(c-d)^{3/2}(c+d)^{9/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{5a^3(4c-3d)\tan(e+fx)}{8f(c-d)(c+d)^4(c+d\sec(e+fx))}+\frac{5(4c-3d)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{24f(c-d)(c+d)^3(c+d\sec(e+fx))^2}-\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3}{4f\left(c^2-d^2\right)(c+d\sec(e+fx))^4}+\frac{a(4c-3d)\tan(e+fx)(a\sec(e+fx)+a)^2}{12f(c-d)(c+d)^2(c+d\sec(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (-5*a^4*(4*c - 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(4*(c - d)^(3/2)*(c + d)^(9/2)
*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*(a + a*Sec[e + f
*x])^3*Tan[e + f*x])/(4*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^4) + (a*(4*c - 3
*d)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(12*(c - d)*(c + d)^2*f*(c + d*Sec
[e + f*x])^3) + (5*(4*c - 3*d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(24*(
c - d)*(c + d)^3*f*(c + d*Sec[e + f*x])^2) + (5*a^3*(4*c - 3*d)*Tan[e + f*x
])/(8*(c - d)*(c + d)^4*f*(c + d*Sec[e + f*x]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
```

```
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}\,(c+dx)^5}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} - \frac{(a^2(4c-3d)\tan(e+fx))\,\text{S}}{4(c^2-d^2)f\sqrt{a-}}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+f}{12(c-d)(c+d)^2f(c+d}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+f}{12(c-d)(c+d)^2f(c+d}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+f}{12(c-d)(c+d)^2f(c+d}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+f}{12(c-d)(c+d)^2f(c+d}$$
$$-\frac{5a^4(4c-3d)\tan^{-1}\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx}{4(c-d)^{3/2}(c+d)^{9/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 9.06, size = 274, normalized size = 1.03

$$\frac{a^3\left(-\frac{120(4c-3d)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \frac{(-72c^4-478c^3d+336c^2d^2+28cd^3+336d^4+(-296c^4-84c^3d-577c^2d^2+984cd^3+198d^4)\cos(e+fx)+(-72c^4-470c^3d+384c^2d^2+200cd^3+48d^4)\cos(2(e+fx))-88c^4\cos(3(e+fx))+36c^3d\cos(3(e+fx))+37c^2d^2\cos(3(e+fx))+24cd^3\cos(3(e+fx))+6d^4\cos(3(e+fx)))\sin(e+fx)}{(d+c\cos(e+fx))^4}\right)}{96(c-d)(c+d)^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^3*((-120*(4*c - 3*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/Sqrt[c^2 - d^2] - ((-72*c^4 - 478*c^3*d + 336*c^2*d^2 + 28*c*d^3 + 336*d
^4 + (-296*c^4 - 84*c^3*d - 577*c^2*d^2 + 984*c*d^3 + 198*d^4)*Cos[e + f*x]
 + (-72*c^4 - 470*c^3*d + 384*c^2*d^2 + 200*c*d^3 + 48*d^4)*Cos[2*(e + f*x)
] - 88*c^4*Cos[3*(e + f*x)] + 36*c^3*d*Cos[3*(e + f*x)] + 37*c^2*d^2*Cos[3*
(e + f*x)] + 24*c*d^3*Cos[3*(e + f*x)] + 6*d^4*Cos[3*(e + f*x)])*Sin[e + f*
x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)*(c + d)^4*f)
```

Maple [A]
time = 0.90, size = 303, normalized size = 1.14

method	result
derivativedivides	$\frac{16a^3\left(\frac{-\frac{5(4c-3d)\left(c^2-2cd+d^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64\left(c^4+4c^3d+6c^2d^2+4c\,d^3+d^4\right)}+\frac{55(c-d)(4c-3d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{192\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{73(4c-3d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{192\left(c^2+2cd+d^2\right)}+\frac{(44c-49d)\tan\left(\frac{f}{2}\right.}{64(c+d)(c-}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^4}\right.}{f}$
default	$\frac{16a^3\left(\frac{-\frac{5(4c-3d)\left(c^2-2cd+d^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{64\left(c^4+4c^3d+6c^2d^2+4c\,d^3+d^4\right)}+\frac{55(c-d)(4c-3d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{192\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{73(4c-3d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{192\left(c^2+2cd+d^2\right)}+\frac{(44c-49d)\tan\left(\frac{f}{2}\right.}{64(c+d)(c-}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^4}\right.}{f}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/f*a^3*((-5/64*(4*c-3*d)*(c^2-2*c*d+d^2)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+d
^4)*tan(1/2*f*x+1/2*e)^7+55/192*(c-d)*(4*c-3*d)/(c^3+3*c^2*d+3*c*d^2+d^3)*t
an(1/2*f*x+1/2*e)^5-73/192*(4*c-3*d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1
/64*(44*c-49*d)/(c+d)/(c-d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*t
an(1/2*f*x+1/2*e)^2-c-d)^4+5/64*(4*c-3*d)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-
3*c*d^4-d^5)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c
-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(258) = 516$.
time = 2.55, size = 1746, normalized size = 6.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] [1/48*(15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e)
^4 + 4*(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 - 3*
a^3*c^2*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x + e))
*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2
*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(
f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3*c^5*d + 12*a^3*c^4*d^2 +
 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a^3*c^6
- 36*a^3*c^5*d - 125*a^3*c^4*d^2 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4 + 24*a^3
*c*d^5 + 6*a^3*d^6)*cos(f*x + e)^3 + (36*a^3*c^6 + 235*a^3*c^5*d - 228*a^3*
c^4*d^2 - 335*a^3*c^3*d^3 + 168*a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*a^3*d^6)*c
os(f*x + e)^2 + (8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d
^3 - 217*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e))*sin(f*x +
e))/((c^11 + 3*c^10*d + c^9*d^2 - 5*c^8*d^3 - 5*c^7*d^4 + c^6*d^5 + 3*c^5*d
^6 + c^4*d^7)*f*cos(f*x + e)^4 + 4*(c^10*d + 3*c^9*d^2 + c^8*d^3 - 5*c^7*d^
4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*d^7 + c^3*d^8)*f*cos(f*x + e)^3 + 6*(c^9*d^
2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*d^5 - 5*c^5*d^6 + c^4*d^7 + 3*c^3*d^8 + c^2
*d^9)*f*cos(f*x + e)^2 + 4*(c^8*d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c
^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x + e) + (c^7*d^4 + 3*c^6*d^
5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f), 1/24*(
15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e)^4 + 4*
(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 - 3*a^3*c^2
*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x + e))*sqrt(-
c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (2*a^3*c^5*d + 12*a^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4
 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a^3*c^6 - 36*a^3*c^5*d - 125*a^3*c^4*d^2
 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4 + 24*a^3*c*d^5 + 6*a^3*d^6)*cos(f*x + e)
^3 + (36*a^3*c^6 + 235*a^3*c^5*d - 228*a^3*c^4*d^2 - 335*a^3*c^3*d^3 + 168*
a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*a^3*d^6)*cos(f*x + e)^2 + (8*a^3*c^6 + 48*
a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d^3 - 217*a^3*c^2*d^4 + 228*a^3*c
*d^5 + 45*a^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 + 3*c^10*d + c^9*d^2
- 5*c^8*d^3 - 5*c^7*d^4 + c^6*d^5 + 3*c^5*d^6 + c^4*d^7)*f*cos(f*x + e)^4 +
 4*(c^10*d + 3*c^9*d^2 + c^8*d^3 - 5*c^7*d^4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*
d^7 + c^3*d^8)*f*cos(f*x + e)^3 + 6*(c^9*d^2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*
d^5 - 5*c^5*d^6 + c^4*d^7 + 3*c^3*d^8 + c^2*d^9)*f*cos(f*x + e)^2 + 4*(c^8*
d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c
*d^10)*f*cos(f*x + e) + (c^7*d^4 + 3*c^6*d^5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*
d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{\sec{\left(e + f x \right)}}{c^{5} + 5 c^{4} d \sec{\left(e + f x \right)} + 10 c^{3} d^{2} \sec^{2}{\left(e + f x \right)} + 10 c^{2} d^{3} \sec^{3}{\left(e + f x \right)} + 5 c d^{4} \sec^{4}{\left(e + f x \right)} + d^{5} \sec^{5}{\left(e + f x \right)}}\, dx + \int \frac{3 \sec^{2}{\left(e + f x \right)}}{c^{5} + 5 c^{4} d \sec{\left(e + f x \right)} + 10 c^{3} d^{2} \sec^{2}{\left(e + f x \right)} + 10 c^{2} d^{3} \sec^{3}{\left(e + f x \right)} + 5 c d^{4} \sec^{4}{\left(e + f x \right)} + d^{5} \sec^{5}{\left(e + f x \right)}}\, dx + \int \frac{3 \sec^{3}{\left(e + f x \right)}}{c^{5} + 5 c^{4} d \sec{\left(e + f x \right)} + 10 c^{3} d^{2} \sec^{2}{\left(e + f x \right)} + 10 c^{2} d^{3} \sec^{3}{\left(e + f x \right)} + 5 c d^{4} \sec^{4}{\left(e + f x \right)} + d^{5} \sec^{5}{\left(e + f x \right)}}\, dx + \int \frac{\sec^{4}{\left(e + f x \right)}}{c^{5} + 5 c^{4} d \sec{\left(e + f x \right)} + 10 c^{3} d^{2} \sec^{2}{\left(e + f x \right)} + 10 c^{2} d^{3} \sec^{3}{\left(e + f x \right)} + 5 c d^{4} \sec^{4}{\left(e + f x \right)} + d^{5} \sec^{5}{\left(e + f x \right)}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*se
c(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d
**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x)**2/(c**5 + 5*c**4*d*sec(
e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*
c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x
)**3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2
*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x
) + Integral(sec(e + f*x)**4/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*s
ec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 +
d**5*sec(e + f*x)**5), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(247) = 494$.
time = 0.65, size = 601, normalized size = 2.26

$$\frac{\frac{15\,(4\,a^3c-3\,a^3d)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^5+3\,c^4d+2\,c^3d^2-2\,c^2d^3-3\,cd^4-d^5)\sqrt{-c^2+d^2}}-\frac{60\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-225\,a^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+315\,a^3c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-195\,a^3cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+45\,a^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-220\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+385\,a^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+55\,a^3c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-385\,a^3cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+165\,a^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+292\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+73\,a^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-511\,a^3c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-73\,a^3cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+219\,a^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-132\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-249\,a^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+45\,a^3c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+309\,a^3cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+147\,a^3d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{(c^5+3\,c^4d+2\,c^3d^2-2\,c^2d^3-3\,cd^4-d^5)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)^4}}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/12*(15*(4*a^3*c - 3*a^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2
*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
 d^2)))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*sqrt(-c^2
+ d^2)) - (60*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 225*a^3*c^3*d*tan(1/2*f*x +
1/2*e)^7 + 315*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 195*a^3*c*d^3*tan(1/2*f
*x + 1/2*e)^7 + 45*a^3*d^4*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c^4*tan(1/2*f*x
 + 1/2*e)^5 + 385*a^3*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 55*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e)^5 - 385*a^3*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 165*a^3*d^4*tan(1/
2*f*x + 1/2*e)^5 + 292*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 + 73*a^3*c^3*d*tan(1/
2*f*x + 1/2*e)^3 - 511*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 73*a^3*c*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 219*a^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 132*a^3*c^4*tan
(1/2*f*x + 1/2*e) - 249*a^3*c^3*d*tan(1/2*f*x + 1/2*e) + 45*a^3*c^2*d^2*tan
(1/2*f*x + 1/2*e) + 309*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + 147*a^3*d^4*tan(1/
2*f*x + 1/2*e))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*(c
*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

Mupad [B]
time = 5.08, size = 385, normalized size = 1.45

$$\frac{\frac{55\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(4a^3c^2-7a^3cd+3a^3d^2)}{12(c+d)^3}-\frac{73\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(4a^3c-3a^3d)}{12(c+d)^2}-\frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7(4a^3c^3-11a^3c^2d+10a^3cd^2-3a^3d^3)}{4(c+d)^4}+\frac{a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(44c-49d)}{4(c+d)(c-d)}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(6c^4-12c^2d^2+6d^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(-4c^4-8c^3d+8cd^3+4d^4)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(4c^4-8c^3d+8cd^3-4d^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(c^4-4c^3d+6c^2d^2-4cd^3+d^4)+4cd^3+4c^3d+c^4+d^4+6c^2d^2\right)}+\frac{5a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)(4c-3d)}{4f(c+d)^{9/2}(c-d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^5),x)
```

```
[Out] ((55*tan(e/2 + (f*x)/2)^5*(4*a^3*c^2 + 3*a^3*d^2 - 7*a^3*c*d))/(12*(c + d)^
3) - (73*tan(e/2 + (f*x)/2)^3*(4*a^3*c - 3*a^3*d))/(12*(c + d)^2) - (5*tan(
e/2 + (f*x)/2)^7*(4*a^3*c^3 - 3*a^3*d^3 + 10*a^3*c*d^2 - 11*a^3*c^2*d))/(4*
(c + d)^4) + (a^3*tan(e/2 + (f*x)/2)*(44*c - 49*d))/(4*(c + d)*(c - d)))/(f
*(tan(e/2 + (f*x)/2)^4*(6*c^4 + 6*d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/2)^2*
(8*c*d^3 - 8*c^3*d - 4*c^4 + 4*d^4) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 - 8*c^3
*d + 4*c^4 - 4*d^4) + tan(e/2 + (f*x)/2)^8*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 +
 6*c^2*d^2) + 4*c*d^3 + 4*c^3*d + c^4 + d^4 + 6*c^2*d^2)) + (5*a^3*atanh((t
an(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(4*c - 3*d))/(4*f*(c + d)^(
9/2)*(c - d)^(3/2))
```

3.210 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=183

$$\frac{d(8c^3-12c^2d+12cd^2-3d^3)\tanh^{-1}(\sin(e+fx))}{2af}-\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af}+\frac{(c-d)(c+}{f}$$

```
[Out] 1/2*d*(8*c^3-12*c^2*d+12*c*d^2-3*d^3)*arctanh(sin(f*x+e))/a/f-1/3*(3*c-4*d)
*d*(c+d*sec(f*x+e))^2*tan(f*x+e)/a/f+(c-d)*(c+d*sec(f*x+e))^3*tan(f*x+e)/f/
(a+a*sec(f*x+e))-1/6*d*(12*c^3-64*c^2*d+48*c*d^2-16*d^3+d*(6*c^2-20*c*d+9*d
^2)*sec(f*x+e))*tan(f*x+e)/a/f
```

Rubi [A]
time = 0.22, antiderivative size = 236, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 100, 158, 152, 65, 223, 209}

$$\frac{d(8c^3-12c^2d+12cd^2-3d^3)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)\left(d(6c^2-20cd+9d^2)\sec(e+fx)+4(3c^3-16c^2d+12cd^2-4d^3)\right)}{6af}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{d(3c-4d)\tan(e+fx)(c+d\sec(e+fx))^2}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
+ a*Sec[e + f*x]]) - ((3*c - 4*d)*d*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3
*a*f) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x
])) - (d*(4*(3*c^3 - 16*c^2*d + 12*c*d^2 - 4*d^3) + d*(6*c^2 - 20*c*d + 9*d
^2)*Sec[e + f*x])*Tan[e + f*x])/(6*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}\,(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+}{}\right.}{af\sqrt{a-as}}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e}{f(a+a\sec}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e}{f(a+a\sec}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e}{f(a+a\sec}$$

$$= -\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e}{f(a+a\sec}$$

$$= \frac{d(8c^3-12c^2d+12cd^2-3d^3)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1243 vs. $2(183) = 366$.
time = 6.49, size = 1243, normalized size = 6.79

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((-8*c^3*d + 12*c^2*d^2 - 12*c*d^3 + 3*d^4)*Cos[e/2 + (f*x)/2]^2*Cos[e + f*
x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^4)/(
f*(d + c*Cos[e + f*x])^4*(a + a*Sec[e + f*x])) + ((8*c^3*d - 12*c^2*d^2 + 1
2*c*d^3 - 3*d^4)*Cos[e/2 + (f*x)/2]^2*Cos[e + f*x]^3*Log[Cos[e/2 + (f*x)/2]
 + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^4)/(f*(d + c*Cos[e + f*x])^4*(a
 + a*Sec[e + f*x])) + (Cos[e/2 + (f*x)/2]*Sec[e/2]*Sec[e]*(c + d*Sec[e + f*
```

```
x])^4*(-18*c^4*Sin[(f*x)/2] + 72*c^3*d*Sin[(f*x)/2] - 36*c^2*d^2*Sin[(f*x)/
2] + 24*c*d^3*Sin[(f*x)/2] + 6*d^4*Sin[(f*x)/2] + 18*c^4*Sin[(3*f*x)/2] - 7
2*c^3*d*Sin[(3*f*x)/2] + 180*c^2*d^2*Sin[(3*f*x)/2] - 108*c*d^3*Sin[(3*f*x)
/2] + 39*d^4*Sin[(3*f*x)/2] - 72*c^2*d^2*Sin[e - (f*x)/2] + 48*c*d^3*Sin[e
- (f*x)/2] - 24*d^4*Sin[e - (f*x)/2] - 36*c^2*d^2*Sin[e + (f*x)/2] + 24*c*d
^3*Sin[e + (f*x)/2] - 6*d^4*Sin[e + (f*x)/2] - 18*c^4*Sin[2*e + (f*x)/2] +
72*c^3*d*Sin[2*e + (f*x)/2] - 144*c^2*d^2*Sin[2*e + (f*x)/2] + 96*c*d^3*Sin
[2*e + (f*x)/2] - 24*d^4*Sin[2*e + (f*x)/2] + 72*c^2*d^2*Sin[e + (3*f*x)/2]
 - 36*c*d^3*Sin[e + (3*f*x)/2] + 21*d^4*Sin[e + (3*f*x)/2] + 18*c^4*Sin[2*e
 + (3*f*x)/2] - 72*c^3*d*Sin[2*e + (3*f*x)/2] + 72*c^2*d^2*Sin[2*e + (3*f*x
)/2] - 36*c*d^3*Sin[2*e + (3*f*x)/2] + 9*d^4*Sin[2*e + (3*f*x)/2] - 36*c^2*
d^2*Sin[3*e + (3*f*x)/2] + 36*c*d^3*Sin[3*e + (3*f*x)/2] - 9*d^4*Sin[3*e +
(3*f*x)/2] + 36*c^2*d^2*Sin[e + (5*f*x)/2] - 12*c*d^3*Sin[e + (5*f*x)/2] +
7*d^4*Sin[e + (5*f*x)/2] - 6*c^4*Sin[2*e + (5*f*x)/2] + 24*c^3*d*Sin[2*e +
(5*f*x)/2] + 12*c*d^3*Sin[2*e + (5*f*x)/2] + d^4*Sin[2*e + (5*f*x)/2] + 12*
c*d^3*Sin[3*e + (5*f*x)/2] - 3*d^4*Sin[3*e + (5*f*x)/2] - 6*c^4*Sin[4*e + (
5*f*x)/2] + 24*c^3*d*Sin[4*e + (5*f*x)/2] - 36*c^2*d^2*Sin[4*e + (5*f*x)/2]
 + 36*c*d^3*Sin[4*e + (5*f*x)/2] - 9*d^4*Sin[4*e + (5*f*x)/2] + 6*c^4*Sin[2
*e + (7*f*x)/2] - 24*c^3*d*Sin[2*e + (7*f*x)/2] + 72*c^2*d^2*Sin[2*e + (7*f
*x)/2] - 48*c*d^3*Sin[2*e + (7*f*x)/2] + 16*d^4*Sin[2*e + (7*f*x)/2] + 36*c
^2*d^2*Sin[3*e + (7*f*x)/2] - 24*c*d^3*Sin[3*e + (7*f*x)/2] + 10*d^4*Sin[3*
e + (7*f*x)/2] + 6*c^4*Sin[4*e + (7*f*x)/2] - 24*c^3*d*Sin[4*e + (7*f*x)/2]
 + 36*c^2*d^2*Sin[4*e + (7*f*x)/2] - 24*c*d^3*Sin[4*e + (7*f*x)/2] + 6*d^4*
Sin[4*e + (7*f*x)/2]))/(48*f*(d + c*Cos[e + f*x])^4*(a + a*Sec[e + f*x]))
```

Maple [A]
time = 0.22, size = 309, normalized size = 1.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(c^4*tan(1/2*f*x+1/2*e)-4*c^3*d*tan(1/2*f*x+1/2*e)+6*c^2*d^2*tan(1/2*
f*x+1/2*e)-4*c*d^3*tan(1/2*f*x+1/2*e)+d^4*tan(1/2*f*x+1/2*e)-1/3*d^4/(tan(1
/2*f*x+1/2*e)+1)^3+1/2*d*(8*c^3-12*c^2*d+12*c*d^2-3*d^3)*ln(tan(1/2*f*x+1/2
*e)+1)-1/2*d^2*(12*c^2-12*c*d+5*d^2)/(tan(1/2*f*x+1/2*e)+1)-d^3*(2*c-d)/(ta
n(1/2*f*x+1/2*e)+1)^2-1/3*d^4/(tan(1/2*f*x+1/2*e)-1)^3-1/2*d*(8*c^3-12*c^2*
d+12*c*d^2-3*d^3)*ln(tan(1/2*f*x+1/2*e)-1)-1/2*d^2*(12*c^2-12*c*d+5*d^2)/(t
an(1/2*f*x+1/2*e)-1)+d^3*(2*c-d)/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(182) = 364$.
time = 0.29, size = 646, normalized size = 3.53

$$\frac{d^4\left(\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{16\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a-\frac{3a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{a\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}-\frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{6\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-12\,cd^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-36\,c^2d^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+24\,c^3d\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+\frac{6c^4\sin(fx+e)}{a(\cos(fx+e)+1)}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(d^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(a*(co
s(f*x + e) + 1))) - 12*c*d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x +
e)/(a*(cos(f*x + e) + 1))) - 36*c^2*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a
- a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)
/(a*(cos(f*x + e) + 1))) + 24*c^3*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*
x + e) + 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 2.51, size = 309, normalized size = 1.69

$$\frac{3\left(\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3\right)\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2\,d^4+2\left(3\,c^4-12\,c^3d+36\,c^2d^2-24\,cd^3+8\,d^4\right)\cos\left(fx+e\right)^3+\left(36\,c^2d^2-12\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^2+\left(12\,cd^3-d^4\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{12\left(af\cos\left(fx+e\right)^4+af\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*
d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) -
3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*d - 12
*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*
d^4 + 2*(3*c^4 - 12*c^3*d + 36*c^2*d^2 - 24*c*d^3 + 8*d^4)*cos(f*x + e)^3 +
 (36*c^2*d^2 - 12*c*d^3 + 7*d^4)*cos(f*x + e)^2 + (12*c*d^3 - d^4)*cos(f*x
+ e))*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^4\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{d^4\sec^5\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{4cd^3\sec^4\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{6c^2d^2\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int \frac{4c^3d\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**4*sec(e +
f*x)**5/(sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(sec(e +
```

```
f*x) + 1), x) + Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x) + 1), x
) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/a
```

Giac [A]
time = 0.56, size = 344, normalized size = 1.88

$$\frac{\frac{3\left(8c^3d-12c^2d^2+12cd^3-3d^4\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{3\left(8c^3d-12c^2d^2+12cd^3-3d^4\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}+\frac{6\left(c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-4c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+6c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-4cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a}-\frac{2\left(36c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-36cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+15d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-72c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+48cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-16d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+36c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-12cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+9d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*x + 1/2*
e) + 1))/a - 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*
x + 1/2*e) - 1))/a + 6*(c^4*tan(1/2*f*x + 1/2*e) - 4*c^3*d*tan(1/2*f*x + 1/
2*e) + 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 4*c*d^3*tan(1/2*f*x + 1/2*e) + d^4*
tan(1/2*f*x + 1/2*e))/a - 2*(36*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 36*c*d^3*t
an(1/2*f*x + 1/2*e)^5 + 15*d^4*tan(1/2*f*x + 1/2*e)^5 - 72*c^2*d^2*tan(1/2*
f*x + 1/2*e)^3 + 48*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 16*d^4*tan(1/2*f*x + 1/2
*e)^3 + 36*c^2*d^2*tan(1/2*f*x + 1/2*e) - 12*c*d^3*tan(1/2*f*x + 1/2*e) + 9
*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

Mupad [B]
time = 2.45, size = 211, normalized size = 1.15

$$\frac{\left(12c^2d^2-12cd^3+5d^4\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\left(-24c^2d^2+16cd^3-\frac{16d^4}{3}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+\left(12c^2d^2-4cd^3+3d^4\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(-a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+3a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-3a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a\right)}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c-d)^4}{af}+\frac{d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\left(8c^3-12c^2d+12cd^2-3d^3\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3*d^4 - 4*c*d^3 + 12*c^2*d^2) + tan(e/2 + (f*x)/2)^5*(
5*d^4 - 12*c*d^3 + 12*c^2*d^2) - tan(e/2 + (f*x)/2)^3*((16*d^4)/3 - 16*c*d^
3 + 24*c^2*d^2))/(f*(a - 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x)/2)^
4 - a*tan(e/2 + (f*x)/2)^6)) + (tan(e/2 + (f*x)/2)*(c - d)^4)/(a*f) + (d*at
anh(tan(e/2 + (f*x)/2))*(12*c*d^2 - 12*c^2*d + 8*c^3 - 3*d^3))/(a*f)
```

3.211 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{3d(2c^2-2cd+d^2)\tanh^{-1}(\sin(e+fx))}{2af}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{d(4(c^2-3cd+d^2)+(2c$$

```
[Out] 3/2*d*(2*c^2-2*c*d+d^2)*arctanh(sin(f*x+e))/a/f+(c-d)*(c+d*sec(f*x+e))^2*ta
n(f*x+e)/f/(a+a*sec(f*x+e))-1/2*d*(4*c^2-12*c*d+4*d^2+(2*c-3*d)*d*sec(f*x+e
))*tan(f*x+e)/a/f
```

Rubi [A]
time = 0.16, antiderivative size = 171, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 100, 152, 65, 223, 209}

$$\frac{3d(2c^2-2cd+d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)\left(4(c^2-3cd+d^2)+d(2c-3d)\sec(e+fx)\right)}{2af}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^2}{f(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[
e + f*x])]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]]) + ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]
)) - (d*(4*(c^2 - 3*c*d + d^2) + (2*c - 3*d)*d*Sec[e + f*x])*Tan[e + f*x])/
(2*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}\,(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+}{}\right.}{af\sqrt{a-a\,\text{se}}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d(4(c^2-3cd+d^2)+(2c}{}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d(4(c^2-3cd+d^2)+(2c}{}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d(4(c^2-3cd+d^2)+(2c}{}$$

$$= \frac{3d(2c^2-2cd+d^2)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 275 vs. $2(117) = 234$.
time = 2.64, size = 275, normalized size = 2.35

$$\frac{\cos^6\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\left(16d^3\csc^3(e+fx)\sin^4\left(\frac{1}{2}(e+fx)\right)+\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\left(3d(2c^2-2cd+d^2)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-2(c^3-3c^2d+9cd^2-3d^3)\tan\left(\frac{1}{2}(e+fx)\right)-3d(2c^2-2cd+d^2)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)+2(c-d)^3\tan^3\left(\frac{1}{2}(e+fx)\right)\right)\right)}{af(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (Cos[(e + f*x)/2]^6*Sec[e + f*x]^2*(16*d^3*Csc[e + f*x]^3*Sin[(e + f*x)/2]^
4 + (-1 + Tan[(e + f*x)/2]^2)*(3*d*(2*c^2 - 2*c*d + d^2)*(Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 2*(c^
3 - 3*c^2*d + 9*c*d^2 - 3*d^3)*Tan[(e + f*x)/2] - 3*d*(2*c^2 - 2*c*d + d^2)
*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2]])*Tan[(e + f*x)/2]^2 + 2*(c - d)^3*Tan[(e + f*x)/2]^3)))/(a*f*(1
+ Cos[e + f*x]))
```

Maple [A]
time = 0.19, size = 208, normalized size = 1.78

method	result
derivativedivides	$\dfrac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{d^3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3d\left(2c^2-2cd+d^2\right)\ln\left(\tan\left(\cdots\right.\right.}{2}}{fa}$
default	$\dfrac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{d^3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{3d\left(2c^2-2cd+d^2\right)\ln\left(\tan\left(\cdots\right.\right.}{2}}{fa}$
norman	$\dfrac{\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(3c^3-9c^2d+21c\,d^2-7d^3\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{3\left(c^3-3c^2d+5c\,d^2-2d^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\left(\cdots\right.}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$
risch	$\dfrac{i\left(2c^3e^{4i(fx+e)}-6c^2d\,e^{4i(fx+e)}+6c\,d^2e^{4i(fx+e)}-3d^3e^{4i(fx+e)}+6c\,d^2e^{3i(fx+e)}-3d^3e^{3i(fx+e)}+4c^3e^{2i(fx+e)}-12c^2d\,e^{2i(\cdots}\right.}{fa\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}+1\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(c^3*tan(1/2*f*x+1/2*e)-3*c^2*d*tan(1/2*f*x+1/2*e)+3*c*d^2*tan(1/2*f*
x+1/2*e)-d^3*tan(1/2*f*x+1/2*e)+1/2*d^3/(tan(1/2*f*x+1/2*e)-1)^2-3/2*d*(2*c
^2-2*c*d+d^2)*ln(tan(1/2*f*x+1/2*e)-1)-3/2*d^2*(2*c-d)/(tan(1/2*f*x+1/2*e)-
1)-1/2*d^3/(tan(1/2*f*x+1/2*e)+1)^2+3/2*d*(2*c^2-2*c*d+d^2)*ln(tan(1/2*f*x+
1/2*e)+1)-3/2*d^2*(2*c-d)/(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(120) = 240.
time = 0.29, size = 420, normalized size = 3.59

$$-\frac{d^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+6cd^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-6c^2d\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-\frac{2c^3\sin(fx+e)}{a(\cos(fx+e)+1)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x + e)
 + 1))) + 6*c*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1)
)) - 6*c^2*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c^3*s
in(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 2.28, size = 227, normalized size = 1.94

$$\frac{3\left(\left(2c^2d-2cd^2+d^3\right)\cos(fx+e)^3+\left(2c^2d-2cd^2+d^3\right)\cos(fx+e)^2\right)\log(\sin(fx+e)+1)-3\left(\left(2c^2d-2cd^2+d^3\right)\cos(fx+e)^3+\left(2c^2d-2cd^2+d^3\right)\cos(fx+e)^2\right)\log(-\sin(fx+e)+1)+2\left(d^3+2\left(c^3-3c^2d+6cd^2-2d^3\right)\cos(fx+e)^2+\left(6cd^2-d^3\right)\cos(fx+e)\right)\sin(fx+e)}{4\left(af\cos(fx+e)^3+af\cos(fx+e)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/4*(3*((2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3
)*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 3*((2*c^2*d - 2*c*d^2 + d^3)*cos(
f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^2)*log(-sin(f*x + e) +
1) + 2*(d^3 + 2*(c^3 - 3*c^2*d + 6*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (6*c*d^2
 - d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3 \sec (e+fx)}{\sec (e+fx)+1}\, dx + \int \frac{d^3 \sec^4 (e+fx)}{\sec (e+fx)+1}\, dx + \int \frac{3cd^2 \sec^3 (e+fx)}{\sec (e+fx)+1}\, dx + \int \frac{3c^2 d \sec^2 (e+fx)}{\sec (e+fx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**3*sec(e +
f*x)**4/(sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(sec(e +
 f*x) + 1), x) + Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/
a
```

Giac [A]
time = 0.52, size = 219, normalized size = 1.87

$$\frac{\frac{3\,(2\,c^2 d-2\,c d^2+d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{3\,(2\,c^2 d-2\,c d^2+d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}+\frac{2\,\left(c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,c^2 d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,c d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a}-\frac{2\,\left(6\,c d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,c d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2 a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*(3*(2*c^2*d - 2*c*d^2 + d^3)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 3*(
2*c^2*d - 2*c*d^2 + d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a + 2*(c^3*tan(
1/2*f*x + 1/2*e) - 3*c^2*d*tan(1/2*f*x + 1/2*e) + 3*c*d^2*tan(1/2*f*x + 1/2
*e) - d^3*tan(1/2*f*x + 1/2*e))/a - 2*(6*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*d
^3*tan(1/2*f*x + 1/2*e)^3 - 6*c*d^2*tan(1/2*f*x + 1/2*e) + d^3*tan(1/2*f*x
+ 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a))/f
```

Mupad [B]
time = 1.94, size = 139, normalized size = 1.19

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,c\,d^2-d^3\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(6\,c\,d^2-3\,d^3\right)}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\right)}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(c-d\right)}^3}{a\,f}+\frac{3\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(6*c*d^2 - d^3) - tan(e/2 + (f*x)/2)^3*(6*c*d^2 - 3*d^3
))/(f*(a - 2*a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4)) + (tan(e/2 +
 (f*x)/2)*(c - d)^3)/(a*f) + (3*d*atanh(tan(e/2 + (f*x)/2))*(2*c^2 - 2*c*d
+ d^2))/(a*f)
```

3.212 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=68

$$\frac{(2c-d)d\tanh^{-1}(\sin(e+fx))}{af}+\frac{d^2\tan(e+fx)}{af}+\frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] (2*c-d)*d*arctanh(sin(f*x+e))/a/f+d^2*tan(f*x+e)/a/f+(c-d)^2*tan(f*x+e)/f/(
a+a*sec(f*x+e))
```

Rubi [A]
time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.84, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 91, 81, 65, 223, 209}

$$\frac{2d(2c-d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)^2\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d^2\tan(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d^2*Tan[e + f*x])/(a*f) + ((c - d)^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])
) + (2*(2*c - d)*d*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
```

```
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}\,(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\tan(e+fx)\mathrm{Subst}\left(\int \frac{a^3(2c-d)d+a^3d^2x}{\sqrt{a-ax}\,\sqrt{a+ax}}\,dx \right.}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(a(2c-d)d\tan(e+fx))\mathrm{S}}{f\sqrt{a-a}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(2(2c-d)d\tan(e+fx))\mathrm{S}}{f\sqrt{a-a}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(2(2c-d)d\tan(e+fx))\mathrm{S}}{f\sqrt{a-a\sec}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{2(2c-d)d\tan^{-1}\left(\frac{\sqrt{a-}}{\sqrt{a+}}\right.}{f\sqrt{a-a\sec(e+fx}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(68) = 136.
time = 1.75, size = 237, normalized size = 3.49

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos(e+fx)(c+d\sec(e+fx))^2\left((c-d)^2\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+d\cos\left(\frac{1}{2}(e+fx)\right)\left(-\left((2c-d)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)+\frac{d\sin(fx)}{\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)\right)}{af(d+c\cos(e+fx))^2(1+\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*Cos[e + f*x]*(c + d*Sec[e + f*x])^2*((c - d)^2*Sec[e/2]
*Sin[(f*x)/2] + d*Cos[(e + f*x)/2]*(-((2*c - d)*(Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + (d*Sin[f*x])/
((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))))/(a*f*(d + c*Cos[e + f*x])
^2*(1 + Sec[e + f*x]))
```

Maple [A]
time = 0.19, size = 127, normalized size = 1.87

method	result
derivativedivides	$\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+d(2c-d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{d^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-d(}{fa}$
default	$\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+d(2c-d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{d^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-d(}{fa}$
norman	$\frac{\frac{\left(c^2-2cd+d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}+\frac{\left(c^2-2cd+3d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{2\left(c^2-2cd+2d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{d(2c-d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}{af}$
risch	$\frac{2i\left(c^2 e^{2i(fx+e)}-2cd\, e^{2i(fx+e)}+d^2 e^{2i(fx+e)}+d^2 e^{i(fx+e)}+c^2-2cd+2d^2\right)}{fa\left(e^{2i(fx+e)}+1\right)\left(e^{i(fx+e)}+1\right)}+\frac{2d\ln\left(e^{i(fx+e)}+i\right)c}{af}-\frac{d^2\ln\left(e^{i(fx+e)}+i\right)}{af}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(c^2*tan(1/2*f*x+1/2*e)-2*c*d*tan(1/2*f*x+1/2*e)+d^2*tan(1/2*f*x+1/2*
e)-d^2/(tan(1/2*f*x+1/2*e)+1)+d*(2*c-d)*ln(tan(1/2*f*x+1/2*e)+1)-d^2/(tan(1
/2*f*x+1/2*e)-1)-d*(2*c-d)*ln(tan(1/2*f*x+1/2*e)-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(72) = 144$.
time = 0.27, size = 241, normalized size = 3.54

$$-\frac{d^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-2cd\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)-\frac{c^2\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c*d*(
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)/(a*(
cos(f*x + e) + 1)))/f
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(72) = 144$.
time = 2.03, size = 165, normalized size = 2.43

$$\frac{\left((2cd-d^2)\cos(fx+e)^2+(2cd-d^2)\cos(fx+e)\right)\log(\sin(fx+e)+1)-\left((2cd-d^2)\cos(fx+e)^2+(2cd-d^2)\cos(fx+e)\right)\log(-\sin(fx+e)+1)+2\left(d^2+(c^2-2cd+2d^2)\cos(fx+e)\right)\sin(fx+e)}{2\left(af\cos(fx+e)^2+af\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/2*(((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*log(sin(f*
x + e) + 1) - ((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*l
og(-sin(f*x + e) + 1) + 2*(d^2 + (c^2 - 2*c*d + 2*d^2)*cos(f*x + e))*sin(f*
x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2 \sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{d^2 \sec^3(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{2cd \sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**2*sec(e +
f*x)**3/(sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e + f*
x) + 1), x))/a
```

Giac [A]
time = 0.54, size = 136, normalized size = 2.00

$$\frac{\frac{(2cd-d^2)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a} - \frac{(2cd-d^2)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a} - \frac{2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a} + \frac{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((2*c*d - d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - (2*c*d - d^2)*log(abs
(tan(1/2*f*x + 1/2*e) - 1))/a - 2*d^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x +
1/2*e)^2 - 1)*a) + (c^2*tan(1/2*f*x + 1/2*e) - 2*c*d*tan(1/2*f*x + 1/2*e) +
 d^2*tan(1/2*f*x + 1/2*e))/a)/f
```

Mupad [B]
time = 1.82, size = 85, normalized size = 1.25

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{(c-d)}^2}{a\,f}+\frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\right)}+\frac{2\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,c-d\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c - d)^2)/(a*f) + (2*d^2*tan(e/2 + (f*x)/2))/(f*(a - a
*tan(e/2 + (f*x)/2)^2)) + (2*d*atanh(tan(e/2 + (f*x)/2))*(2*c - d))/(a*f)
```

3.213 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal. Leaf size=43

$$\frac{d\tanh^{-1}(\sin(e+fx))}{af}+\frac{(c-d)\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] d*arctanh(sin(f*x+e))/a/f+(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4083, 3855, 3879}

$$\frac{(c-d)\tan(e+fx)}{f(a\sec(e+fx)+a)}+\frac{d\tanh^{-1}(\sin(e+fx))}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(a*f) + ((c - d)*Tan[e + f*x])/(f*(a + a*Sec[e +
f*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = (c-d)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx + \frac{d\int \sec(e+fx)\,dx}{a}$$
$$= \frac{d\tanh^{-1}(\sin(e+fx))}{af} + \frac{(c-d)\tan(e+fx)}{f(a+a\sec(e+fx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(43) = 86.
time = 0.28, size = 109, normalized size = 2.53

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(d\cos\left(\frac{1}{2}(e+fx)\right)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+(c-d)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\right)}{af(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
[Out] (2*Cos[(e + f*x)/2]*(d*Cos[(e + f*x)/2]*(-Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Sec[e/2]*Sin[
(f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

Maple [A]
time = 0.17, size = 61, normalized size = 1.42

method	result	size
derivativedivides	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{fa}$	61
default	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{fa}$	61
risch	$\frac{2ic}{fa\left(e^{i(fx+e)}+1\right)}-\frac{2id}{fa\left(e^{i(fx+e)}+1\right)}-\frac{d\ln\left(e^{i(fx+e)}-i\right)}{af}+\frac{d\ln\left(e^{i(fx+e)}+i\right)}{af}$	91
norman	$\frac{\frac{(c-d)\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}}{\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f/a*(c*tan(1/2*f*x+1/2*e)-d*tan(1/2*f*x+1/2*e)-d*ln(tan(1/2*f*x+1/2*e)-1)
+d*ln(tan(1/2*f*x+1/2*e)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.
time = 0.28, size = 107, normalized size = 2.49

$$\frac{d\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] (d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*sin(f*x + e)/(a
*(cos(f*x + e) + 1)))/f
```

Fricas [A]
time = 3.25, size = 80, normalized size = 1.86

$$\frac{(d\cos(fx+e)+d)\log(\sin(fx+e)+1)-(d\cos(fx+e)+d)\log(-\sin(fx+e)+1)+2(c-d)\sin(fx+e)}{2(af\cos(fx+e)+af)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/2*((d*cos(f*x + e) + d)*log(sin(f*x + e) + 1) - (d*cos(f*x + e) + d)*log(
-sin(f*x + e) + 1) + 2*(c - d)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sec(e+fx)}{\sec(e+fx)+1}\,dx+\int \frac{d\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d*sec(e + f*x)**
2/(sec(e + f*x) + 1), x))/a
```

Giac [A]
time = 0.49, size = 70, normalized size = 1.63

$$\frac{\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}+\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - d*log(abs(tan(1/2*f*x + 1/2*e) -
1))/a + (c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/a)/f
```

Mupad [B]
time = 1.73, size = 41, normalized size = 0.95

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c-d\right)}{a\,f}+\frac{2\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
[Out] (tan(e/2 + (f*x)/2)*(c - d))/(a*f) + (2*d*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

3.214 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=83

$$-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a(c-d)^{3/2}\sqrt{c+d}\,f}+\frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))}$$

```
[Out] -2*d*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a/(c-d)^(3/2)/f/(c
+d)^(1/2)+tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.61, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4072, 98, 95, 211}

$$\frac{2d\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{3/2}\sqrt{c+d}\,\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{\tan(e+fx)}{f(c-d)(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/((c - d)*f*(a + a*Sec[e + f*x])) + (2*d*ArcTan[(Sqrt[c + d]*Sq
rt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x
])/((c - d)^(3/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{3/2}(c+dx)}\,dx, x, \sec \ldots\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{(ad\tan(e+fx))\text{Subst}\left(\int \ldots \right)}{(c-d)f\sqrt{a-a\sec \ldots}}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{(2ad\tan(e+fx))\text{Subst}\left(\int \frac{\ldots}{ac- \ldots}\right)}{(c-d)f\sqrt{a-a\sec \ldots}}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{2d\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+a \ldots}}{\sqrt{c-d}\,\sqrt{a-a \ldots}}\right)}{(c-d)^{3/2}\sqrt{c+d}\,f\sqrt{a-a\sec \ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.70, size = 160, normalized size = 1.93

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\frac{2d\text{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos\left(\frac{1}{2}(e+fx)\right)(i\cos(e)+\sin(e))}{\sqrt{c^2-d^2}\,\sqrt{(\cos(e)-i\sin(e))^2}}+\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\right)}{a(c-d)f(1+\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*((2*d*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*C
os[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e
 + f*x)/2]*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2
]) + Sec[e/2]*Sin[(f*x)/2]))/(a*(c - d)*f*(1 + Cos[e + f*x]))
```

Maple [A]
time = 0.21, size = 74, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c-d}-\frac{2d\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)\sqrt{(c+d)(c-d)}}}{fa}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c-d}-\frac{2d\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)\sqrt{(c+d)(c-d)}}}{fa}$
risch	$\frac{2i}{fa(c-d)\left(\mathrm{e}^{i(fx+e)}+1\right)}+\frac{d\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}\,(c-d)fa}-\frac{d\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c-d)fa}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(1/(c-d)*tan(1/2*f*x+1/2*e)-2*d/(c-d)/((c+d)*(c-d))^(1/2)*arctanh((c-
d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.08, size = 367, normalized size = 4.42

$$\left[-\frac{\sqrt{c^2-d^2}\,(d\cos(fx+e)+d)\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}\,(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)-2\,(c^2-d^2)\sin(fx+e)}{2\,((ac^3-ac^2d-acd^2+ad^3)f\cos(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f)},\ -\frac{\sqrt{-c^2+d^2}\,(d\cos(fx+e)+d)\arctan\left(-\frac{\sqrt{-c^2+d^2}\,(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)-(c^2-d^2)\sin(fx+e)}{(ac^3-ac^2d-acd^2+ad^3)f\cos(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*(sqrt(c^2 - d^2)*(d*cos(f*x + e) + d)*log((2*c*d*cos(f*x + e) - (c^2
- 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x +
e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^2
 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) +
 (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f), -(sqrt(-c^2 + d^2)*(d*cos(f*x + e)
 + d)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +
e))) - (c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos
(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c\sec(e+fx)+c+d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x) + c + d*sec(e + f*x)**2 + d*sec(e + f
*x)), x)/a
```

Giac [A]
time = 0.47, size = 110, normalized size = 1.33

$$\frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d}{(ac-ad)\sqrt{-c^2+d^2}}+\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac-ad}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x
 + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d/((a*c - a*d)*sqrt(
-c^2 + d^2)) + tan(1/2*f*x + 1/2*e)/(a*c - a*d))/f
```

Mupad [B]
time = 1.95, size = 110, normalized size = 1.33

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\,(c-d)}-\frac{2\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)c^2-2\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)c\,d+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)d^2}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c+d}\,(c-d)^{3/2}}\right)}{af\,\sqrt{c+d}\,(c-d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)) - (2*d*atanh((c^2*sin(e/2 + (f*x)/2) + d^2
*sin(e/2 + (f*x)/2) - 2*c*d*sin(e/2 + (f*x)/2))/(cos(e/2 + (f*x)/2)*(c + d)
^(1/2)*(c - d)^(3/2))))/(a*f*(c + d)^(1/2)*(c - d)^(3/2))
```

3.215 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=145

$$-\frac{2d(2c+d)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a(c-d)^{5/2}(c+d)^{3/2}f}+\frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))}-\frac{d\tan(e}{(c^2-d^2)f(a+a\sec(e+}$$

```
[Out] -2*d*(2*c+d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a/(c-d)^(5
/2)/(c+d)^(3/2)/f+(c+2*d)*tan(f*x+e)/(c-d)^2/(c+d)/f/(a+a*sec(f*x+e))-d*tan
(f*x+e)/(c^2-d^2)/f/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.17, antiderivative size = 196, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 105, 157, 12, 95, 211}

$$\frac{2d(2c+d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{5/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)}{f(c^2-d^2)(a\sec(e+fx)+a)(c+d\sec(e+fx))}+\frac{(c+2d)\tan(e+fx)}{f(c-d)^2(c+d)(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 2*d)*Tan[e + f*x])/((c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])) + (2*d*
(2*c + d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a
 - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(5/2)*(c + d)^(3/2)*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*Tan[e + f*x])/((c^2 - d^2)*f
*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{3/2}(c+dx)^2}\,dx, x, \text{se}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))} - \frac{\tan(e}{}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\,\text{ta}}{(c^2-d^2)\,f(a+a\,\text{sec}(}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\,\text{ta}}{(c^2-d^2)\,f(a+a\,\text{sec}(}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\,\text{ta}}{(c^2-d^2)\,f(a+a\,\text{sec}(}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} + \frac{2d(2c+d)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{(c-d)^{5/2}(c+d)^{3/2}f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.25, size = 286, normalized size = 1.97

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^3(e+fx)\left(\frac{2d(2c+d)\text{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))(i\cos(e)+\sin(e))}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+(d+c\cos(e+fx))\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+\frac{d^2\cos\left(\frac{1}{2}(e+fx)\right)(-d\sin(e)+c\sin(fx))}{c(c+d)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)}\right)}{a(c-d)^2f(1+\sec(e+fx))(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^3*((2*d*(2*c + d)*Arc
Tan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c
^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x
])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2
]) + (d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (d^2*Cos[(e + f*x)/2]*(-(
d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e
/2]))))/(a*(c - d)^2*f*(1 + Sec[e + f*x])*(c + d*Sec[e + f*x])^2)
```

Maple [A]
time = 0.31, size = 146, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^2-2cd+d^2}+\frac{4d\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{(2c+d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^2}}{fa}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^2-2cd+d^2}+\frac{4d\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{(2c+d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^2}}{fa}$
risch	$\frac{2i\left(c^3e^{2i(fx+e)}+c^2d\,e^{2i(fx+e)}+d^3e^{2i(fx+e)}+2c^2d\,e^{i(fx+e)}+3c\,d^2e^{i(fx+e)}+d^3e^{i(fx+e)}+c^3+c^2d+c\,d^2\right)}{\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)f(c-d)^2ac(c+d)}$ + 2d ln(e^{i(fx- … / √c …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(1/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e)+4*d/(c-d)^2*(-1/2*d/(c+d)*tan(1
/2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)-1/2*(2*c+
d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))
^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(142) = 284$.
time = 2.05, size = 711, normalized size = 4.90

$$\left[\frac{\left(2cd^2+d^3+(2c^2d+cd^2)\cos(fx+e)^2+(2c^2d+3cd^2+d^3)\cos(fx+e)\right)\sqrt{c^2-d^2}\log\left(\frac{2cd\cos(fx+e)-(c^2-2d^2)\cos(fx+e)^2-2\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2c^2-d^2}{c^2\cos(fx+e)^2+2cd\cos(fx+e)+d^2}\right)+2\left(c^3d+2c^2d^2-cd^3-2d^4+(c^4+c^3d-cd^3-d^4)\cos(fx+e)\right)\sin(fx+e)}{2\left((ac^6-ac^5d-2ac^4d^2+2ac^3d^3+ac^2d^4-acd^5)f\cos(fx+e)^2+(ac^6-3ac^4d^2+3ac^2d^4-ad^6)f\cos(fx+e)+(ac^5d-ac^4d^2-2ac^3d^3+2ac^2d^4+acd^5-ad^6)f\right)},-\frac{\left(2cd^2+d^3+(2c^2d+cd^2)\cos(fx+e)^2+(2c^2d+3cd^2+d^3)\cos(fx+e)\right)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)-\left(c^3d+2c^2d^2-cd^3-2d^4+(c^4+c^3d-cd^3-d^4)\cos(fx+e)\right)\sin(fx+e)}{(ac^6-ac^5d-2ac^4d^2+2ac^3d^3+ac^2d^4-acd^5)f\cos(fx+e)^2+(ac^6-3ac^4d^2+3ac^2d^4-ad^6)f\cos(fx+e)+(ac^5d-ac^4d^2-2ac^3d^3+2ac^2d^4+acd^5-ad^6)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d^
2 + d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d
^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(c^3*d +
2*c^2*d^2 - c*d^3 - 2*d^4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f
*x + e))/((a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^
5)*f*cos(f*x + e)^2 + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x
 + e) + (a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)
*f), -((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d
^2 + d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*
x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^
4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f*x + e))/((a*c^6 - a*c^5
*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*x + e)^2 + (a
*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x + e) + (a*c^5*d - a*c^4
*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^2\sec(e+fx)+c^2+2cd\sec^2(e+fx)+2cd\sec(e+fx)+d^2\sec^3(e+fx)+d^2\sec^2(e+fx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**2 + 2
*c*d*sec(e + f*x) + d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a
```

Giac [A]
time = 0.46, size = 221, normalized size = 1.52

$$-\frac{\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{(ac^3-ac^2d-acd^2+ad^3)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}-\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)(2\,cd+d^2)}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{-c^2+d^2}}-\frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac^2-2\,acd+ad^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -(2*d^2*tan(1/2*f*x + 1/2*e)/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*(c*tan(1/
2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) - 2*(pi*floor(1/2*(f*
x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/
```

```
2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*c*d + d^2)/((a*c^3 - a*c^2*d - a*c*d^
2 + a*d^3)*sqrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^2 - 2*a*c*d + a*d^
2))/f
```

Mupad [B]
time = 2.04, size = 187, normalized size = 1.29

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a\,f\,(c-d)^2}-\frac{2\,d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,(c+d)\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,\left(a\,c^3-3\,a\,c^2\,d+3\,a\,c\,d^2-a\,d^3\right)-a\,d^3-a\,c^3+a\,c\,d^2+a\,c^2\,d\right)}-\frac{2\,d\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,c-2\,d)\,\left(a\,c^2-2\,a\,c\,d+a\,d^2\right)}{2\,a\,\sqrt{c+d}\,(c-d)^{5/2}}\right)\,(2\,c+d)}{a\,f\,(c+d)^{3/2}\,(c-d)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)^2) - (2*d^2*tan(e/2 + (f*x)/2))/(f*(c + d)*
(tan(e/2 + (f*x)/2)^2*(a*c^3 - a*d^3 + 3*a*c*d^2 - 3*a*c^2*d) - a*d^3 - a*c
^3 + a*c*d^2 + a*c^2*d)) - (2*d*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(a*c^
2 + a*d^2 - 2*a*c*d))/(2*a*(c + d)^(1/2)*(c - d)^(5/2)))*(2*c + d))/(a*f*(c
 + d)^(3/2)*(c - d)^(5/2))
```

3.216 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=207

$$-\frac{3d(2c^2+2cd+d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan(\frac{1}{2}(e+fx))}{\sqrt{c+d}}\right)}{a(c-d)^{7/2}(c+d)^{5/2}f}+\frac{d(2c+3d)\tan(e+fx)}{2a(c-d)^2(c+d)f(c+d\sec(e+fx))^2}+\frac{}{(c-d)f(a+}$$

```
[Out] -3*d*(2*c^2+2*c*d+d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/
a/(c-d)^(7/2)/(c+d)^(5/2)/f+1/2*d*(2*c+3*d)*tan(f*x+e)/a/(c-d)^2/(c+d)/f/(c
+d*sec(f*x+e))^2+tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2+1/2
*d*(2*c+d)*(c+4*d)*tan(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.28, antiderivative size = 268, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 105, 156, 157, 12, 95, 211}

$$\frac{3d(2c^2+2cd+d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{7/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{d(4c+d)\tan(e+fx)}{2f\,(c^2-d^2)^2\,(a\sec(e+fx)+a)(c+d\sec(e+fx))}-\frac{d\tan(e+fx)}{2f\,(c^2-d^2)\,(a\sec(e+fx)+a)(c+d\sec(e+fx))^2}+\frac{(2c+d)(c+4d)\tan(e+fx)}{2f(c-d)^3(c+d)^2(a\sec(e+fx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c + d)*(c + 4*d)*Tan[e + f*x])/(2*(c - d)^3*(c + d)^2*f*(a + a*Sec[e +
f*x])) + (3*d*(2*c^2 + 2*c*d + d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(7/2)
*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*Ta
n[e + f*x])/(2*(c^2 - d^2)*f*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2) -
 (d*(4*c + d)*Tan[e + f*x])/(2*(c^2 - d^2)^2*f*(a + a*Sec[e + f*x])*(c + d*
Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{3/2}(c+dx)^3}\,dx, x, \mathrm{se}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{\tan}{}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{}{2\,(c}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{}{2\left(c^2-d^2\right)f(a+a}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{}{2\left(c^2-d^2\right)f(a+a}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} - \frac{}{2\left(c^2-d^2\right)f(a+a}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} + \frac{3d(2c^2+2cd+d^2)}{(c-d)^{7/2}(c+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.84, size = 1422, normalized size = 6.87

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 2*c*d + d^2)*Cos[e/2 + (f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e +
f*x]^4*(((-6*I)*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e
] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]])
)*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*
Sqrt[Cos[2*e] - I*Sin[2*e]]) - (6*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 -
 d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2
*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(
Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^3*(c + d)^2*(a +
```

```
 a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d + c*Cos[e
 + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^4*(8*c^5*d*Sin[(f*x)/2] + 10*c^4*d^2*
Sin[(f*x)/2] - 11*c^3*d^3*Sin[(f*x)/2] - 17*c^2*d^4*Sin[(f*x)/2] - 2*c*d^5*
Sin[(f*x)/2] + 2*d^6*Sin[(f*x)/2] - 8*c^5*d*Sin[(3*f*x)/2] - 22*c^4*d^2*Sin
[(3*f*x)/2] - 27*c^3*d^3*Sin[(3*f*x)/2] - 5*c^2*d^4*Sin[(3*f*x)/2] + 2*c*d^
5*Sin[(3*f*x)/2] + 4*c^6*Sin[e - (f*x)/2] + 8*c^5*d*Sin[e - (f*x)/2] + 18*c
^4*d^2*Sin[e - (f*x)/2] + 35*c^3*d^3*Sin[e - (f*x)/2] + 25*c^2*d^4*Sin[e -
(f*x)/2] + 2*c*d^5*Sin[e - (f*x)/2] - 2*d^6*Sin[e - (f*x)/2] - 4*c^6*Sin[e
+ (f*x)/2] - 8*c^5*d*Sin[e + (f*x)/2] - 6*c^4*d^2*Sin[e + (f*x)/2] - 7*c^3*
d^3*Sin[e + (f*x)/2] + 5*c^2*d^4*Sin[e + (f*x)/2] + 2*c*d^5*Sin[e + (f*x)/2
] - 2*d^6*Sin[e + (f*x)/2] + 8*c^5*d*Sin[2*e + (f*x)/2] + 22*c^4*d^2*Sin[2*
e + (f*x)/2] + 17*c^3*d^3*Sin[2*e + (f*x)/2] + 13*c^2*d^4*Sin[2*e + (f*x)/2
] + 2*c*d^5*Sin[2*e + (f*x)/2] - 2*d^6*Sin[2*e + (f*x)/2] + 2*c^6*Sin[e + (
3*f*x)/2] + 4*c^5*d*Sin[e + (3*f*x)/2] - 4*c^4*d^2*Sin[e + (3*f*x)/2] - 19*
c^3*d^3*Sin[e + (3*f*x)/2] - 5*c^2*d^4*Sin[e + (3*f*x)/2] + 2*c*d^5*Sin[e +
 (3*f*x)/2] - 8*c^5*d*Sin[2*e + (3*f*x)/2] - 16*c^4*d^2*Sin[2*e + (3*f*x)/2
] - c^3*d^3*Sin[2*e + (3*f*x)/2] + 2*c^2*d^4*Sin[2*e + (3*f*x)/2] - 2*c*d^5
*Sin[2*e + (3*f*x)/2] + 2*c^6*Sin[3*e + (3*f*x)/2] + 4*c^5*d*Sin[3*e + (3*f
*x)/2] + 2*c^4*d^2*Sin[3*e + (3*f*x)/2] + 7*c^3*d^3*Sin[3*e + (3*f*x)/2] +
2*c^2*d^4*Sin[3*e + (3*f*x)/2] - 2*c*d^5*Sin[3*e + (3*f*x)/2] - 2*c^6*Sin[e
 + (5*f*x)/2] - 4*c^5*d*Sin[e + (5*f*x)/2] - 8*c^4*d^2*Sin[e + (5*f*x)/2] -
 2*c^3*d^3*Sin[e + (5*f*x)/2] + c^2*d^4*Sin[e + (5*f*x)/2] - 6*c^4*d^2*Sin[
2*e + (5*f*x)/2] - 2*c^3*d^3*Sin[2*e + (5*f*x)/2] + c^2*d^4*Sin[2*e + (5*f*
x)/2] - 2*c^6*Sin[3*e + (5*f*x)/2] - 4*c^5*d*Sin[3*e + (5*f*x)/2] - 2*c^4*d
^2*Sin[3*e + (5*f*x)/2]))/(8*c^2*(-c + d)^3*(c + d)^2*f*(a + a*Sec[e + f*x]
)*(c + d*Sec[e + f*x])^3)
```

Maple [A]
time = 0.42, size = 221, normalized size = 1.07

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^3-3c^2d+3c\,d^2-d^3}+\frac{2d\left(\frac{-\frac{3d\left(2c^2-cd-d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)}+\frac{d(6c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c+2d}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}-\frac{3\left(2c^2+2cd+d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{f}{}\right.}{\sqrt{(c+d)\,(}}\right.}{2\left(c^2+2cd+d^2\right)\sqrt{(c+d)\,(c-}}\right.}{(c-d)^3}}{fa}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^3-3c^2d+3c\,d^2-d^3}+\frac{2d\left(\frac{-\frac{3d\left(2c^2-cd-d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2\left(c^2+2cd+d^2\right)}+\frac{d(6c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c+2d}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}-\frac{3\left(2c^2+2cd+d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{f}{}\right.}{\sqrt{(c+d)\,(}}\right.}{2\left(c^2+2cd+d^2\right)\sqrt{(c+d)\,(c-}}\right.}{(c-d)^3}}{fa}$

risch	$\frac{i\left(-4c^5d-2c^3d^3+c^2d^4-2c\,d^5\mathrm{e}^{3i(fx+e)}-8c^5d\,\mathrm{e}^{2i(fx+e)}-18c^4d^2\mathrm{e}^{2i(fx+e)}-35c^3d^3\mathrm{e}^{2i(fx+e)}-4c^5d\,\mathrm{e}^{4i(fx+e)}-2c^4d^2\mathrm{e}^{4i(fx\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(1/(c^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e)+2*d/(c-d)^3*((-3/2*d*
(2*c^2-c*d-d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(6*c+d)/(c+d)*ta
n(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)^2-3/2
*(2*c^2+2*c*d+d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/
2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(202) = 404$.
time = 2.47, size = 1357, normalized size = 6.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*(3*(2*c^2*d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f
*x + e)^3 + (2*c^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (4
*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*
c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(
f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f
*x + e) + d^2)) - 2*(2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 +
(2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3 - 9*c^2*d^4 - 2*c*d^5 + d^6)*cos(f
*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6
```

```
)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a*c^8*d - 3*a*c^7*d^2 + 3*a*c^6*d^3
 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6 + a*c^2*d^7)*f*cos(f*x + e)^3 + (a
*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^3 + 9*a*c^5*d^4 + 3*a*c^4*d^5 - 7*
a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(f*x + e)^2 + (2*a*c^8*d - a*c^7*d^
2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a
*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a
*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f), -1/2*(3*(2*c^2*
d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f*x + e)^3 + (2*c
^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (4*c^3*d^2 + 6*c^2
*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^
2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^4*d^2 + 9*c^3*d^
3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 + (2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3
- 9*c^2*d^4 - 2*c*d^5 + d^6)*cos(f*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3
*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a
*c^8*d - 3*a*c^7*d^2 + 3*a*c^6*d^3 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6
+ a*c^2*d^7)*f*cos(f*x + e)^3 + (a*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^
3 + 9*a*c^5*d^4 + 3*a*c^4*d^5 - 7*a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(
f*x + e)^2 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d
^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d
^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*
c*d^8 + a*d^9)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^3\sec(e+fx)+c^3+3c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^3(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^4(e+fx)+d^3\sec^3(e+fx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**2
+ 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e + f*x)*
*2 + d**3*sec(e + f*x)**4 + d**3*sec(e + f*x)**3), x)/a
```

Giac [A]
time = 0.55, size = 362, normalized size = 1.75

$$-\frac{\frac{3\left(2c^2d+2cd^2+d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5\right)\sqrt{-c^2+d^2}}-\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^3-3ac^2d+3acd^2-ad^3}+\frac{6c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-7cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(ac^5-ac^4d-2ac^3d^2+2ac^2d^3+acd^4-ad^5\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] -(3*(2*c^2*d + 2*c*d^2 + d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
```

```
+ d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*s
qrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*
d^3) + (6*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*c*d^3*tan(1/2*f*x + 1/2*e)^3 -
 3*d^4*tan(1/2*f*x + 1/2*e)^3 - 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 7*c*d^3*ta
n(1/2*f*x + 1/2*e) - d^4*tan(1/2*f*x + 1/2*e))/((a*c^5 - a*c^4*d - 2*a*c^3*
d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*
f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B]
time = 2.97, size = 379, normalized size = 1.83

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,{\left(c-d\right)}^3}-\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(d^3+6\,c\,d^2\right)}{c+d}+\frac{3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(-2\,c^2\,d^2+c\,d^3+d^4\right)}{{\left(c+d\right)}^2}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,a\,c^5-6\,a\,c^4\,d+4\,a\,c^3\,d^2+4\,a\,c^2\,d^3-6\,a\,c\,d^4+2\,a\,d^5\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(a\,c^5-5\,a\,c^4\,d+10\,a\,c^3\,d^2-10\,a\,c^2\,d^3+5\,a\,c\,d^4-a\,d^5\right)-a\,c^5+a\,d^5-2\,a\,c^2\,d^3+2\,a\,c^3\,d^2-a\,c\,d^4+a\,c^4\,d\right)}+\frac{d\,\mathrm{atan}\left(\frac{1{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^4-4{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,d+6{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d^2-4{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^3+1{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^4}{\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)\,3{}\mathrm{i}}{a\,f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)^3) - ((tan(e/2 + (f*x)/2)*(6*c*d^2 + d^3))/
(c + d) + (3*tan(e/2 + (f*x)/2)^3*(c*d^3 + d^4 - 2*c^2*d^2))/(c + d)^2)/(f*
(tan(e/2 + (f*x)/2)^2*(2*a*c^5 + 2*a*d^5 + 4*a*c^2*d^3 + 4*a*c^3*d^2 - 6*a*
c*d^4 - 6*a*c^4*d) - tan(e/2 + (f*x)/2)^4*(a*c^5 - a*d^5 - 10*a*c^2*d^3 + 1
0*a*c^3*d^2 + 5*a*c*d^4 - 5*a*c^4*d) - a*c^5 + a*d^5 - 2*a*c^2*d^3 + 2*a*c^
3*d^2 - a*c*d^4 + a*c^4*d)) + (d*atan((c^4*tan(e/2 + (f*x)/2)*1i + d^4*tan(
e/2 + (f*x)/2)*1i - c*d^3*tan(e/2 + (f*x)/2)*4i - c^3*d*tan(e/2 + (f*x)/2)*
4i + c^2*d^2*tan(e/2 + (f*x)/2)*6i)/((c + d)^(1/2)*(c - d)^(7/2)))*(2*c*d +
 2*c^2 + d^2)*3i)/(a*f*(c + d)^(5/2)*(c - d)^(7/2))
```

3.217 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=258

$$\frac{5(2c-d)d^2(2c^2-3cd+2d^2)\tanh^{-1}(\sin(e+fx))}{2a^2f}-\frac{d(c^2+10cd-12d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f}+$$

```
[Out] 5/2*(2*c-d)*d^2*(2*c^2-3*c*d+2*d^2)*arctanh(sin(f*x+e))/a^2/f-1/3*d*(c^2+10
*c*d-12*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/a^2/f+1/3*(c-d)*(c+10*d)*(c+d*se
c(f*x+e))^3*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+1/3*(c-d)*(c+d*sec(f*x+e))^4*
tan(f*x+e)/f/(a+a*sec(f*x+e))^2-1/6*d*(4*c^4+40*c^3*d-176*c^2*d^2+160*c*d^3
-48*d^4+d*(2*c^3+20*c^2*d-57*c*d^2+30*d^3)*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Rubi [A]
time = 0.29, antiderivative size = 315, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4072, 100, 155, 158, 152, 65, 223, 209}

$$-\frac{d(c^2+10cd-12d^2)\tan(e+fx)(c+d\sec(e+fx))^2}{3a^2f}-\frac{d\tan(e+fx)\left(d(2c^3+20c^2d-57cd^2+30d^3)\sec(e+fx)+4(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)\right)}{6a^2f}+\frac{(c-d)(c+10d)\tan(e+fx)(c+d\sec(e+fx))^3}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{5d^2(2c-d)\left(2c^2-3cd+2d^2\right)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[
a + a*Sec[e + f*x]]) - (d*(c^2 + 10*c*d - 12*d^2)*(c + d*Sec[e + f*x])^2*Ta
n[e + f*x])/(3*a^2*f) + ((c - d)*(c + 10*d)*(c + d*Sec[e + f*x])^3*Tan[e +
f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^4*Tan[
e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (d*(4*(c^4 + 10*c^3*d - 44*c^2*d^2
 + 40*c*d^3 - 12*d^4) + d*(2*c^3 + 20*c^2*d - 57*c*d^2 + 30*d^3)*Sec[e + f*
x])*Tan[e + f*x])/(6*a^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^5}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c \cdots}{\cdots}\right.}{3af\sqrt{a-\cdots}}$$

$$= \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\,\text{s}\cdots}{3f(a\cdots}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(\cdots}{\cdots}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(\cdots}{\cdots}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(\cdots}{\cdots}$$

$$= -\frac{d\left(c^2+10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(\cdots}{\cdots}$$

$$= \frac{5(2c-d)d^2\left(2c^2-3cd+2d^2\right)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e\cdots}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 4.18, size = 446, normalized size = 1.73

$$\frac{240d^2(-4c^3+8c^2d-7cd^2+2d^3)\cos^4\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+2\cos\left(\frac{1}{2}(e+fx)\right)\left(6c^5+15c^4d-120c^3d^2+420c^2d^3-300cd^4+104d^5+(6c^5+60c^4d-300c^3d^2+840c^2d^3-585cd^4+190d^5)\cos(e+fx)+4(2c^5+5c^4d-40c^3d^2+130c^2d^3-95cd^4+30d^5)\cos(2(e+fx))+2c^5\cos(3(e+fx))+20c^4d\cos(3(e+fx))-100c^3d^2\cos(3(e+fx))+280c^2d^3\cos(3(e+fx))-215cd^4\cos(3(e+fx))+66d^5\cos(3(e+fx))+2c^5\cos(4(e+fx))+5c^4d\cos(4(e+fx))-40c^3d^2\cos(4(e+fx))+100c^2d^3\cos(4(e+fx))-80cd^4\cos(4(e+fx))+24d^5\cos(4(e+fx))\right)\sec^3(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)}{24a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (240*d^2*(-4*c^3 + 8*c^2*d - 7*c*d^2 + 2*d^3)*Cos[(e + f*x)/2]^4*(Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])
 + 2*Cos[(e + f*x)/2]*(6*c^5 + 15*c^4*d - 120*c^3*d^2 + 420*c^2*d^3 - 300*c
*d^4 + 104*d^5 + (6*c^5 + 60*c^4*d - 300*c^3*d^2 + 840*c^2*d^3 - 585*c*d^4
+ 190*d^5)*Cos[e + f*x] + 4*(2*c^5 + 5*c^4*d - 40*c^3*d^2 + 130*c^2*d^3 - 9
5*c*d^4 + 30*d^5)*Cos[2*(e + f*x)] + 2*c^5*Cos[3*(e + f*x)] + 20*c^4*d*Cos[
3*(e + f*x)] - 100*c^3*d^2*Cos[3*(e + f*x)] + 280*c^2*d^3*Cos[3*(e + f*x)]
- 215*c*d^4*Cos[3*(e + f*x)] + 66*d^5*Cos[3*(e + f*x)] + 2*c^5*Cos[4*(e + f
*x)] + 5*c^4*d*Cos[4*(e + f*x)] - 40*c^3*d^2*Cos[4*(e + f*x)] + 100*c^2*d^3
*Cos[4*(e + f*x)] - 80*c*d^4*Cos[4*(e + f*x)] + 24*d^5*Cos[4*(e + f*x)])*Se
c[e + f*x]^3*Sin[(e + f*x)/2])/(24*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A]
time = 0.28, size = 436, normalized size = 1.69 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f/a^2*(-1/3*c^5*tan(1/2*f*x+1/2*e)^3+5/3*c^4*d*tan(1/2*f*x+1/2*e)^3-10/
3*c^3*d^2*tan(1/2*f*x+1/2*e)^3+10/3*c^2*d^3*tan(1/2*f*x+1/2*e)^3-5/3*c*d^4*
tan(1/2*f*x+1/2*e)^3+1/3*d^5*tan(1/2*f*x+1/2*e)^3+c^5*tan(1/2*f*x+1/2*e)+5*
c^4*d*tan(1/2*f*x+1/2*e)-30*c^3*d^2*tan(1/2*f*x+1/2*e)+50*c^2*d^3*tan(1/2*f
*x+1/2*e)-35*c*d^4*tan(1/2*f*x+1/2*e)+9*d^5*tan(1/2*f*x+1/2*e)-2/3*d^5/(tan
(1/2*f*x+1/2*e)+1)^3+5*d^2*(4*c^3-8*c^2*d+7*c*d^2-2*d^3)*ln(tan(1/2*f*x+1/2
*e)+1)-5*d^3*(4*c^2-5*c*d+2*d^2)/(tan(1/2*f*x+1/2*e)+1)-d^4*(5*c-3*d)/(tan(
1/2*f*x+1/2*e)+1)^2-2/3*d^5/(tan(1/2*f*x+1/2*e)-1)^3-5*d^2*(4*c^3-8*c^2*d+7
*c*d^2-2*d^3)*ln(tan(1/2*f*x+1/2*e)-1)-5*d^3*(4*c^2-5*c*d+2*d^2)/(tan(1/2*f
*x+1/2*e)-1)+d^4*(5*c-3*d)/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(258) = 516.
time = 0.29, size = 836, normalized size = 3.24

$$\frac{d^5\left(\frac{4\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^2-\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{a^2\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}+\frac{\frac{27\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-5cd^4\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+10c^2d^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)-10c^3d^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{5c^4d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{c^5\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(d^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x + e) +
 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(cos(f
*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) -
5*c*d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*
x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1)
 + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10*c^2*d^3*(
(15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/
a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 10*c^3*d^2*((9*sin(f*x + e)/(co
s(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x
+ e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) -
1)/a^2) + 5*c^4*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(
f*x + e) + 1)^3)/a^2 + c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [**A**]
time = 1.62, size = 472, normalized size = 1.83

$$\frac{15\left(\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\cos\left(fx+e\right)^5+2\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\cos\left(fx+e\right)^4+\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\cos\left(fx+e\right)^5+2\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\cos\left(fx+e\right)^4+\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\cos\left(fx+e\right)^3\right)\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2d^5+2\left(2c^5+5c^4d-40c^3d^2+100c^2d^3-80cd^4+24d^5\right)\cos\left(fx+e\right)^4+\left(2c^5+20c^4d-100c^3d^2+280c^2d^3-215cd^4+66d^5\right)\cos\left(fx+e\right)^3+6\left(10c^2d^3-5cd^4+2d^5\right)\cos\left(fx+e\right)^2+\left(15cd^4-2d^5\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{12\left(a^2f\cos\left(fx+e\right)^5+2a^2f\cos\left(fx+e\right)^4+a^2f\cos\left(fx+e\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(15*((4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c
^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d
^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((4*c^3*d^
2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c^3*d^2 - 8*c^2*d^3
+ 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^
5)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*d^5 + 2*(2*c^5 + 5*c^4*d -
 40*c^3*d^2 + 100*c^2*d^3 - 80*c*d^4 + 24*d^5)*cos(f*x + e)^4 + (2*c^5 + 20
*c^4*d - 100*c^3*d^2 + 280*c^2*d^3 - 215*c*d^4 + 66*d^5)*cos(f*x + e)^3 + 6
*(10*c^2*d^3 - 5*c*d^4 + 2*d^5)*cos(f*x + e)^2 + (15*c*d^4 - 2*d^5)*cos(f*x
 + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f
*cos(f*x + e)^3)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^5 \sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^5 \sec^6(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{5cd^4 \sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{10c^2d^3 \sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{10c^3d^2 \sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{5c^4d \sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**5*sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(5*c*d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(10*c**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) +
 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*se
c(e + f*x) + 1), x))/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(247) = 494.
time = 0.59, size = 506, normalized size = 1.96

$$\frac{\frac{15\,(4c^3d^2-8c^2d^3+7cd^4-2d^5)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} - \frac{15\,(4c^3d^2-8c^2d^3+7cd^4-2d^5)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} - \frac{2\left(60c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-75cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+30d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-120c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+120cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-40d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+60c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-45cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+18d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a^2} - \frac{a^4c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5a^4c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+10a^4c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-10a^4c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+5a^4cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-a^4d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^4c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-15a^4c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+90a^4c^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-150a^4c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+105a^4cd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-27a^4d^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*log(abs(tan(1/2*f*x + 1/2
*e) + 1))/a^2 - 15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a^2 - 2*(60*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 75*c*d^4*
tan(1/2*f*x + 1/2*e)^5 + 30*d^5*tan(1/2*f*x + 1/2*e)^5 - 120*c^2*d^3*tan(1/
2*f*x + 1/2*e)^3 + 120*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 40*d^5*tan(1/2*f*x +
1/2*e)^3 + 60*c^2*d^3*tan(1/2*f*x + 1/2*e) - 45*c*d^4*tan(1/2*f*x + 1/2*e)
+ 18*d^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a^2) - (a^4*
c^5*tan(1/2*f*x + 1/2*e)^3 - 5*a^4*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 10*a^4*c^
3*d^2*tan(1/2*f*x + 1/2*e)^3 - 10*a^4*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 5*a^
4*c*d^4*tan(1/2*f*x + 1/2*e)^3 - a^4*d^5*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^5
*tan(1/2*f*x + 1/2*e) - 15*a^4*c^4*d*tan(1/2*f*x + 1/2*e) + 90*a^4*c^3*d^2*
tan(1/2*f*x + 1/2*e) - 150*a^4*c^2*d^3*tan(1/2*f*x + 1/2*e) + 105*a^4*c*d^4
*tan(1/2*f*x + 1/2*e) - 27*a^4*d^5*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B]
time = 1.97, size = 268, normalized size = 1.04

$$\frac{5\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,c-d\right)\,\left(2\,c^2-3\,c\,d+2\,d^2\right)}{a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{2\,{\left(c-d\right)}^5}{a^2}-\frac{5\,\left(c+d\right)\,{\left(c-d\right)}^4}{2\,a^2}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,{\left(c-d\right)}^5}{6\,a^2\,f}-\frac{\left(20\,c^2\,d^3-25\,c\,d^4+10\,d^5\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(-40\,c^2\,d^3+40\,c\,d^4-\frac{40\,d^5}{3}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(20\,c^2\,d^3-15\,c\,d^4+6\,d^5\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)

[Out] (5*d^2*atanh(tan(e/2 + (f*x)/2))*(2*c - d)*(2*c^2 - 3*c*d + 2*d^2))/(a^2*f)
 - (tan(e/2 + (f*x)/2)*((2*(c - d)^5)/a^2 - (5*(c + d)*(c - d)^4)/(2*a^2)))
/f - (tan(e/2 + (f*x)/2)^3*(c - d)^5)/(6*a^2*f) - (tan(e/2 + (f*x)/2)*(6*d^
5 - 15*c*d^4 + 20*c^2*d^3) + tan(e/2 + (f*x)/2)^5*(10*d^5 - 25*c*d^4 + 20*c
^2*d^3) - tan(e/2 + (f*x)/2)^3*((40*d^5)/3 - 40*c*d^4 + 40*c^2*d^3))/(f*(3*
a^2*tan(e/2 + (f*x)/2)^2 - 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)
/2)^6 - a^2))
```

3.218 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=193

$$\frac{d^2(12c^2-16cd+7d^2)\tanh^{-1}(\sin(e+fx))}{2a^2f}+\frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}+\frac{(c-d)(c+d}{3f(a}$$

```
[Out] 1/2*d^2*(12*c^2-16*c*d+7*d^2)*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)*(c+8*d)*(
c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+1/3*(c-d)*(c+d*sec(f*x+
e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-1/6*d*(4*c^3+32*c^2*d-80*c*d^2+32*d^3
+d*(2*c^2+16*c*d-21*d^2)*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Rubi [A]
time = 0.21, antiderivative size = 249, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 100, 155, 152, 65, 223, 209}

$$-\frac{d\tan(e+fx)\left(d(2c^2+16cd-21d^2)\sec(e+fx)+4(c^3+8c^2d-20cd^2+8d^3)\right)}{6a^2f}+\frac{(c-d)(c+8d)\tan(e+fx)(c+d\sec(e+fx))^2}{3f\,(a^2\sec(e+fx)+a^2)}+\frac{d^2(12c^2-16cd+7d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 +
Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]]) + ((c - d)*(c + 8*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(
a^2 + a^2*Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(3
*f*(a + a*Sec[e + f*x])^2) - (d*(4*(c^3 + 8*c^2*d - 20*c*d^2 + 8*d^3) + d*(
2*c^2 + 16*c*d - 21*d^2)*Sec[e + f*x])*Tan[e + f*x])/(6*a^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
```

```
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+}{}\right.}{3af\sqrt{a-}}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec}{3f(a+a}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec}{3f(a+a}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec}{3f(a+a}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec}{3f(a+a}$$

$$= \frac{d^2(12c^2-16cd+7d^2)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} +$$

Mathematica [A]
time = 2.88, size = 310, normalized size = 1.61

$$\frac{-24d^2(12c^2-16cd+7d^2)\cos^4\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+2\cos\left(\frac{1}{2}(e+fx)\right)\left(2c^4+16c^3d-60c^2d^2+112cd^3-37d^4+6(c^4+2c^3d-12c^2d^2+28cd^3-10d^4)\cos(e+fx)+(2c^4+16c^3d-60c^2d^2+112cd^3-43d^4)\cos(2(e+fx))+2c^4\cos(3(e+fx))+4c^3d\cos(3(e+fx))-24c^2d^2\cos(3(e+fx))+40cd^3\cos(3(e+fx))-16d^4\cos(3(e+fx))\right)\sec^2(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)}{12a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-24*d^2*(12*c^2 - 16*c*d + 7*d^2)*Cos[(e + f*x)/2]^4*(Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[(e
 + f*x)/2]*(2*c^4 + 16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 37*d^4 + 6*(c^4 + 2
*c^3*d - 12*c^2*d^2 + 28*c*d^3 - 10*d^4)*Cos[e + f*x] + (2*c^4 + 16*c^3*d -
```

```
 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*Cos[2*(e + f*x)] + 2*c^4*Cos[3*(e + f*x)]
 + 4*c^3*d*Cos[3*(e + f*x)] - 24*c^2*d^2*Cos[3*(e + f*x)] + 40*c*d^3*Cos[3*
(e + f*x)] - 16*d^4*Cos[3*(e + f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(12*
a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A]
time = 0.24, size = 317, normalized size = 1.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f/a^2*(-1/3*c^4*tan(1/2*f*x+1/2*e)^3+4/3*c^3*d*tan(1/2*f*x+1/2*e)^3-2*c
^2*d^2*tan(1/2*f*x+1/2*e)^3+4/3*c*d^3*tan(1/2*f*x+1/2*e)^3-1/3*d^4*tan(1/2*
f*x+1/2*e)^3+c^4*tan(1/2*f*x+1/2*e)+4*c^3*d*tan(1/2*f*x+1/2*e)-18*c^2*d^2*t
an(1/2*f*x+1/2*e)+20*c*d^3*tan(1/2*f*x+1/2*e)-7*d^4*tan(1/2*f*x+1/2*e)+d^2*
(12*c^2-16*c*d+7*d^2)*ln(tan(1/2*f*x+1/2*e)+1)-d^3*(8*c-5*d)/(tan(1/2*f*x+1
/2*e)+1)-d^4/(tan(1/2*f*x+1/2*e)+1)^2-d^3*(8*c-5*d)/(tan(1/2*f*x+1/2*e)-1)+
d^4/(tan(1/2*f*x+1/2*e)-1)^2-d^2*(12*c^2-16*c*d+7*d^2)*ln(tan(1/2*f*x+1/2*e
)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(193) = 386.
time = 0.29, size = 580, normalized size = 3.01

$$-\frac{d^4\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-4cd^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+6c^2d^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-\frac{4c^3d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c^4\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 4*c*d^3*((
15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a
^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^2*d^2*((9*sin(f*x + e)/(cos(
f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x +
e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
/a^2) - 4*c^3*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 2.32, size = 376, normalized size = 1.95

$$\frac{3\left(\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^4+2\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^3+\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^4+2\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^3+\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\cos\left(fx+e\right)^2\right)\log\left(-\sin\left(fx+e\right)+1\right)+2\left(3\,d^4+4\left(c^4+2\,c^3d-12\,c^2d^2+20\,cd^3-8\,d^4\right)\cos\left(fx+e\right)^3+\left(2\,c^4+16\,c^3d-60\,c^2d^2+112\,cd^3-43\,d^4\right)\cos\left(fx+e\right)^2+6\left(4\,cd^3-d^4\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{12\left(a^2f\cos\left(fx+e\right)^4+2\,a^2f\cos\left(fx+e\right)^3+a^2f\cos\left(fx+e\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^4 + 2*(12*c^2*d^2 - 1
6*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x +
 e)^2)*log(sin(f*x + e) + 1) - 3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x +
 e)^4 + 2*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16
*c*d^3 + 7*d^4)*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(3*d^4 + 4*(c^4
+ 2*c^3*d - 12*c^2*d^2 + 20*c*d^3 - 8*d^4)*cos(f*x + e)^3 + (2*c^4 + 16*c^3
*d - 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*cos(f*x + e)^2 + 6*(4*c*d^3 - d^4)*co
s(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 +
a^2*f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^4\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d^4\sec^5\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{4cd^3\sec^4\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{6c^2d^2\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{4c^3d\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(4*c*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) +
1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1), x))/a**2
```

Giac [A]
time = 0.56, size = 359, normalized size = 1.86

$$\frac{\frac{3\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{3\left(12\,c^2d^2-16\,cd^3+7\,d^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}-\frac{6\left(8\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-5\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-8\,cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^2}-\frac{a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,a^4c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,a^4c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,a^4cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^4d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,a^4c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+54\,a^4c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-60\,a^4cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+21\,a^4d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a
^2 - 3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a
```

```
^2 - 6*(8*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 5*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*c
*d^3*tan(1/2*f*x + 1/2*e) + 3*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2
*e)^2 - 1)^2*a^2) - (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(1/2*f
*x + 1/2*e)^3 + 6*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*tan(1/2*
f*x + 1/2*e)^3 + a^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*f*x + 1
/2*e) - 12*a^4*c^3*d*tan(1/2*f*x + 1/2*e) + 54*a^4*c^2*d^2*tan(1/2*f*x + 1/
2*e) - 60*a^4*c*d^3*tan(1/2*f*x + 1/2*e) + 21*a^4*d^4*tan(1/2*f*x + 1/2*e))
/a^6)/f
```

Mupad [B]
time = 1.91, size = 193, normalized size = 1.00

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c\,d^3-3\,d^4\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(8\,c\,d^3-5\,d^4\right)}{f\,\left(a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,{\left(c-d\right)}^4}{2\,a^2}-\frac{2\,\left(c+d\right)\,{\left(c-d\right)}^3}{a^2}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,{\left(c-d\right)}^4}{6\,a^2\,f}+\frac{d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(12\,c^2-16\,c\,d+7\,d^2\right)}{a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(8*c*d^3 - 3*d^4) - tan(e/2 + (f*x)/2)^3*(8*c*d^3 - 5*d
^4))/(f*(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) - (t
an(e/2 + (f*x)/2)*((3*(c - d)^4)/(2*a^2) - (2*(c + d)*(c - d)^3)/a^2))/f -
(tan(e/2 + (f*x)/2)^3*(c - d)^4)/(6*a^2*f) + (d^2*atanh(tan(e/2 + (f*x)/2))
*(12*c^2 - 16*c*d + 7*d^2))/(a^2*f)
```

3.219 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{(3c-2d)d^2\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c^3+4c^2d-12cd^2+10d^3-(c-}{3f\left(a^2+a^2\text{s}\right.}$$

```
[Out] (3*c-2*d)*d^2*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)*(c+d*sec(f*x+e))^2*tan(f*
x+e)/f/(a+a*sec(f*x+e))^2+1/3*(c^3+4*c^2*d-12*c*d^2+10*d^3-(c-4*d)*d^2*sec(
f*x+e))*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A]
time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 100, 148, 65, 223, 209}

$$\frac{\tan(e+fx)\left(c^3+4c^2d-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2d^2(3c-2d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(3*c - 2*d)*d^2*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) +
 ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
 + ((c^3 + 4*c^2*d - 12*c*d^2 + 10*d^3 - (c - 4*d)*d^2*Sec[e + f*x])*Tan[e
+ f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+}{}\right.}{3af\sqrt{a-a}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^3+4c^2d-12cd^2+10a}{3f\,(a}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^3+4c^2d-12cd^2+10a}{3f\,(a}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^3+4c^2d-12cd^2+10a}{3f\,(a}$$

$$= \frac{2(3c-2d)d^2\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 294 vs. 2(133) = 266.
time = 1.64, size = 294, normalized size = 2.21

$$\frac{2\cos^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(6d^2(-3c+2d)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-8(c-d)^3\csc^3(e+fx)\sin^4\left(\frac{1}{2}(e+fx)\right)+32(c-d)^3\csc^5(e+fx)\sin^8\left(\frac{1}{2}(e+fx)\right)+2(2c^3+3c^2d-12cd^2+13d^3)\tan\left(\frac{1}{2}(e+fx)\right)+6(3c-2d)d^2\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)-2(c-d)^2(2c+7d)\tan^3\left(\frac{1}{2}(e+fx)\right)\right)}{3a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]^6*Sec[e + f*x]*(6*d^2*(-3*c + 2*d)*(Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 8*(c -
d)^3*Csc[e + f*x]^3*Sin[(e + f*x)/2]^4 + 32*(c - d)^3*Csc[e + f*x]^5*Sin[(e
 + f*x)/2]^8 + 2*(2*c^3 + 3*c^2*d - 12*c*d^2 + 13*d^3)*Tan[(e + f*x)/2] + 6
*(3*c - 2*d)*d^2*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]])*Tan[(e + f*x)/2]^2 - 2*(c - d)^2*(2*c + 7*d)*Ta
n[(e + f*x)/2]^3))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A]
time = 0.22, size = 216, normalized size = 1.62

method	result
derivativedivides	$\frac{-\frac{c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c^2d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c\,d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{d^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\cdots}{\cdots}$
default	$\frac{-\frac{c^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c^2d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c\,d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{d^3\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\cdots}{\cdots}$
norman	$\frac{\frac{\left(c^3-3c\,d^2+2d^3\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}-\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}-\frac{\left(c^3+3c^2d-9c\,d^2+9d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}-\frac{\left(2c^3+3c^2d-1\cdots\right.}{}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3a}$
risch	$\frac{2i\left(3c^3\mathrm{e}^{4i(fx+e)}-9c\,d^2\mathrm{e}^{4i(fx+e)}+6d^3\mathrm{e}^{4i(fx+e)}+3c^3\mathrm{e}^{3i(fx+e)}+9c^2d\,\mathrm{e}^{3i(fx+e)}-27c\,d^2\mathrm{e}^{3i(fx+e)}+18d^3\mathrm{e}^{3i(fx+e)}+5c^3\mathrm{e}^{2i\cdots}\right.}{3f\,a^2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f/a^2*(-1/3*c^3*tan(1/2*f*x+1/2*e)^3+c^2*d*tan(1/2*f*x+1/2*e)^3-c*d^2*t
an(1/2*f*x+1/2*e)^3+1/3*d^3*tan(1/2*f*x+1/2*e)^3+c^3*tan(1/2*f*x+1/2*e)+3*c
^2*d*tan(1/2*f*x+1/2*e)-9*c*d^2*tan(1/2*f*x+1/2*e)+5*d^3*tan(1/2*f*x+1/2*e)
-2*d^3/(tan(1/2*f*x+1/2*e)+1)+2*d^2*(3*c-2*d)*ln(tan(1/2*f*x+1/2*e)+1)-2*d^
3/(tan(1/2*f*x+1/2*e)-1)-2*d^2*(3*c-2*d)*ln(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(137) = 274$.
time = 0.29, size = 370, normalized size = 2.78

$$\frac{d^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)-3\,cd^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{3c^2d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{c^3\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(d^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 3*c*d^2*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(s
in(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e)
+ 1) - 1)/a^2) + 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2 + c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(137) = 274$.

time = 1.48, size = 282, normalized size = 2.12

$$\frac{3\left((3cd^2-2d^3)\cos(fx+e)^3+2(3cd^2-2d^3)\cos(fx+e)^2+(3cd^2-2d^3)\cos(fx+e)\right)\log(\sin(fx+e)+1)-3\left((3cd^2-2d^3)\cos(fx+e)^3+2(3cd^2-2d^3)\cos(fx+e)^2+(3cd^2-2d^3)\cos(fx+e)\right)\log(-\sin(fx+e)+1)+2\left(3d^3+(2c^3+3c^2d-12cd^2+10d^3)\cos(fx+e)^2+(c^3+6c^2d-15cd^2+14d^3)\cos(fx+e)\right)\sin(fx+e)}{6\left(a^2f\cos(fx+e)^3+2a^2f\cos(fx+e)^2+a^2f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*((3*c*d^2 - 2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)
^2 + (3*c*d^2 - 2*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*((3*c*d^2 -
2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (3*c*d^2 - 2*d
^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(3*d^3 + (2*c^3 + 3*c^2*d - 12
*c*d^2 + 10*d^3)*cos(f*x + e)^2 + (c^3 + 6*c^2*d - 15*c*d^2 + 14*d^3)*cos(f
*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2
*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int \frac{d^3\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int \frac{3cd^2\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int \frac{3c^2d\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(3*c*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
 x))/a**2
```

Giac [A]
time = 0.51, size = 250, normalized size = 1.88

$$-\frac{\frac{12d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^2}-\frac{6(3cd^2-2d^3)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2}+\frac{6(3cd^2-2d^3)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2}+\frac{a^4c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^4c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3a^4cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-a^4d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^4c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-9a^4c^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+27a^4cd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-15a^4d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) - 6*(3
*c*d^2 - 2*d^3)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 6*(3*c*d^2 - 2*d^3
)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 + (a^4*c^3*tan(1/2*f*x + 1/2*e)^3
- 3*a^4*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^4*c*d^2*tan(1/2*f*x + 1/2*e)^3 -
 a^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^3*tan(1/2*f*x + 1/2*e) - 9*a^4*c^
```

```
2*d*tan(1/2*f*x + 1/2*e) + 27*a^4*c*d^2*tan(1/2*f*x + 1/2*e) - 15*a^4*d^3*t
an(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B]
time = 1.88, size = 136, normalized size = 1.02

$$\frac{2\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(3\,c-2\,d\right)}{a^2\,f}-\frac{2\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-a^2\right)}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,{\left(c-d\right)}^3}{6\,a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{{\left(c-d\right)}^3}{a^2}-\frac{3\,\left(c+d\right)\,{\left(c-d\right)}^2}{2\,a^2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*atanh(tan(e/2 + (f*x)/2))*(3*c - 2*d))/(a^2*f) - (2*d^3*tan(e/2 + (f
*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2)) - (tan(e/2 + (f*x)/2)^3*(c - d
)^3)/(6*a^2*f) - (tan(e/2 + (f*x)/2)*((c - d)^3/a^2 - (3*(c + d)*(c - d)^2)
/(2*a^2)))/f
```

3.220 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{d^2\tanh^{-1}(\sin(e+fx))}{a^2f}+\frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

```
[Out] d^2*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^2+1
/3*(c-d)*(c+5*d)*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.67, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.194$, Rules used = {4072, 91, 79, 65, 223, 209}

$$\frac{(c+5d)(c-d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{2d^2\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)^2\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - d)^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + (2*d^2*ArcTan[Sqrt[
a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a -
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c - d)*(c + 5*d)*Tan[e + f*x
])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}\,(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^3(c^2+4cd-2d^2)+3a^3d^2x}{\sqrt{a-ax}\,(a+ax)^{3/2}}\right.}{3a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} - \frac{(d^2\tan(e+}{}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(2d^2\tan(e}{}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(2d^2\tan(e}{a}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{2d^2\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+}{af\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(89) = 178.
time = 0.78, size = 181, normalized size = 2.03

$$-\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(6d^2\cos^3\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+(c-d)^2\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-4(c^2+cd-2d^2)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+(c-d)^2\cos\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)\right)}{3a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-2*Cos[(e + f*x)/2]*(6*d^2*Cos[(e + f*x)/2]^3*(Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)^2*Sec[e
/2]*Sin[(f*x)/2] - 4*(c^2 + c*d - 2*d^2)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f
*x)/2] + (c - d)^2*Cos[(e + f*x)/2]*Tan[e/2]))/(3*a^2*f*(1 + Cos[e + f*x])^
2)
```

Maple [A]
time = 0.21, size = 131, normalized size = 1.47

method	result

derivativedivides	$\frac{2cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{2cd\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{c^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{d^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d^{2}\ln}{2f\,a^{2}}$
default	$\frac{2cd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{2cd\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{c^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{d^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d^{2}\ln}{2f\,a^{2}}$
risch	$\frac{2i\left(3c^{2}e^{2i(fx+e)}-3d^{2}e^{2i(fx+e)}+3c^{2}e^{i(fx+e)}+6d\,e^{i(fx+e)}c-9d^{2}e^{i(fx+e)}+2c^{2}+2cd-4d^{2}\right)}{3f\,a^{2}\left(e^{i(fx+e)}+1\right)^{3}}-\frac{d^{2}\ln\left(e^{i(fx+e)}-i\right)}{a^{2}f}+\frac{d^{2}}{}$
norman	$\frac{-\frac{\left(c^{2}-2cd+d^{2}\right)\left(\tan^{7}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}+\frac{\left(c^{2}+2cd-3d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{\left(5c^{2}+2cd-7d^{2}\right)\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}-\frac{\left(7c^{2}+10cd-17d^{2}\right)\left(\tan^{3}\left(\frac{fx}{2}\right.\right.}{6af}}{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f/a^2*(2*c*d*tan(1/2*f*x+1/2*e)+2/3*c*d*tan(1/2*f*x+1/2*e)^3-1/3*c^2*ta
n(1/2*f*x+1/2*e)^3-1/3*d^2*tan(1/2*f*x+1/2*e)^3+c^2*tan(1/2*f*x+1/2*e)-3*d^
2*tan(1/2*f*x+1/2*e)+2*d^2*ln(tan(1/2*f*x+1/2*e)+1)-2*d^2*ln(tan(1/2*f*x+1/
2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(90) = 180$.

time = 0.28, size = 211, normalized size = 2.37

$$-\frac{d^{2}\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}{a^{2}}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{2}}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^{2}}\right)-\frac{2cd\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}}-\frac{c^{2}\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a^{2}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 2*c*d*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/(cos
(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Fricas [A]

time = 2.87, size = 165, normalized size = 1.85

$$\frac{3\left(d^{2}\cos\left(fx+e\right)^{2}+2\,d^{2}\cos\left(fx+e\right)+d^{2}\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(d^{2}\cos\left(fx+e\right)^{2}+2\,d^{2}\cos\left(fx+e\right)+d^{2}\right)\log\left(-\sin\left(fx+e\right)+1\right)+2\left(c^{2}+4\,cd-5\,d^{2}+2\left(c^{2}+cd-2\,d^{2}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{6\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+a^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(sin(f*x + e) + 1
) - 3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(-sin(f*x + e) + 1
) + 2*(c^2 + 4*c*d - 5*d^2 + 2*(c^2 + c*d - 2*d^2)*cos(f*x + e))*sin(f*x +
e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2 \sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^2 \sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{2cd\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(2*c*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a*
*2
```

Giac [A]
time = 0.48, size = 158, normalized size = 1.78

$$\frac{\frac{6\,d^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} - \frac{6\,d^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} - \frac{a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,a^4cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^4d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-6\,a^4cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+9\,a^4d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*d^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 6*d^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/a^2 - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^4*c*d*tan(1/2
*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*tan(1/2*f*x +
1/2*e) - 6*a^4*c*d*tan(1/2*f*x + 1/2*e) + 9*a^4*d^2*tan(1/2*f*x + 1/2*e))/a
^6)/f
```

Mupad [B]
time = 1.77, size = 89, normalized size = 1.00

$$\frac{2\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{{\left(c-d\right)}^2}{2\,a^2}-\frac{c^2-d^2}{a^2}\right)}{f} - \frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,{\left(c-d\right)}^2}{6\,a^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (tan(e/2 + (f*x)/2)*((c - d)^2/
(2*a^2) - (c^2 - d^2)/a^2))/f - (tan(e/2 + (f*x)/2)^3*(c - d)^2)/(6*a^2*f)
```

3.221 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c+2d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

```
[Out] 1/3*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2+1/3*(c+2*d)*tan(f*x+e)/f/(a^2+a^2
*sec(f*x+e))
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {4085, 3879}

$$\frac{(c+2d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)}+\frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - d)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((c + 2*d)*Tan[e + f*
x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx &= \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c+2d)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{3a}\\ &= \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c+2d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 76, normalized size = 1.17

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(3(c+d)\sin\left(\frac{fx}{2}\right)-3c\sin\left(e+\frac{fx}{2}\right)+(2c+d)\sin\left(e+\frac{3fx}{2}\right)\right)}{3a^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(3*(c + d)*Sin[(f*x)/2] - 3*c*Sin[e + (f*x)/2] +
 (2*c + d)*Sin[e + (3*f*x)/2]))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A]
time = 0.17, size = 60, normalized size = 0.92

method	result	size
derivativedivides	$\frac{-\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\,a^2}$	60
default	$\frac{-\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\,a^2}$	60
risch	$\frac{2i\left(3\,\mathrm{e}^{2i(fx+e)}c+3\,\mathrm{e}^{i(fx+e)}c+3d\,\mathrm{e}^{i(fx+e)}+2c+d\right)}{3f\,a^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3}$	64
norman	$\frac{-\frac{(c-d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}-\frac{(c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{(2c+d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3af}}{a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/a^2*(-1/3*c*tan(1/2*f*x+1/2*e)^3+1/3*d*tan(1/2*f*x+1/2*e)^3+c*tan(1/2
*f*x+1/2*e)+d*tan(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.27, size = 101, normalized size = 1.55

$$\frac{\frac{d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/6*(d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/a^2)/f
```

Fricas [A]
time = 2.23, size = 62, normalized size = 0.95

$$\frac{((2c+d)\cos(fx+e)+c+2d)\sin(fx+e)}{3\left(a^2f\cos(fx+e)^2+2a^2f\cos(fx+e)+a^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*((2*c + d)*cos(f*x + e) + c + 2*d)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 +
 2*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int \frac{d\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Giac [A]
time = 0.45, size = 60, normalized size = 0.92

$$-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{6a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/6*(c*tan(1/2*f*x + 1/2*e)^3 - d*tan(1/2*f*x + 1/2*e)^3 - 3*c*tan(1/2*f*x
 + 1/2*e) - 3*d*tan(1/2*f*x + 1/2*e))/(a^2*f)
```

Mupad [B]
time = 1.71, size = 45, normalized size = 0.69

$$\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(c+d)}{2a^2f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(c-d)}{6a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c + d))/(2*a^2*f) - (tan(e/2 + (f*x)/2)^3*(c - d))/(6*
a^2*f)
```

3.222 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=129

$$\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{5/2}\sqrt{c+d}\,f}+\frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2}+\frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)}$$

```
[Out] 2*d^2*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^2/(c-d)^(5/2)/f
/(c+d)^(1/2)+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2+1/3*(c-4*d)*tan(f*x+
e)/(c-d)^2/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A]
time = 0.17, antiderivative size = 183, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 106, 157, 12, 95, 211}

$$\frac{(c-4d)\tan(e+fx)}{3f(c-d)^2\left(a^2\sec(e+fx)+a^2\right)}-\frac{2d^2\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{5/2}\sqrt{c+d}\,\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}+\frac{\tan(e+fx)}{3f(c-d)(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(3*(c - d)*f*(a + a*Sec[e + f*x])^2) - (2*d^2*ArcTan[(Sqrt[c +
 d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e
 + f*x])/(a*(c - d)^(5/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a
*Sec[e + f*x]]) + ((c - 4*d)*Tan[e + f*x])/(3*(c - d)^2*f*(a^2 + a^2*Sec[e
+ f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{5/2}(c+dx)}\,dx, x, \text{se}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{}{\sqrt{a-}}\right.}{3a(c-d)f\sqrt{a-a\,\text{se}}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\,(a^2+a^2\sec(e+}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\,(a^2+a^2\sec(e+}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\,(a^2+a^2\sec(e+}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} - \frac{2d^2\tan^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{c}}{\sqrt{c-d}\,\sqrt{c}}\right.}{a(c-d)^{5/2}\sqrt{c+d}\,f\sqrt{a-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.72, size = 209, normalized size = 1.62

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(-\frac{24id^2\text{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\sec\left(\frac{e}{2}\right)\left(3(c-3d)\sin\left(\frac{fx}{2}\right)-3(c-2d)\sin\left(e+\frac{fx}{2}\right)+(2c-5d)\sin\left(e+\frac{3fx}{2}\right)\right)\right)}{3a^2(c-d)^2f(1+\cos(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]*(((-24*I)*d^2*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d
 + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*
Cos[(e + f*x)/2]^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*S
in[e])^2]) + Sec[e/2]*(3*(c - 3*d)*Sin[(f*x)/2] - 3*(c - 2*d)*Sin[e + (f*x)
/2] + (2*c - 5*d)*Sin[e + (3*f*x)/2])))/(3*a^2*(c - d)^2*f*(1 + Cos[e + f*x
])^2)
```

Maple [A]
time = 0.24, size = 122, normalized size = 0.95

method	result
derivativedivides	$-\frac{-\frac{\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c-d)^2}+\frac{4d^2\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)^2\sqrt{(c+d)(c-d)}}}{2fa^2}$
default	$-\frac{-\frac{\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c-d)^2}+\frac{4d^2\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)^2\sqrt{(c+d)(c-d)}}}{2fa^2}$
risch	$\frac{2i\left(3\,e^{2i(fx+e)}c-6d\,e^{2i(fx+e)}+3\,e^{i(fx+e)}c-9d\,e^{i(fx+e)}+2c-5d\right)}{3fa^2(c-d)^2\left(e^{i(fx+e)}+1\right)^3}+\frac{d^2\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c-d)^2fa^2}-\ -\ d^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/a^2*(-1/(c-d)^2*(1/3*c*tan(1/2*f*x+1/2*e)^3-1/3*d*tan(1/2*f*x+1/2*e)^
3-c*tan(1/2*f*x+1/2*e)+3*d*tan(1/2*f*x+1/2*e))+4*d^2/(c-d)^2/((c+d)*(c-d))^
(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(121) = 242.
time = 2.36, size = 618, normalized size = 4.79

$$\left[\frac{3\left(d^2\cos(fx+e)^2+2d^2\cos(fx+e)+d^2\right)\sqrt{c^2-d^2}\log\left(\frac{2cd\cos(fx+e)-(c^2-2d^2)\cos(fx+e)^2+2\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2c^2-d^2}{c^2\cos(fx+e)^2+2cd\cos(fx+e)+d^2}\right)+2\left(c^3-4c^2d-cd^2+4d^3+(2c^3-5c^2d-2cd^2+5d^3)\cos(fx+e)\right)\sin(fx+e)}{6\left((a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)^2+2(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)+(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\right)},\ \frac{3\left(d^2\cos(fx+e)^2+2d^2\cos(fx+e)+d^2\right)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)+\left(c^3-4c^2d-cd^2+4d^3+(2c^3-5c^2d-2cd^2+5d^3)\cos(fx+e)\right)\sin(fx+e)}{3\left((a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)^2+2(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\cos(fx+e)+(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d -
 2*c*d^2 + 5*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a
^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d
^3 - a^2*d^4)*f*cos(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d
^4)*f), 1/3*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(-c^2 +
d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e
))) + (c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d - 2*c*d^2 + 5*d^3)*
cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4
)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*co
s(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c\sec^2(e+fx)+2c\sec(e+fx)+c+d\sec^3(e+fx)+2d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x)**2 + 2*c*sec(e + f*x) + c + d*sec(e +
 f*x)**3 + 2*d*sec(e + f*x)**2 + d*sec(e + f*x)), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(116) = 232.
time = 0.50, size = 249, normalized size = 1.93

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{(a^2c^2-2a^2cd+a^2d^2)\sqrt{-c^2+d^2}}+\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^4cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+12a^4cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-9a^4d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3-3a^6c^2d+3a^6cd^2-a^6d^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1
/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((a^2*c^2
- 2*a^2*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + (a^4*c^2*tan(1/2*f*x + 1/2*e)^3
- 2*a^4*c*d*tan(1/2*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4
*c^2*tan(1/2*f*x + 1/2*e) + 12*a^4*c*d*tan(1/2*f*x + 1/2*e) - 9*a^4*d^2*tan
(1/2*f*x + 1/2*e))/(a^6*c^3 - 3*a^6*c^2*d + 3*a^6*c*d^2 - a^6*d^3))/f
```

Mupad [B]
time = 1.92, size = 168, normalized size = 1.30

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{1}{a^2\,(c-d)}-\frac{c+d}{2\,a^2\,(c-d)^2}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{6\,a^2\,f\,\left(c-d\right)}-\frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3-3\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d+3\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^2-1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^3}{\sqrt{c+d}\,{(c-d)}^{5/2}}\right)\,2\mathrm{i}}{a^2\,f\,\sqrt{c+d}\,{\left(c-d\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(1/(a^2*(c - d)) - (c + d)/(2*a^2*(c - d)^2)))/f - tan(
e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)) - (d^2*atan((c^3*tan(e/2 + (f*x)/2)*1i -
 d^3*tan(e/2 + (f*x)/2)*1i + c*d^2*tan(e/2 + (f*x)/2)*3i - c^2*d*tan(e/2 +
(f*x)/2)*3i)/((c + d)^(1/2)*(c - d)^(5/2)))*2i)/(a^2*f*(c + d)^(1/2)*(c - d
)^(5/2))
```

3.223 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=211

$$\frac{2d^2(3c+2d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{7/2}(c+d)^{3/2}f}+\frac{d(c^2-6cd-10d^2)\tan(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sec(e+fx))}+\frac{(c-6d}{3a^2(c-d)^2f(1+\sec(}$$

```
[Out] 2*d^2*(3*c+2*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^2/(c-
d)^(7/2)/(c+d)^(3/2)/f+1/3*d*(c^2-6*c*d-10*d^2)*tan(f*x+e)/a^2/(c-d)^3/(c+d
)/f/(c+d*sec(f*x+e))+1/3*(c-6*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(c
+d*sec(f*x+e))+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.25, antiderivative size = 260, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 105, 157, 12, 95, 211}

$$\frac{(c^2-6cd-10d^2)\tan(e+fx)}{3f(c-d)^3(c+d)\left(a^2\sec(e+fx)+a^2\right)}-\frac{2d^2(3c+2d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{7/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))}+\frac{(c+4d)\tan(e+fx)}{3f(c-d)^2(c+d)(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 4*d)*Tan[e + f*x])/(3*(c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])^2) - (
2*d^2*(3*c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d
]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a*(c - d)^(7/2)*(c + d)^(3/2)*f
*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c^2 - 6*c*d - 10*d^
2)*Tan[e + f*x])/(3*(c - d)^3*(c + d)*f*(a^2 + a^2*Sec[e + f*x])) - (d*Tan[
e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{5/2}(c+dx)^2}\,dx, x, \mathrm{s}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{\tan(}{}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} - \frac{}{(c^2-d^2)\,f(a+a\,\mathrm{s}}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{(c^2-6cd-1}{3(c-d)^3(c+d)f}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{(c^2-6cd-1}{3(c-d)^3(c+d)f}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{(c^2-6cd-1}{3(c-d)^3(c+d)f}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} - \frac{2d^2(3c+2d)\tan^{-}}{a(c-d)^{7/2}(c+d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.93, size = 376, normalized size = 1.78

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^4(e+fx)\left(\frac{12d^2(3c+2d)\mathrm{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))(i\cos(e)+\sin(e))}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+(c-d)(d+c\cos(e+fx))\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-4(c-4d)\cos^2\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+\frac{6d^3\cos^3\left(\frac{1}{2}(e+fx)\right)(-d\sin(e)+c\sin(fx))}{c(c+d)\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)}+(c-d)\cos\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\tan\left(\frac{e}{2}\right)\right)}{3a^2(-c+d)^3f(1+\sec(e+fx))^2(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^4*((12*d^2*(3*c + 2*d
)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(S
qrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]^3*(d + c*Cos[
e + f*x])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Si
n[e])^2]) + (c - d)*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] - 4*(c - 4*d
)*Cos[(e + f*x)/2]^2*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (6*d^3*Co
s[(e + f*x)/2]^3*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2
```

```
])*(Cos[e/2] + Sin[e/2])) + (c - d)*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*T
an[e/2]))/(3*a^2*(-c + d)^3*f*(1 + Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2)
```

Maple [A]
time = 0.36, size = 203, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+5d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-2cd+d^2\right)(c-d)}-\frac{4d^2\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c\right.}\right.}{(c-}}{2fa^2}$
default	$\frac{-\frac{\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+5d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-2cd+d^2\right)(c-d)}-\frac{4d^2\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c\right.}\right.}{(c-}}{2fa^2}$
risch	$\frac{2i\left(-3c^4e^{4i(fx+e)}+6c^3d\,e^{4i(fx+e)}+9c^2d^2e^{4i(fx+e)}+3d^4e^{4i(fx+e)}-3c^4e^{3i(fx+e)}+6c^3d\,e^{3i(fx+e)}+27c^2d^2e^{3i(fx+e)}+21\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f/a^2*(-1/(c^2-2*c*d+d^2)/(c-d)*(1/3*c*tan(1/2*f*x+1/2*e)^3-1/3*d*tan(1
/2*f*x+1/2*e)^3-c*tan(1/2*f*x+1/2*e)+5*d*tan(1/2*f*x+1/2*e))-4*d^2/(c-d)^3*
(-d/(c+d)*tan(1/2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2
-c-d)-(3*c+2*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/
((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(205) = 410$.
time = 2.84, size = 1268, normalized size = 6.01

$$\left[-\frac{3 \left(3 c d^3 + 2 d^4 + (3 c^2 d^2 + 2 c d^3) \cos(fx+e)^3 + (6 c^2 d^2 + 7 c d^3 + 2 d^4) \cos(fx+e)^2 + (3 c^2 d^2 + 8 c d^3 + 4 d^4) \cos(fx+e)\right) \sqrt{c^2 - d^2} \log\left(\frac{2 c d \cos(fx+e) - (c^2 - 2 d^2) \cos(fx+e)^2 - 2 \sqrt{c^2 - d^2} (d \cos(fx+e) + c) \sin(fx+e) + 2 c^2 - d^2}{c^2 \cos(fx+e)^2 + 2 c d \cos(fx+e) + d^2}\right) - 2 \left(c^4 d - 6 c^3 d^2 - 11 c^2 d^3 + 6 c d^4 + 10 d^5 + (2 c^5 - 6 c^4 d - 10 c^3 d^2 + 3 c^2 d^3 + 8 c d^4 + 3 d^5) \cos(fx+e)^2 + (c^5 - 4 c^4 d - 14 c^3 d^2 - 10 c^2 d^3 + 13 c d^4 + 14 d^5) \cos(fx+e)\right) \sin(fx+e)}{6 \left((a^2 c^7 - 2 a^2 c^6 d - a^2 c^5 d^2 + 4 a^2 c^4 d^3 - a^2 c^3 d^4 - 2 a^2 c^2 d^5 + a^2 c d^6) f \cos(fx+e)^3 + (2 a^2 c^7 - 3 a^2 c^6 d - 4 a^2 c^5 d^2 + 7 a^2 c^4 d^3 + 2 a^2 c^3 d^4 - 5 a^2 c^2 d^5 + a^2 d^7) f \cos(fx+e)^2 + (a^2 c^7 - 5 a^2 c^5 d^2 + 2 a^2 c^4 d^3 + 7 a^2 c^3 d^4 - 4 a^2 c^2 d^5 - 3 a^2 c d^6 + 2 a^2 d^7) f \cos(fx+e) + (a^2 c^6 d - 2 a^2 c^5 d^2 - a^2 c^4 d^3 + 4 a^2 c^3 d^4 - a^2 c^2 d^5 - 2 a^2 c d^6 + a^2 d^7) f\right)}, \frac{3 \left(3 c d^3 + 2 d^4 + (3 c^2 d^2 + 2 c d^3) \cos(fx+e)^3 + (6 c^2 d^2 + 7 c d^3 + 2 d^4) \cos(fx+e)^2 + (3 c^2 d^2 + 8 c d^3 + 4 d^4) \cos(fx+e)\right) \sqrt{-c^2 + d^2} \arctan\left(-\frac{\sqrt{-c^2 + d^2} (d \cos(fx+e) + c)}{(c^2 - d^2) \sin(fx+e)}\right) + \left(c^4 d - 6 c^3 d^2 - 11 c^2 d^3 + 6 c d^4 + 10 d^5 + (2 c^5 - 6 c^4 d - 10 c^3 d^2 + 3 c^2 d^3 + 8 c d^4 + 3 d^5) \cos(fx+e)^2 + (c^5 - 4 c^4 d - 14 c^3 d^2 - 10 c^2 d^3 + 13 c d^4 + 14 d^5) \cos(fx+e)\right) \sin(fx+e)}{3 \left((a^2 c^7 - 2 a^2 c^6 d - a^2 c^5 d^2 + 4 a^2 c^4 d^3 - a^2 c^3 d^4 - 2 a^2 c^2 d^5 + a^2 c d^6) f \cos(fx+e)^3 + (2 a^2 c^7 - 3 a^2 c^6 d - 4 a^2 c^5 d^2 + 7 a^2 c^4 d^3 + 2 a^2 c^3 d^4 - 5 a^2 c^2 d^5 + a^2 d^7) f \cos(fx+e)^2 + (a^2 c^7 - 5 a^2 c^5 d^2 + 2 a^2 c^4 d^3 + 7 a^2 c^3 d^4 - 4 a^2 c^2 d^5 - 3 a^2 c d^6 + 2 a^2 d^7) f \cos(fx+e) + (a^2 c^6 d - 2 a^2 c^5 d^2 - a^2 c^4 d^3 + 4 a^2 c^3 d^4 - a^2 c^2 d^5 - 2 a^2 c d^6 + a^2 d^7) f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/6*(3*(3*c*d^3 + 2*d^4 + (3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d
^2 + 7*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*
x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e
)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c
^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^4*d - 6*c^3*d^2 - 11*
c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^3*d^2 + 3*c^2*d^3 + 8*
c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2*d^3 +
13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*c^7 - 2*a^2*c^6*d - a^
2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d^5 + a^2*c*d^6)*f*cos(
f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2 + 7*a^2*c^4*d^3 + 2*a
^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^2 + (a^2*c^7 - 5*a^2*c
^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^
2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^
3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f), 1/3*(3*(3*c*d^3 + 2*d^4 +
(3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d^2 + 7*c*d^3 + 2*d^4)*cos(f*
x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*x + e))*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (c
^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^
3*d^2 + 3*c^2*d^3 + 8*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c
^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*
c^7 - 2*a^2*c^6*d - a^2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d
^5 + a^2*c*d^6)*f*cos(f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2
 + 7*a^2*c^4*d^3 + 2*a^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^
2 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^
5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 -
a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^2 \sec^2(e+fx) + 2c^2 \sec(e+fx) + c^2 + 2cd \sec^3(e+fx) + 4cd \sec^2(e+fx) + 2cd \sec(e+fx) + d^2 \sec^4(e+fx) + 2d^2 \sec^3(e+fx) + d^2 \sec^2(e+fx)}\, dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x)**2 + 2*c**2*sec(e + f*x) + c**2 +
2*c*d*sec(e + f*x)**3 + 4*c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2*s
ec(e + f*x)**4 + 2*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(196) = 392.
time = 0.53, size = 474, normalized size = 2.25

$$\frac{\frac{12\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)}+\frac{12\,\left(3\,cd^2+2\,d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\sqrt{-c^2+d^2}}-\frac{a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,a^4c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,a^4c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,a^4cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^4d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+24\,a^4c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-54\,a^4c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+48\,a^4cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-15\,a^4d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6c^6-6\,a^6c^5d+15\,a^6c^4d^2-20\,a^6c^3d^3+15\,a^6c^2d^4-6\,a^6cd^5+a^6d^6}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^
2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) + 12*
(3*c*d^2 + 2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arcta
n(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a
^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)) - (a^4*c^4*
tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^2*d^2
*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*tan(1/2*f*x + 1/2*e)^3 + a^4*d^4*tan(
1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*f*x + 1/2*e) + 24*a^4*c^3*d*tan(1/2*
f*x + 1/2*e) - 54*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e) + 48*a^4*c*d^3*tan(1/2*f
*x + 1/2*e) - 15*a^4*d^4*tan(1/2*f*x + 1/2*e))/(a^6*c^6 - 6*a^6*c^5*d + 15*
a^6*c^4*d^2 - 20*a^6*c^3*d^3 + 15*a^6*c^2*d^4 - 6*a^6*c*d^5 + a^6*d^6))/f
```

Mupad [**B**]
time = 2.18, size = 314, normalized size = 1.49

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3}{2\,a^2\,{\left(c-d\right)}^2}-\frac{c^2-d^2}{a^2\,{\left(c-d\right)}^4}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{6\,a^2\,f\,{\left(c-d\right)}^2}+\frac{2\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(c+d\right)\,\left(a^2\,d^4-a^2\,c^4+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(a^2\,c^4-4\,a^2\,c^3\,d+6\,a^2\,c^2\,d^2-4\,a^2\,c\,d^3+a^2\,d^4\right)-2\,a^2\,c\,d^3+2\,a^2\,c^3\,d\right)}-\frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^4-4\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,d+6\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d^2-4\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^3+1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^4}{\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}\right)\,\left(3\,c+2\,d\right)\,2\mathrm{i}}{a^2\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3/(2*a^2*(c - d)^2) - (c^2 - d^2)/(a^2*(c - d)^4)))/f
- tan(e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)^2) + (2*d^3*tan(e/2 + (f*x)/2))/(f*
(c + d)*(a^2*d^4 - a^2*c^4 + tan(e/2 + (f*x)/2)^2*(a^2*c^4 + a^2*d^4 - 4*a^
2*c*d^3 - 4*a^2*c^3*d + 6*a^2*c^2*d^2) - 2*a^2*c*d^3 + 2*a^2*c^3*d)) - (d^2
*atan((c^4*tan(e/2 + (f*x)/2)*1i + d^4*tan(e/2 + (f*x)/2)*1i - c*d^3*tan(e/
2 + (f*x)/2)*4i - c^3*d*tan(e/2 + (f*x)/2)*4i + c^2*d^2*tan(e/2 + (f*x)/2)*
6i)/((c + d)^(1/2)*(c - d)^(7/2)))*(3*c + 2*d)*2i)/(a^2*f*(c + d)^(3/2)*(c
- d)^(7/2))
```

3.224 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=284

$$\frac{d^2(12c^2+16cd+7d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{9/2}(c+d)^{5/2}f}+\frac{d(2c^2-16cd-21d^2)\tan(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sec(e+fx))^2}+\frac{}{3a^2(c-d)^2}$$

```
[Out] d^2*(12*c^2+16*c*d+7*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2
))/a^2/(c-d)^(9/2)/(c+d)^(5/2)/f+1/6*d*(2*c^2-16*c*d-21*d^2)*tan(f*x+e)/a^2
/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))^2+1/3*(c-8*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1
+sec(f*x+e))/(c+d*sec(f*x+e))^2+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2/(
c+d*sec(f*x+e))^2+1/6*d*(2*c^3-16*c^2*d-59*c*d^2-32*d^3)*tan(f*x+e)/a^2/(c-
d)^4/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A]

time = 0.38, antiderivative size = 346, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 105, 156, 157, 12, 95, 211}

$$\frac{(2c^3-16c^2d-59cd^2-32d^3)\tan(e+fx)}{6f(c-d)^4(c+d)^2\left(a^2\sec(e+fx)+a^2\right)}-\frac{d^2(12c^2+16cd+7d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{9/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{d(5c+2d)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(a\sec(e+fx)+a)^2(c+d\sec(e+fx))}-\frac{d\tan(e+fx)}{2f\left(c^2-d^2\right)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^2}+\frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6f(c-d)^3(c+d)^2(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 22*c*d + 11*d^2)*Tan[e + f*x])/(6*(c - d)^3*(c + d)^2*f*(a + a*Se
c[e + f*x])^2) - (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a
+ a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a
*(c - d)^(9/2)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]) + ((2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Tan[e + f*x])/(6*(c - d)^4
*(c + d)^2*f*(a^2 + a^2*Sec[e + f*x])) - (d*Tan[e + f*x])/(2*(c^2 - d^2)*f*
(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2) - (d*(5*c + 2*d)*Tan[e + f*x
])/(2*(c^2 - d^2)^2*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
```

```
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{5/2}(c+dx)^3}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \text{ta}$$

$$= -\frac{d\tan(e+fx)}{2(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{}{2}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} - \frac{}{2(c^2-d^2)\,f(a+}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^3-16c^2d-5}{6(c-d)^4(c+d)}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^3-16c^2d-5}{6(c-d)^4(c+d)}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^3-16c^2d-5}{6(c-d)^4(c+d)}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} - \frac{d^2(12c^2+16cd+}{a(c-d)^{9/2}(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.39, size = 2220, normalized size = 7.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
[Out] ((12*c^2 + 16*c*d + 7*d^2)*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[
e + f*x]^5*(((-4*I)*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[C
```

os[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[
2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d
^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (4*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sq
rt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sq
rt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*S
in[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d
)^2*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d
 + c*Cos[e + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-16*c^7*Sin[(f*x)/2] + 1
4*c^6*d*Sin[(f*x)/2] + 220*c^5*d^2*Sin[(f*x)/2] + 334*c^4*d^3*Sin[(f*x)/2]
+ 54*c^3*d^4*Sin[(f*x)/2] - 156*c^2*d^5*Sin[(f*x)/2] - 48*c*d^6*Sin[(f*x)/2
] + 18*d^7*Sin[(f*x)/2] + 14*c^7*Sin[(3*f*x)/2] - 16*c^6*d*Sin[(3*f*x)/2] -
 226*c^5*d^2*Sin[(3*f*x)/2] - 532*c^4*d^3*Sin[(3*f*x)/2] - 583*c^3*d^4*Sin[
(3*f*x)/2] - 232*c^2*d^5*Sin[(3*f*x)/2] - 6*c*d^6*Sin[(3*f*x)/2] + 6*d^7*Si
n[(3*f*x)/2] - 12*c^7*Sin[e - (f*x)/2] + 20*c^6*d*Sin[e - (f*x)/2] + 236*c^
5*d^2*Sin[e - (f*x)/2] + 628*c^4*d^3*Sin[e - (f*x)/2] + 778*c^3*d^4*Sin[e -
 (f*x)/2] + 420*c^2*d^5*Sin[e - (f*x)/2] + 48*c*d^6*Sin[e - (f*x)/2] - 18*d
^7*Sin[e - (f*x)/2] + 12*c^7*Sin[e + (f*x)/2] - 20*c^6*d*Sin[e + (f*x)/2] -
 236*c^5*d^2*Sin[e + (f*x)/2] - 460*c^4*d^3*Sin[e + (f*x)/2] - 310*c^3*d^4*
Sin[e + (f*x)/2] + 39*c^2*d^5*Sin[e + (f*x)/2] + 48*c*d^6*Sin[e + (f*x)/2]
- 18*d^7*Sin[e + (f*x)/2] - 16*c^7*Sin[2*e + (f*x)/2] + 14*c^6*d*Sin[2*e +
(f*x)/2] + 220*c^5*d^2*Sin[2*e + (f*x)/2] + 502*c^4*d^3*Sin[2*e + (f*x)/2]
+ 522*c^3*d^4*Sin[2*e + (f*x)/2] + 303*c^2*d^5*Sin[2*e + (f*x)/2] + 48*c*d^
6*Sin[2*e + (f*x)/2] - 18*d^7*Sin[2*e + (f*x)/2] - 6*c^7*Sin[e + (3*f*x)/2]
 + 6*c^6*d*Sin[e + (3*f*x)/2] + 126*c^5*d^2*Sin[e + (3*f*x)/2] + 114*c^4*d^
3*Sin[e + (3*f*x)/2] - 159*c^3*d^4*Sin[e + (3*f*x)/2] - 144*c^2*d^5*Sin[e +
 (3*f*x)/2] - 6*c*d^6*Sin[e + (3*f*x)/2] + 6*d^7*Sin[e + (3*f*x)/2] + 14*c^
7*Sin[2*e + (3*f*x)/2] - 16*c^6*d*Sin[2*e + (3*f*x)/2] - 226*c^5*d^2*Sin[2*
e + (3*f*x)/2] - 412*c^4*d^3*Sin[2*e + (3*f*x)/2] - 235*c^3*d^4*Sin[2*e + (
3*f*x)/2] - 7*c^2*d^5*Sin[2*e + (3*f*x)/2] + 6*c*d^6*Sin[2*e + (3*f*x)/2] -
 6*d^7*Sin[2*e + (3*f*x)/2] - 6*c^7*Sin[3*e + (3*f*x)/2] + 6*c^6*d*Sin[3*e
+ (3*f*x)/2] + 126*c^5*d^2*Sin[3*e + (3*f*x)/2] + 234*c^4*d^3*Sin[3*e + (3*
f*x)/2] + 189*c^3*d^4*Sin[3*e + (3*f*x)/2] + 81*c^2*d^5*Sin[3*e + (3*f*x)/2
] + 6*c*d^6*Sin[3*e + (3*f*x)/2] - 6*d^7*Sin[3*e + (3*f*x)/2] + 6*c^7*Sin[e
 + (5*f*x)/2] - 14*c^6*d*Sin[e + (5*f*x)/2] - 134*c^5*d^2*Sin[e + (5*f*x)/2
] - 274*c^4*d^3*Sin[e + (5*f*x)/2] - 193*c^3*d^4*Sin[e + (5*f*x)/2] - 27*c^
2*d^5*Sin[e + (5*f*x)/2] + 6*c*d^6*Sin[e + (5*f*x)/2] - 6*c^7*Sin[2*e + (5*
f*x)/2] + 12*c^6*d*Sin[2*e + (5*f*x)/2] + 42*c^5*d^2*Sin[2*e + (5*f*x)/2] -
 48*c^4*d^3*Sin[2*e + (5*f*x)/2] - 105*c^3*d^4*Sin[2*e + (5*f*x)/2] - 27*c^
2*d^5*Sin[2*e + (5*f*x)/2] + 6*c*d^6*Sin[2*e + (5*f*x)/2] + 6*c^7*Sin[3*e +
 (5*f*x)/2] - 14*c^6*d*Sin[3*e + (5*f*x)/2] - 134*c^5*d^2*Sin[3*e + (5*f*x)
/2] - 202*c^4*d^3*Sin[3*e + (5*f*x)/2] - 61*c^3*d^4*Sin[3*e + (5*f*x)/2] +
12*c^2*d^5*Sin[3*e + (5*f*x)/2] - 6*c*d^6*Sin[3*e + (5*f*x)/2] - 6*c^7*Sin[
4*e + (5*f*x)/2] + 12*c^6*d*Sin[4*e + (5*f*x)/2] + 42*c^5*d^2*Sin[4*e + (5*
f*x)/2] + 24*c^4*d^3*Sin[4*e + (5*f*x)/2] + 27*c^3*d^4*Sin[4*e + (5*f*x)/2]
 + 12*c^2*d^5*Sin[4*e + (5*f*x)/2] - 6*c*d^6*Sin[4*e + (5*f*x)/2] + 4*c^7*S

```
in[2*e + (7*f*x)/2] - 14*c^6*d*Sin[2*e + (7*f*x)/2] - 40*c^5*d^2*Sin[2*e +
(7*f*x)/2] - 46*c^4*d^3*Sin[2*e + (7*f*x)/2] - 12*c^3*d^4*Sin[2*e + (7*f*x)
/2] + 3*c^2*d^5*Sin[2*e + (7*f*x)/2] - 24*c^4*d^3*Sin[3*e + (7*f*x)/2] - 12
*c^3*d^4*Sin[3*e + (7*f*x)/2] + 3*c^2*d^5*Sin[3*e + (7*f*x)/2] + 4*c^7*Sin[
4*e + (7*f*x)/2] - 14*c^6*d*Sin[4*e + (7*f*x)/2] - 40*c^5*d^2*Sin[4*e + (7*
f*x)/2] - 22*c^4*d^3*Sin[4*e + (7*f*x)/2]))/(48*c^2*(-c + d)^4*(c + d)^2*f*
(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3)
```

Maple [A]
time = 0.48, size = 280, normalized size = 0.99

method	result
derivativedivides	$\dfrac{-\dfrac{\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+7d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^3-3c^2d+3c\,d^2-d^3\right)(c-d)}-\dfrac{8d^2\left(\dfrac{-\frac{d\left(8c^2-3cd-5d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4\left(c^2+2cd+d^2\right)}+\frac{d(8c+3d)\tan}{4c+4}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right.}\right.}{}}{2f\,a^2}$
default	$\dfrac{-\dfrac{\frac{c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{d\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+7d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^3-3c^2d+3c\,d^2-d^3\right)(c-d)}-\dfrac{8d^2\left(\dfrac{-\frac{d\left(8c^2-3cd-5d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4\left(c^2+2cd+d^2\right)}+\frac{d(8c+3d)\tan}{4c+4}}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right.}\right.}{}}{2f\,a^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f/a^2*(-1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c-d)*(1/3*c*tan(1/2*f*x+1/2*e)^3-1
/3*d*tan(1/2*f*x+1/2*e)^3-c*tan(1/2*f*x+1/2*e)+7*d*tan(1/2*f*x+1/2*e))-8*d^
2/(c-d)^4*((-1/4*d*(8*c^2-3*c*d-5*d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3
+1/4*d*(8*c+3*d)/(c+d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/
2*f*x+1/2*e)^2-c-d)^2-1/4*(12*c^2+16*c*d+7*d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d
))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(278) = 556$.
time = 2.37, size = 2062, normalized size = 7.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2*
d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*co
s(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)*co
s(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e))
*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2
*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(
f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d^2 - 16*c^4*d^3 - 61*c^
3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5*d^2 - 3
2*c^4*d^3 + 28*c^3*d^4 + 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x + e)^3 + (2
*c^7 - 8*c^6*d - 68*c^5*d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*d^5 + 89*c
*d^6 + 6*d^7)*cos(f*x + e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^
3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*
c^10 - 2*a^2*c^9*d - 2*a^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5*d^5 + 2*a^2*
c^4*d^6 + 2*a^2*c^3*d^7 - a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(a^2*c^10 - a^2
*c^9*d - 4*a^2*c^8*d^2 + 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a^2*c^5*d^5 - 4*
a^2*c^4*d^6 + 4*a^2*c^3*d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*cos(f*x + e)^3 + (
a^2*c^10 + 2*a^2*c^9*d - 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 22*a^2*c^6*d^4 - 2
2*a^2*c^4*d^6 + 4*a^2*c^3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d^9 - a^2*d^10)*f*c
os(f*x + e)^2 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4
+ 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9
 - a^2*d^10)*f*cos(f*x + e) + (a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4
+ 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f
), 1/6*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2
*d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*c
os(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)*c
os(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)
)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^
2)*sin(f*x + e))) + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*
c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5*d^2 - 32*c^4*d^3 + 28*c^3*d^4 +
 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x + e)^3 + (2*c^7 - 8*c^6*d - 68*c^5*
```

```
d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*d^5 + 89*c*d^6 + 6*d^7)*cos(f*x +
e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134
*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^10 - 2*a^2*c^9*d - 2*a
^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5*d^5 + 2*a^2*c^4*d^6 + 2*a^2*c^3*d^7
- a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(a^2*c^10 - a^2*c^9*d - 4*a^2*c^8*d^2 +
 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a^2*c^5*d^5 - 4*a^2*c^4*d^6 + 4*a^2*c^3*
d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*cos(f*x + e)^3 + (a^2*c^10 + 2*a^2*c^9*d -
 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 22*a^2*c^6*d^4 - 22*a^2*c^4*d^6 + 4*a^2*c^
3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 + 2*(a^2*c
^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x +
 e) + (a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^3\sec^2(e+fx)+2c^3\sec(e+fx)+c^3+3c^2d\sec^3(e+fx)+6c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^4(e+fx)+6cd^2\sec^3(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^5(e+fx)+2d^3\sec^4(e+fx)+d^3\sec^3(e+fx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x)**2 + 2*c**3*sec(e + f*x) + c**3 +
3*c**2*d*sec(e + f*x)**3 + 6*c**2*d*sec(e + f*x)**2 + 3*c**2*d*sec(e + f*x)
 + 3*c*d**2*sec(e + f*x)**4 + 6*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e + f
*x)**2 + d**3*sec(e + f*x)**5 + 2*d**3*sec(e + f*x)**4 + d**3*sec(e + f*x)*
*3), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 751 vs. 2(267) = 534.
time = 0.56, size = 751, normalized size = 2.64

$$\frac{\frac{6\,(12\,c^2d^2+16\,cd^3+7\,d^4)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(a^2c^6-2\,a^2c^5d-a^2c^4d^2+4\,a^2c^3d^3-a^2c^2d^4-2\,a^2cd^5+a^2d^6)\sqrt{-c^2+d^2}}-\frac{a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,a^4c^5d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^4c^4d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-20\,a^4c^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^4c^2d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,a^4cd^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^4d^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+36\,a^4c^5d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-135\,a^4c^4d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+240\,a^4c^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-225\,a^4c^2d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+108\,a^4cd^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-21\,a^4d^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^6c^9-9\,a^6c^8d+36\,a^6c^7d^2-84\,a^6c^6d^3+126\,a^6c^5d^4-126\,a^6c^4d^5+84\,a^6c^3d^6-36\,a^6c^2d^7+9\,a^6cd^8-a^6d^9}+\frac{6\left(8\,c^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-5\,d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-8\,c^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-11\,cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{(a^2c^6-2\,a^2c^5d-a^2c^4d^2+4\,a^2c^3d^3-a^2c^2d^4-2\,a^2cd^5+a^2d^6)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)^2}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(12*c^2*d^2 + 16*c*d^3 + 7*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sg
n(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/s
qrt(-c^2 + d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3 - a
^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(-c^2 + d^2)) - (a^4*c^6*tan(1/2*f*
x + 1/2*e)^3 - 6*a^4*c^5*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^4*d^2*tan(1/2*
f*x + 1/2*e)^3 - 20*a^4*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^2*d^4*tan
(1/2*f*x + 1/2*e)^3 - 6*a^4*c*d^5*tan(1/2*f*x + 1/2*e)^3 + a^4*d^6*tan(1/2*
```

```
f*x + 1/2*e)^3 - 3*a^4*c^6*tan(1/2*f*x + 1/2*e) + 36*a^4*c^5*d*tan(1/2*f*x
+ 1/2*e) - 135*a^4*c^4*d^2*tan(1/2*f*x + 1/2*e) + 240*a^4*c^3*d^3*tan(1/2*f
*x + 1/2*e) - 225*a^4*c^2*d^4*tan(1/2*f*x + 1/2*e) + 108*a^4*c*d^5*tan(1/2*
f*x + 1/2*e) - 21*a^4*d^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9 - 9*a^6*c^8*d + 36
*a^6*c^7*d^2 - 84*a^6*c^6*d^3 + 126*a^6*c^5*d^4 - 126*a^6*c^4*d^5 + 84*a^6*
c^3*d^6 - 36*a^6*c^2*d^7 + 9*a^6*c*d^8 - a^6*d^9) + 6*(8*c^2*d^3*tan(1/2*f*
x + 1/2*e)^3 - 3*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 5*d^5*tan(1/2*f*x + 1/2*e)^
3 - 8*c^2*d^3*tan(1/2*f*x + 1/2*e) - 11*c*d^4*tan(1/2*f*x + 1/2*e) - 3*d^5*
tan(1/2*f*x + 1/2*e))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3
 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1
/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B]
time = 2.28, size = 505, normalized size = 1.78

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(-8\,c^2\,d^3+3\,c\,d^4+5\,d^5\right)}{{\left(c+d\right)}^2}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(3\,d^4+8\,c\,d^3\right)}{c+d}}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,a^2\,c^6-8\,a^2\,c^5\,d+10\,a^2\,c^4\,d^2-10\,a^2\,c^2\,d^4+8\,a^2\,c\,d^5-2\,a^2\,d^6\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(a^2\,c^6-6\,a^2\,c^5\,d+15\,a^2\,c^4\,d^2-20\,a^2\,c^3\,d^3+15\,a^2\,c^2\,d^4-6\,a^2\,c\,d^5+a^2\,d^6\right)-a^2\,c^6-a^2\,d^6+2\,a^2\,c\,d^5+2\,a^2\,c^5\,d+a^2\,c^2\,d^4-4\,a^2\,c^3\,d^3+a^2\,c^4\,d^2\right)}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{2}{a^2\,{\left(c-d\right)}^3}-\frac{3\,\left(c+d\right)}{2\,a^2\,{\left(c-d\right)}^4}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3}{6\,a^2\,f\,{\left(c-d\right)}^3}-\frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^5-5\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^4\,d+10\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,d^2-10\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d^3+5\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^4-1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^5}{\sqrt{c+d}\,{\left(c-d\right)}^{9/2}}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)\,1\mathrm{i}}{a^2\,f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^3),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^3*(3*c*d^4 + 5*d^5 - 8*c^2*d^3))/(c + d)^2 + (tan(e/2
+ (f*x)/2)*(8*c*d^3 + 3*d^4))/(c + d))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2*c^6
- 2*a^2*d^6 + 8*a^2*c*d^5 - 8*a^2*c^5*d - 10*a^2*c^2*d^4 + 10*a^2*c^4*d^2)
- tan(e/2 + (f*x)/2)^4*(a^2*c^6 + a^2*d^6 - 6*a^2*c*d^5 - 6*a^2*c^5*d + 15*
a^2*c^2*d^4 - 20*a^2*c^3*d^3 + 15*a^2*c^4*d^2) - a^2*c^6 - a^2*d^6 + 2*a^2*
c*d^5 + 2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)) + (tan(e/
2 + (f*x)/2)*(2/(a^2*(c - d)^3) - (3*(c + d))/(2*a^2*(c - d)^4)))/f - tan(e
/2 + (f*x)/2)^3/(6*a^2*f*(c - d)^3) - (d^2*atan((c^5*tan(e/2 + (f*x)/2)*1i
- d^5*tan(e/2 + (f*x)/2)*1i + c*d^4*tan(e/2 + (f*x)/2)*5i - c^4*d*tan(e/2 +
 (f*x)/2)*5i - c^2*d^3*tan(e/2 + (f*x)/2)*10i + c^3*d^2*tan(e/2 + (f*x)/2)*
10i)/((c + d)^(1/2)*(c - d)^(9/2)))*(16*c*d + 12*c^2 + 7*d^2)*1i)/(a^2*f*(c
 + d)^(5/2)*(c - d)^(9/2))
```

3.225 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=363

$$\frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{2d(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136}{15a^3f}$$

```
[Out] 1/2*d^3*(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*arctanh(sin(f*x+e))/a^3/f-2/15*d*
(2*c^5+18*c^4*d+107*c^3*d^2-472*c^2*d^3+456*c*d^4-136*d^5)*tan(f*x+e)/a^3/f
-1/30*d^2*(4*c^4+36*c^3*d+216*c^2*d^2-626*c*d^3+345*d^4)*sec(f*x+e)*tan(f*x
+e)/a^3/f-1/15*d*(2*c^3+18*c^2*d+111*c*d^2-136*d^3)*(c+d*sec(f*x+e))^2*tan(
f*x+e)/a^3/f+1/15*(c-d)*(2*c^2+18*c*d+115*d^2)*(c+d*sec(f*x+e))^3*tan(f*x+e
)/f/(a^3+a^3*sec(f*x+e))+1/15*(c-d)*(2*c+13*d)*(c+d*sec(f*x+e))^4*tan(f*x+e
)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*sec(f*x+e))^5*tan(f*x+e)/f/(a+a*sec
(f*x+e))^3
```

Rubi [A]
time = 0.35, antiderivative size = 405, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4072, 100, 155, 158, 152, 65, 223, 209}

$$\frac{(c-d)\left(2c^2+18cd+115d^2\right)\tan(e+fx)(c+d\sec(e+fx))^3}{15f\left(a^3\sec(e+fx)+a^3\right)}-\frac{d(2c^3+18c^2d+111cd^2-136d^3)\tan(e+fx)(c+d\sec(e+fx))^2}{15a^3f}-\frac{d\tan(e+fx)\left(d(4c^4+36c^3d+216c^2d^2-626cd^3+345d^4)\sec(e+fx)+4(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)\right)}{30a^3f}+\frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}+\frac{(c-d)(2c+13d)\tan(e+fx)(c+d\sec(e+fx))^4}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]
]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (d*(2*c^3 + 18*c^2*d + 111*c*d^2 - 136*d^3)*(c
 + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a^3*f) + ((c - d)*(2*c^2 + 18*c*d +
115*d^2)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]
)) + ((c - d)*(2*c + 13*d)*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(15*a*f*(a
+ a*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f*x])^5*Tan[e + f*x])/(5*f*(
a + a*Sec[e + f*x])^3) - (d*(4*(2*c^5 + 18*c^4*d + 107*c^3*d^2 - 472*c^2*d^
3 + 456*c*d^4 - 136*d^5) + d*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c*d^3 +
345*d^4)*Sec[e + f*x])*Tan[e + f*x])/(30*a^3*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{(c+dx)^6}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\mathrm{Subst}\left(\int \frac{(c}{}\right.}{5af\sqrt{a}}$$

$$= \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d}{5f(a}$$

$$= \frac{(c-d)\left(2c^2+18cd+115d^2\right)(c+d\sec(e+fx))^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c}{}$$

$$= -\frac{d(2c^3+18c^2d+111cd^2-136d^3)\,(c+d\sec(e+fx))^2\tan(e+fx}{15a^3f}$$

$$= -\frac{d(2c^3+18c^2d+111cd^2-136d^3)\,(c+d\sec(e+fx))^2\tan(e+fx}{15a^3f}$$

$$= -\frac{d(2c^3+18c^2d+111cd^2-136d^3)\,(c+d\sec(e+fx))^2\tan(e+fx}{15a^3f}$$

$$= -\frac{d(2c^3+18c^2d+111cd^2-136d^3)\,(c+d\sec(e+fx))^2\tan(e+fx}{15a^3f}$$

$$= \frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\mathrm{ta}}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1338 vs. $2(363) = 726$.
time = 6.72, size = 1338, normalized size = 3.69

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (4*(-40*c^3*d^3 + 90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[
e + f*x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x]
)^6)/(f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*(-40*c^3*d^3 +
90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[e + f*x]^3*Log[Cos
[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^6)/(f*(d + c*Cos
[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]^2*Cos[e + f*x]
^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[e/2] - 6*c^5*d*Sin[e/2] + 15*c^
4*d^2*Sin[e/2] - 20*c^3*d^3*Sin[e/2] + 15*c^2*d^4*Sin[e/2] - 6*c*d^5*Sin[e/
2] + d^6*Sin[e/2]))/(5*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (
8*Cos[e/2 + (f*x)/2]^4*Cos[e + f*x]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(-4*c
^6*Sin[e/2] + 9*c^5*d*Sin[e/2] + 15*c^4*d^2*Sin[e/2] - 70*c^3*d^3*Sin[e/2]
+ 90*c^2*d^4*Sin[e/2] - 51*c*d^5*Sin[e/2] + 11*d^6*Sin[e/2]))/(15*f*(d + c*
Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]*Cos[e + f*x
]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[(f*x)/2] - 6*c^5*d*Sin[(f*x)/2
] + 15*c^4*d^2*Sin[(f*x)/2] - 20*c^3*d^3*Sin[(f*x)/2] + 15*c^2*d^4*Sin[(f*x
)/2] - 6*c*d^5*Sin[(f*x)/2] + d^6*Sin[(f*x)/2]))/(5*f*(d + c*Cos[e + f*x])^
6*(a + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^3*Cos[e + f*x]^3*Sec[e/2]
*(c + d*Sec[e + f*x])^6*(-4*c^6*Sin[(f*x)/2] + 9*c^5*d*Sin[(f*x)/2] + 15*c^
4*d^2*Sin[(f*x)/2] - 70*c^3*d^3*Sin[(f*x)/2] + 90*c^2*d^4*Sin[(f*x)/2] - 51
*c*d^5*Sin[(f*x)/2] + 11*d^6*Sin[(f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^6*(a
 + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^5*Cos[e + f*x]^3*Sec[e/2]*(c
+ d*Sec[e + f*x])^6*(7*c^6*Sin[(f*x)/2] + 18*c^5*d*Sin[(f*x)/2] + 30*c^4*d^
2*Sin[(f*x)/2] - 440*c^3*d^3*Sin[(f*x)/2] + 855*c^2*d^4*Sin[(f*x)/2] - 642*
c*d^5*Sin[(f*x)/2] + 172*d^6*Sin[(f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^6*(a
 + a*Sec[e + f*x])^3) + (8*d^6*Cos[e/2 + (f*x)/2]^6*Sec[e]*(c + d*Sec[e + f
*x])^6*Sin[f*x])/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*C
os[e/2 + (f*x)/2]^6*Cos[e + f*x]^2*Sec[e]*(c + d*Sec[e + f*x])^6*(-18*c*d^5
*Sin[e] + 9*d^6*Sin[e] - 90*c^2*d^4*Sin[f*x] + 108*c*d^5*Sin[f*x] - 40*d^6*
Sin[f*x]))/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (4*Cos[e/2
 + (f*x)/2]^6*Cos[e + f*x]*Sec[e]*(c + d*Sec[e + f*x])^6*(2*d^6*Sin[e] + 18
*c*d^5*Sin[f*x] - 9*d^6*Sin[f*x]))/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e
 + f*x])^3)
```

Maple [A]

time = 0.27, size = 579, normalized size = 1.60

method	result
derivativedivides	$\frac{-\frac{2d^4\left(30c^2-42cd+17d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4d^5(3c-2d)}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+6c^5d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+15c^4d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+255c^2d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-186c\,d^5\tan}{}$
default	$\frac{-\frac{2d^4\left(30c^2-42cd+17d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4d^5(3c-2d)}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+6c^5d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+15c^4d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+255c^2d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-186c\,d^5\tan}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f/a^3*(-2*d^4*(30*c^2-42*c*d+17*d^2)/(tan(1/2*f*x+1/2*e)+1)-4*d^5*(3*c-
2*d)/(tan(1/2*f*x+1/2*e)+1)^2-6/5*c^5*d*tan(1/2*f*x+1/2*e)^5+3*c^4*d^2*tan(
1/2*f*x+1/2*e)^5+3*c^2*d^4*tan(1/2*f*x+1/2*e)^5-6/5*c*d^5*tan(1/2*f*x+1/2*e
)^5+10*c^4*d^2*tan(1/2*f*x+1/2*e)^3+30*c^2*d^4*tan(1/2*f*x+1/2*e)^3+6*c^5*d
*tan(1/2*f*x+1/2*e)+15*c^4*d^2*tan(1/2*f*x+1/2*e)+255*c^2*d^4*tan(1/2*f*x+1
/2*e)-186*c*d^5*tan(1/2*f*x+1/2*e)-2*d^3*(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*
ln(tan(1/2*f*x+1/2*e)-1)-2*d^4*(30*c^2-42*c*d+17*d^2)/(tan(1/2*f*x+1/2*e)-1
)+4*d^5*(3*c-2*d)/(tan(1/2*f*x+1/2*e)-1)^2-80/3*c^3*d^3*tan(1/2*f*x+1/2*e)^
3-16*c*d^5*tan(1/2*f*x+1/2*e)^3-140*c^3*d^3*tan(1/2*f*x+1/2*e)-4*c^3*d^3*ta
n(1/2*f*x+1/2*e)^5+2*d^3*(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*ln(tan(1/2*f*x+1
/2*e)+1)-4/3*d^6/(tan(1/2*f*x+1/2*e)-1)^3+1/5*c^6*tan(1/2*f*x+1/2*e)^5+1/5*
d^6*tan(1/2*f*x+1/2*e)^5-2/3*c^6*tan(1/2*f*x+1/2*e)^3+10/3*d^6*tan(1/2*f*x+
1/2*e)^3+c^6*tan(1/2*f*x+1/2*e)+49*d^6*tan(1/2*f*x+1/2*e)-4/3*d^6/(tan(1/2*
f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(364) = 728$.
time = 0.32, size = 1026, normalized size = 2.83

$$\frac{d^6\left(\frac{20\left(\frac{33\sin(fx+e)}{\cos(fx+e)+1}-\frac{76\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{51\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3-\frac{3a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{a^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}+\frac{\frac{735\sin(fx+e)}{\cos(fx+e)+1}+\frac{50\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{690\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{690\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)-6cd^5\left(\frac{60\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^3-\frac{2a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{465\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+45c^2d^4\left(\frac{40\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)-20c^3d^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+\frac{15c^4d^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^6\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{18c^5d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(d^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*x +
e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 690
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 6*c*d^5*(60*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 - 2*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(sin(f*x + e
)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)/a^3) + 45*c^2*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^3) - 20*c^3*d^3*((105*sin(f*x + e)/(cos(f*x + e)
+ 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
```

```
 e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^4*d^2*(15*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3 + c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3 + 18*c^5*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 1.78, size = 640, normalized size = 1.76

$$\frac{15\left((40c^3d^3-90c^2d^4+78cd^5-23d^6)\cos(fx+e)^6+3(40c^3d^3-90c^2d^4+78cd^5-23d^6)\cos(fx+e)^5+3(40c^3d^3-90c^2d^4+78cd^5-23d^6)\cos(fx+e)^4+(40c^3d^3-90c^2d^4+78cd^5-23d^6)\cos(fx+e)^3\right)\log(\sin(fx+e)+1)-15\left((40c^3d^3-90c^2d^4+78cd^5-23d^6)\cos(fx+e)^6+3(40c^3d^3-90c^2d^4+78cd^5-23d^6)\cos(fx+e)^5+3(40c^3d^3-90c^2d^4+78cd^5-23d^6)\cos(fx+e)^4+(40c^3d^3-90c^2d^4+78cd^5-23d^6)\cos(fx+e)^3\right)\log(-\sin(fx+e)+1)+2\left(10d^6+2(7c^6+18c^5d+30c^4d^2-440c^3d^3+1080c^2d^4-912cd^5+272d^6)\cos(fx+e)^5+3(4c^6+36c^5d+60c^4d^2-680c^3d^3+1710c^2d^4-1434cd^5+429d^6)\cos(fx+e)^4+(4c^6+36c^5d+210c^4d^2-1280c^3d^3+3510c^2d^4-2874cd^5+869d^6)\cos(fx+e)^3+5(90c^2d^4-54cd^5+19d^6)\cos(fx+e)^2+15(6cd^5-d^6)\cos(fx+e)\right)\sin(fx+e)}{60\left(a^3f\cos(fx+e)^6+3a^3f\cos(fx+e)^5+3a^3f\cos(fx+e)^4+a^3f\cos(fx+e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*
(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^
3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d
^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((40*c^3
*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*(40*c^3*d^3 - 90*
c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^3 - 90*c^2*d^4 +
78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 2
3*d^6)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(10*d^6 + 2*(7*c^6 + 18*c
^5*d + 30*c^4*d^2 - 440*c^3*d^3 + 1080*c^2*d^4 - 912*c*d^5 + 272*d^6)*cos(f
*x + e)^5 + 3*(4*c^6 + 36*c^5*d + 60*c^4*d^2 - 680*c^3*d^3 + 1710*c^2*d^4 -
 1434*c*d^5 + 429*d^6)*cos(f*x + e)^4 + (4*c^6 + 36*c^5*d + 210*c^4*d^2 - 1
280*c^3*d^3 + 3510*c^2*d^4 - 2874*c*d^5 + 869*d^6)*cos(f*x + e)^3 + 5*(90*c
^2*d^4 - 54*c*d^5 + 19*d^6)*cos(f*x + e)^2 + 15*(6*c*d^5 - d^6)*cos(f*x + e
))*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3*a^3*f*c
os(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^6\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int \frac{d^6\sec^7(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int \frac{6cd^5\sec^6(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int \frac{15c^2d^4\sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int \frac{20c^3d^3\sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int \frac{15c^4d^2\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int \frac{6c^5d\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**6*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**6*sec(e + f*x)**7/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*d**5*sec(e + f*x)**6/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c
**2*d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
```

```
*x) + 1), x) + Integral(20*c**3*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*s
ec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c**4*d**2*sec(e + f*
x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(6*c**5*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.73, size = 672, normalized size = 1.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*log(abs(tan(1/2*f*x
+ 1/2*e) + 1))/a^3 - 30*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 20*(90*c^2*d^4*tan(1/2*f*x + 1/2*e)^5 -
 126*c*d^5*tan(1/2*f*x + 1/2*e)^5 + 51*d^6*tan(1/2*f*x + 1/2*e)^5 - 180*c^2
*d^4*tan(1/2*f*x + 1/2*e)^3 + 216*c*d^5*tan(1/2*f*x + 1/2*e)^3 - 76*d^6*tan
(1/2*f*x + 1/2*e)^3 + 90*c^2*d^4*tan(1/2*f*x + 1/2*e) - 90*c*d^5*tan(1/2*f*
x + 1/2*e) + 33*d^6*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a
^3) + (3*a^12*c^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^12*c^5*d*tan(1/2*f*x + 1/2*
e)^5 + 45*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^12*c^3*d^3*tan(1/2*f*x
 + 1/2*e)^5 + 45*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 18*a^12*c*d^5*tan(1/
2*f*x + 1/2*e)^5 + 3*a^12*d^6*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^6*tan(1/2*
f*x + 1/2*e)^3 + 150*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 400*a^12*c^3*d^3
*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 240*a^1
2*c*d^5*tan(1/2*f*x + 1/2*e)^3 + 50*a^12*d^6*tan(1/2*f*x + 1/2*e)^3 + 15*a^
12*c^6*tan(1/2*f*x + 1/2*e) + 90*a^12*c^5*d*tan(1/2*f*x + 1/2*e) + 225*a^12
*c^4*d^2*tan(1/2*f*x + 1/2*e) - 2100*a^12*c^3*d^3*tan(1/2*f*x + 1/2*e) + 38
25*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e) - 2790*a^12*c*d^5*tan(1/2*f*x + 1/2*e)
 + 735*a^12*d^6*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.91, size = 327, normalized size = 0.90

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,{\left(c-d\right)}^6}{2\,a^3}-\frac{6\,\left(c+d\right)\,{\left(c-d\right)}^5}{a^3}+\frac{15\,{\left(c+d\right)}^2\,{\left(c-d\right)}^4}{4\,a^3}\right)}{f}-\frac{\left(30\,c^2\,d^4-42\,c\,d^5+17\,d^6\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5+\left(-60\,c^2\,d^4+72\,c\,d^5-\frac{76\,d^6}{3}\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(30\,c^2\,d^4-30\,c\,d^5+11\,d^6\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^6-3\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4+3\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-a^3\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{{\left(c-d\right)}^6}{3\,a^3}-\frac{\left(c+d\right)\,{\left(c-d\right)}^5}{2\,a^3}\right)}{f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^6}{20\,a^3\,f}+\frac{d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(40\,c^3-90\,c^2\,d+78\,c\,d^2-23\,d^3\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^6/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((5*(c - d)^6)/(2*a^3) - (6*(c + d)*(c - d)^5)/a^3 + (1
5*(c + d)^2*(c - d)^4)/(4*a^3)))/f - (tan(e/2 + (f*x)/2)*(11*d^6 - 30*c*d^5
 + 30*c^2*d^4) + tan(e/2 + (f*x)/2)^5*(17*d^6 - 42*c*d^5 + 30*c^2*d^4) - ta
n(e/2 + (f*x)/2)^3*((76*d^6)/3 - 72*c*d^5 + 60*c^2*d^4))/(f*(3*a^3*tan(e/2
```

```
+ (f*x)/2)^2 - 3*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^6 - a^3)
) + (tan(e/2 + (f*x)/2)^3*((c - d)^6/(3*a^3) - ((c + d)*(c - d)^5)/(2*a^3))
)/f + (tan(e/2 + (f*x)/2)^5*(c - d)^6)/(20*a^3*f) + (d^3*atanh(tan(e/2 + (f
*x)/2))*(78*c*d^2 - 90*c^2*d + 40*c^3 - 23*d^3))/(a^3*f)
```

3.226 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=287

$$\frac{d^3(20c^2-30cd+13d^2)\tanh^{-1}(\sin(e+fx))}{2a^3f}-\frac{2d(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)\tan(e+fx)}{15a^3f}-d^2(4$$

```
[Out] 1/2*d^3*(20*c^2-30*c*d+13*d^2)*arctanh(sin(f*x+e))/a^3/f-2/15*d*(2*c^4+15*c
^3*d+72*c^2*d^2-180*c*d^3+76*d^4)*tan(f*x+e)/a^3/f-1/30*d^2*(4*c^3+30*c^2*d
+146*c*d^2-195*d^3)*sec(f*x+e)*tan(f*x+e)/a^3/f+1/15*(c-d)*(2*c^2+15*c*d+76
*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+1/15*(c-d)*(2*c+
11*d)*(c+d*sec(f*x+e))^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*s
ec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A]
time = 0.28, antiderivative size = 329, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 100, 155, 152, 65, 223, 209}

$$\frac{(c-d)\left(2c^2+15cd+76d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{15f\left(a^3\sec(e+fx)+a^3\right)}-\frac{d\tan(e+fx)\left(d(4c^3+30c^2d+146cd^2-195d^3)\sec(e+fx)+4(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)\right)}{30a^3f}+\frac{d^3(20c^2-30cd+13d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}+\frac{(c-d)(2c+11d)\tan(e+fx)(c+d\sec(e+fx))^3}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 +
 Sec[e + f*x])]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) + ((c - d)*(2*c^2 + 15*c*d + 76*d^2)*(c + d*Sec[e + f*x])^2*T
an[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])) + ((c - d)*(2*c + 11*d)*(c + d
*Sec[e + f*x])^3*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((c - d)*(
c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (d*(4*(2
*c^4 + 15*c^3*d + 72*c^2*d^2 - 180*c*d^3 + 76*d^4) + d*(4*c^3 + 30*c^2*d +
146*c*d^2 - 195*d^3)*Sec[e + f*x])*Tan[e + f*x])/(30*a^3*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
```

```
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
```

```
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^5}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c \ldots}{\ldots}\right)}{5af\sqrt{a- \ldots}}$$

$$= \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d \ldots}{5f(a \ldots}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c \ldots}{\ldots}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c \ldots}{\ldots}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c \ldots}{\ldots}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c \ldots}{\ldots}$$

$$= \frac{d^3(20c^2-30cd+13d^2)\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx \ldots}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A]
time = 2.12, size = 439, normalized size = 1.53

$$\frac{-480d^3(20c^2-30cd+13d^2)\cos^6\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+2\cos\left(\frac{1}{2}(e+fx)\right)\left(29c^5+105c^4d+340c^3d^2-1940c^2d^3+3420cd^4-1354d^5+[illegible]1910cd^4-777d^5)\cos(e+fx)+6(6c^5+20c^4d+60c^3d^2-360c^2d^3+630cd^4-261d^5)\cos(2(e+fx))+12c^5\cos(3(e+fx))+90c^4d\cos(3(e+fx))+120c^3d^2\cos(3(e+fx))-1020c^2d^3\cos(3(e+fx))+1710cd^4\cos(3(e+fx))-717d^5\cos(3(e+fx))+7c^5\cos(4(e+fx))+15c^4d\cos(4(e+fx))+20c^3d^2\cos(4(e+fx))-220c^2d^3\cos(4(e+fx))+360cd^4\cos(4(e+fx))-152d^5\cos(4(e+fx))\right)\sec^2(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)}{120a^3f(1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

[Out] (-480*d^3*(20*c^2 - 30*c*d + 13*d^2)*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[
(e + f*x)/2]*(29*c^5 + 105*c^4*d + 340*c^3*d^2 - 1940*c^2*d^3 + 3420*c*d^4
- 1354*d^5 + 3*(12*c^5 + 90*c^4*d + 120*c^3*d^2 - 1020*c^2*d^3 + 1910*c*d^4
 - 777*d^5)*Cos[e + f*x] + 6*(6*c^5 + 20*c^4*d + 60*c^3*d^2 - 360*c^2*d^3 +
 630*c*d^4 - 261*d^5)*Cos[2*(e + f*x)] + 12*c^5*Cos[3*(e + f*x)] + 90*c^4*d
Cos[3(e + f*x)] + 120*c^3*d^2*Cos[3*(e + f*x)] - 1020*c^2*d^3*Cos[3*(e +
f*x)] + 1710*c*d^4*Cos[3*(e + f*x)] - 717*d^5*Cos[3*(e + f*x)] + 7*c^5*Cos[
4*(e + f*x)] + 15*c^4*d*Cos[4*(e + f*x)] + 20*c^3*d^2*Cos[4*(e + f*x)] - 22
0*c^2*d^3*Cos[4*(e + f*x)] + 360*c*d^4*Cos[4*(e + f*x)] - 152*d^5*Cos[4*(e
+ f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(120*a^3*f*(1 + Cos[e + f*x])^3)

Maple [A]
time = 0.28, size = 441, normalized size = 1.54 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)

[Out] 1/4/f/a^3*(1/5*c^5*tan(1/2*f*x+1/2*e)^5-c^4*d*tan(1/2*f*x+1/2*e)^5+2*c^3*d^
2*tan(1/2*f*x+1/2*e)^5-2*c^2*d^3*tan(1/2*f*x+1/2*e)^5+c*d^4*tan(1/2*f*x+1/2
*e)^5-1/5*d^5*tan(1/2*f*x+1/2*e)^5-2/3*c^5*tan(1/2*f*x+1/2*e)^3+20/3*c^3*d^
2*tan(1/2*f*x+1/2*e)^3-40/3*c^2*d^3*tan(1/2*f*x+1/2*e)^3+10*c*d^4*tan(1/2*f
*x+1/2*e)^3-8/3*d^5*tan(1/2*f*x+1/2*e)^3+c^5*tan(1/2*f*x+1/2*e)+5*c^4*d*tan
(1/2*f*x+1/2*e)+10*c^3*d^2*tan(1/2*f*x+1/2*e)-70*c^2*d^3*tan(1/2*f*x+1/2*e)
+85*c*d^4*tan(1/2*f*x+1/2*e)-31*d^5*tan(1/2*f*x+1/2*e)-2*d^5/(tan(1/2*f*x+1
/2*e)+1)^2+2*d^3*(20*c^2-30*c*d+13*d^2)*ln(tan(1/2*f*x+1/2*e)+1)-2*d^4*(10*
c-7*d)/(tan(1/2*f*x+1/2*e)+1)+2*d^5/(tan(1/2*f*x+1/2*e)-1)^2-2*d^3*(20*c^2-
30*c*d+13*d^2)*ln(tan(1/2*f*x+1/2*e)-1)-2*d^4*(10*c-7*d)/(tan(1/2*f*x+1/2*e
)-1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(288) = 576$.
time = 0.30, size = 747, normalized size = 2.60

$$-\frac{d^{5}\left(\frac{60\left(\frac{5\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{7\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}\right)}{a^{3}-\frac{2a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{a^{3}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}}+\frac{\frac{465\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{40\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}}{a^{3}}-\frac{390\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{3}}+\frac{390\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^{3}}\right)-15cd^{4}\left(\frac{40\sin\left(fx+e\right)}{\left(a^{3}-\frac{a^{3}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\left(\cos\left(fx+e\right)+1\right)}+\frac{\frac{85\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}}{a^{3}}-\frac{60\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{3}}+\frac{60\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^{3}}\right)+10c^{2}d^{3}\left(\frac{\frac{105\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{20\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}}{a^{3}}-\frac{60\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)}{a^{3}}+\frac{60\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)}{a^{3}}\right)-\frac{10c^{3}d^{2}\left(\frac{15\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{10\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}}-\frac{c^{5}\left(\frac{15\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{10\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{3\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}}-\frac{15c^{4}d\left(\frac{5\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}\right)}{a^{3}}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")

[Out] -1/60*(d^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 4
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*

```
x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 15*c*d^4*(40*sin(f*x + e)/((a^3 - a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e
)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e
)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^2*d^3*((105*
sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 10*c
^3*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^5*(15*sin(f*x + e
)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5)/a^3 - 15*c^4*d*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 1.86, size = 521, normalized size = 1.82

$$\frac{15\left(\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\cos\left(fx+e\right)^{5}+3\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\cos\left(fx+e\right)^{4}+3\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\cos\left(fx+e\right)^{3}+\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\cos\left(fx+e\right)^{2}\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\cos\left(fx+e\right)^{5}+3\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\cos\left(fx+e\right)^{4}+3\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\cos\left(fx+e\right)^{3}+\left(20\,c^{2}d^{3}-30\,cd^{4}+13\,d^{5}\right)\cos\left(fx+e\right)^{2}\right)\log\left(-\sin\left(fx+e\right)+1\right)+2\left(15\,d^{5}+2\left(7\,c^{5}+15\,c^{4}d+20\,c^{3}d^{2}-220\,c^{2}d^{3}+360\,cd^{4}-152\,d^{5}\right)\cos\left(fx+e\right)^{4}+3\left(4\,c^{5}+30\,c^{4}d+40\,c^{3}d^{2}-340\,c^{2}d^{3}+570\,cd^{4}-239\,d^{5}\right)\cos\left(fx+e\right)^{3}+\left(4\,c^{5}+30\,c^{4}d+140\,c^{3}d^{2}-640\,c^{2}d^{3}+1170\,cd^{4}-479\,d^{5}\right)\cos\left(fx+e\right)^{2}+15\left(10\,cd^{4}-3\,d^{5}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{60\left(a^{3}f\cos\left(fx+e\right)^{5}+3\,a^{3}f\cos\left(fx+e\right)^{4}+3\,a^{3}f\cos\left(fx+e\right)^{3}+a^{3}f\cos\left(fx+e\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2*d^3 -
 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos
(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(sin(f*x
+ e) + 1) - 15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2
*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^
5)*cos(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(-s
in(f*x + e) + 1) + 2*(15*d^5 + 2*(7*c^5 + 15*c^4*d + 20*c^3*d^2 - 220*c^2*d
^3 + 360*c*d^4 - 152*d^5)*cos(f*x + e)^4 + 3*(4*c^5 + 30*c^4*d + 40*c^3*d^2
 - 340*c^2*d^3 + 570*c*d^4 - 239*d^5)*cos(f*x + e)^3 + (4*c^5 + 30*c^4*d +
140*c^3*d^2 - 640*c^2*d^3 + 1170*c*d^4 - 479*d^5)*cos(f*x + e)^2 + 15*(10*c
*d^4 - 3*d^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^5 + 3*a^3*f*c
os(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^{5}\sec\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{d^{5}\sec^{6}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{5cd^{4}\sec^{5}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{10c^{2}d^{3}\sec^{4}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{10c^{3}d^{2}\sec^{3}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int \frac{5c^{4}d\sec^{2}\left(e+fx\right)}{\sec^{3}\left(e+fx\right)+3\sec^{2}\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**5*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e
```

```
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c*d**4*sec(e + f*x)**5/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*c
**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*s
ec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**
2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.61, size = 504, normalized size = 1.76

$$\frac{\frac{30\,(20\,c^2d^3-30\,cd^4+13\,d^5)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{30\,(20\,c^2d^3-30\,cd^4+13\,d^5)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{60\left(10\,cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-7\,d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-10\,cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+5\,d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^3}+\frac{3\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-15\,a^{12}c^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+30\,a^{12}c^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-30\,a^{12}c^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+15\,a^{12}cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-3\,a^{12}d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+100\,a^{12}c^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-200\,a^{12}c^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+150\,a^{12}cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-40\,a^{12}d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,a^{12}c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+75\,a^{12}c^4d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+150\,a^{12}c^3d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1050\,a^{12}c^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1275\,a^{12}cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-465\,a^{12}d^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*log(abs(tan(1/2*f*x + 1/2*e) + 1)
)/a^3 - 30*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*log(abs(tan(1/2*f*x + 1/2*e) -
1))/a^3 - 60*(10*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 7*d^5*tan(1/2*f*x + 1/2*e)^
3 - 10*c*d^4*tan(1/2*f*x + 1/2*e) + 5*d^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f
*x + 1/2*e)^2 - 1)^2*a^3) + (3*a^12*c^5*tan(1/2*f*x + 1/2*e)^5 - 15*a^12*c^
4*d*tan(1/2*f*x + 1/2*e)^5 + 30*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 30*a^
12*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 15*a^12*c*d^4*tan(1/2*f*x + 1/2*e)^5 -
3*a^12*d^5*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 10
0*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 200*a^12*c^2*d^3*tan(1/2*f*x + 1/2*
e)^3 + 150*a^12*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 40*a^12*d^5*tan(1/2*f*x + 1/
2*e)^3 + 15*a^12*c^5*tan(1/2*f*x + 1/2*e) + 75*a^12*c^4*d*tan(1/2*f*x + 1/2
*e) + 150*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e) - 1050*a^12*c^2*d^3*tan(1/2*f*x
 + 1/2*e) + 1275*a^12*c*d^4*tan(1/2*f*x + 1/2*e) - 465*a^12*d^5*tan(1/2*f*x
 + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.87, size = 252, normalized size = 0.88

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,{\left(c-d\right)}^5}{2\,a^3}-\frac{15\,\left(c+d\right)\,{\left(c-d\right)}^4}{4\,a^3}+\frac{5\,{\left(c+d\right)}^2\,{\left(c-d\right)}^3}{2\,a^3}\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(10\,c\,d^4-5\,d^5\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(10\,c\,d^4-7\,d^5\right)}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a^3\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{{\left(c-d\right)}^5}{4\,a^3}-\frac{5\,\left(c+d\right)\,{\left(c-d\right)}^4}{12\,a^3}\right)}{f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^5}{20\,a^3\,f}+\frac{d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(20\,c^2-30\,c\,d+13\,d^2\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*(c - d)^5)/(2*a^3) - (15*(c + d)*(c - d)^4)/(4*a^3)
 + (5*(c + d)^2*(c - d)^3)/(2*a^3)))/f + (tan(e/2 + (f*x)/2)*(10*c*d^4 - 5*
d^5) - tan(e/2 + (f*x)/2)^3*(10*c*d^4 - 7*d^5))/(f*(a^3*tan(e/2 + (f*x)/2)^
4 - 2*a^3*tan(e/2 + (f*x)/2)^2 + a^3)) + (tan(e/2 + (f*x)/2)^3*((c - d)^5/(
4*a^3) - (5*(c + d)*(c - d)^4)/(12*a^3)))/f + (tan(e/2 + (f*x)/2)^5*(c - d)
^5)/(20*a^3*f) + (d^3*atanh(tan(e/2 + (f*x)/2))*(20*c^2 - 30*c*d + 13*d^2))
/(a^3*f)
```

3.227 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=205

$$\frac{(4c-3d)d^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{(c-d)(c+d\sec(e+\ldots}{5f(a+a\sec(\ldots}$$

```
[Out] (4*c-3*d)*d^3*arctanh(sin(f*x+e))/a^3/f+1/15*(c-d)*(2*c+9*d)*(c+d*sec(f*x+e
))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*sec(f*x+e))^3*tan(f*x
+e)/f/(a+a*sec(f*x+e))^3+1/15*(2*c^4+8*c^3*d+21*c^2*d^2-88*c*d^3+72*d^4-d^2
*(2*c^2+10*c*d-27*d^2)*sec(f*x+e))*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A]
time = 0.19, antiderivative size = 265, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 100, 155, 148, 65, 223, 209}

$$\frac{\tan(e+fx)\left(2c^4+8c^3d-d^2(2c^2+10cd-27d^2)\sec(e+fx)+21c^2d^2-88cd^3+72d^4\right)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{2d^3(4c-3d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}+\frac{(c-d)(2c+9d)\tan(e+fx)(c+d\sec(e+fx))^2}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*(4*c - 3*d)*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 + ((c - d)*(2*c + 9*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a
*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a +
 a*Sec[e + f*x])^3) + ((2*c^4 + 8*c^3*d + 21*c^2*d^2 - 88*c*d^3 + 72*d^4 -
d^2*(2*c^2 + 10*c*d - 27*d^2)*Sec[e + f*x])*Tan[e + f*x])/(15*f*(a^3 + a^3*
Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c}{}\right.}{5af\sqrt{a-}}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\,\text{s}}{5f(a-}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\,\text{s}}{5f(a-}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\,\text{s}}{5f(a-}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\,\text{s}}{5f(a-}$$

$$= \frac{2(4c-3d)d^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d}{}$$

Mathematica [A]
time = 2.40, size = 292, normalized size = 1.42

$$\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(3(c-d)^4\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-8(c-d)^3(2c+3d)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+4(c-d)^2\left(7c^2+26cd+57d^2\right)\cos^4\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-60d^3\cos^5\left(\frac{1}{2}(e+fx)\right)\left((4c-3d)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-d\sec(e)\sec(e+fx)\sin(fx)\right)+3(c-d)^4\cos\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)-8(c-d)^3(2c+3d)\cos^3\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{e}{2}\right)\right)}{15a^3f(1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(3*(c - d)^4*Sec[e/2]*Sin[(f*x)/2] - 8*(c - d)^3*(2*c +
 3*d)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f*x)/2] + 4*(c - d)^2*(7*c^2 + 26*c*
d + 57*d^2)*Cos[(e + f*x)/2]^4*Sec[e/2]*Sin[(f*x)/2] - 60*d^3*Cos[(e + f*x)
/2]^5*((4*c - 3*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]) - d*Sec[e]*Sec[e + f*x]*Sin[f*x]) + 3*(c - d)
^4*Cos[(e + f*x)/2]*Tan[e/2] - 8*(c - d)^3*(2*c + 3*d)*Cos[(e + f*x)/2]^3*T
an[e/2]))/(15*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A]
time = 0.24, size = 321, normalized size = 1.57 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f/a^3*(1/5*c^4*tan(1/2*f*x+1/2*e)^5-4/5*c^3*d*tan(1/2*f*x+1/2*e)^5+6/5*
c^2*d^2*tan(1/2*f*x+1/2*e)^5-4/5*c*d^3*tan(1/2*f*x+1/2*e)^5+1/5*d^4*tan(1/2
*f*x+1/2*e)^5-2/3*c^4*tan(1/2*f*x+1/2*e)^3+4*c^2*d^2*tan(1/2*f*x+1/2*e)^3-1
6/3*c*d^3*tan(1/2*f*x+1/2*e)^3+2*d^4*tan(1/2*f*x+1/2*e)^3+c^4*tan(1/2*f*x+1
/2*e)+4*c^3*d*tan(1/2*f*x+1/2*e)+6*c^2*d^2*tan(1/2*f*x+1/2*e)-28*c*d^3*tan(
1/2*f*x+1/2*e)+17*d^4*tan(1/2*f*x+1/2*e)-4*d^4/(tan(1/2*f*x+1/2*e)+1)+4*d^3
*(4*c-3*d)*ln(tan(1/2*f*x+1/2*e)+1)-4*d^4/(tan(1/2*f*x+1/2*e)-1)-4*d^3*(4*c
-3*d)*ln(tan(1/2*f*x+1/2*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(209) = 418$.
time = 0.29, size = 515, normalized size = 2.51

$$\frac{3\,d^4\left(\frac{40\,\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)-4\,cd^3\left(\frac{\frac{105\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+\frac{6\,c^2d^2\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^4\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{12\,c^3d\left(\frac{5\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^3) - 4*c*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e
)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^2*d^2*(15*sin(f*x + e)/(cos(f*x + e) +
 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 12*c^
3*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^
5)/a^3)/f
```

Fricas [A]
time = 1.35, size = 403, normalized size = 1.97

$$\frac{15\left((4\,cd^3-3\,d^4)\cos(fx+e)^4+3\,(4\,cd^3-3\,d^4)\cos(fx+e)^3+3\,(4\,cd^3-3\,d^4)\cos(fx+e)^2+(4\,cd^3-3\,d^4)\cos(fx+e)\right)\log(\sin(fx+e)+1)-15\left((4\,cd^3-3\,d^4)\cos(fx+e)^4+3\,(4\,cd^3-3\,d^4)\cos(fx+e)^3+3\,(4\,cd^3-3\,d^4)\cos(fx+e)^2+(4\,cd^3-3\,d^4)\cos(fx+e)\right)\log(-\sin(fx+e)+1)+2\left(15\,d^4+(7\,c^4+12\,c^3d+12\,c^2d^2-88\,cd^3+72\,d^4)\cos(fx+e)^3+3\,(2\,c^4+12\,c^3d+12\,c^2d^2-68\,cd^3+57\,d^4)\cos(fx+e)^2+(2\,c^4+12\,c^3d+42\,c^2d^2-128\,cd^3+117\,d^4)\cos(fx+e)\right)\sin(fx+e)}{30\left(a^3f\cos(fx+e)^4+3\,a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2+a^3f\cos(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3 - 3*d^4)*cos(f*x +
e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3*d^4)*cos(f*x + e))
*log(sin(f*x + e) + 1) - 15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3
- 3*d^4)*cos(f*x + e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3
*d^4)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(15*d^4 + (7*c^4 + 12*c^3*d
+ 12*c^2*d^2 - 88*c*d^3 + 72*d^4)*cos(f*x + e)^3 + 3*(2*c^4 + 12*c^3*d + 12
*c^2*d^2 - 68*c*d^3 + 57*d^4)*cos(f*x + e)^2 + (2*c^4 + 12*c^3*d + 42*c^2*d
^2 - 128*c*d^3 + 117*d^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^4
 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^4 \sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^4 \sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{4cd^3 \sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{6c^2d^2 \sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{4c^3 d \sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*
*2*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*
x) + 1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.58, size = 374, normalized size = 1.82

$$-\frac{\frac{120\,d^4 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3} - \frac{60\,(4\,cd^3-3\,d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} + \frac{60\,(4\,cd^3-3\,d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} - \frac{3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 12\,a^{12}c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 18\,a^{12}c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 12\,a^{12}cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 3\,a^{12}d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 10\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 60\,a^{12}c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 80\,a^{12}cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,a^{12}d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 60\,a^{12}c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 90\,a^{12}c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - 420\,a^{12}cd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 255\,a^{12}d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 60
*(4*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 + 60*(4*c*d^3 - 3
*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - (3*a^12*c^4*tan(1/2*f*x + 1/
2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12*c^2*d^2*tan(1/2*f*x
 + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^4*tan(1/2*f*x
 + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 60*a^12*c^2*d^2*tan(1/2*
f*x + 1/2*e)^3 - 80*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*d^4*tan(1/2
*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e) + 60*a^12*c^3*d*tan(1/2*
```

```
f*x + 1/2*e) + 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 420*a^12*c*d^3*tan(1/
2*f*x + 1/2*e) + 255*a^12*d^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B]
time = 1.82, size = 195, normalized size = 0.95

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,{\left(c-d\right)}^4}{4\,a^3}+\frac{3\,{\left(c^2-d^2\right)}^2}{2\,a^3}-\frac{2\,\left(c+d\right)\,{\left(c-d\right)}^3}{a^3}\right)}{f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{{\left(c-d\right)}^4}{6\,a^3}-\frac{\left(c+d\right)\,{\left(c-d\right)}^3}{3\,a^3}\right)}{f}-\frac{2\,d^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2-a^3\right)}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^4}{20\,a^3\,f}+\frac{2\,d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(4\,c-3\,d\right)}{a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*(c - d)^4)/(4*a^3) + (3*(c^2 - d^2)^2)/(2*a^3) - (2
*(c + d)*(c - d)^3)/a^3))/f + (tan(e/2 + (f*x)/2)^3*((c - d)^4/(6*a^3) - ((
c + d)*(c - d)^3)/(3*a^3)))/f - (2*d^4*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2
+ (f*x)/2)^2 - a^3)) + (tan(e/2 + (f*x)/2)^5*(c - d)^4)/(20*a^3*f) + (2*d^3
*atanh(tan(e/2 + (f*x)/2))*(4*c - 3*d))/(a^3*f)
```

3.228 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{d^3\tanh^{-1}(\sin(e+fx))}{a^3f}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(c-d)\left(2(2c^2+8cd+11d^2)+(2c^2+11\right.}{15af(a+a\sec}$$

```
[Out] d^3*arctanh(sin(f*x+e))/a^3/f+1/5*(c-d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+
a*sec(f*x+e))^3+1/15*(c-d)*(4*c^2+16*c*d+22*d^2+(2*c^2+11*c*d+29*d^2)*sec(f
*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A]
time = 0.14, antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 100, 150, 65, 223, 209}

$$\frac{2d^3\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(c-d)\tan(e+fx)\left((2c^2+11cd+29d^2)\sec(e+fx)+2(2c^2+8cd+11d^2)\right)}{15af(a\sec(e+fx)+a)^2}+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c - d)*
(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - d
)*(2*(2*c^2 + 8*c*d + 11*d^2) + (2*c^2 + 11*c*d + 29*d^2)*Sec[e + f*x])*Tan
[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
```

```
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c}{} \right.}{5af\sqrt{a-a}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2(2c^2+8cd+1\right.}{}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2(2c^2+8cd+1\right.}{}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-d)\left(2(2c^2+8cd+1\right.}{}$$

$$= \frac{2d^3\tan^{-1}\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec}{5f(a+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 295 vs. 2(133) = 266.

time = 1.47, size = 295, normalized size = 2.22

$$\frac{-240d^3\cos^6\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+(c-d)\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(5(8c^2+17cd+29d^2)\sin\left(\frac{fx}{2}\right)-15(2c^2+5cd+5d^2)\sin\left(e+\frac{fx}{2}\right)+20c^2\sin\left(e+\frac{3fx}{2}\right)+65cd\sin\left(e+\frac{3fx}{2}\right)+95d^2\sin\left(e+\frac{3fx}{2}\right)-15c^2\sin\left(2e+\frac{3fx}{2}\right)-15cd\sin\left(2e+\frac{3fx}{2}\right)-15d^2\sin\left(2e+\frac{3fx}{2}\right)+7c^2\sin\left(2e+\frac{5fx}{2}\right)+16cd\sin\left(2e+\frac{5fx}{2}\right)+22d^2\sin\left(2e+\frac{5fx}{2}\right)\right)}{30a^3f(1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-240*d^3*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Cos[(e + f*x)/2]*Sec[e/2]
*(5*(8*c^2 + 17*c*d + 29*d^2)*Sin[(f*x)/2] - 15*(2*c^2 + 5*c*d + 5*d^2)*Sin
[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] + 65*c*d*Sin[e + (3*f*x)/2] + 95*
d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] - 15*c*d*Sin[2*e + (3*
f*x)/2] - 15*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (5*f*x)/2] + 16*c*d
*Sin[2*e + (5*f*x)/2] + 22*d^2*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(1 + Cos[e
+ f*x])^3)
```

Maple [A]

time = 0.22, size = 216, normalized size = 1.62

method	result
derivativedivides	$\frac{4d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+2c\,d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{c^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{d^3\left(\tan^5\left(\frac{fx}{2}\ldots\right.\right.}{5}\ldots}{\ldots}$
default	$\frac{4d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+2c\,d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{c^3\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{d^3\left(\tan^5\left(\frac{fx}{2}\ldots\right.\right.}{5}\ldots}{\ldots}$
risch	$\frac{2i(15c^3\mathrm{e}^{4i(fx+e)}-15d^3\mathrm{e}^{4i(fx+e)}+30c^3\mathrm{e}^{3i(fx+e)}+45c^2d\,\mathrm{e}^{3i(fx+e)}-75d^3\mathrm{e}^{3i(fx+e)}+40c^3\mathrm{e}^{2i(fx+e)}+45c^2d\,\mathrm{e}^{2i(fx+e)}+60c\ldots}{15f\,a^3(\mathrm{e}^{i(fx\ldots}}$
norman	$\frac{\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\left(\tan^{11}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{20af}-\frac{\left(c^3+3c^2d+3c\,d^2-7d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4fa}+\frac{3\left(3c^3+c^2d-c\,d^2-3d^3\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{10fa}+\frac{(11c^3+2\ldots}{\ldots}}{\left(\tan^2\left(\frac{fx}{2}+\ldots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f/a^3*(4*d^3*ln(tan(1/2*f*x+1/2*e)+1)+2*c*d^2*tan(1/2*f*x+1/2*e)^3+3*c^
2*d*tan(1/2*f*x+1/2*e)+3*c*d^2*tan(1/2*f*x+1/2*e)+1/5*c^3*tan(1/2*f*x+1/2*e
)^5-1/5*d^3*tan(1/2*f*x+1/2*e)^5-3/5*c^2*d*tan(1/2*f*x+1/2*e)^5+3/5*c*d^2*t
an(1/2*f*x+1/2*e)^5-2/3*c^3*tan(1/2*f*x+1/2*e)^3-4/3*d^3*tan(1/2*f*x+1/2*e)
^3+c^3*tan(1/2*f*x+1/2*e)-7*d^3*tan(1/2*f*x+1/2*e)-4*d^3*ln(tan(1/2*f*x+1/2
*e)-1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(133) = 266$.
time = 0.30, size = 333, normalized size = 2.50

$$-\frac{d^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)-\frac{3cd^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{9c^2d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^3) - 3*c*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^3*(
15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^2*d*(5*sin(f*x + e)/(co
s(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 2.34, size = 262, normalized size = 1.97

$$\frac{15\left(d^3\cos(fx+e)^3+3d^3\cos(fx+e)^2+3d^3\cos(fx+e)+d^3\right)\log(\sin(fx+e)+1)-15\left(d^3\cos(fx+e)^3+3d^3\cos(fx+e)^2+3d^3\cos(fx+e)+d^3\right)\log(-\sin(fx+e)+1)+2\left(2c^3+9c^2d+21cd^2-32d^3+(7c^3+9c^2d+6cd^2-22d^3)\cos(fx+e)^2+3(2c^3+9c^2d+6cd^2-17d^3)\cos(fx+e)\right)\sin(fx+e)}{30\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e) +
d^3)*log(sin(f*x + e) + 1) - 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2
+ 3*d^3*cos(f*x + e) + d^3)*log(-sin(f*x + e) + 1) + 2*(2*c^3 + 9*c^2*d + 2
1*c*d^2 - 32*d^3 + (7*c^3 + 9*c^2*d + 6*c*d^2 - 22*d^3)*cos(f*x + e)^2 + 3*
(2*c^3 + 9*c^2*d + 6*c*d^2 - 17*d^3)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3 \sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^3 \sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{3cd^2 \sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{3c^2 d \sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*
*2*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1), x))/a**3
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(126) = 252.
time = 0.54, size = 259, normalized size = 1.95

$$\frac{\frac{60\,d^3 \log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} - \frac{60\,d^3 \log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} + \frac{3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 9\,a^{12}c^2 d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 9\,a^{12}cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 3\,a^{12}d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 10\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,a^{12}cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 20\,a^{12}d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 45\,a^{12}c^2 d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 45\,a^{12}cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) - 105\,a^{12}d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a^{15}}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*d^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 60*d^3*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a^3 + (3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 9*a^12*c^2*
d*tan(1/2*f*x + 1/2*e)^5 + 9*a^12*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 3*a^12*d^3
*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^3*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c*d^
2*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3
*tan(1/2*f*x + 1/2*e) + 45*a^12*c^2*d*tan(1/2*f*x + 1/2*e) + 45*a^12*c*d^2*
tan(1/2*f*x + 1/2*e) - 105*a^12*d^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [**B**]
time = 1.80, size = 147, normalized size = 1.11

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{{\left(c-d\right)}^3}{4\,a^3}-\frac{3\,\left(c+d\right)\,{\left(c-d\right)}^2}{4\,a^3}+\frac{3\,{\left(c+d\right)}^2\,\left(c-d\right)}{4\,a^3}\right)}{f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{{\left(c-d\right)}^3}{12\,a^3}-\frac{\left(c+d\right)\,{\left(c-d\right)}^2}{4\,a^3}\right)}{f}+\frac{2\,d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^3}{20\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((c - d)^3/(4*a^3) - (3*(c + d)*(c - d)^2)/(4*a^3) + (3
*(c + d)^2*(c - d))/(4*a^3)))/f + (tan(e/2 + (f*x)/2)^3*((c - d)^3/(12*a^3)
 - ((c + d)*(c - d)^2)/(4*a^3)))/f + (2*d^3*atanh(tan(e/2 + (f*x)/2)))/(a^3
*f) + (tan(e/2 + (f*x)/2)^5*(c - d)^3)/(20*a^3*f)
```

3.229 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal. Leaf size=115

$$\frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{(2c^2+6cd+7d^2)\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))}$$

```
[Out] 1/5*(c-d)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+2/15*(c-d)*(c+4*d)*tan(f*x+e)/a
/f/(a+a*sec(f*x+e))^2+1/15*(2*c^2+6*c*d+7*d^2)*tan(f*x+e)/f/(a^3+a^3*sec(f*
x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4072, 91, 79, 37}

$$\frac{(2c^2+6cd+7d^2)\tan(e+fx)}{15f\,(a^3\sec(e+fx)+a^3)}+\frac{(c-d)^2\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}+\frac{2(c+4d)(c-d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - d)^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (2*(c - d)*(c + 4*d
)*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c^2 + 6*c*d + 7*d^2)*
Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
```

```
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^2}{\sqrt{a-ax}\,(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^3(2c^2+6cd-3d^2)+5a^3d^2x}{\sqrt{a-ax}\,(a+ax)^{5/2}} \cdots\right.}{5a^2f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a \cdots}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{((2c^2+6 \cdots}{\cdots}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c^2+6c \cdots}{15f\,(a^3 \cdots}$$

Mathematica [A]
time = 0.49, size = 180, normalized size = 1.57

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(10(4c^2+3cd+2d^2)\sin\left(\frac{fx}{2}\right)-30c(c+d)\sin\left(e+\frac{fx}{2}\right)+20c^2\sin\left(e+\frac{3fx}{2}\right)+30cd\sin\left(e+\frac{3fx}{2}\right)+10d^2\sin\left(e+\frac{3fx}{2}\right)-15c^2\sin\left(2e+\frac{3fx}{2}\right)+7c^2\sin\left(2e+\frac{5fx}{2}\right)+6cd\sin\left(2e+\frac{5fx}{2}\right)+2d^2\sin\left(2e+\frac{5fx}{2}\right)\right)}{30a^3f(1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(10*(4*c^2 + 3*c*d + 2*d^2)*Sin[(f*x)/2] - 30*c*
(c + d)*Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] + 30*c*d*Sin[e + (3*f*
```

```
x)/2] + 10*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin
[2*e + (5*f*x)/2] + 6*c*d*Sin[2*e + (5*f*x)/2] + 2*d^2*Sin[2*e + (5*f*x)/2]
))/(30*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A]
time = 0.20, size = 74, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{(c-d)^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{2(-c-d)(c-d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+(-c-d)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4fa^3}$
default	$\frac{\frac{(c-d)^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{2(-c-d)(c-d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+(-c-d)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4fa^3}$
risch	$\frac{2i\left(15c^2e^{4i(fx+e)}+30c^2e^{3i(fx+e)}+30cd\,e^{3i(fx+e)}+40c^2e^{2i(fx+e)}+30cd\,e^{2i(fx+e)}+20d^2e^{2i(fx+e)}+20c^2e^{i(fx+e)}+30d\,e\right.}{15fa^3\left(e^{i(fx+e)}+1\right)^5}$
norman	$\frac{\frac{\left(c^2-2cd+d^2\right)\left(\tan^9\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{20af}+\frac{\left(c^2+2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4fa}-\frac{\left(2c^2+3cd+d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3fa}-\frac{\left(4c^2-3cd-d^2\right)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15fa}+}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f/a^3*(1/5*(c-d)^2*tan(1/2*f*x+1/2*e)^5+2/3*(-c-d)*(c-d)*tan(1/2*f*x+1/
2*e)^3+(-c-d)^2*tan(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.28, size = 200, normalized size = 1.74

$$\frac{\frac{d^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{6\,cd\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*x
+ e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 6*c*d*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 1.44, size = 119, normalized size = 1.03

$$\frac{\left((7\,c^2+6\,cd+2\,d^2)\cos(fx+e)^2+2\,c^2+6\,cd+7\,d^2+6\,(c^2+3\,cd+d^2)\cos(fx+e)\right)\sin(fx+e)}{15\,\left(a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2+3\,a^3f\cos(fx+e)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((7*c^2 + 6*c*d + 2*d^2)*cos(f*x + e)^2 + 2*c^2 + 6*c*d + 7*d^2 + 6*(c
^2 + 3*c*d + d^2)*cos(f*x + e))*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2 \sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^2 \sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{2cd \sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e
 + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.51, size = 129, normalized size = 1.12

$$\frac{3\,c^2 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 6\,cd \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 3\,d^2 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 10\,c^2 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 10\,d^2 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,c^2 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 30\,cd \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 15\,d^2 \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{60\,a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(3*c^2*tan(1/2*f*x + 1/2*e)^5 - 6*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*d^2*t
an(1/2*f*x + 1/2*e)^5 - 10*c^2*tan(1/2*f*x + 1/2*e)^3 + 10*d^2*tan(1/2*f*x
+ 1/2*e)^3 + 15*c^2*tan(1/2*f*x + 1/2*e) + 30*c*d*tan(1/2*f*x + 1/2*e) + 15
*d^2*tan(1/2*f*x + 1/2*e))/(a^3*f)
```

Mupad [B]
time = 1.83, size = 79, normalized size = 0.69

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(c+d\right)}^2}{4\,a^3\,f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,c^2-2\,d^2\right)}{12\,a^3\,f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5\,{\left(c-d\right)}^2}{20\,a^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c + d)^2)/(4*a^3*f) - (tan(e/2 + (f*x)/2)^3*(2*c^2 - 2
*d^2))/(12*a^3*f) + (tan(e/2 + (f*x)/2)^5*(c - d)^2)/(20*a^3*f)
```

3.230 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3} dx$

Optimal. Leaf size=102

$$\frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{(2c+3d)\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}$$

```
[Out] 1/5*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(2*c+3*d)*tan(f*x+e)/a/f/(a+
a*sec(f*x+e))^2+1/15*(2*c+3*d)*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4085, 3881, 3879}

$$\frac{(2c+3d)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)}+\frac{(2c+3d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}+\frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - d)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((2*c + 3*d)*Tan[e +
f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c + 3*d)*Tan[e + f*x])/(15*f*(a
^3 + a^3*Sec[e + f*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
```

```
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx}{5a}$$
$$= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c+3d)\int \frac{}{a+}}{15a}$$
$$= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c+3d)\tan}{15f(a^3+a^3\mathrm{s}}$$

Mathematica [A]
time = 0.35, size = 135, normalized size = 1.32

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(5(8c+3d)\sin\left(\frac{fx}{2}\right)-15(2c+d)\sin\left(e+\frac{fx}{2}\right)+20c\sin\left(e+\frac{3fx}{2}\right)+15d\sin\left(e+\frac{3fx}{2}\right)-15c\sin\left(2e+\frac{3fx}{2}\right)+7c\sin\left(2e+\frac{5fx}{2}\right)+3d\sin\left(2e+\frac{5fx}{2}\right)\right)}{30a^3f(1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(5*(8*c + 3*d)*Sin[(f*x)/2] - 15*(2*c + d)*Sin[e
 + (f*x)/2] + 20*c*Sin[e + (3*f*x)/2] + 15*d*Sin[e + (3*f*x)/2] - 15*c*Sin[
2*e + (3*f*x)/2] + 7*c*Sin[2*e + (5*f*x)/2] + 3*d*Sin[2*e + (5*f*x)/2]))/(3
0*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A]
time = 0.22, size = 64, normalized size = 0.63

method	result	siz
derivativedivides	$\frac{\frac{(c-d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f\,a^3}$	64
default	$\frac{\frac{(c-d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f\,a^3}$	64
risch	$\frac{2i\left(15c\,e^{4i(fx+e)}+30c\,e^{3i(fx+e)}+15d\,e^{3i(fx+e)}+40\,e^{2i(fx+e)}c+15d\,e^{2i(fx+e)}+20\,e^{i(fx+e)}c+15d\,e^{i(fx+e)}+7c+3d\right)}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}$	11
norman	$\frac{\frac{(c-d)\left(\tan^7\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{20af}-\frac{(c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4af}+\frac{(5c+3d)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{12fa}-\frac{(13c-3d)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{60fa}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)a^2}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a^3*(1/5*(c-d)*tan(1/2*f*x+1/2*e)^5-2/3*c*tan(1/2*f*x+1/2*e)^3+c*tan(
1/2*f*x+1/2*e)+d*tan(1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.28, size = 125, normalized size = 1.23

$$\frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{3d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 3*d*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Fricas [A]
time = 3.15, size = 99, normalized size = 0.97

$$\frac{\left((7c+3d)\cos(fx+e)^2+3(2c+3d)\cos(fx+e)+2c+3d\right)\sin(fx+e)}{15\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((7*c + 3*d)*cos(f*x + e)^2 + 3*(2*c + 3*d)*cos(f*x + e) + 2*c + 3*d)*
sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f
*x + e) + a^3*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\frac{d\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Giac [A]
time = 0.48, size = 75, normalized size = 0.74

$$\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}-10\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+15\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{60\,a^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(3*c*tan(1/2*f*x + 1/2*e)^5 - 3*d*tan(1/2*f*x + 1/2*e)^5 - 10*c*tan(1/
2*f*x + 1/2*e)^3 + 15*c*tan(1/2*f*x + 1/2*e) + 15*d*tan(1/2*f*x + 1/2*e))/(
a^3*f)
```

Mupad [B]
time = 1.74, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(15\,c+15\,d-10\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-3\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}\right)}{60\,a^{3}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(15*c + 15*d - 10*c*tan(e/2 + (f*x)/2)^2 + 3*c*tan(e/2
+ (f*x)/2)^4 - 3*d*tan(e/2 + (f*x)/2)^4))/(60*a^3*f)
```

3.231 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=181

$$-\frac{2d^3\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{7/2}\sqrt{c+d}\,f}+\frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3}+\frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2}+1$$

```
[Out] -2*d^3*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^3/(c-d)^(7/2)/
f/(c+d)^(1/2)+1/5*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^3+1/15*(2*c-7*d)*tan(
f*x+e)/a/(c-d)^2/f/(a+a*sec(f*x+e))^2+1/15*(2*c^2-9*c*d+22*d^2)*tan(f*x+e)/
(c-d)^3/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A]
time = 0.23, antiderivative size = 235, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 106, 157, 12, 95, 211}

$$\frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15f(c-d)^3\left(a^3\sec(e+fx)+a^3\right)}+\frac{2d^3\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{7/2}\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{(2c-7d)\tan(e+fx)}{15af(c-d)^2(a\sec(e+fx)+a)^2}+\frac{\tan(e+fx)}{5f(c-d)(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(5*(c - d)*f*(a + a*Sec[e + f*x])^3) + ((2*c - 7*d)*Tan[e + f*
x])/(15*a*(c - d)^2*f*(a + a*Sec[e + f*x])^2) + (2*d^3*ArcTan[(Sqrt[c + d]*
Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f
*x])/(a^2*(c - d)^(7/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) + ((2*c^2 - 9*c*d + 22*d^2)*Tan[e + f*x])/(15*(c - d)^3*f*(a^
3 + a^3*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{7/2}(c+dx)}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{5a(c-d)f\sqrt{a-a}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+f}{15a(c-d)^2f(a+a\sec(e}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+f}{15a(c-d)^2f(a+a\sec(e}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+f}{15a(c-d)^2f(a+a\sec(e}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+f}{15a(c-d)^2f(a+a\sec(e}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+f}{15a(c-d)^2f(a+a\sec(e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.06, size = 345, normalized size = 1.91

$$\frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(\frac{480d^3\text{ArcTan}\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos^5\left(\frac{1}{2}(e+fx)\right)(i\cos(e)+\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\sec\left(\frac{e}{2}\right)\left(5(8c^2-27cd+37d^2)\sin\left(\frac{fx}{2}\right)-15(2c^2-7cd+9d^2)\sin\left(e+\frac{fx}{2}\right)+20c^2\sin\left(e+\frac{3fx}{2}\right)-75cd\sin\left(e+\frac{3fx}{2}\right)+115d^2\sin\left(e+\frac{3fx}{2}\right)-15c^2\sin\left(2e+\frac{3fx}{2}\right)+45cd\sin\left(2e+\frac{3fx}{2}\right)-45d^2\sin\left(2e+\frac{3fx}{2}\right)+7c^2\sin\left(2e+\frac{5fx}{2}\right)-24cd\sin\left(2e+\frac{5fx}{2}\right)+32d^2\sin\left(2e+\frac{5fx}{2}\right)\right)\right)}{30a^3(c-d)^3f(1+\cos(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]*((480*d^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c
*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[
(e + f*x)/2]^5*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e
])^2]) + Sec[e/2]*(5*(8*c^2 - 27*c*d + 37*d^2)*Sin[(f*x)/2] - 15*(2*c^2 - 7
*c*d + 9*d^2)*Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] - 75*c*d*Sin[e +
 (3*f*x)/2] + 115*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] + 45
*c*d*Sin[2*e + (3*f*x)/2] - 45*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (
```

```
5*f*x)/2] - 24*c*d*Sin[2*e + (5*f*x)/2] + 32*d^2*Sin[2*e + (5*f*x)/2])))/(3
0*a^3*(c - d)^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A]
time = 0.30, size = 203, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{c^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2cd\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{d^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2c^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+2cd\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{4d^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c^{2}}{(c-d)^{3}}}{4f\,a^{3}}$
default	$\frac{\frac{\frac{c^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2cd\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{d^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2c^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+2cd\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{4d^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+c^{2}}{(c-d)^{3}}}{4f\,a^{3}}$
risch	$\frac{2i\left(15c^{2}e^{4i(fx+e)}-45cd\,e^{4i(fx+e)}+45d^{2}e^{4i(fx+e)}+30c^{2}e^{3i(fx+e)}-105cd\,e^{3i(fx+e)}+135d^{2}e^{3i(fx+e)}+40c^{2}e^{2i(fx+e)}-135\right.}{15f\,a^{3}(c-d)^{3}\left(e^{i(fx+e)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a^3*(1/(c-d)^3*(1/5*c^2*tan(1/2*f*x+1/2*e)^5-2/5*c*d*tan(1/2*f*x+1/2*
e)^5+1/5*d^2*tan(1/2*f*x+1/2*e)^5-2/3*c^2*tan(1/2*f*x+1/2*e)^3+2*c*d*tan(1/
2*f*x+1/2*e)^3-4/3*d^2*tan(1/2*f*x+1/2*e)^3+c^2*tan(1/2*f*x+1/2*e)-4*c*d*ta
n(1/2*f*x+1/2*e)+7*d^2*tan(1/2*f*x+1/2*e))-8*d^3/(c-d)^3/((c+d)*(c-d))^(1/2
)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(173) = 346.
time = 3.02, size = 1027, normalized size = 5.67

$$\left[-\frac{15\left(d^{3}\cos\left(fx+e\right)^{3}+3d^{3}\cos\left(fx+e\right)^{2}+3d^{3}\cos\left(fx+e\right)+d^{3}\right)\sqrt{c^{2}-d^{2}}\log\left(\frac{2cd\cos(fx+e)-(c^{2}-2d^{2})\cos(fx+e)^{2}+2\sqrt{c^{2}-d^{2}}(d\cos(fx+e)+c)\sin(fx+e)+2c^{2}-d^{2}}{c^{2}\cos(fx+e)^{2}+2cd\cos(fx+e)+d^{2}}\right)-2\left(2c^{4}-9c^{3}d+20c^{2}d^{2}+9cd^{3}-22d^{4}+\left(7c^{4}-24c^{3}d+25c^{2}d^{2}+24cd^{3}-32d^{4}\right)\cos\left(fx+e\right)^{2}+3\left(2c^{4}-9c^{3}d+15c^{2}d^{2}+9cd^{3}-17d^{4}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{30\left(\left(a^{3}c^{5}-3a^{3}c^{4}d+2a^{3}c^{3}d^{2}+2a^{3}c^{2}d^{3}-3a^{3}cd^{4}+a^{3}d^{5}\right)f\cos\left(fx+e\right)^{3}+3\left(a^{3}c^{5}-3a^{3}c^{4}d+2a^{3}c^{3}d^{2}+2a^{3}c^{2}d^{3}-3a^{3}cd^{4}+a^{3}d^{5}\right)f\cos\left(fx+e\right)^{2}+3\left(a^{3}c^{5}-3a^{3}c^{4}d+2a^{3}c^{3}d^{2}+2a^{3}c^{2}d^{3}-3a^{3}cd^{4}+a^{3}d^{5}\right)f\cos\left(fx+e\right)+\left(a^{3}c^{5}-3a^{3}c^{4}d+2a^{3}c^{3}d^{2}+2a^{3}c^{2}d^{3}-3a^{3}cd^{4}+a^{3}d^{5}\right)f\right)},\ -\frac{15\left(d^{3}\cos\left(fx+e\right)^{3}+3d^{3}\cos\left(fx+e\right)^{2}+3d^{3}\cos\left(fx+e\right)+d^{3}\right)\sqrt{-c^{2}+d^{2}}\arctan\left(-\frac{\sqrt{-c^{2}+d^{2}}(d\cos(fx+e)+c)}{(c^{2}-d^{2})\sin(fx+e)}\right)-\left(2c^{4}-9c^{3}d+20c^{2}d^{2}+9cd^{3}-22d^{4}+\left(7c^{4}-24c^{3}d+25c^{2}d^{2}+24cd^{3}-32d^{4}\right)\cos\left(fx+e\right)^{2}+3\left(2c^{4}-9c^{3}d+15c^{2}d^{2}+9cd^{3}-17d^{4}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{15\left(\left(a^{3}c^{5}-3a^{3}c^{4}d+2a^{3}c^{3}d^{2}+2a^{3}c^{2}d^{3}-3a^{3}cd^{4}+a^{3}d^{5}\right)f\cos\left(fx+e\right)^{3}+3\left(a^{3}c^{5}-3a^{3}c^{4}d+2a^{3}c^{3}d^{2}+2a^{3}c^{2}d^{3}-3a^{3}cd^{4}+a^{3}d^{5}\right)f\cos\left(fx+e\right)^{2}+3\left(a^{3}c^{5}-3a^{3}c^{4}d+2a^{3}c^{3}d^{2}+2a^{3}c^{2}d^{3}-3a^{3}cd^{4}+a^{3}d^{5}\right)f\cos\left(fx+e\right)+\left(a^{3}c^{5}-3a^{3}c^{4}d+2a^{3}c^{3}d^{2}+2a^{3}c^{2}d^{3}-3a^{3}cd^{4}+a^{3}d^{5}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e)
+ d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)
^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^
2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(2*c^4 - 9*c^3*d + 20*c^2
*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3 - 32*d^
4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*d^4)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^
2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d
+ 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 +
 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a
^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2
*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f), -1/15*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos
(f*x + e)^2 + 3*d^3*cos(f*x + e) + d^3)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2
+ d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^4 - 9*c^3*d
+ 20*c^2*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3
 - 32*d^4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*
d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 +
2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^
3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x
+ e)^2 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c
*d^4 + a^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2
*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c\sec^3(e+fx)+3c\sec^2(e+fx)+3c\sec(e+fx)+c+d\sec^4(e+fx)+3d\sec^3(e+fx)+3d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x)**3 + 3*c*sec(e + f*x)**2 + 3*c*sec(e
+ f*x) + c + d*sec(e + f*x)**4 + 3*d*sec(e + f*x)**3 + 3*d*sec(e + f*x)**2
+ d*sec(e + f*x)), x)/a**3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 471 vs. 2(166) = 332.
time = 0.53, size = 471, normalized size = 2.60

$$-\frac{\frac{120\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^3}{\left(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3\right)\sqrt{-c^2+d^2}}-\frac{3a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12a^{12}c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+18a^{12}c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12a^{12}cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+3a^{12}d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-10a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+50a^{12}c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-90a^{12}c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+70a^{12}cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-20a^{12}d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+15a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-90a^{12}c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+240a^{12}c^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-270a^{12}cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+105a^{12}d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^5-5a^{15}c^4d+10a^{15}c^3d^2-10a^{15}c^2d^3+5a^{15}cd^4-a^{15}d^5}}{60f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/60*(120*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*t
an(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^3/((a^3*
c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*sqrt(-c^2 + d^2)) - (3*a^12*c^4*
tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12*c^2
*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a^12
*d^4*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 50*a^12*
c^3*d*tan(1/2*f*x + 1/2*e)^3 - 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 70*
a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^4*tan(1/2*f*x + 1/2*e)^3 + 15
*a^12*c^4*tan(1/2*f*x + 1/2*e) - 90*a^12*c^3*d*tan(1/2*f*x + 1/2*e) + 240*a
^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 270*a^12*c*d^3*tan(1/2*f*x + 1/2*e) + 10
5*a^12*d^4*tan(1/2*f*x + 1/2*e))/(a^15*c^5 - 5*a^15*c^4*d + 10*a^15*c^3*d^2
 - 10*a^15*c^2*d^3 + 5*a^15*c*d^4 - a^15*d^5))/f
```

Mupad [B]
time = 1.95, size = 228, normalized size = 1.26

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3}{4\,a^3\,\left(c-d\right)}-\frac{\left(c+d\right)\,\left(\frac{3}{4\,a^3\,\left(c-d\right)}-\frac{c+d}{4\,a^3\,{\left(c-d\right)}^2}\right)}{c-d}\right)}{f}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(\frac{1}{4\,a^3\,\left(c-d\right)}-\frac{c+d}{12\,a^3\,{\left(c-d\right)}^2}\right)}{f}+\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^5}{20\,a^3\,f\,\left(c-d\right)}-\frac{2\,d^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-2\,d\right)\,\left(a^3\,c^3-3\,a^3\,c^2\,d+3\,a^3\,c\,d^2-a^3\,d^3\right)}{2\,a^3\,\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}\right)}{a^3\,f\,\sqrt{c+d}\,{\left(c-d\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3/(4*a^3*(c - d)) - ((c + d)*(3/(4*a^3*(c - d)) - (c +
 d)/(4*a^3*(c - d)^2)))/(c - d)))/f - (tan(e/2 + (f*x)/2)^3*(1/(4*a^3*(c -
d)) - (c + d)/(12*a^3*(c - d)^2)))/f + tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c -
d)) - (2*d^3*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(a^3*c^3 - a^3*d^3 + 3*a
^3*c*d^2 - 3*a^3*c^2*d))/(2*a^3*(c + d)^(1/2)*(c - d)^(7/2))))/(a^3*f*(c +
d)^(1/2)*(c - d)^(7/2))
```

3.232 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=288

$$-\frac{2d^3(4c+3d)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{9/2}(c+d)^{3/2}f}+\frac{d(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sec(e+fx))}+\frac{}{5(c-d)f(a+}$$

```
[Out] -2*d^3*(4*c+3*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^3/(c
-d)^(9/2)/(c+d)^(3/2)/f+1/15*d*(2*c^3-12*c^2*d+43*c*d^2+72*d^3)*tan(f*x+e)/
a^3/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))+1/5*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e)
)^3/(c+d*sec(f*x+e))+1/15*(2*c-9*d)*tan(f*x+e)/a/(c-d)^2/f/(a+a*sec(f*x+e))
^2/(c+d*sec(f*x+e))+1/15*(2*c^2-12*c*d+45*d^2)*tan(f*x+e)/(c-d)^3/f/(a^3+a^
3*sec(f*x+e))/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.34, antiderivative size = 325, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 105, 157, 12, 95, 211}

$$\frac{(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15f(c-d)^4(c+d)\left(a^3\sec(e+fx)+a^3\right)}+\frac{2d^3(4c+3d)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\,\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{9/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\,\sqrt{a\sec(e+fx)+a}}-\frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))}+\frac{(2c^2-10cd-27d^2)\tan(e+fx)}{15af(c-d)^3(c+d)(a\sec(e+fx)+a)^2}+\frac{(c+6d)\tan(e+fx)}{5f(c-d)^2(c+d)(a\sec(e+fx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 6*d)*Tan[e + f*x])/(5*(c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])^3) + (
(2*c^2 - 10*c*d - 27*d^2)*Tan[e + f*x])/(15*a*(c - d)^3*(c + d)*f*(a + a*Se
c[e + f*x])^2) + (2*d^3*(4*c + 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a^2*(c - d)^(
9/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (
(2*c^3 - 12*c^2*d + 43*c*d^2 + 72*d^3)*Tan[e + f*x])/(15*(c - d)^4*(c + d)*
f*(a^3 + a^3*Sec[e + f*x])) - (d*Tan[e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e
+ f*x])^3*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{\frac{1}{(a+ax)^{7/2}(c+dx)^2}}{\sqrt{a-ax}}\,dx, x\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} - \frac{\tan}{}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} - \frac{}{(c^2-d^2)\,f(a+a}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{(2c^2-10cd}{15a(c-d)^3(c+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{(2c^2-10cd}{15a(c-d)^3(c+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{(2c^2-10cd}{15a(c-d)^3(c+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{(2c^2-10cd}{15a(c-d)^3(c+}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{(2c^2-10cd}{15a(c-d)^3(c+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.10, size = 1772, normalized size = 6.15

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((4*c + 3*d)*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^5*(((
16*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Si
n[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*
d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos
```

```
[2*e] - I*Sin[2*e]]) + (16*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]
*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] -
 I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[
c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d)*(a + a*Sec[
e + f*x])^3*(c + d*Sec[e + f*x])^2) + (Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*
x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-55*c^5*Sin[(f*x)/2] + 135*c^4*d*Sin[(f
*x)/2] - 20*c^3*d^2*Sin[(f*x)/2] - 810*c^2*d^3*Sin[(f*x)/2] - 450*c*d^4*Sin
[(f*x)/2] + 150*d^5*Sin[(f*x)/2] + 47*c^5*Sin[(3*f*x)/2] - 137*c^4*d*Sin[(3
*f*x)/2] + 88*c^3*d^2*Sin[(3*f*x)/2] + 812*c^2*d^3*Sin[(3*f*x)/2] + 690*c*d
^4*Sin[(3*f*x)/2] + 75*d^5*Sin[(3*f*x)/2] - 50*c^5*Sin[e - (f*x)/2] + 130*c
^4*d*Sin[e - (f*x)/2] - 10*c^3*d^2*Sin[e - (f*x)/2] - 1030*c^2*d^3*Sin[e -
(f*x)/2] - 990*c*d^4*Sin[e - (f*x)/2] - 150*d^5*Sin[e - (f*x)/2] + 50*c^5*S
in[e + (f*x)/2] - 130*c^4*d*Sin[e + (f*x)/2] + 10*c^3*d^2*Sin[e + (f*x)/2]
+ 1030*c^2*d^3*Sin[e + (f*x)/2] + 765*c*d^4*Sin[e + (f*x)/2] - 150*d^5*Sin[
e + (f*x)/2] - 55*c^5*Sin[2*e + (f*x)/2] + 135*c^4*d*Sin[2*e + (f*x)/2] - 2
0*c^3*d^2*Sin[2*e + (f*x)/2] - 810*c^2*d^3*Sin[2*e + (f*x)/2] - 675*c*d^4*S
in[2*e + (f*x)/2] - 150*d^5*Sin[2*e + (f*x)/2] - 30*c^5*Sin[e + (3*f*x)/2]
+ 90*c^4*d*Sin[e + (3*f*x)/2] - 60*c^3*d^2*Sin[e + (3*f*x)/2] - 360*c^2*d^3
*Sin[e + (3*f*x)/2] - 30*c*d^4*Sin[e + (3*f*x)/2] + 75*d^5*Sin[e + (3*f*x)/
2] + 47*c^5*Sin[2*e + (3*f*x)/2] - 137*c^4*d*Sin[2*e + (3*f*x)/2] + 88*c^3*
d^2*Sin[2*e + (3*f*x)/2] + 812*c^2*d^3*Sin[2*e + (3*f*x)/2] + 525*c*d^4*Sin
[2*e + (3*f*x)/2] - 75*d^5*Sin[2*e + (3*f*x)/2] - 30*c^5*Sin[3*e + (3*f*x)/
2] + 90*c^4*d*Sin[3*e + (3*f*x)/2] - 60*c^3*d^2*Sin[3*e + (3*f*x)/2] - 360*
c^2*d^3*Sin[3*e + (3*f*x)/2] - 195*c*d^4*Sin[3*e + (3*f*x)/2] - 75*d^5*Sin[
3*e + (3*f*x)/2] + 20*c^5*Sin[e + (5*f*x)/2] - 76*c^4*d*Sin[e + (5*f*x)/2]
+ 106*c^3*d^2*Sin[e + (5*f*x)/2] + 346*c^2*d^3*Sin[e + (5*f*x)/2] + 219*c*d
^4*Sin[e + (5*f*x)/2] + 15*d^5*Sin[e + (5*f*x)/2] - 15*c^5*Sin[2*e + (5*f*x
)/2] + 45*c^4*d*Sin[2*e + (5*f*x)/2] - 30*c^3*d^2*Sin[2*e + (5*f*x)/2] - 90
*c^2*d^3*Sin[2*e + (5*f*x)/2] + 75*c*d^4*Sin[2*e + (5*f*x)/2] + 15*d^5*Sin[
2*e + (5*f*x)/2] + 20*c^5*Sin[3*e + (5*f*x)/2] - 76*c^4*d*Sin[3*e + (5*f*x)
/2] + 106*c^3*d^2*Sin[3*e + (5*f*x)/2] + 346*c^2*d^3*Sin[3*e + (5*f*x)/2] +
 144*c*d^4*Sin[3*e + (5*f*x)/2] - 15*d^5*Sin[3*e + (5*f*x)/2] - 15*c^5*Sin[
4*e + (5*f*x)/2] + 45*c^4*d*Sin[4*e + (5*f*x)/2] - 30*c^3*d^2*Sin[4*e + (5*
f*x)/2] - 90*c^2*d^3*Sin[4*e + (5*f*x)/2] - 15*d^5*Sin[4*e + (5*f*x)/2] + 7
*c^5*Sin[2*e + (7*f*x)/2] - 27*c^4*d*Sin[2*e + (7*f*x)/2] + 38*c^3*d^2*Sin[
2*e + (7*f*x)/2] + 72*c^2*d^3*Sin[2*e + (7*f*x)/2] + 15*c*d^4*Sin[2*e + (7*
f*x)/2] + 15*c*d^4*Sin[3*e + (7*f*x)/2] + 7*c^5*Sin[4*e + (7*f*x)/2] - 27*c
^4*d*Sin[4*e + (7*f*x)/2] + 38*c^3*d^2*Sin[4*e + (7*f*x)/2] + 72*c^2*d^3*Si
n[4*e + (7*f*x)/2]))/(120*c*(-c + d)^4*(c + d)*f*(a + a*Sec[e + f*x])^3*(c
+ d*Sec[e + f*x])^2)
```

Maple [A]

time = 0.40, size = 284, normalized size = 0.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f/a^3*(1/(c^2-2*c*d+d^2)/(c-d)^2*(1/5*c^2*tan(1/2*f*x+1/2*e)^5-2/5*c*d*
tan(1/2*f*x+1/2*e)^5+1/5*d^2*tan(1/2*f*x+1/2*e)^5-2/3*c^2*tan(1/2*f*x+1/2*e
)^3+8/3*c*d*tan(1/2*f*x+1/2*e)^3-2*d^2*tan(1/2*f*x+1/2*e)^3+c^2*tan(1/2*f*x
+1/2*e)-6*c*d*tan(1/2*f*x+1/2*e)+17*d^2*tan(1/2*f*x+1/2*e))+16*d^3/(c-d)^4*
(-1/2*d/(c+d)*tan(1/2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*
e)^2-c-d)-1/2*(4*c+3*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x
+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 832 vs. 2(283) = 566.
time = 3.52, size = 1725, normalized size = 5.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/30*(15*(4*c*d^4 + 3*d^5 + (4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2
*d^3 + 13*c*d^4 + 3*d^5)*cos(f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*c
os(f*x + e)^2 + (4*c^2*d^3 + 15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(c^2 - d^2
)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2
)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*
c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^
4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2 + 99*c^3*d^3 - 23*c^
2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 29*c^5*d + 51*c^4*d^2
+ 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(f*x + e)^2 + (2*c^6 -
 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141*c*d^5 - 171*d^6)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3*c^6*d^2 + 5*a^3*c^5*
```

d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^3*c*d^7)*f*cos(f*x +
e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*a^3*c^4*d^4 - 8*a^3*c
^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3*(a^3*c^8 - 2*a^3*c^7
*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*
c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^
3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x +
e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d
^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f), -1/15*(15*(4*c*d^4 + 3*d^5 +
(4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2*d^3 + 13*c*d^4 + 3*d^5)*cos(
f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (4*c^2*d^3 +
15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(
d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^5*d - 12*c^4*d^2 + 4
1*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2
 + 99*c^3*d^3 - 23*c^2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 2
9*c^5*d + 51*c^4*d^2 + 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(
f*x + e)^2 + (2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141
*c*d^5 - 171*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3
*c^6*d^2 + 5*a^3*c^5*d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^
3*c*d^7)*f*cos(f*x + e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*
a^3*c^4*d^4 - 8*a^3*c^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3
*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2
*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c
^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*
a^3*d^8)*f*cos(f*x + e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*
c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^2\sec^3(e+fx)+3c^2\sec^2(e+fx)+3c^2\sec(e+fx)+c^2+2cd\sec^4(e+fx)+6cd\sec^3(e+fx)+6cd\sec^2(e+fx)+2cd\sec(e+fx)+d^2\sec^5(e+fx)+3d^2\sec^4(e+fx)+3d^2\sec^3(e+fx)+d^2\sec^2(e+fx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)

[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x)**3 + 3*c**2*sec(e + f*x)**2 + 3*c*
*2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**4 + 6*c*d*sec(e + f*x)**3 + 6*
c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**5 + 3*d**2*se
c(e + f*x)**4 + 3*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**3

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 918 vs. 2(271) = 542.
time = 0.58, size = 918, normalized size = 3.19

$$-\frac{120\,d^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a^3c^5-3a^3c^4d+2a^3c^3d^2+2a^3c^2d^3-3a^3cd^4+a^3d^5\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)}+\frac{120\left(4cd^5+3d^6\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(a^3c^5-3a^3c^4d+2a^3c^3d^2+2a^3c^2d^3-3a^3cd^4+a^3d^5\right)\sqrt{-c^2+d^2}}-\ldots$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan
(1/2*f*x + 1/2*e)^2 - c - d)) + 120*(4*c*d^3 + 3*d^4)*(pi*floor(1/2*(f*x +
e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*
f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 +
2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(-c^2 + d^2)) - (3*a^12*c^8*tan(
1/2*f*x + 1/2*e)^5 - 24*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 84*a^12*c^6*d^2
*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 + 210*a^1
2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^5
+ 84*a^12*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 24*a^12*c*d^7*tan(1/2*f*x + 1/2*
e)^5 + 3*a^12*d^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8*tan(1/2*f*x + 1/2*e)
^3 + 100*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 420*a^12*c^6*d^2*tan(1/2*f*x +
 1/2*e)^3 + 980*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 1400*a^12*c^4*d^4*tan
(1/2*f*x + 1/2*e)^3 + 1260*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 700*a^12*c
^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 220*a^12*c*d^7*tan(1/2*f*x + 1/2*e)^3 - 30*
a^12*d^8*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^8*tan(1/2*f*x + 1/2*e) - 180*a^
12*c^7*d*tan(1/2*f*x + 1/2*e) + 1020*a^12*c^6*d^2*tan(1/2*f*x + 1/2*e) - 31
80*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e) + 5850*a^12*c^4*d^4*tan(1/2*f*x + 1/2*
e) - 6540*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e) + 4380*a^12*c^2*d^6*tan(1/2*f*x
 + 1/2*e) - 1620*a^12*c*d^7*tan(1/2*f*x + 1/2*e) + 255*a^12*d^8*tan(1/2*f*x
 + 1/2*e))/(a^15*c^10 - 10*a^15*c^9*d + 45*a^15*c^8*d^2 - 120*a^15*c^7*d^3
+ 210*a^15*c^6*d^4 - 252*a^15*c^5*d^5 + 210*a^15*c^4*d^6 - 120*a^15*c^3*d^7
 + 45*a^15*c^2*d^8 - 10*a^15*c*d^9 + a^15*d^10))/f
```

Mupad [B]
time = 2.12, size = 464, normalized size = 1.61

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20\,a^3\,f\,{\left(c-d\right)}^2}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{2\,\left(c^2-d^2\right)\,\left(\frac{1}{a^3\,{\left(c-d\right)}^2}-\frac{c^2-d^2}{2\,a^3\,{\left(c-d\right)}^4}\right)}{{\left(c-d\right)}^2}-\frac{3}{2\,a^3\,{\left(c-d\right)}^2}+\frac{{\left(c+d\right)}^2}{4\,a^3\,{\left(c-d\right)}^4}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{1}{3\,a^3\,{\left(c-d\right)}^2}-\frac{c^2-d^2}{6\,a^3\,{\left(c-d\right)}^4}\right)}{f}+\frac{2\,d^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(c+d\right)\,\left(a^3\,c^5-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(a^3\,c^5-5\,a^3\,c^4\,d+10\,a^3\,c^3\,d^2-10\,a^3\,c^2\,d^3+5\,a^3\,c\,d^4-a^3\,d^5\right)+a^3\,d^5-3\,a^3\,c\,d^4-3\,a^3\,c^4\,d+2\,a^3\,c^2\,d^3+2\,a^3\,c^3\,d^2\right)}+\frac{d^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^5-5\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^4\,d+10\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,d^2-10\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d^3+5\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^4-1\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^5}{\sqrt{c+d}\,{\left(c-d\right)}^{9/2}}\right)\,\left(4\,c+3\,d\right)\,2\mathrm{i}}{a^3\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c - d)^2) - (tan(e/2 + (f*x)/2)*((2*(c^2 -
d^2)*(1/(a^3*(c - d)^2) - (c^2 - d^2)/(2*a^3*(c - d)^4)))/(c - d)^2 - 3/(2*
a^3*(c - d)^2) + (c + d)^2/(4*a^3*(c - d)^4)))/f - (tan(e/2 + (f*x)/2)^3*(1
/(3*a^3*(c - d)^2) - (c^2 - d^2)/(6*a^3*(c - d)^4)))/f + (2*d^4*tan(e/2 + (
f*x)/2))/(f*(c + d)*(a^3*c^5 - tan(e/2 + (f*x)/2)^2*(a^3*c^5 - a^3*d^5 + 5*
a^3*c*d^4 - 5*a^3*c^4*d - 10*a^3*c^2*d^3 + 10*a^3*c^3*d^2) + a^3*d^5 - 3*a^
3*c*d^4 - 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2)) + (d^3*atan((c^5*ta
n(e/2 + (f*x)/2)*1i - d^5*tan(e/2 + (f*x)/2)*1i + c*d^4*tan(e/2 + (f*x)/2)*
5i - c^4*d*tan(e/2 + (f*x)/2)*5i - c^2*d^3*tan(e/2 + (f*x)/2)*10i + c^3*d^2
*tan(e/2 + (f*x)/2)*10i)/((c + d)^(1/2)*(c - d)^(9/2)))*(4*c + 3*d)*2i)/(a^
3*f*(c + d)^(3/2)*(c - d)^(9/2))
```

3.233 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=368

$$-\frac{d^3(20c^2+30cd+13d^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{11/2}(c+d)^{5/2}f}+\frac{d(4c^3-30c^2d+146cd^2+195d^3)\tan(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sec(e+fx))^2}+\frac{}{5(c}$$

```
[Out] -d^3*(20*c^2+30*c*d+13*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1
/2))/a^3/(c-d)^(11/2)/(c+d)^(5/2)/f+1/30*d*(4*c^3-30*c^2*d+146*c*d^2+195*d^
3)*tan(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))^2+1/5*tan(f*x+e)/(c-d)/f
/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2+1/15*(2*c-11*d)*tan(f*x+e)/a/(c-d)^2
/f/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2+1/15*(2*c^2-15*c*d+76*d^2)*tan(f*x
+e)/(c-d)^3/f/(a^3+a^3*sec(f*x+e))/(c+d*sec(f*x+e))^2+1/30*d*(4*c^4-30*c^3*
d+142*c^2*d^2+525*c*d^3+304*d^4)*tan(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sec(
f*x+e))
```

Rubi [A]

time = 0.50, antiderivative size = 414, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 105, 156, 157, 12, 95, 211}

$$\frac{(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\tan(e+fx)}{30f(c-d)^5(c+d)^2(a^3\sec(e+fx)+a^3)}+\frac{d^3(20c^2+30cd+13d^2)\tan(e+fx)\text{ArcTan}\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{11/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}-\frac{3d(2c+d)\tan(e+fx)}{2f(c^2-d^2)^2(a\sec(e+fx)+a)^3(c+d\sec(e+fx))}-\frac{d\tan(e+fx)}{2f(c^2-d^2)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))^2}+\frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10f(c-d)^3(c+d)^2(a\sec(e+fx)+a)^3}+\frac{(4c^3-26c^2d-184cd^2-109d^3)\tan(e+fx)}{30af(c-d)^4(c+d)^2(a\sec(e+fx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 39*c*d + 22*d^2)*Tan[e + f*x])/(10*(c - d)^3*(c + d)^2*f*(a + a*S
ec[e + f*x])^3) + ((4*c^3 - 26*c^2*d - 184*c*d^2 - 109*d^3)*Tan[e + f*x])/(
30*a*(c - d)^4*(c + d)^2*f*(a + a*Sec[e + f*x])^2) + (d^3*(20*c^2 + 30*c*d
+ 13*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a
 - a*Sec[e + f*x]])]*Tan[e + f*x])/(a^2*(c - d)^(11/2)*(c + d)^(5/2)*f*Sqrt
[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((4*c^4 - 30*c^3*d + 142*c
^2*d^2 + 525*c*d^3 + 304*d^4)*Tan[e + f*x])/(30*(c - d)^5*(c + d)^2*f*(a^3
+ a^3*Sec[e + f*x])) - (d*Tan[e + f*x])/(2*(c^2 - d^2)*f*(a + a*Sec[e + f*x
])^3*(c + d*Sec[e + f*x])^2) - (3*d*(2*c + d)*Tan[e + f*x])/(2*(c^2 - d^2)^
2*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
```

```
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\,(a+ax)^{7/2}(c+dx)^3}\,dx, x, \text{s}\right.}{f\sqrt{a-a\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{\text{ta}}{}$$

$$= -\frac{d\tan(e+fx)}{2(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{}{2}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} - \frac{}{2(c^2-d^2)\,f(a+}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{(4c^3-26c^2d-}{30a(c-d)^4(c}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{(4c^3-26c^2d-}{30a(c-d)^4(c}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{(4c^3-26c^2d-}{30a(c-d)^4(c}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{(4c^3-26c^2d-}{30a(c-d)^4(c}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{(4c^3-26c^2d-}{30a(c-d)^4(c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.88, size = 1096, normalized size = 2.98

Warning: Unable to verify antiderivative.

[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]

[Out] (4*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8*c*Sin[e/2] + 23*d*Sin[e/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) + ((20*c^2 + 30*c*d + 13*d^2)*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^6*(((-8*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (8*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^5*(c + d)^2*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (2*Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*Sin[(f*x)/2])/(5*(-c + d)^3*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) + (4*Cos[e/2 + (f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8*c*Sin[(f*x)/2] + 23*d*Sin[(f*x)/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (8*Cos[e/2 + (f*x)/2]^5*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(7*c^2*Sin[(f*x)/2] - 44*c*d*Sin[(f*x)/2] + 127*d^2*Sin[(f*x)/2]))/(15*(-c + d)^5*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) + (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])*Sec[e]*Sec[e + f*x]^6*(d^6*Sin[e] - c*d^5*Sin[f*x]))/(c^2*(-c + d)^4*(c + d)*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*Sec[e]*Sec[e + f*x]^6*(-11*c^2*d^5*Sin[e] - 6*c*d^6*Sin[e] + 2*d^7*Sin[e] + 10*c^3*d^4*Sin[f*x] + 6*c^2*d^5*Sin[f*x] - c*d^6*Sin[f*x]))/(c^2*(-c + d)^5*(c + d)^2*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (2*Cos[e/2 + (f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^6*Tan[e/2])/(5*(-c + d)^3*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3)

Maple [A]

time = 0.55, size = 365, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{c^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2cd\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{d^{2}\left(\tan^{5}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2c^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{10cd\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{8d^{2}\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)\left(c^{2}-2cd+d^{2}\right)}$

default	$\frac{\frac{c^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2cd\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}+\frac{d^2\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5}-\frac{2c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+\frac{10cd\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}-\frac{8d^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3}+}{\left(c^3-3c^2d+3c\,d^2-d^3\right)\left(c^2-2cd+d^2\right)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f/a^3*(1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c^2-2*c*d+d^2)*(1/5*c^2*tan(1/2*f*x
+1/2*e)^5-2/5*c*d*tan(1/2*f*x+1/2*e)^5+1/5*d^2*tan(1/2*f*x+1/2*e)^5-2/3*c^2
*tan(1/2*f*x+1/2*e)^3+10/3*c*d*tan(1/2*f*x+1/2*e)^3-8/3*d^2*tan(1/2*f*x+1/2
*e)^3+c^2*tan(1/2*f*x+1/2*e)-8*c*d*tan(1/2*f*x+1/2*e)+31*d^2*tan(1/2*f*x+1/
2*e))+16*d^3/(c-d)^5*((-1/4*d*(10*c^2-3*c*d-7*d^2)/(c^2+2*c*d+d^2)*tan(1/2*
f*x+1/2*e)^3+5/4*d*(2*c+d)/(c+d)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^
2-d*tan(1/2*f*x+1/2*e)^2-c-d)^2-1/4*(20*c^2+30*c*d+13*d^2)/(c^2+2*c*d+d^2)/
((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1327 vs. 2(363) = 726.
time = 4.29, size = 2715, normalized size = 7.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/60*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13*
c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d^6
)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 + 13
*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*d^6
+ 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)*
cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f
*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d
^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(4*c^6*d^2 - 30*c^
5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8 + (14
*c^8 - 60*c^7*d + 78*c^6*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^3*d^5 - 41
9*c^2*d^6 - 90*c*d^7 + 15*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7*d + 114*c^
6*d^2 + 1056*c^5*d^3 + 1626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6 - 913*c*d^7
 - 45*d^8)*cos(f*x + e)^3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 828*c^5*d^3 + 2
400*c^4*d^4 + 1197*c^3*d^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*d^8)*cos(f*x +
 e)^2 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*d^5 -
 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^11
 - 3*a^3*c^10*d + 8*a^3*c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^6*d^5 + 8*a^3*c^5
*d^6 - 3*a^3*c^3*d^8 + a^3*c^2*d^9)*f*cos(f*x + e)^5 + (3*a^3*c^11 - 7*a^3*
c^10*d - 6*a^3*c^9*d^2 + 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4 - 30*a^3*c^6*d^5 +
12*a^3*c^5*d^6 + 16*a^3*c^4*d^7 - 9*a^3*c^3*d^8 - 3*a^3*c^2*d^9 + 2*a^3*c*d
^10)*f*cos(f*x + e)^4 + (3*a^3*c^11 - 3*a^3*c^10*d - 17*a^3*c^9*d^2 + 21*a^
3*c^8*d^3 + 30*a^3*c^7*d^4 - 46*a^3*c^6*d^5 - 18*a^3*c^5*d^6 + 42*a^3*c^4*d
^7 - a^3*c^3*d^8 - 15*a^3*c^2*d^9 + 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)
^3 + (a^3*c^11 + 3*a^3*c^10*d - 15*a^3*c^9*d^2 - a^3*c^8*d^3 + 42*a^3*c^7*d
^4 - 18*a^3*c^6*d^5 - 46*a^3*c^5*d^6 + 30*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 17
*a^3*c^2*d^9 - 3*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^2 + (2*a^3*c^10*d
- 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*
c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3
*a^3*d^11)*f*cos(f*x + e) + (a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 -
6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f)
, -1/30*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13
*c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d^
6)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 + 1
3*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*d^6
 + 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)
*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) +
c)/((c^2 - d^2)*sin(f*x + e))) - (4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 55
5*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8 + (14*c^8 - 60*c^7*d + 78*c^6
*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^3*d^5 - 419*c^2*d^6 - 90*c*d^7 + 1
5*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7*d + 114*c^6*d^2 + 1056*c^5*d^3 + 1
626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6 - 913*c*d^7 - 45*d^8)*cos(f*x + e)^
3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 828*c^5*d^3 + 2400*c^4*d^4 + 1197*c^3*d
^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*d^8)*cos(f*x + e)^2 + (8*c^7*d - 48*c^
6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*d^5 - 459*c^2*d^6 - 1733*c*d^
7 - 717*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^11 - 3*a^3*c^10*d + 8*a^3*
```

```
c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^6*d^5 + 8*a^3*c^5*d^6 - 3*a^3*c^3*d^8 + a
^3*c^2*d^9)*f*cos(f*x + e)^5 + (3*a^3*c^11 - 7*a^3*c^10*d - 6*a^3*c^9*d^2 +
 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4 - 30*a^3*c^6*d^5 + 12*a^3*c^5*d^6 + 16*a^3*
c^4*d^7 - 9*a^3*c^3*d^8 - 3*a^3*c^2*d^9 + 2*a^3*c*d^10)*f*cos(f*x + e)^4 +
(3*a^3*c^11 - 3*a^3*c^10*d - 17*a^3*c^9*d^2 + 21*a^3*c^8*d^3 + 30*a^3*c^7*d
^4 - 46*a^3*c^6*d^5 - 18*a^3*c^5*d^6 + 42*a^3*c^4*d^7 - a^3*c^3*d^8 - 15*a^
3*c^2*d^9 + 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 + (a^3*c^11 + 3*a^3*c
^10*d - 15*a^3*c^9*d^2 - a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 18*a^3*c^6*d^5 - 46
*a^3*c^5*d^6 + 30*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 3*a^3*c*d
^10 + 3*a^3*d^11)*f*cos(f*x + e)^2 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*
c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7
+ 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e
) + (a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^
4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(e+fx)}{c^3\sec^3(e+fx)+3c^3\sec^2(e+fx)+3c^3\sec(e+fx)+c^3+3c^2d\sec^4(e+fx)+9c^2d\sec^3(e+fx)+9c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^5(e+fx)+9cd^2\sec^4(e+fx)+9cd^2\sec^3(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^6(e+fx)+3d^3\sec^5(e+fx)+3d^3\sec^4(e+fx)+d^3\sec^3(e+fx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x)**3 + 3*c**3*sec(e + f*x)**2 + 3*c*
*3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**4 + 9*c**2*d*sec(e + f*x)**
3 + 9*c**2*d*sec(e + f*x)**2 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x
)**5 + 9*c*d**2*sec(e + f*x)**4 + 9*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**6 + 3*d**3*sec(e + f*x)**5 + 3*d**3*sec(e +
 f*x)**4 + d**3*sec(e + f*x)**3), x)/a**3
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. 2(349) = 698.
time = 0.64, size = 1369, normalized size = 3.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(20*c^2*d^3 + 30*c*d^4 + 13*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2
)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e
))/sqrt(-c^2 + d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4*d^3
 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*sqrt(-c^2 + d^2)) -
 (3*a^12*c^12*tan(1/2*f*x + 1/2*e)^5 - 36*a^12*c^11*d*tan(1/2*f*x + 1/2*e)^
5 + 198*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^5 - 660*a^12*c^9*d^3*tan(1/2*f*x
```

```
 + 1/2*e)^5 + 1485*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^5 - 2376*a^12*c^7*d^5*
tan(1/2*f*x + 1/2*e)^5 + 2772*a^12*c^6*d^6*tan(1/2*f*x + 1/2*e)^5 - 2376*a^
12*c^5*d^7*tan(1/2*f*x + 1/2*e)^5 + 1485*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e)^
5 - 660*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^5 + 198*a^12*c^2*d^10*tan(1/2*f*x
 + 1/2*e)^5 - 36*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^12*tan(1/2*f
*x + 1/2*e)^5 - 10*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 150*a^12*c^11*d*tan(1
/2*f*x + 1/2*e)^3 - 990*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^3 + 3850*a^12*c^
9*d^3*tan(1/2*f*x + 1/2*e)^3 - 9900*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^3 + 1
7820*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e)^3 - 23100*a^12*c^6*d^6*tan(1/2*f*x +
 1/2*e)^3 + 21780*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e)^3 - 14850*a^12*c^4*d^8*
tan(1/2*f*x + 1/2*e)^3 + 7150*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^3 - 2310*a^
12*c^2*d^10*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^3
 - 40*a^12*d^12*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^12*tan(1/2*f*x + 1/2*e)
- 270*a^12*c^11*d*tan(1/2*f*x + 1/2*e) + 2340*a^12*c^10*d^2*tan(1/2*f*x + 1
/2*e) - 11850*a^12*c^9*d^3*tan(1/2*f*x + 1/2*e) + 38475*a^12*c^8*d^4*tan(1/
2*f*x + 1/2*e) - 84780*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e) + 131040*a^12*c^6*
d^6*tan(1/2*f*x + 1/2*e) - 144180*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e) + 11272
5*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e) - 61350*a^12*c^3*d^9*tan(1/2*f*x + 1/2*
e) + 22140*a^12*c^2*d^10*tan(1/2*f*x + 1/2*e) - 4770*a^12*c*d^11*tan(1/2*f*
x + 1/2*e) + 465*a^12*d^12*tan(1/2*f*x + 1/2*e))/(a^15*c^15 - 15*a^15*c^14*
d + 105*a^15*c^13*d^2 - 455*a^15*c^12*d^3 + 1365*a^15*c^11*d^4 - 3003*a^15*
c^10*d^5 + 5005*a^15*c^9*d^6 - 6435*a^15*c^8*d^7 + 6435*a^15*c^7*d^8 - 5005
*a^15*c^6*d^9 + 3003*a^15*c^5*d^10 - 1365*a^15*c^4*d^11 + 455*a^15*c^3*d^12
 - 105*a^15*c^2*d^13 + 15*a^15*c*d^14 - a^15*d^15) + 60*(10*c^2*d^4*tan(1/2
*f*x + 1/2*e)^3 - 3*c*d^5*tan(1/2*f*x + 1/2*e)^3 - 7*d^6*tan(1/2*f*x + 1/2*
e)^3 - 10*c^2*d^4*tan(1/2*f*x + 1/2*e) - 15*c*d^5*tan(1/2*f*x + 1/2*e) - 5*
d^6*tan(1/2*f*x + 1/2*e))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4
*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*(c*tan(1/2*f*x
+ 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B]
time = 2.36, size = 655, normalized size = 1.78

$$\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{20\,a^3\,f\,{\left(c-d\right)}^3}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,{\left(c+d\right)}^2}{4\,a^3\,{\left(c-d\right)}^5}-\frac{5}{2\,a^3\,{\left(c-d\right)}^3}+\frac{3\,\left(c+d\right)\,\left(\frac{5}{4\,a^3\,{\left(c-d\right)}^3}-\frac{3\,\left(c+d\right)}{4\,a^3\,{\left(c-d\right)}^4}\right)}{c-d}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{5}{12\,a^3\,{\left(c-d\right)}^3}-\frac{c+d}{4\,a^3\,{\left(c-d\right)}^4}\right)}{f}-\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(-10\,c^2\,d^4+3\,c\,d^5+7\,d^6\right)}{{\left(c+d\right)}^2}+\frac{5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(d^5+2\,c\,d^4\right)}{c+d}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,a^3\,c^7-10\,a^3\,c^6\,d+18\,a^3\,c^5\,d^2-10\,a^3\,c^4\,d^3-10\,a^3\,c^3\,d^4+18\,a^3\,c^2\,d^5-10\,a^3\,c\,d^6+2\,a^3\,d^7\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(a^3\,c^7-7\,a^3\,c^6\,d+21\,a^3\,c^5\,d^2-35\,a^3\,c^4\,d^3+35\,a^3\,c^3\,d^4-21\,a^3\,c^2\,d^5+7\,a^3\,c\,d^6-a^3\,d^7\right)-a^3\,c^7+a^3\,d^7-3\,a^3\,c\,d^6+3\,a^3\,c^6\,d+a^3\,c^2\,d^5+5\,a^3\,c^3\,d^4-5\,a^3\,c^4\,d^3-a^3\,c^5\,d^2\right)}+\frac{d^3\,\mathrm{atan}\left(\frac{1{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^6-6{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^5\,d+15{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^4\,d^2-20{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^3\,d^3+15{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c^2\,d^4-6{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,c\,d^5+1{}\mathrm{i}\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,d^6}{\sqrt{c+d}\,{\left(c-d\right)}^{11/2}}\right)\,\left(20\,c^2+30\,c\,d+13\,d^2\right)\,1{}\mathrm{i}}{a^3\,f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c - d)^3) - (tan(e/2 + (f*x)/2)*((3*(c + d)
^2)/(4*a^3*(c - d)^5) - 5/(2*a^3*(c - d)^3) + (3*(c + d)*(5/(4*a^3*(c - d)^
3) - (3*(c + d))/(4*a^3*(c - d)^4)))/(c - d)))/f - (tan(e/2 + (f*x)/2)^3*(5
/(12*a^3*(c - d)^3) - (c + d)/(4*a^3*(c - d)^4)))/f - ((tan(e/2 + (f*x)/2)^
3*(3*c*d^5 + 7*d^6 - 10*c^2*d^4))/(c + d)^2 + (5*tan(e/2 + (f*x)/2)*(2*c*d^
4 + d^5))/(c + d))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^3*c^7 + 2*a^3*d^7 - 10*a^3
*c*d^6 - 10*a^3*c^6*d + 18*a^3*c^2*d^5 - 10*a^3*c^3*d^4 - 10*a^3*c^4*d^3 +
```

```
18*a^3*c^5*d^2) - tan(e/2 + (f*x)/2)^4*(a^3*c^7 - a^3*d^7 + 7*a^3*c*d^6 - 7
*a^3*c^6*d - 21*a^3*c^2*d^5 + 35*a^3*c^3*d^4 - 35*a^3*c^4*d^3 + 21*a^3*c^5*
d^2) - a^3*c^7 + a^3*d^7 - 3*a^3*c*d^6 + 3*a^3*c^6*d + a^3*c^2*d^5 + 5*a^3*
c^3*d^4 - 5*a^3*c^4*d^3 - a^3*c^5*d^2)) + (d^3*atan((c^6*tan(e/2 + (f*x)/2)
*1i + d^6*tan(e/2 + (f*x)/2)*1i - c*d^5*tan(e/2 + (f*x)/2)*6i - c^5*d*tan(e
/2 + (f*x)/2)*6i + c^2*d^4*tan(e/2 + (f*x)/2)*15i - c^3*d^3*tan(e/2 + (f*x)
/2)*20i + c^4*d^2*tan(e/2 + (f*x)/2)*15i)/((c + d)^(1/2)*(c - d)^(11/2)))*(
30*c*d + 20*c^2 + 13*d^2)*1i)/(a^3*f*(c + d)^(5/2)*(c - d)^(11/2))
```

3.234 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}\,f}$$

```
[Out] 2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))*a^(1/2)/f/d^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {4065, 212}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4065

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-2*(b/f), Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+}}\right.}{f}$$

$$= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

Mathematica [A]
time = 0.25, size = 102, normalized size = 1.67

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{d+c\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{d}f\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],
x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(S
qrt[d]*f*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(49) = 98$.
time = 3.58, size = 302, normalized size = 4.95

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\cos(fx+e)\left(\ln\left(-\frac{2\left(\sqrt{2}\sqrt{-d}\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\sin(fx+e)+c\cos(fx+e)-d\cos\right.}{\cos(fx+e)-1-\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2
)*cos(f*x+e)*(ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1)
)^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c+d)
```

```
/(cos(f*x+e)-1-sin(f*x+e)))-ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/
(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c*sin(f*x+e)-d*s
in(f*x+e)+c-d)/(cos(f*x+e)-1+sin(f*x+e))))*(-1+cos(f*x+e))/sin(f*x+e)^2/(-2
*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*2^(1/2)/(-d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(52) = 104$.
time = 2.53, size = 327, normalized size = 5.36

$$\left[\frac{\sqrt{\frac{a}{d}}\log\left(-\frac{8\,acd\cos(fx+e)+(ac^2-6\,acd+ad^2)\cos(fx+e)^3+4\left(2\,d^2\cos(fx+e)+(cd-d^2)\cos(fx+e)^2\right)\sqrt{\frac{a}{d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\sin(fx+e)+8\,ad^2+(ac^2+2\,acd-7\,ad^2)\cos(fx+e)^2}{\cos(fx+e)^3+\cos(fx+e)^2}\right)}{2\,f}, \frac{\sqrt{-\frac{a}{d}}\arctan\left(-\frac{2\,d\sqrt{-\frac{a}{d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{(ac-ad)\cos(fx+e)^2+2\,ad+(ac+ad)\cos(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(a/d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f
*x + e)^3 + 4*(2*d^2*cos(f*x + e) + (c*d - d^2)*cos(f*x + e)^2)*sqrt(a/d)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x +
e))*sin(f*x + e) + 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(c
os(f*x + e)^3 + cos(f*x + e)^2))/f, sqrt(-a/d)*arctan(-2*d*sqrt(-a/d)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e)/((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)
*cos(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\,\sec\left(e+fx\right)}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)), x
)
```

3.235 $\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\sqrt{2}\sqrt{c-d}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{a+a\sec(e+fx)}}\right.}{\sqrt{a}\,f}$$

```
[Out] arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^(1/2)+2*arctanh(a^(1/2)*d^(1/
2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*d^(1/2)/f/a^(1
/2)
```

Rubi [A]

time = 0.30, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4066, 4068, 209, 4065, 212}

$$\frac{\sqrt{2}\sqrt{c-d}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f}+\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) + (2*Sqrt[d]*
ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d
*Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4065

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-2*(b/f), Subst
```

```
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4066

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-(b*c - a*d)/d,
 Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x],
x] + Dist[b/d, Int[Csc[e + f*x]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - (-c+d)\int \frac{\text{se}}{\sqrt{a+a\sec(e+}}$$

$$= -\frac{(2(c-d))\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+}}\right.}{f}$$

$$= \frac{\sqrt{2}\sqrt{c-d}\,\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\,\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f}$$

Mathematica [A]
time = 18.49, size = 187, normalized size = 1.34

$$\frac{\sqrt{c}\left(-\sqrt{2}\sqrt{c-d}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{c-d}\sqrt{c-c\cos(e+fx)}}\right)+2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{d}\sqrt{c-c\cos(e+fx)}}\right)\right)\sqrt{c+d\sec(e+fx)}\sin(e+fx)}{f\sqrt{c-c\cos(e+fx)}\sqrt{d+c\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],
x]
```

```
[Out] (Sqrt[c]*(-(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + c*Cos[e +
f*x]])/(Sqrt[c - d]*Sqrt[c - c*Cos[e + f*x]])]) + 2*Sqrt[d]*ArcTanh[(Sqrt[c
]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c - c*Cos[e + f*x]])])*Sqrt[c + d
*Sec[e + f*x]]*Sin[e + f*x])/(f*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e +
 f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(113) = 226$.
time = 3.64, size = 503, normalized size = 3.59

method	result
default	$\frac{\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\,\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\cos(fx+e)(-1+\cos(fx+e))\left(\ln\left(\frac{\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\,\sqrt{c-d}\,\sin(fx+e)-c\cos(f}{\sin(fx+e)\sqrt{c-d}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)
*cos(f*x+e)*(-1+cos(f*x+e))*(ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
*(c-d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(1/
2))*2^(1/2)*(-d)^(1/2)*c-ln(((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c-
d)^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/sin(f*x+e)/(c-d)^(1/2))*
2^(1/2)*(-d)^(1/2)*d+d*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(
f*x+e)+1))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*
x+e)+c-d)/(cos(f*x+e)-1+sin(f*x+e)))*(c-d)^(1/2)-d*ln(-2*(2^(1/2)*(-d)^(1/2
)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(
f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)-c+d)/(cos(f*x+e)-1-sin(f*x+e)))*(c-d)^(1/2
))/sin(f*x+e)^2/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/a/(c-d)^(1/2)*2^
(1/2)/(-d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

Fricas [A]
time = 5.32, size = 1120, normalized size = 8.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*cos(f*x + e) + c
 - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(d/a)*log(-((c^2 - 6*c
*d + d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sq
rt(d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/c
os(f*x + e))*sin(f*x + e) + 8*c*d*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(
f*x + e)^2 + 8*d^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/f, 1/2*(2*sqrt(2)*s
qrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*s
in(f*x + e))) + sqrt(d/a)*log(-((c^2 - 6*c*d + d^2)*cos(f*x + e)^3 + 4*((c
- d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(d/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8*c*d
*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(f*x + e)^2 + 8*d^2)/(cos(f*x + e)
^3 + cos(f*x + e)^2)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqr
t(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2
*(c + d)*cos(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2
*sqrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((c - d)*co
s(f*x + e)^2 + (c + d)*cos(f*x + e) + 2*d)))/f, (sqrt(2)*sqrt((c - d)/a)*ar
ctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + s
qrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((c - d)*cos(
f*x + e)^2 + (c + d)*cos(f*x + e) + 2*d)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + d\sec{(e + fx)}}\,\sec{(e + fx)}}{\sqrt{a(\sec{(e + fx)} + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*sec(e + f*x))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c + d/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

3.236 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}\,f}$$

[Out] arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {4068, 209}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f\sqrt{c-d}}$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]

[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c}}\right.}{f}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx}}\right.}{\sqrt{a}\sqrt{c-d}f}$$

Mathematica [A]
time = 0.22, size = 107, normalized size = 1.37

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+fx)}\sec(e+fx)}{\sqrt{c-d}f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (2*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]]*Cos[(e +
 f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(Sqrt[c - d]*f*Sqrt[a*(1 +
Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(63) = 126$.
time = 3.49, size = 170, normalized size = 2.18

method	result
default	$\frac{2\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\cos(fx+e)(-1+\cos(fx+e))\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\ln\left(-\frac{\sqrt{c-d}\cos(fx+e)-\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\sin(}{\sin(fx+e)}\right.}{f\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\sin(fx+e)^2a\sqrt{c-d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*cos(f*x+e)*(-1+cos(f*x+e))*((d+c*co
s(f*x+e))/cos(f*x+e))^(1/2)*ln(-((c-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+e)
)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e))/(-2*(d+c*cos(f*
x+e))/(cos(f*x+e)+1))^(1/2)/sin(f*x+e)^2/a/(c-d)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Fricas [A]
time = 4.10, size = 262, normalized size = 3.36

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{ac-ad}}\log\left(-\frac{2\sqrt{2}(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\sqrt{-\frac{1}{ac-ad}}\cos(fx+e)\sin(fx+e)-(3c-d)\cos(fx+e)^2-2(c+d)\cos(fx+e)+c-3d}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)}{2f}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{ac-ad}\sin(fx+e)}\right)}{\sqrt{ac-ad}\,f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/(a*c - a*d))*log(-(2*sqrt(2)*(c - d)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a*c
 - a*d))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*c
os(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))/f, -sqrt(2)*a
rctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/(sqrt(a*c -
 a*d)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\sqrt{c+d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
 x)
```

3.237 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,\sqrt{c-d}\,f}+\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,\sqrt{d}\,f}$$

```
[Out] -arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(
c+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)+2*arctanh(a^(1/2)*d^(1
/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/f/a^(1/2)/d^(
1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {4070, 4068, 209, 4065, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,\sqrt{d}\,f}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)) + (2*ArcTa
nh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[
e + f*x]])])/(Sqrt[a]*Sqrt[d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4065

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-2*(b/f), Subst
```

```
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4070

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-a/b, Int[Csc
[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dis
t[1/b, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \int \frac{\text{se}}{\sqrt{a+a\sec(e+}}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d}}\right.}{f}$$

$$= -\frac{\sqrt{2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-d}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}\,\sqrt{c+d\sec(e+f}}\right.}{\sqrt{a}\,\sqrt{c-d}\,f}$$

Mathematica [A]
time = 0.28, size = 171, normalized size = 1.21

$$\frac{2\left(-\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{c-d}\,\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)+\sqrt{2}\,\sqrt{c-d}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{d}\,\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+fx)}\,\sec(e+fx)}{\sqrt{c-d}\,\sqrt{d}\,f\sqrt{a(1+\sec(e+fx))}\,\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]
),x]
```

[Out] (2*(-(Sqrt[d]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]) + Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d
 + c*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x]
)/(Sqrt[c - d]*Sqrt[d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]
])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(114) = 228$.
time = 3.40, size = 403, normalized size = 2.86

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\,\cos(fx+e)(-1+\cos(fx+e))\left(\ln\left(-\frac{\sqrt{c-d}\,\cos(fx+e)-\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\,\sin}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)

[Out] -1/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2
)*cos(f*x+e)*(-1+cos(f*x+e))*(ln(-((c-d)^(1/2)*cos(f*x+e)-(-2*(d+c*cos(f*x+
e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-(c-d)^(1/2))/sin(f*x+e))*2^(1/2)*(-d)^
(1/2)-ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*
sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c*sin(f*x+e)-d*sin(f*x+e)+c-d)/(cos(f*
x+e)-1+sin(f*x+e)))*(c-d)^(1/2)+ln(2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e
))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c*sin(f*x+e)-
d*sin(f*x+e)-c+d)/(-cos(f*x+e)+1+sin(f*x+e)))*(c-d)^(1/2))/sin(f*x+e)^2/(-2
*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/a/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="maxima")

[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)

Fricas [A]
time = 3.90, size = 1172, normalized size = 8.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/
(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c +
d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(a*
d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 +
4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) +
 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + co
s(f*x + e)^2)))/(a*d*f), 1/2*(2*sqrt(2)*a*d*arctan(sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a*c - a*d) + sqrt(a*d)*log(-(8*a*c*d
*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f
*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8*a*d^2 + (a*c^
2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/
(a*d*f), 1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*
sqrt(-1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 +
 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) +
 2*sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c - a
*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))))/(a*d*f), (sqrt(2)*
a*d*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a
*c - a*d) + sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f
*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/
((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))))/(a*d*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,\sqrt{c+d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x
))), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
), x)
```

3.238 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{d}\,\sqrt{c+d}\,f}$$

```
[Out] 2*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(
1/2)/f/d^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {4052, 211}

$$\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}\,f\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Se
c[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = -\frac{(2a)\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}\,f}$$

Mathematica [A]
time = 0.22, size = 94, normalized size = 1.54

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{d}\sqrt{c+d}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e
+ f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(
Sqrt[d]*Sqrt[c + d]*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(47) = 94.
time = 8.22, size = 432, normalized size = 7.08

method	result
default	$-\frac{\left(\ln\left(\frac{2\sqrt{2}\sqrt{\frac{d}{c-d}}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,c\sin(fx+e)-2\sqrt{2}\sqrt{\frac{d}{c-d}}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\,d\sin(fx+e)-2c\sin(fx+e)+2d\sin(fx+e)}{\sqrt{(c+d)(c-d)}\sin(fx+e)-c\cos(fx+e)+d\cos(fx+e)+c-}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2/f*(ln(2*(2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
c*sin(f*x+e)-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d
*sin(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)
*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-
d))-ln(2*(-2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*s
```

```
in(f*x+e)+2^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*si
n(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c
-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))
)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*
2^(1/2)/(d/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(49) = 98$.
time = 2.70, size = 360, normalized size = 5.90

$$\left[\frac{\sqrt{-\frac{a}{cd+d^2}}\log\left(-\frac{(ac^2+8acd+8ad^2)\cos(fx+e)^3+ad^2+(ac^2+2acd)\cos(fx+e)^2-4\left((c^2d+3cd^2+2d^3)\cos(fx+e)^2-(cd^2+d^3)\cos(fx+e)\right)\sqrt{-\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sin(fx+e)-(6acd+7ad^2)\cos(fx+e)}{c^2\cos(fx+e)^3+(c^2+2cd)\cos(fx+e)^2+d^2+(2cd+d^2)\cos(fx+e)}\right)}{2f}, \frac{\sqrt{\frac{a}{cd+d^2}}\arctan\left(\frac{2(cd+d^2)\sqrt{\frac{a}{cd+d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{(ac+2ad)\cos(fx+e)^2-ad+(ac+ad)\cos(fx+e)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*sqrt(-a/(c*d + d^2))*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3
+ a*d^2 + (a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*((c^2*d + 3*c*d^2 + 2*d^3)*c
os(f*x + e)^2 - (c*d^2 + d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*x +
e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2
)*cos(f*x + e)))/f, sqrt(a/(c*d + d^2))*arctan(2*(c*d + d^2)*sqrt(a/(c*d +
d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a
*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e)))/f]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sec(e+fx)+1)}\sec(e+fx)}{c+d\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(47) = 94.
time = 0.87, size = 139, normalized size = 2.28

$$\frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{2}\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2c-\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2d+ac+3\,ad\right)}{4\sqrt{-cd-d^2}\,a}\right)\mathrm{sgn}(\cos(fx+e))}{\sqrt{-cd-d^2}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] 2*sqrt(-a)*arctan(1/4*sqrt(2)*((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan
(1/2*f*x + 1/2*e)^2 + a))^2*c - (sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*ta
n(1/2*f*x + 1/2*e)^2 + a))^2*d + a*c + 3*a*d)/(sqrt(-c*d - d^2)*a))*sgn(cos
(f*x + e))/(sqrt(-c*d - d^2)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

3.239 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=149

$$\frac{2\sqrt{a}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{df}-\frac{2\sqrt{a}\,\sqrt{c}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{d\sqrt{c+d}\,f}$$

```
[Out] 2*g^(3/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2))*a^(1/2)/d/f-2*g^(3/2)*arctanh(a^(1/2)*c^(1/2)*g^(1/2)*tan(f*
x+e)/(c+d)^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)*c^(1/
2)/d/f/(c+d)^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4055, 3887, 214, 4050}

$$\frac{2\sqrt{a}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{df}-\frac{2\sqrt{a}\,\sqrt{c}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{df\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),
x]
```

```
[Out] (2*Sqrt[a]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]])])/(d*f) - (2*Sqrt[a]*Sqrt[c]*g^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]])])/(d*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3887

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/f), Subst[Int[1/(b - d*x^2), x], x, b*(C
ot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[a*(d/b), 0]
```

Rule 4050

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*b*(g
```

```
/f), Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc[
e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4055

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[g/d,
Int[Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c*(g/d), I
nt[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx &= \frac{g\int\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx}{d} - \frac{(cg)\int \frac{\sqrt{g\sec}}{}}{} \\ &= -\frac{(2ag^2)\,\mathrm{Subst}\left(\int\frac{1}{a-gx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(}}\right.}{df} \\ &= \frac{2\sqrt{a}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df} -\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.40, size = 427, normalized size = 2.87

$$\frac{\left(-2i+\sqrt{2}\right)g^2\left(2\sqrt{c+d}\,\mathrm{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(e+fx)\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{1}{4}(e+fx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)}\right)+2\sqrt{c+d}\,\mathrm{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(e+fx)\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{1}{4}(e+fx)\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)}\right)+i\left(2\sqrt{c+d}\log\left(\sqrt{2}+2\sin\left(\frac{1}{2}(e+fx)\right)\right)-\sqrt{c+d}\log\left(2-\sqrt{2}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)-\sqrt{c+d}\log\left(2+\sqrt{2}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)+2\sqrt{c}\log\left(\sqrt{2}\sqrt{c+d}-2\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)\right)-2\sqrt{c}\log\left(\sqrt{2}\sqrt{c+d}+2\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{4\left(i+\sqrt{2}\right)d\sqrt{c+d}\,f\sqrt{g\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I + Sqrt[2])*g^2*(2*Sqrt[c + d]*ArcTan[(Cos[(e + f*x)/4] - (-1 + Sqrt[
2])*Sin[(e + f*x)/4])/((1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]
+ 2*Sqrt[c + d]*ArcTan[(Cos[(e + f*x)/4] - (1 + Sqrt[2])*Sin[(e + f*x)/4])/
((-1 + Sqrt[2])*Cos[(e + f*x)/4] - Sin[(e + f*x)/4])] + I*(2*Sqrt[c + d]*Lo
g[Sqrt[2] + 2*Sin[(e + f*x)/2]] - Sqrt[c + d]*Log[2 - Sqrt[2]*Cos[(e + f*x)
/2] - Sqrt[2]*Sin[(e + f*x)/2]] - Sqrt[c + d]*Log[2 + Sqrt[2]*Cos[(e + f*x)
/2] - Sqrt[2]*Sin[(e + f*x)/2]] + 2*Sqrt[c]*Log[Sqrt[2]*Sqrt[c + d] - 2*Sqr
```

```
t[c]*Sin[(e + f*x)/2]] - 2*Sqrt[c]*Log[Sqrt[2]*Sqrt[c + d] + 2*Sqrt[c]*Sin[
(e + f*x)/2]]))*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(4*(I + Sqrt[2
])*d*Sqrt[c + d]*f*Sqrt[g*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(117) = 234$.
time = 13.20, size = 568, normalized size = 3.81

method	result
default	$\frac{2\left(\frac{g}{\cos(fx+e)}\right)^{\frac{3}{2}}(\cos^2(fx+e))(-1+\cos(fx+e))^2\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\left(\sqrt{\frac{c}{c-d}}\ \operatorname{arctanh}\left(\frac{\sqrt{\frac{1}{\cos(fx+e)+1}}\ (1+\cos(fx+e)+\sin(fx}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] 2/f*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2*(-1+cos(f*x+e))^2*(a*(cos(f*x+e)+1)/c
os(f*x+e))^(1/2)*((1/(c-d)*c)^(1/2)*arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(1
+cos(f*x+e)+sin(f*x+e)))*((c+d)*(c-d))^(1/2)-(1/(c-d)*c)^(1/2)*arctanh(1/2*
(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)-sin(f*x+e)))*((c+d)*(c-d))^(1/2)-c*l
n(-2*(2*(1/(c-d)*c)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2*(1/(c-d)*
c)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-((c+d)*(c-d))^(1/2)*cos(f*x+
e)+c*sin(f*x+e)-d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(
f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))+c*ln(-2*(2*(1/(c-d)*c)^(1/2)*(1/(cos
(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2*(1/(c-d)*c)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)
*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+
d)*(c-d))^(1/2))/(c*cos(f*x+e)-d*cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-
c+d)))/sin(f*x+e)^4/(1/(cos(f*x+e)+1))^(3/2)*(c-d)/(1/(c-d)*c)^(1/2)/(-c+d+
((c+d)*(c-d))^(1/2))/(c-d+((c+d)*(c-d))^(1/2))/((c+d)*(c-d))^(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 8.05, size = 1196, normalized size = 8.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*c*g/(c + d))*g*log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*
d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)
*cos(f*x + e))*sqrt(a*c*g/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) + (8*a
*c^2 + 8*a*c*d + a*d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)
^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + sqrt(a*g)*g*log((a*g*cos(f*x + e)
^3 - 7*a*g*cos(f*x + e)^2 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) +
8*a*g)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/(d*f), -1/2*(2*sqrt(-a*c*g/(c +
d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(a*c*g*
sin(f*x + e))) - sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2
 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 +
cos(f*x + e)^2)))/(d*f), 1/2*(2*sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(a*g*cos(f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g)) + sqrt(a*c*g/(c + d))*g*
log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*d)*g*cos(f*x + e)^2 + 4*((c^
2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x
 + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) + (8*a*c^2 + 8*a*c*d + a*d^2)*g)/(
c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos
(f*x + e))))/(d*f), (sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*g*cos(
f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g)) - sqrt(-a*c*g/(c + d))*g*arctan(1/2
*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-a*c*g/(c + d))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(a*c*g*sin(f*x + e))))/
(d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{c+d\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(g*sec(e + f*x))**(3/2)/(c + d*sec(e +
f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) +
 c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c+\frac{d}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
),x)
```

```
[Out] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
), x)
```

3.240 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=122

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f}-\frac{2\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)\sqrt{c+d}\,f}$$

```
[Out] arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)
/f/a^(1/2)-2*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))
^(1/2))*d^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4057, 3880, 209, 4052, 211}

$$\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)}-\frac{2\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])
/(Sqrt[a]*(c - d)*f) - (2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 4057

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[
Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[Csc
[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^
2 - d^2, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+ ...}}{c+d\sec(e+fx)}}{a(c-d)}$$

$$= -\frac{2\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} + \frac{(2d)\mathrm{S} ...}{...}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f} - \frac{2\sqrt{d}\tan^{-1}\left(...\right)}{\sqrt{a} ...}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 33.85, size = 229015, normalized size = 1877.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. 2(99) = 198.
time = 9.03, size = 520, normalized size = 4.26

method	result
default	$\frac{\left(2\sqrt{(c+d)(c-d)}\,\ln\left(-\frac{-\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}+\cos(fx+e)-1}{\sin(fx+e)}\right)\sqrt{\frac{d}{c-d}}+d\sqrt{2}\,\ln\left(-\frac{2\left(\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\sqrt{-\frac{2}{\mathrm{co}}}\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/f*(2*((c+d)*(c-d))^(1/2)*ln(-(-sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)+cos(f*x+e)-1)/sin(f*x+e))*(d/(c-d))^(1/2)+d*2^(1/2)*ln(-2*(2^(1/2)*
(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(
d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+
e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(
f*x+e)-d*cos(f*x+e)-((c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))-d*2^(1/2)*ln(2*(-2
^(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2^
(1/2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c
+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/
(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))*(-2*cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)/(d/(c-d))^(1
/2)/(c-d)/((c+d)*(c-d))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Fricas [A]
time = 3.72, size = 1021, normalized size = 8.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x +
 e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(-d/(a*c + a*d))*log(
-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 - 4*(
(c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(-d/(a
*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + d^2 - (6*
c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)
^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*(sqrt(2)*sqrt(-1
/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f
*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(d/(a*c + a*d))*arctan(2*(c + d)*sqrt(d/(
a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)))/((c - d)*f), -1/
2*(sqrt(-d/(a*c + a*d))*log(-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 +
 2*c*d)*cos(f*x + e)^2 - 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d
^2)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3
+ (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqr
t(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f), -(sqrt(d/(a*c + a*d))*arctan(2*
(c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)) +
 sqrt(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,(c+d\sec(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+fx)\sqrt{a+\frac{a}{\cos(e+fx)}}\left(c+\frac{d}{\cos(e+fx)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.241 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=124

$$-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f}+\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)\sqrt{d}\,\sqrt{c+d}\,f}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d
)/f/a^(1/2)+2*c*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+
e))^(1/2))/(c-d)/f/a^(1/2)/d^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4061, 3880, 209, 4052, 211}

$$\frac{2c\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{d}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,\sqrt{d}\,f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\,f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[a]*(c - d)*f)) + (2*c*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[
c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[d]*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 4061

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[-a/(b*c - a*d), I
nt[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[c/(b*c - a*d), Int[
Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ
[c^2 - d^2, 0])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{c\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+\ldots}}{c+d\sec(e+fx)}}{a(c-d)}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} - \frac{(2c)\text{Subs}\ldots}{\ldots}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f} + \frac{2c\tan^{-1}\left(\frac{\ldots}{\sqrt{c\ldots}}\right.}{\sqrt{a}\,(c\ldots}$$

Mathematica [A]
time = 0.41, size = 141, normalized size = 1.14

$$-\frac{2\left(\sqrt{d}\sqrt{c+d}\,\text{ArcTan}\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)-\sqrt{2}\,c\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)}{(c-d)\sqrt{d}\sqrt{c+d}\,f\sqrt{\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*(Sqrt[d]*Sqrt[c + d]*ArcTan[Sin[(e + f*x)/2]/Sqrt[Cos[e + f*x]]] - Sqrt
[2]*c*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f
```

```
*x]])])*Cos[(e + f*x)/2])/((c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[Cos[e + f*x]]
*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(101) = 202$.
time = 9.42, size = 518, normalized size = 4.18

method	result
default	$-\frac{\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}\left(2\ln\left(\frac{\sin(fx+e)\sqrt{-\frac{2\cos(fx+e)}{\cos(fx+e)+1}}-\cos(fx+e)+1}{\sin(fx+e)}\right)\sqrt{(c+d)(c-d)}\sqrt{\frac{d}{c-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2/f*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*(2*ln((sin(f*x+e)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)+1)/s
in(f*x+e))*((c+d)*(c-d))^(1/2)*(d/(c-d))^(1/2)+c*2^(1/2)*ln(2*(2^(1/2)*(d/(
c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2)*(d/(c
-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-c*sin(f*x+e)+d
*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-((c+d)*(c-d))^(1/2))/(((c+d)*(c-
d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)+c-d))-c*2^(1/2)*ln(-2*(2^(1/
2)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2^(1/2
)*(d/(c-d))^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-((c+d)*
(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)+((c+d)*(c-d))^(1/2))/(((c
+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d)))/a/((c+d)*(c-d)
)^(1/2)/(c-d)/(d/(c-d))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

Fricas [A]
time = 3.29, size = 1099, normalized size = 8.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a*
c*d - a*d^2)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (
a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*cos(f*x
 + e)^2 - d*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
 e) - (6*a*c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)
*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f)
, -1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x +
e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt(
a*c*d + a*d^2)*c*arctan(2*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d +
(a*c + a*d)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f), 1/2*(sqrt(-a*c*d - a*d^2
)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (a*c^2 + 2*a
*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*cos(f*x + e)^2 - d
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*
c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x +
e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqrt(2)*(a*c*d + a*d^2)*arcta
n(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin
(f*x + e)))/sqrt(a))/((a*c^2*d - a*d^3)*f), (sqrt(a*c*d + a*d^2)*c*arctan(2
*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))
) + sqrt(2)*(a*c*d + a*d^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((a*c^2*d - a*d^3)*f)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,(c+d\sec(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))),
 x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.242 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=167

$$-\frac{\sqrt{2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f}+\frac{2\sqrt{c}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)\sqrt{c+d}\,f}$$

```
[Out] -g^(3/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2^(1/2)/(g*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)+2*g^(3/2)*arctanh(a^(1/2)
*c^(1/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e
))^(1/2))*c^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4059, 3893, 214, 4050}

$$\frac{2\sqrt{c}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{c}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{c+d}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{a\sec(e+fx)+a}\,\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] -((Sqrt[2]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*S
ec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[c]*
g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4050

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*b*(g
/f), Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc[
e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4059

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(-a)*
(g/(b*c - a*d)), Int[Sqrt[g*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Dist[c*(g/(b*c - a*d)), Int[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*x]
]/(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = -\frac{g\int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{(cg)\int \frac{\sqrt{g\sec(e+fx)}\,\sqrt{\cdots}}{c+d\sec\cdots}}{a(c-\cdots}$$

$$= \frac{(2g^2)\,\text{Subst}\left(\int \frac{1}{2a-gx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec\cdots}}\right.}{(c-d)f}$$

$$= -\frac{\sqrt{2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{g}\,\tan(e+fx)}{\sqrt{2}\,\sqrt{g\sec(e+fx)}\,\sqrt{a+a\sec(e+f\cdots}}\right.}{\sqrt{a}\,(c-d)f}$$

Mathematica [A]
time = 0.41, size = 198, normalized size = 1.19

$$\frac{g\cos\left(\frac{1}{2}(e+fx)\right)\left(2\sqrt{c+d}\,\log\left(\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)\right)-2\sqrt{c+d}\,\log\left(\cos\left(\frac{1}{4}(e+fx)\right)+\sin\left(\frac{1}{4}(e+fx)\right)\right)+\sqrt{2}\,\sqrt{c}\left(-\log\left(\sqrt{2}\,\sqrt{c+d}-2\sqrt{c}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\sqrt{2}\,\sqrt{c+d}+2\sqrt{c}\,\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\sqrt{g\sec(e+fx)}}{(c-d)\sqrt{c+d}\,f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (g*Cos[(e + f*x)/2]*(2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] - Sin[(e + f*x)/4]]
 - 2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] + Sin[(e + f*x)/4]] + Sqrt[2]*Sqrt[c]
*(-Log[Sqrt[2]*Sqrt[c + d] - 2*Sqrt[c]*Sin[(e + f*x)/2]] + Log[Sqrt[2]*Sqrt
[c + d] + 2*Sqrt[c]*Sin[(e + f*x)/2]]))*Sqrt[g*Sec[e + f*x]])/((c - d)*Sqrt
[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(132) = 264$.

time = 13.86, size = 473, normalized size = 2.83

method	result
default	$\frac{\left(\frac{g}{\cos(fx+e)}\right)^{\frac{3}{2}} \sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\, (\cos^2(fx+e))(-1+\cos(fx+e))^2 \left(\operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right)\sqrt{2}\, \sqrt{\frac{c}{c-d}}\, \sqrt{(c+d)\,(c-d)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/f*(g/cos(f*x+e))^(3/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*cos(f*x+e)^2*(
-1+cos(f*x+e))^2*(arcsinh((-1+cos(f*x+e))/sin(f*x+e))*2^(1/2)*(1/(c-d)*c)^(
1/2)*((c+d)*(c-d))^(1/2)+c*ln(-2*(2*(1/(c-d)*c)^(1/2)*(1/(cos(f*x+e)+1))^(1
/2)*c*sin(f*x+e)-2*(1/(c-d)*c)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)-
((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+((c+d)*(c-d))^(1/2
))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c+d))-c*ln(-2*
(2*(1/(c-d)*c)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2*(1/(c-d)*c)^(1
/2)*(1/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)+c*
sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f*x+e)-d*cos(f*x+e)-((c
+d)*(c-d))^(1/2)*sin(f*x+e)-c+d)))/sin(f*x+e)^4/(1/(cos(f*x+e)+1))^(3/2)/a/
(1/(c-d)*c)^(1/2)/(c-d)/((c+d)*(c-d))^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

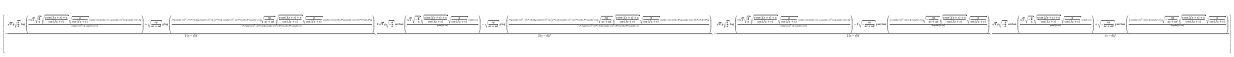
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*c*f*g*integrate((((c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x + 2*e
)^2 - 2*(c*d - 2*d^2)*cos(f*x + e)^2 - (c^2 - 4*c*d)*sin(2*f*x + 2*e)*sin(f
*x + e) - 2*(c*d - 2*d^2)*sin(f*x + e)^2 + (c^2 - (c^2 - 4*c*d)*cos(f*x + e
))*cos(2*f*x + 2*e) - (c^2 - 2*c*d)*cos(f*x + e))*cos(1/2*arctan2(sin(f*x +
 e), cos(f*x + e))) - (c^2*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2*cos(f*x + e
) + c^2)*sin(2*f*x + 2*e) + (c^2 - 2*c*d)*sin(f*x + e))*sin(1/2*arctan2(sin
(f*x + e), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2
+ c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin(
f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2*
```

```
f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2*cos(2*f*x
 + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x
 + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*
(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x + e),
cos(f*x + e)))^2), x) + sqrt(2)*c*f*g*integrate(((2*c*d*cos(f*x + e)^2 + 2*
c*d*sin(f*x + e)^2 - (c^2 - 2*c*d)*cos(2*f*x + 2*e)^2 + c^2*cos(f*x + e) -
(c^2 - 2*c*d)*sin(2*f*x + 2*e)^2 + (c^2 - 2*c*d + 4*d^2)*sin(2*f*x + 2*e)*s
in(f*x + e) - (c^2 - 2*c*d - (c^2 - 2*c*d + 4*d^2)*cos(f*x + e))*cos(2*f*x
+ 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + (c^2*sin(f*x + e) +
(c^2 + 2*c*d - 4*d^2)*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2 - 2*c*d + (c^2 +
 2*c*d - 4*d^2)*cos(f*x + e))*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x + e
), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*si
n(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)
^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*
e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2*cos(2*f*x + 2*e)^
2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*
sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*c
os(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e)))^2), x) - sqrt(2)*d*g*log(cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))
)^2 + sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(1/2*arctan2(si
n(f*x + e), cos(f*x + e))) + 1) + sqrt(2)*d*g*log(cos(1/2*arctan2(sin(f*x +
 e), cos(f*x + e)))^2 + sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*
sin(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + 1))*sqrt(g)/((c*d - d^2)*sqr
t(a)*f)
```

Fricas [A]
time = 4.42, size = 1167, normalized size = 6.99



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f*x
 + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) +
sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f
*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x +
 e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/c
os(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 + 8*c*d +
 d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d
+ d^2)*cos(f*x + e))))/((c - d)*f), 1/2*(2*sqrt(2)*g*sqrt(-g/a)*arctan(sqrt
(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))
*cos(f*x + e)/(g*sin(f*x + e))) - sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*
```

```
x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2
 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)
*g*cos(f*x + e) + (8*c^2 + 8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c
*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*
(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f*x + e)^
2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt
(-c*g/(a*c + a*d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))
*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(
f*x + e))/(c*g*sin(f*x + e))))/((c - d)*f), (sqrt(2)*g*sqrt(-g/a)*arctan(sq
rt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e
))*cos(f*x + e)/(g*sin(f*x + e))) + sqrt(-c*g/(a*c + a*d))*g*arctan(1/2*(c*
cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(c*g*sin(f*x + e))))/((c
- d)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a(\sec(e+fx)+1)}\,(c+d\sec(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e +
 f*x))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

3.243 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=231

$$\frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,df}+\frac{\sqrt{2}\,g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,(c-d)f}$$

```
[Out] 2*g^(5/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2))/d/f/a^(1/2)+g^(5/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2
^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)
-2*c^(3/2)*g^(5/2)*arctanh(a^(1/2)*c^(1/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(
g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/(c-d)/d/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4063, 4050, 214, 4108, 3893, 3887}

$$-\frac{2c^{3/2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,df(c-d)\sqrt{c+d}}+\frac{\sqrt{2}\,g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,f(c-d)}+\frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}\,df}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]])])/(Sqrt[a]*d*f) + (Sqrt[2]*g^(5/2)*ArcTanh[(Sqrt[a]*
Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*f) - (2*c^(3/2)*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt
[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*d*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3887

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/f), Subst[Int[1/(b - d*x^2), x], x, b*(C
ot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && !GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4050

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*b*(g
/f), Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc[
e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4063

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(-c^2
)*(g^2/(d*(b*c - a*d))), Int[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*x]]
/(c + d*Csc[e + f*x])), x], x] + Dist[g^2/(d*(b*c - a*d)), Int[Sqrt[g*Csc[e
 + f*x]]*((a*c + (b*c - a*d)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x
] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\sec(e+fx)}\,(ac+(ac-ad)\sec(e+fx))}{\sqrt{a+a\sec(e+fx)}}dx}{a(c-d)d} - \frac{(c^2g^2)\int \frac{\sqrt{g}}{}}{}$$

$$= \frac{g^2\int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}dx}{c-d} + \frac{g^2\int \sqrt{g\sec(e+fx)}\,\sqrt{a+}}{ad}$$

$$= -\frac{2c^{3/2}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e}}\right.}{\sqrt{a}(c-d)d\sqrt{c+d}\,f}$$

$$= \frac{2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}\,df} + \frac{\sqrt{}}{}$$

Mathematica [A]
time = 0.36, size = 155, normalized size = 0.67

$$\frac{2g^2\left(d\sqrt{c+d}\tanh^{-1}\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{2}\left((c-d)\sqrt{c+d}\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)-c^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{g\sec(e+fx)}}{(c-d)d\sqrt{c+d}\,f\sqrt{a(1+\sec(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (2*g^2*(d*Sqrt[c + d]*ArcTanh[Sin[(e + f*x)/2]] + Sqrt[2]*((c - d)*Sqrt[c +
 d]*ArcTanh[Sqrt[2]*Sin[(e + f*x)/2]] - c^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[c]*Si
n[(e + f*x)/2])/Sqrt[c + d]]))*Cos[(e + f*x)/2]*Sqrt[g*Sec[e + f*x]])/((c -
 d)*d*Sqrt[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(184) = 368.
time = 13.78, size = 727, normalized size = 3.15

method	result
default	$\frac{2\left(\frac{g}{\cos(fx+e)}\right)^{\frac{5}{2}}\sqrt{\frac{a(\cos(fx+e)+1)}{\cos(fx+e)}}\,(\cos^3(fx+e))(-1+\cos(fx+e))^3\left(\sqrt{2}\,\sqrt{\frac{c}{c-d}}\,\sqrt{(c+d)(c-d)}\,\operatorname{arcsinh}\left(\frac{-1+\cos(fx+}{\sin(fx+e)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 2/f*(g/cos(f*x+e))^(5/2)*(a*(cos(f*x+e)+1)/cos(f*x+e))^(1/2)*cos(f*x+e)^3*(
-1+cos(f*x+e))^3*(2^(1/2)*(1/(c-d)*c)^(1/2)*((c+d)*(c-d))^(1/2)*arcsinh((-1
+cos(f*x+e))/sin(f*x+e))*d+(1/(c-d)*c)^(1/2)*((c+d)*(c-d))^(1/2)*arctanh(1/
2*(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)-sin(f*x+e)))*c-(1/(c-d)*c)^(1/2)*(
(c+d)*(c-d))^(1/2)*arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)-sin(f
*x+e)))*d-(1/(c-d)*c)^(1/2)*((c+d)*(c-d))^(1/2)*arctanh(1/2*(1/(cos(f*x+e)+
1))^(1/2)*(1+cos(f*x+e)+sin(f*x+e)))*c+(1/(c-d)*c)^(1/2)*((c+d)*(c-d))^(1/2
)*arctanh(1/2*(1/(cos(f*x+e)+1))^(1/2)*(1+cos(f*x+e)+sin(f*x+e)))*d-c^2*ln(
-2*(2*(1/(c-d)*c)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*c*sin(f*x+e)-2*(1/(c-d)*c)
^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)
+c*sin(f*x+e)-d*sin(f*x+e)-((c+d)*(c-d))^(1/2))/(c*cos(f*x+e)-d*cos(f*x+e)-
((c+d)*(c-d))^(1/2)*sin(f*x+e)-c+d))+c^2*ln(2*(-2*(1/(c-d)*c)^(1/2)*(1/(cos
(f*x+e)+1))^(1/2)*c*sin(f*x+e)+2*(1/(c-d)*c)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)
*d*sin(f*x+e)+((c+d)*(c-d))^(1/2)*cos(f*x+e)-c*sin(f*x+e)+d*sin(f*x+e)-((c+
d)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-
c+d)))/sin(f*x+e)^6/(1/(cos(f*x+e)+1))^(5/2)/a/(1/(c-d)*c)^(1/2)/((c+d)*(c-
d))^(1/2)/(-c+d+((c+d)*(c-d))^(1/2))/(c-d+((c+d)*(c-d))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] -1/2*(sqrt(2)*c^2*f*g^2*integrate(((c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x
+ 2*e)^2 - 2*(c*d - 2*d^2)*cos(f*x + e)^2 - (c^2 - 4*c*d)*sin(2*f*x + 2*e)*
sin(f*x + e) - 2*(c*d - 2*d^2)*sin(f*x + e)^2 + (c^2 - (c^2 - 4*c*d)*cos(f*
x + e))*cos(2*f*x + 2*e) - (c^2 - 2*c*d)*cos(f*x + e))*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) - (c^2*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2*cos(f*
x + e) + c^2)*sin(2*f*x + 2*e) + (c^2 - 2*c*d)*sin(f*x + e))*sin(1/2*arctan
2(sin(f*x + e), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x +
e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2
*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*c
os(2*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2*cos(
2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(
2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2
 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x +
 e), cos(f*x + e)))^2), x) + sqrt(2)*c^2*f*g^2*integrate(((2*c*d*cos(f*x +
e)^2 + 2*c*d*sin(f*x + e)^2 - (c^2 - 2*c*d)*cos(2*f*x + 2*e)^2 + c^2*cos(f*
x + e) - (c^2 - 2*c*d)*sin(2*f*x + 2*e)^2 + (c^2 - 2*c*d + 4*d^2)*sin(2*f*x
```

```
 + 2*e)*sin(f*x + e) - (c^2 - 2*c*d - (c^2 - 2*c*d + 4*d^2)*cos(f*x + e))*c
os(2*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + (c^2*sin(f*
x + e) + (c^2 + 2*c*d - 4*d^2)*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2 - 2*c*d
 + (c^2 + 2*c*d - 4*d^2)*cos(f*x + e))*sin(2*f*x + 2*e))*sin(1/2*arctan2(si
n(f*x + e), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2
 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin
(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2
*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2*cos(2*f*
x + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*
x + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2
*(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x + e),
 cos(f*x + e)))^2), x) - sqrt(2)*d^2*g^2*log(cos(1/2*arctan2(sin(f*x + e),
cos(f*x + e)))^2 + sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(1
/2*arctan2(sin(f*x + e), cos(f*x + e))) + 1) + sqrt(2)*d^2*g^2*log(cos(1/2*
arctan2(sin(f*x + e), cos(f*x + e)))^2 + sin(1/2*arctan2(sin(f*x + e), cos(
f*x + e)))^2 - 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + 1) - (c*d -
 d^2)*g^2*log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(1/2*
arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(f*x
+ e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)
)) + 2) + (c*d - d^2)*g^2*log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))
)^2 + 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*
arctan2(sin(f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x +
e), cos(f*x + e))) + 2) - (c*d - d^2)*g^2*log(2*cos(1/2*arctan2(sin(f*x + e
), cos(f*x + e)))^2 + 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*
sqrt(2)*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*ar
ctan2(sin(f*x + e), cos(f*x + e))) + 2) + (c*d - d^2)*g^2*log(2*cos(1/2*arc
tan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(1/2*arctan2(sin(f*x + e), cos(f
*x + e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) - 2*sq
rt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + 2))*sqrt(g)/((c*d^2 -
d^3)*sqrt(a)*f)
```

Fricas [A]
time = 110.12, size = 1695, normalized size = 7.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*d*g^2*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*co
s(f*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
)) + c*sqrt(c*g/(a*c + a*d))*g^2*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d
)*g*cos(f*x + e)^2 - 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*
```

```
cos(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2
+ 8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2
+ (2*c*d + d^2)*cos(f*x + e))) - (c - d)*g^2*sqrt(g/a)*log((g*cos(f*x + e)^
3 - 4*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g
)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/((c*d - d^2)*f), -1/2*(sqrt(2)*d*g^2*
sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos(f*x + e)^2 - 2*g*co
s(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*c*sqrt(-c*g/(a
*c + a*d))*g^2*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(
-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x +
e))/(c*g*sin(f*x + e))) - (c - d)*g^2*sqrt(g/a)*log((g*cos(f*x + e)^3 - 4*(
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g)/(cos(
f*x + e)^3 + cos(f*x + e)^2)))/((c*d - d^2)*f), -1/2*(2*sqrt(2)*d*g^2*sqrt(
-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e))) - 2*(c - d)*g^2*sqrt(-g/a)
*arctan(2*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(f*x + e) - 2*g))
 + c*sqrt(c*g/(a*c + a*d))*g^2*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*
g*cos(f*x + e)^2 - 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*co
s(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 +
8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 +
(2*c*d + d^2)*cos(f*x + e))))/((c*d - d^2)*f), -(sqrt(2)*d*g^2*sqrt(-g/a)*a
rctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos
(f*x + e))*cos(f*x + e)/(g*sin(f*x + e))) - (c - d)*g^2*sqrt(-g/a)*arctan(2
*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(f*x + e) - 2*g)) + c*sqrt
(-c*g/(a*c + a*d))*g^2*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e
))*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/co
s(f*x + e))/(c*g*sin(f*x + e))))/((c*d - d^2)*f)]
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [sign(cos(sageVARf*sa
geVARx+sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

3.244 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal. Leaf size=250

$$\frac{(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(12bc^4+95ac^3d+112bc^2d^2+80acd^3+1}{30f}$$

```
[Out] 1/8*(8*a*c^4+24*a*c^2*d^2+3*a*d^4+16*b*c^3*d+12*b*c*d^3)*arctanh(sin(f*x+e)
)/f+1/30*(95*a*c^3*d+80*a*c*d^3+12*b*c^4+112*b*c^2*d^2+16*b*d^4)*tan(f*x+e)
/f+1/120*d*(130*a*c^2*d+45*a*d^3+24*b*c^3+116*b*c*d^2)*sec(f*x+e)*tan(f*x+e
)/f+1/60*(35*a*c*d+12*b*c^2+16*b*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/20*
(5*a*d+4*b*c)*(c+d*sec(f*x+e))^3*tan(f*x+e)/f+1/5*b*(c+d*sec(f*x+e))^4*tan(
f*x+e)/f
```

Rubi [A]
time = 0.34, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{(35acd+12bc^2+16bd^2)\tan(e+fx)(c+d\sec(e+fx))^2}{60f}+\frac{d(130ac^2d+45ad^3+24bc^3+116bcd^2)\tan(e+fx)\sec(e+fx)}{120f}+\frac{(95ac^3d+80acd^3+12bc^4+112bc^2d^2+16bd^4)\tan(e+fx)}{30f}+\frac{(8ac^4+24ac^2d^2+3ad^4+16bc^3d+12bcd^3)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(5ad+4bc)\tan(e+fx)(c+d\sec(e+fx))^3}{20f}+\frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*ArcTanh[Sin[e
 + f*x]])/(8*f) + ((12*b*c^4 + 95*a*c^3*d + 112*b*c^2*d^2 + 80*a*c*d^3 + 16
*b*d^4)*Tan[e + f*x])/(30*f) + (d*(24*b*c^3 + 130*a*c^2*d + 116*b*c*d^2 + 4
5*a*d^3)*Sec[e + f*x]*Tan[e + f*x])/(120*f) + ((12*b*c^2 + 35*a*c*d + 16*b*
d^2)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + ((4*b*c + 5*a*d)*(c + d*
Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + (b*(c + d*Sec[e + f*x])^4*Tan[e + f*
x])/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f}+\frac{1}{5}\int \sec(e+$$

$$= \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{b(}{}$$

$$= \frac{(12bc^2+35acd+16bd^2)\,(c+d\sec(e+fx))^2\tan(}{60f}$$

$$= \frac{d(24bc^3+130ac^2d+116bcd^2+45ad^3)\sec(e+fx}{120f}$$

$$= \frac{d(24bc^3+130ac^2d+116bcd^2+45ad^3)\sec(e+fx}{120f}$$

$$= \frac{(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4)\tanh^{-}}{8f}$$

$$= \frac{(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4)\tanh^{-}}{8f}$$

Mathematica [A]
time = 4.10, size = 201, normalized size = 0.80

$$\frac{15(4bcd(4c^2+3d^2)+a(8c^4+24c^2d^2+3d^4))\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(15d(3ad(8c^2+d^2)+4b(4c^3+3cd^2))\sec(e+fx)+30d^3(4bc+ad)\sec^3(e+fx)+8(15(4acd(c^2+d^2)+b(c^4+6c^2d^2+d^4))+10d^2(2acd+b(3c^2+d^2))\tan^2(e+fx)+3bd^4\tan^4(e+fx))\right)}{120f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (15*(4*b*c*d*(4*c^2 + 3*d^2) + a*(8*c^4 + 24*c^2*d^2 + 3*d^4))*ArcTanh[Sin[
e + f*x]] + Tan[e + f*x]*(15*d*(3*a*d*(8*c^2 + d^2) + 4*b*(4*c^3 + 3*c*d^2)
)*Sec[e + f*x] + 30*d^3*(4*b*c + a*d)*Sec[e + f*x]^3 + 8*(15*(4*a*c*d*(c^2
+ d^2) + b*(c^4 + 6*c^2*d^2 + d^4)) + 10*d^2*(2*a*c*d + b*(3*c^2 + d^2))*Ta
n[e + f*x]^2 + 3*b*d^4*Tan[e + f*x]^4)))/(120*f)
```

Maple [A]
time = 0.40, size = 313, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^4*ln(sec(f*x+e)+tan(f*x+e))+4*a*c^3*d*tan(f*x+e)+6*a*c^2*d^2*(1/2*
sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-4*a*c*d^3*(-2/3-1/3*se
c(f*x+e)^2)*tan(f*x+e)+a*d^4*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e
)+3/8*ln(sec(f*x+e)+tan(f*x+e)))+b*c^4*tan(f*x+e)+4*b*c^3*d*(1/2*sec(f*x+e)
*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-6*b*c^2*d^2*(-2/3-1/3*sec(f*x+e)
^2)*tan(f*x+e)+4*b*c*d^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/
8*ln(sec(f*x+e)+tan(f*x+e)))-b*d^4*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^
2)*tan(f*x+e))
```

Maxima [A]
time = 0.29, size = 410, normalized size = 1.64

$$\frac{480\left(\tan(fx+e)^3+3\tan(fx+e)\right)bc^2d^2+320\left(\tan(fx+e)^3+3\tan(fx+e)\right)acd^3+16\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)bd^4-60bcd^3\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log(\sin(fx+e)-1)\right)-15ad^4\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log(\sin(fx+e)-1)\right)-240bc^3d\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)-360ac^2d^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+240ac^4\log(\sec(fx+e)+\tan(fx+e))+240bc^4\tan(fx+e)+960ac^3d\tan(fx+e)}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c^2*d^2 + 320*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15
*tan(f*x + e))*b*d^4 - 60*b*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) - 240*b*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
 e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(sin(f*x
```

```
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*lo
g(sec(f*x + e) + tan(f*x + e)) + 240*b*c^4*tan(f*x + e) + 960*a*c^3*d*tan(f
*x + e))/f
```

Fricas [A]
time = 3.06, size = 291, normalized size = 1.16

$$\frac{15\left(8ac^{4}+16bc^{3}d+24ac^{2}d^{2}+12bcd^{3}+3ad^{4}\right)\cos\left(fx+e\right)^{5}\log\left(\sin\left(fx+e\right)+1\right)-15\left(8ac^{4}+16bc^{3}d+24ac^{2}d^{2}+12bcd^{3}+3ad^{4}\right)\cos\left(fx+e\right)^{5}\log\left(-\sin\left(fx+e\right)+1\right)+2\left(24bd^{4}+8\left(15bc^{4}+60ac^{3}d+60bc^{2}d^{2}+40acd^{3}+8bd^{4}\right)\cos\left(fx+e\right)^{4}+15\left(16bc^{3}d+24ac^{2}d^{2}+12bcd^{3}+3ad^{4}\right)\cos\left(fx+e\right)^{3}+16\left(15bc^{2}d^{2}+10acd^{3}+2bd^{4}\right)\cos\left(fx+e\right)^{2}+30\left(4bcd^{3}+ad^{4}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{240\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(
f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2
+ 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*b*d^4
 + 8*(15*b*c^4 + 60*a*c^3*d + 60*b*c^2*d^2 + 40*a*c*d^3 + 8*b*d^4)*cos(f*x
+ e)^4 + 15*(16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)
^3 + 16*(15*b*c^2*d^2 + 10*a*c*d^3 + 2*b*d^4)*cos(f*x + e)^2 + 30*(4*b*c*d^
3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^{4}\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**4*sec(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 850 vs. 2(238) = 476.
time = 0.59, size = 850, normalized size = 3.40

$$15\left(8ac^{4}+16bc^{3}d+24ac^{2}d^{2}+12bcd^{3}+3ad^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15\left(8ac^{4}+16bc^{3}d+24ac^{2}d^{2}+12bcd^{3}+3ad^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*log(
abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 +
12*b*c*d^3 + 3*a*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(120*b*c^4*tan
(1/2*f*x + 1/2*e)^9 + 480*a*c^3*d*tan(1/2*f*x + 1/2*e)^9 - 240*b*c^3*d*tan(
```

```
1/2*f*x + 1/2*e)^9 - 360*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^9 + 720*b*c^2*d^2*t
an(1/2*f*x + 1/2*e)^9 + 480*a*c*d^3*tan(1/2*f*x + 1/2*e)^9 - 300*b*c*d^3*ta
n(1/2*f*x + 1/2*e)^9 - 75*a*d^4*tan(1/2*f*x + 1/2*e)^9 + 120*b*d^4*tan(1/2*
f*x + 1/2*e)^9 - 480*b*c^4*tan(1/2*f*x + 1/2*e)^7 - 1920*a*c^3*d*tan(1/2*f*
x + 1/2*e)^7 + 480*b*c^3*d*tan(1/2*f*x + 1/2*e)^7 + 720*a*c^2*d^2*tan(1/2*f
*x + 1/2*e)^7 - 1920*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 1280*a*c*d^3*tan(1/
2*f*x + 1/2*e)^7 + 120*b*c*d^3*tan(1/2*f*x + 1/2*e)^7 + 30*a*d^4*tan(1/2*f*
x + 1/2*e)^7 - 160*b*d^4*tan(1/2*f*x + 1/2*e)^7 + 720*b*c^4*tan(1/2*f*x + 1
/2*e)^5 + 2880*a*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 2400*b*c^2*d^2*tan(1/2*f*x
+ 1/2*e)^5 + 1600*a*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 464*b*d^4*tan(1/2*f*x +
1/2*e)^5 - 480*b*c^4*tan(1/2*f*x + 1/2*e)^3 - 1920*a*c^3*d*tan(1/2*f*x + 1/
2*e)^3 - 480*b*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 720*a*c^2*d^2*tan(1/2*f*x + 1
/2*e)^3 - 1920*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 1280*a*c*d^3*tan(1/2*f*x
+ 1/2*e)^3 - 120*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 30*a*d^4*tan(1/2*f*x + 1/
2*e)^3 - 160*b*d^4*tan(1/2*f*x + 1/2*e)^3 + 120*b*c^4*tan(1/2*f*x + 1/2*e)
+ 480*a*c^3*d*tan(1/2*f*x + 1/2*e) + 240*b*c^3*d*tan(1/2*f*x + 1/2*e) + 360
*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 720*b*c^2*d^2*tan(1/2*f*x + 1/2*e) + 480*
a*c*d^3*tan(1/2*f*x + 1/2*e) + 300*b*c*d^3*tan(1/2*f*x + 1/2*e) + 75*a*d^4*
tan(1/2*f*x + 1/2*e) + 120*b*d^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e
)^2 - 1)^5)/f
```

Mupad [B]
time = 5.55, size = 555, normalized size = 2.22

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(ac^4+2bc^3d+3ac^2d^2+\frac{3bcd^3}{2}+\frac{3ad^4}{8}\right)}{4ac^4+8bc^3d+12ac^2d^2+6bcd^3+\frac{3ad^4}{2}}\right)\left(2ac^4+4bc^3d+6ac^2d^2+3bcd^3+\frac{3ad^4}{4}\right)}{f}-\frac{\left(2bc^4-\frac{5ad^4}{4}+2bd^4-6ac^2d^2+12bc^2d^2+8acd^3+8ac^3d-5bcd^3-4bc^3d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+\left(\frac{ad^4}{2}-8bc^4-\frac{8bd^4}{3}+12ac^2d^2-32bc^2d^2-\frac{64acd^3}{3}-32ac^3d+2bcd^3+8bc^3d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+\left(12bc^4+48ac^3d+40bc^2d^2+\frac{80acd^3}{3}+\frac{116bd^4}{15}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\left(-\frac{ad^4}{2}-8bc^4-\frac{8bd^4}{3}-12ac^2d^2-32bc^2d^2-\frac{64acd^3}{3}-32ac^3d-2bcd^3-8bc^3d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+\left(\frac{5ad^4}{4}+2bc^4+2bd^4+6ac^2d^2+12bc^2d^2+8acd^3+8ac^3d+5bcd^3+4bc^3d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^4 + (3*a*d^4)/8 + 3*a*c^2*d^2 + (3*b*c*d^
3)/2 + 2*b*c^3*d))/(4*a*c^4 + (3*a*d^4)/2 + 12*a*c^2*d^2 + 6*b*c*d^3 + 8*b*
c^3*d))*(2*a*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*b*c*d^3 + 4*b*c^3*d))/f -
(tan(e/2 + (f*x)/2)^5*(12*b*c^4 + (116*b*d^4)/15 + 40*b*c^2*d^2 + (80*a*c*d
^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*((5*a*d^4)/4 + 2*b*c^4 + 2*b*d^4 +
 6*a*c^2*d^2 + 12*b*c^2*d^2 + 8*a*c*d^3 + 8*a*c^3*d + 5*b*c*d^3 + 4*b*c^3*d
) + tan(e/2 + (f*x)/2)^9*(2*b*c^4 - (5*a*d^4)/4 + 2*b*d^4 - 6*a*c^2*d^2 + 1
2*b*c^2*d^2 + 8*a*c*d^3 + 8*a*c^3*d - 5*b*c*d^3 - 4*b*c^3*d) - tan(e/2 + (f
*x)/2)^3*((a*d^4)/2 + 8*b*c^4 + (8*b*d^4)/3 + 12*a*c^2*d^2 + 32*b*c^2*d^2 +
 (64*a*c*d^3)/3 + 32*a*c^3*d + 2*b*c*d^3 + 8*b*c^3*d) - tan(e/2 + (f*x)/2)^
7*(8*b*c^4 - (a*d^4)/2 + (8*b*d^4)/3 - 12*a*c^2*d^2 + 32*b*c^2*d^2 + (64*a*
c*d^3)/3 + 32*a*c^3*d - 2*b*c*d^3 - 8*b*c^3*d))/(f*(5*tan(e/2 + (f*x)/2)^2
- 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^
8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.245 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal. Leaf size=180

$$\frac{(8ac^3+12bc^2d+12acd^2+3bd^3)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\tan(e+fx)}{6f}+\frac{d(6bc^2$$

```
[Out] 1/8*(8*a*c^3+12*a*c*d^2+12*b*c^2*d+3*b*d^3)*arctanh(sin(f*x+e))/f+1/6*(4*a*
d*(4*c^2+d^2)+3*b*(c^3+4*c*d^2))*tan(f*x+e)/f+1/24*d*(20*a*c*d+6*b*c^2+9*b*
d^2)*sec(f*x+e)*tan(f*x+e)/f+1/12*(4*a*d+3*b*c)*(c+d*sec(f*x+e))^2*tan(f*x+
e)/f+1/4*b*(c+d*sec(f*x+e))^3*tan(f*x+e)/f
```

Rubi [A]
time = 0.24, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{d(20acd+6bc^2+9bd^2)\tan(e+fx)\sec(e+fx)}{24f}+\frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\tan(e+fx)}{6f}+\frac{(8ac^3+12acd^2+12bc^2d+3bd^3)\tanh^{-1}(\sin(e+fx))}{8f}+\frac{(4ad+3bc)\tan(e+fx)(c+d\sec(e+fx))^2}{12f}+\frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*ArcTanh[Sin[e + f*x]])/(8*f)
 + ((4*a*d*(4*c^2 + d^2) + 3*b*(c^3 + 4*c*d^2))*Tan[e + f*x])/(6*f) + (d*(6
*b*c^2 + 20*a*c*d + 9*b*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + ((3*b*c +
4*a*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + (b*(c + d*Sec[e + f*x]
)^3*Tan[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx &= \frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{1}{4}\int \sec(e\\ &= \frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\\ &= \frac{d(6bc^2+20acd+9bd^2)\sec(e+fx)\tan(e+fx)}{24f}\\ &= \frac{d(6bc^2+20acd+9bd^2)\sec(e+fx)\tan(e+fx)}{24f}\\ &= \frac{(8ac^3+12bc^2d+12acd^2+3bd^3)\tanh^{-1}(\sin(e+}{8f}\\ &= \frac{(8ac^3+12bc^2d+12acd^2+3bd^3)\tanh^{-1}(\sin(e+}{8f}\end{aligned}$$

Mathematica [A]

time = 1.16, size = 143, normalized size = 0.79

$$\frac{3(3bd(4c^2+d^2)+4a(2c^3+3cd^2))\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(9d(4acd+b(4c^2+d^2))\sec(e+fx)+6bd^3\sec^3(e+fx)+8(3ad(3c^2+d^2)+3b(c^3+3cd^2)+d^2(3bc+ad)\tan^2(e+fx))\right)}{24f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (3*(3*b*d*(4*c^2 + d^2) + 4*a*(2*c^3 + 3*c*d^2))*ArcTanh[Sin[e + f*x]] + Ta
n[e + f*x]*(9*d*(4*a*c*d + b*(4*c^2 + d^2))*Sec[e + f*x] + 6*b*d^3*Sec[e +
f*x]^3 + 8*(3*a*d*(3*c^2 + d^2) + 3*b*(c^3 + 3*c*d^2) + d^2*(3*b*c + a*d)*T
an[e + f*x]^2)))/(24*f)
```

Maple [A]
time = 0.34, size = 223, normalized size = 1.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^3*ln(sec(f*x+e)+tan(f*x+e))+3*a*c^2*d*tan(f*x+e)+3*a*c*d^2*(1/2*se
c(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a*d^3*(-2/3-1/3*sec(f*x+
e)^2)*tan(f*x+e)+b*c^3*tan(f*x+e)+3*b*c^2*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*
ln(sec(f*x+e)+tan(f*x+e)))-3*b*c*d^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+b*d
^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*
x+e))))
```

Maxima [A]
time = 0.28, size = 288, normalized size = 1.60

$$\frac{48\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)bcd^{2}+16\left(\tan\left(fx+e\right)^{3}+3\tan\left(fx+e\right)\right)ad^{3}-3\,bd^{3}\left(\frac{2\left(3\sin(fx+e)^{3}-5\sin(fx+e)\right)}{\sin(fx+e)^{4}-2\sin(fx+e)^{2}+1}-3\log\left(\sin\left(fx+e\right)+1\right)+3\log\left(\sin\left(fx+e\right)-1\right)\right)-36\,bc^{2}d\left(\frac{2\sin(fx+e)}{\sin(fx+e)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-36\,acd^{2}\left(\frac{2\sin(fx+e)}{\sin(fx+e)^{2}-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+48\,ac^{3}\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)+48\,bc^{3}\tan\left(fx+e\right)+144\,ac^{2}d\tan\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c*d^2 + 16*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a*d^3 - 3*b*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(
f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 36*b*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c^3*log
(sec(f*x + e) + tan(f*x + e)) + 48*b*c^3*tan(f*x + e) + 144*a*c^2*d*tan(f*x
 + e))/f
```

Fricas [A]
time = 2.39, size = 220, normalized size = 1.22

$$\frac{3\left(8\,ac^{3}+12\,bc^{2}d+12\,acd^{2}+3\,bd^{3}\right)\cos\left(fx+e\right)^{4}\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,ac^{3}+12\,bc^{2}d+12\,acd^{2}+3\,bd^{3}\right)\cos\left(fx+e\right)^{4}\log\left(-\sin\left(fx+e\right)+1\right)+2\left(6\,bd^{3}+8\left(3\,bc^{3}+9\,ac^{2}d+6\,bcd^{2}+2\,ad^{3}\right)\cos\left(fx+e\right)^{3}+9\left(4\,bc^{2}d+4\,acd^{2}+bd^{3}\right)\cos\left(fx+e\right)^{2}+8\left(3\,bcd^{2}+ad^{3}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x + e)^4*log(si
n(f*x + e) + 1) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) + 2*(6*b*d^3 + 8*(3*b*c^3 + 9*a*c^2*d + 6*b*c*
d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*b*c^2*d + 4*a*c*d^2 + b*d^3)*cos(f*x +
 e)^2 + 8*(3*b*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(e + fx))\,(c + d\sec(e + fx))^3 \sec(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**3*sec(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(170) = 340.
time = 0.53, size = 586, normalized size = 3.26

$$\frac{3\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(24bc^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+72ac^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-36bc^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-36acd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+72bcd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+24ad^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-15bd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-72bc^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-216ac^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+36bc^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+36acd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-120bcd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-40ad^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-9bd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+72bc^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+216ac^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+36bc^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+36acd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+120bcd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+40ad^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-9bd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-24bc^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-72ac^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-36bc^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-36acd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-72bcd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-24ad^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-15bd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^4}}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*log(abs(tan(1/2*f*x +
 1/2*e) + 1)) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*log(abs(tan
(1/2*f*x + 1/2*e) - 1)) - 2*(24*b*c^3*tan(1/2*f*x + 1/2*e)^7 + 72*a*c^2*d*t
an(1/2*f*x + 1/2*e)^7 - 36*b*c^2*d*tan(1/2*f*x + 1/2*e)^7 - 36*a*c*d^2*tan(
1/2*f*x + 1/2*e)^7 + 72*b*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 24*a*d^3*tan(1/2*f
*x + 1/2*e)^7 - 15*b*d^3*tan(1/2*f*x + 1/2*e)^7 - 72*b*c^3*tan(1/2*f*x + 1/
2*e)^5 - 216*a*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 36*b*c^2*d*tan(1/2*f*x + 1/2*
e)^5 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 120*b*c*d^2*tan(1/2*f*x + 1/2*e)
^5 - 40*a*d^3*tan(1/2*f*x + 1/2*e)^5 - 9*b*d^3*tan(1/2*f*x + 1/2*e)^5 + 72*
b*c^3*tan(1/2*f*x + 1/2*e)^3 + 216*a*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 36*b*c^
2*d*tan(1/2*f*x + 1/2*e)^3 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 120*b*c*d^
2*tan(1/2*f*x + 1/2*e)^3 + 40*a*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*b*d^3*tan(1/
2*f*x + 1/2*e)^3 - 24*b*c^3*tan(1/2*f*x + 1/2*e) - 72*a*c^2*d*tan(1/2*f*x +
 1/2*e) - 36*b*c^2*d*tan(1/2*f*x + 1/2*e) - 36*a*c*d^2*tan(1/2*f*x + 1/2*e)
 - 72*b*c*d^2*tan(1/2*f*x + 1/2*e) - 24*a*d^3*tan(1/2*f*x + 1/2*e) - 15*b*d
^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B]
time = 5.49, size = 395, normalized size = 2.19

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a\,c^3+\frac{3\,b\,c^2\,d}{2}+\frac{3\,a\,c\,d^2}{2}+\frac{3\,b\,d^3}{8}\right)}{4\,a\,c^3+6\,b\,c^2\,d+6\,a\,c\,d^2+\frac{3\,b\,d^3}{2}}\right)\left(2\,a\,c^3+3\,b\,c^2\,d+3\,a\,c\,d^2+\frac{3\,b\,d^3}{4}\right)}{f}-\frac{\left(2\,a\,d^3+2\,b\,c^3-\frac{5\,b\,d^3}{4}-3\,a\,c\,d^2+6\,a\,c^2\,d+6\,b\,c\,d^2-3\,b\,c^2\,d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+\left(3\,a\,c\,d^2-6\,b\,c^3-\frac{3\,b\,d^3}{4}-\frac{10\,a\,d^3}{3}-18\,a\,c^2\,d-10\,b\,c\,d^2+3\,b\,c^2\,d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\left(\frac{10\,a\,d^3}{3}+6\,b\,c^3-\frac{3\,b\,d^3}{4}+3\,a\,c\,d^2+18\,a\,c^2\,d+10\,b\,c\,d^2+3\,b\,c^2\,d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+\left(-2\,a\,d^3-2\,b\,c^3-\frac{5\,b\,d^3}{4}-3\,a\,c\,d^2-6\,a\,c^2\,d-6\,b\,c\,d^2-3\,b\,c^2\,d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^3 + (3*b*d^3)/8 + (3*a*c*d^2)/2 + (3*b*c^
2*d)/2))/(4*a*c^3 + (3*b*d^3)/2 + 6*a*c*d^2 + 6*b*c^2*d))*(2*a*c^3 + (3*b*d
^3)/4 + 3*a*c*d^2 + 3*b*c^2*d))/f - (tan(e/2 + (f*x)/2)^7*(2*a*d^3 + 2*b*c^
3 - (5*b*d^3)/4 - 3*a*c*d^2 + 6*a*c^2*d + 6*b*c*d^2 - 3*b*c^2*d) + tan(e/2
+ (f*x)/2)^3*((10*a*d^3)/3 + 6*b*c^3 - (3*b*d^3)/4 + 3*a*c*d^2 + 18*a*c^2*d
 + 10*b*c*d^2 + 3*b*c^2*d) - tan(e/2 + (f*x)/2)^5*((10*a*d^3)/3 + 6*b*c^3 +
 (3*b*d^3)/4 - 3*a*c*d^2 + 18*a*c^2*d + 10*b*c*d^2 - 3*b*c^2*d) - tan(e/2 +
 (f*x)/2)*(2*a*d^3 + 2*b*c^3 + (5*b*d^3)/4 + 3*a*c*d^2 + 6*a*c^2*d + 6*b*c*
d^2 + 3*b*c^2*d))/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*t
an(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.246 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal. Leaf size=115

$$\frac{(2bcd+a(2c^2+d^2))\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2(3acd+b(c^2+d^2))\tan(e+fx)}{3f}+\frac{d(2bc+3ad)\sec(e+fx)\tan}{6f}$$

```
[Out] 1/2*(2*b*c*d+a*(2*c^2+d^2))*arctanh(sin(f*x+e))/f+2/3*(3*a*c*d+b*(c^2+d^2))
*tan(f*x+e)/f+1/6*d*(3*a*d+2*b*c)*sec(f*x+e)*tan(f*x+e)/f+1/3*b*(c+d*sec(f*
x+e))^2*tan(f*x+e)/f
```

Rubi [A]

time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{2(3acd+b(c^2+d^2))\tan(e+fx)}{3f}+\frac{(a(2c^2+d^2)+2bcd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{d(3ad+2bc)\tan(e+fx)\sec(e+fx)}{6f}+\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]])/(2*f) + (2*(3*a*c*d + b
*(c^2 + d^2))*Tan[e + f*x])/(3*f) + (d*(2*b*c + 3*a*d)*Sec[e + f*x]*Tan[e +
 f*x])/(6*f) + (b*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx &= \frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f}+\frac{1}{3}\int \sec(e+ \\
&= \frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\,\mathrm{s} \\
&= \frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\,\mathrm{s} \\
&= \frac{(2bcd+a(2c^2+d^2))\tanh^{-1}(\sin(e+fx))}{2f}+\frac{d(2b \\
&= \frac{(2bcd+a(2c^2+d^2))\tanh^{-1}(\sin(e+fx))}{2f}+\frac{2(3a
\end{aligned}
$$

Mathematica [A]
time = 0.62, size = 88, normalized size = 0.77

$$
\frac{3(2bcd+a(2c^2+d^2))\tanh^{-1}(\sin(e+fx))+\tan(e+fx)\left(12acd+6b(c^2+d^2)+3d(2bc+ad)\sec(e+fx)+2bd^2\tan^2(e+fx)\right)}{6f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (3*(2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(12*a*c
*d + 6*b*(c^2 + d^2) + 3*d*(2*b*c + a*d)*Sec[e + f*x] + 2*b*d^2*Tan[e + f*x
]^2))/(6*f)
```

Maple [A]
time = 0.26, size = 143, normalized size = 1.24

method	result
derivativedivides	$\frac{a c^2 \ln(\sec(fx+e)+\tan(fx+e))+2acd\tan(fx+e)+d^2 a\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b c^2\tan(fx+e)}{f}$
default	$\frac{a c^2 \ln(\sec(fx+e)+\tan(fx+e))+2acd\tan(fx+e)+d^2 a\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b c^2\tan(fx+e)}{f}$
norman	$\frac{\frac{4\left(6acd+3b c^2+b d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f}-\frac{\left(4acd-d^2a+2b c^2-2bcd+2b d^2\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}-\frac{\left(4acd+d^2a+2b c^2+2bcd+2b d^2\right)\tan\left(\frac{fx}{2}\right.}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$
risch	$-\frac{i\left(3a d^2 e^{5i(fx+e)}+6bcd\,e^{5i(fx+e)}-12acd\,e^{4i(fx+e)}-6b c^2 e^{4i(fx+e)}-24acd\,e^{2i(fx+e)}-12b c^2 e^{2i(fx+e)}-12b d^2 e^{2i(fx+e)}\right.}{3f\left(e^{2i(fx+e)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c^2*ln(sec(f*x+e)+tan(f*x+e))+2*a*c*d*tan(f*x+e)+d^2*a*(1/2*sec(f*x+
e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+b*c^2*tan(f*x+e)+2*b*c*d*(1/2*
sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-b*d^2*(-2/3-1/3*sec(f*
x+e)^2)*tan(f*x+e))
```

Maxima [A]
time = 0.30, size = 179, normalized size = 1.56

$$\frac{4\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)bd^2-6\,bcd\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-3\,ad^2\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)+12\,ac^2\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)+12\,bc^2\tan\left(fx+e\right)+24\,acd\tan\left(fx+e\right)}{12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*d^2 - 6*b*c*d*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*a*
d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*b*c^2*tan(f
*x + e) + 24*a*c*d*tan(f*x + e))/f
```

Fricas [A]
time = 3.00, size = 158, normalized size = 1.37

$$\frac{3\left(2\,ac^2+2\,bcd+ad^2\right)\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\left(2\,ac^2+2\,bcd+ad^2\right)\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)+2\left(2\,bd^2+2\left(3\,bc^2+6\,acd+2\,bd^2\right)\cos\left(fx+e\right)^2+3\left(2\,bcd+ad^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{12\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*
b*d^2 + 2*(3*b*c^2 + 6*a*c*d + 2*b*d^2)*cos(f*x + e)^2 + 3*(2*b*c*d + a*d^2
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(e + fx))(c + d\sec(e + fx))^2 \sec(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2*sec(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(107) = 214.
time = 0.51, size = 294, normalized size = 2.56

$$\frac{3\,(2\,ac^2 + 2\,bcd + ad^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) - 3\,(2\,ac^2 + 2\,bcd + ad^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right) - \frac{2\left(6\,bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 12\,acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 6\,bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 3\,ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 6\,bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 12\,bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 24\,acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 - 4\,bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 6\,bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 12\,acd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 6\,bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 3\,ad^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 6\,bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 1\right)^3}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(
2*a*c^2 + 2*b*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(6*b*c^2*
tan(1/2*f*x + 1/2*e)^5 + 12*a*c*d*tan(1/2*f*x + 1/2*e)^5 - 6*b*c*d*tan(1/2*
f*x + 1/2*e)^5 - 3*a*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*b*d^2*tan(1/2*f*x + 1/2
*e)^5 - 12*b*c^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*c*d*tan(1/2*f*x + 1/2*e)^3 -
 4*b*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*b*c^2*tan(1/2*f*x + 1/2*e) + 12*a*c*d*t
an(1/2*f*x + 1/2*e) + 6*b*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2*f*x +
1/2*e) + 6*b*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B]
time = 5.21, size = 227, normalized size = 1.97

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(a\,c^2+b\,c\,d+\frac{a\,d^2}{2}\right)}{4\,a\,c^2+4\,b\,c\,d+2\,a\,d^2}\right)\,(2\,a\,c^2+2\,b\,c\,d+a\,d^2)}{f} - \frac{(2\,b\,c^2 - a\,d^2 + 2\,b\,d^2 + 4\,a\,c\,d - 2\,b\,c\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5 + \left(-4\,b\,c^2 - 8\,a\,c\,d - \frac{4\,b\,d^2}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3 + (a\,d^2 + 2\,b\,c^2 + 2\,b\,d^2 + 4\,a\,c\,d + 2\,b\,c\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6 - 3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 + 3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^2 + (a*d^2)/2 + b*c*d))/(4*a*c^2 + 2*a*d^
2 + 4*b*c*d))*(2*a*c^2 + a*d^2 + 2*b*c*d))/f - (tan(e/2 + (f*x)/2)*(a*d^2 +
 2*b*c^2 + 2*b*d^2 + 4*a*c*d + 2*b*c*d) - tan(e/2 + (f*x)/2)^3*(4*b*c^2 + (
4*b*d^2)/3 + 8*a*c*d) + tan(e/2 + (f*x)/2)^5*(2*b*c^2 - a*d^2 + 2*b*d^2 + 4
*a*c*d - 2*b*c*d))/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + ta
n(e/2 + (f*x)/2)^6 - 1))
```

3.247 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal. Leaf size=61

$$\frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{(bc+ad)\tan(e+fx)}{f}+\frac{bd\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*(2*a*c+b*d)*arctanh(sin(f*x+e))/f+(a*d+b*c)*tan(f*x+e)/f+1/2*b*d*sec(f*
x+e)*tan(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4082, 3872, 3855, 3852, 8}

$$\frac{(ad+bc)\tan(e+fx)}{f}+\frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] ((2*a*c + b*d)*ArcTanh[Sin[e + f*x]])/(2*f) + ((b*c + a*d)*Tan[e + f*x])/f
+ (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx &= \frac{bd\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\int \sec(e+fx)(2\\ &= \frac{bd\sec(e+fx)\tan(e+fx)}{2f}+(bc+ad)\int \sec^2(e\\ &= \frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bd\sec(e+fx)}{2f}\\ &= \frac{(2ac+bd)\tanh^{-1}(\sin(e+fx))}{2f}+\frac{(bc+ad)\tan(}{f}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.23

$$\frac{ac\tanh^{-1}(\sin(e+fx))}{f}+\frac{bd\tanh^{-1}(\sin(e+fx))}{2f}+\frac{bc\tan(e+fx)}{f}+\frac{ad\tan(e+fx)}{f}+\frac{bd\sec(e+fx)\tan(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
[Out] (a*c*ArcTanh[Sin[e + f*x]])/f + (b*d*ArcTanh[Sin[e + f*x]])/(2*f) + (b*c*Ta
n[e + f*x])/f + (a*d*Tan[e + f*x])/f + (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f
)
```

Maple [A]
time = 0.19, size = 75, normalized size = 1.23

method	result
derivativedivides	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)+bc\tan(fx+e)+db\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)+bc\tan(fx+e)+db\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$

norman	$\frac{\frac{(2ad+2bc+db)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(2ad+2bc-db)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}}{\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{(2ac+db)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{(2ac+db)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{2f}$
risch	$-\frac{i\left(bd\,e^{3i(fx+e)}-2ad\,e^{2i(fx+e)}-2bc\,e^{2i(fx+e)}-bd\,e^{i(fx+e)}-2ad-2bc\right)}{f\left(e^{2i(fx+e)}+1\right)^2}+\frac{ac\ln\left(e^{i(fx+e)}+i\right)}{f}+\frac{\ln\left(e^{i(fx+e)}+i\right)db}{2f}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c*ln(sec(f*x+e)+tan(f*x+e))+a*d*tan(f*x+e)+b*c*tan(f*x+e)+d*b*(1/2*s
ec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Maxima [A]
time = 0.28, size = 96, normalized size = 1.57

$$-\frac{bd\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right)\right)-4\,bc\tan\left(fx+e\right)-4\,ad\tan\left(fx+e\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -1/4*(b*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*b*c*tan(f
*x + e) - 4*a*d*tan(f*x + e))/f
```

Fricas [A]
time = 2.12, size = 103, normalized size = 1.69

$$\frac{(2\,ac+bd)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,ac+bd)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\left(bd+2\left(bc+ad\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4\,f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*a*c + b*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + b*d)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(b*d + 2*(b*c + a*d)*cos(f*x + e))*s
in(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)\sec\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))*sec(e + f*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(57) = 114$.

time = 0.47, size = 153, normalized size = 2.51

$$\frac{(2\,ac+bd)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-(2\,ac+bd)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(2\,bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2\,ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-bd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a*c + b*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - (2*a*c + b*d)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(2*b*c*tan(1/2*f*x + 1/2*e)^3 + 2*a*d*tan
(1/2*f*x + 1/2*e)^3 - b*d*tan(1/2*f*x + 1/2*e)^3 - 2*b*c*tan(1/2*f*x + 1/2*
e) - 2*a*d*tan(1/2*f*x + 1/2*e) - b*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x +
1/2*e)^2 - 1)^2)/f
```

Mupad [B]

time = 2.79, size = 104, normalized size = 1.70

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,a\,c+b\,d\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,a\,d+2\,b\,c+b\,d\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,a\,d+2\,b\,c-b\,d\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (atanh(tan(e/2 + (f*x)/2))*(2*a*c + b*d))/f + (tan(e/2 + (f*x)/2)*(2*a*d +
2*b*c + b*d) - tan(e/2 + (f*x)/2)^3*(2*a*d + 2*b*c - b*d))/(f*(tan(e/2 + (f
*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

3.248 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{b\tanh^{-1}(\sin(e+fx))}{df}-\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,d\sqrt{c+d}\,f}$$

```
[Out] b*arctanh(sin(f*x+e))/d/f-2*(-a*d+b*c)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*
e)/(c+d)^(1/2))/d/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4083, 3855, 3916, 2738, 214}

$$\frac{b\tanh^{-1}(\sin(e+fx))}{df}-\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c-d}\,\sqrt{c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/(d*f) - (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(
e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*d*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{b\int \sec(e+fx)\,dx}{d} + \frac{(-bc+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{d}$$

$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{(bc-ad)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d^2}$$

$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \text{ta}\right.}{d^2 f}$$

$$= \frac{b\tanh^{-1}(\sin(e+fx))}{df} - \frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,d\sqrt{c+d}\,f}$$

Mathematica [A]
time = 0.19, size = 112, normalized size = 1.47

$$\frac{\frac{2(bc-ad)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + b\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] ((2*(b*c - a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[
c^2 - d^2] + b*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(d*f)
```

Maple [A]
time = 0.27, size = 92, normalized size = 1.21

method	result

derivativedivides	$\frac{-\frac{2(-ad+bc)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{d\sqrt{(c+d)(c-d)}}+\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{d}}{f}$
default	$\frac{-\frac{2(-ad+bc)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{d\sqrt{(c+d)(c-d)}}+\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{d}}{f}$
risch	$\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)a}{\sqrt{c^2-d^2}\,f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)bc}{\sqrt{c^2-d^2}\,fd}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{}}{c\sqrt{c^2}}\right.}{\sqrt{c^2-d^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2/d*(-a*d+b*c)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/(
(c+d)*(c-d))^(1/2))+b/d*ln(tan(1/2*f*x+1/2*e)+1)-b/d*ln(tan(1/2*f*x+1/2*e)-
1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 5.43, size = 328, normalized size = 4.32

$$\left[-\frac{(bc-ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\sqrt{c^2-d^2}\,(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)-(bc^2-bd^2)\log\left(\sin\left(fx+e\right)+1\right)+(bc^2-bd^2)\log\left(-\sin\left(fx+e\right)+1\right)}{2\,(c^2d-d^3)f},-\frac{2\,(bc-ad)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}\,(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)-(bc^2-bd^2)\log\left(\sin\left(fx+e\right)+1\right)+(bc^2-bd^2)\log\left(-\sin\left(fx+e\right)+1\right)}{2\,(c^2d-d^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*
cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^
```

```
2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (b*c^2 - b*d^2)
*log(sin(f*x + e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d -
d^3)*f), -1/2*(2*(b*c - a*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*c
os(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (b*c^2 - b*d^2)*log(sin(f*x
+ e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d - d^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))\sec(e+fx)}{c+d\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Giac [A]
time = 0.54, size = 127, normalized size = 1.67

$$\frac{\frac{b\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d}+\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(bc-ad)}{\sqrt{-c^2+d^2}\,d}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d - b*log(abs(tan(1/2*f*x + 1/2*e) -
1))/d + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(b*c - a*d)/(
sqrt(-c^2 + d^2)*d))/f
```

Mupad [B]
time = 2.73, size = 573, normalized size = 7.54

$$\frac{a c^2 \ln\left(\frac{c\sin\left(\frac{e}{2}+\frac{fx}{2}\right)-d\sin\left(\frac{e}{2}+\frac{fx}{2}\right)+\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{f\,(c^2-d^2)^{3/2}}-\frac{a d^2 \ln\left(\frac{c\sin\left(\frac{e}{2}+\frac{fx}{2}\right)-d\sin\left(\frac{e}{2}+\frac{fx}{2}\right)+\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{f\,(c^2-d^2)^{3/2}}-\frac{2bd\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{f\,(c^2-d^2)}-\frac{a\ln\left(\frac{c\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+d\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\sqrt{(c+d)\,(c-d)}}{f\,(c^2-d^2)}+\frac{bcd\ln\left(\frac{c\sin\left(\frac{e}{2}+\frac{fx}{2}\right)-d\sin\left(\frac{e}{2}+\frac{fx}{2}\right)+\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{f\,(c^2-d^2)^{3/2}}+\frac{2bc^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{d\,f\,(c^2-d^2)}-\frac{bc^3\ln\left(\frac{c\sin\left(\frac{e}{2}+\frac{fx}{2}\right)-d\sin\left(\frac{e}{2}+\frac{fx}{2}\right)+\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)}{d\,f\,(c^2-d^2)^{3/2}}+\frac{bc\ln\left(\frac{c\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+d\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)}\right)\sqrt{(c+d)\,(c-d)}}{d\,f\,(c^2-d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (a*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2
)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (a*d^2*lo
g((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 -
d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*b*d*atanh(sin(e
/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (a*log((c*cos(e/2 + (f
*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e
```

```
/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) + (b*c*d*log((c*sin
(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/
2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*b*c^2*atanh(sin(e/2 + (
f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(c^2 - d^2)) - (b*c^3*log((c*sin(e/2 + (f
*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e
/2 + (f*x)/2)))/(d*f*(c^2 - d^2)^(3/2)) + (b*c*log((c*cos(e/2 + (f*x)/2) +
d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x
)/2))*((c + d)*(c - d))^(1/2))/(d*f*(c^2 - d^2))
```

3.249 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=99

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}f}+\frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/2)
/(c+d)^(3/2)/f+(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$\frac{(bc-ad)\tan(e+fx)}{f\,(c^2-d^2)\,(c+d\sec(e+fx))}+\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d
)^(3/2)*(c + d)^(3/2)*f) + ((b*c - a*d)*Tan[e + f*x])/((c^2 - d^2)*f*(c + d
*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx &= \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{\int \frac{(-ac+bd)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{-c^2+d^2}\\
&= \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{(ac-bd)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c^2-d^2}\\
&= \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{(ac-bd)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d\,(c^2-d^2)}\\
&= \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{(2(ac-bd))\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,d
}{d\,(c^2-d^2)\,f}\\
&= \frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}f}+\frac{(bc-ad)\tan(e+f
}{(c^2-d^2)\,f(c+d\sec(e+
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 97, normalized size = 0.98

$$\frac{-\frac{2(ac-bd)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}}+\frac{(bc-ad)\sin(e+fx)}{(c-d)(c+d)(d+c\cos(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

[Out] ((-2*(a*c - b*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) + ((b*c - a*d)*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e + f*x]))/f

Maple [A]
time = 0.23, size = 132, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}+\frac{2(ac-db)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{f}$
default	$\frac{\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}+\frac{2(ac-db)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{f}$
risch	$\frac{2i(-ad+bc)\left(d\,e^{i(fx+e)}+c\right)}{c(c^2-d^2)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)ac}{\sqrt{c^2-d^2}\,(c+d)(c-d)f}-\frac{\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c}}{\sqrt{c^2-}}\right.}{\sqrt{c^2-d^2}\,(c+d)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(2*(a*d-b*c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)+2*(a*c-b*d)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for more de

Fricas [A]
time = 2.62, size = 403, normalized size = 4.07

$$\left[\frac{\left(acd-bd^2+\left(ac^2-bcd\right)\cos\left(fx+e\right)\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\sqrt{c^2-d^2}\,(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)+2\left(bc^3-ac^2d-bcd^2+ad^3\right)\sin\left(fx+e\right)}{2\left(\left(c^5-2\,c^3d^2+cd^4\right)f\cos\left(fx+e\right)+\left(c^4d-2\,c^2d^3+d^5\right)f\right)},\ \frac{\left(acd-bd^2+\left(ac^2-bcd\right)\cos\left(fx+e\right)\right)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}\,(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)+\left(bc^3-ac^2d-bcd^2+ad^3\right)\sin\left(fx+e\right)}{\left(c^5-2\,c^3d^2+cd^4\right)f\cos\left(fx+e\right)+\left(c^4d-2\,c^2d^3+d^5\right)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*c*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2
*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos
(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(
f*x + e) + d^2)) + 2*(b*c^3 - a*c^2*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^
5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x + e) + (c^4*d - 2*c^2*d^3 + d^5)*f), ((a*c
*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c
^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^3 - a*c^2
*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x +
e) + (c^4*d - 2*c^2*d^3 + d^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(e + fx)})\sec{(e + fx)}}{(c + d\sec{(e + fx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**2, x)
```

Giac [A]
time = 0.48, size = 172, normalized size = 1.74

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(ac-bd)}{(c^2-d^2)\sqrt{-c^2+d^2}}+\frac{bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(c^2-d^2)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - b*d)/((c^2 -
 d^2)*sqrt(-c^2 + d^2)) + (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x + 1/2
*e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c^2 -
d^2)))/f
```

Mupad [B]

time = 2.12, size = 106, normalized size = 1.07

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)\,(a\,c-b\,d)}{f\,(c+d)^{3/2}\,(c-d)^{3/2}}-\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(a\,d-b\,c)}{f\,\left(c+d\right)\,\left(c-d\right)\,\left(\left(d-c\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+c+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(a*c - b*d))/(f*
(c + d)^(3/2)*(c - d)^(3/2)) - (2*tan(e/2 + (f*x)/2)*(a*d - b*c))/(f*(c + d
)*(c - d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
```

3.250 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal. Leaf size=166

$$-\frac{(3bcd-a(2c^2+d^2))\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{5/2}f}+\frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2}-\frac{(3acd-b(c^2+2d^2}{2\left(c^2-d^2\right)^2 f(c+}$$

```
[Out] -(3*b*c*d-a*(2*c^2+d^2))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2)
)/(c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(
f*x+e))^2-1/2*(3*a*c*d-b*(c^2+2*d^2))*tan(f*x+e)/(c^2-d^2)^2/f/(c+d*sec(f*x
+e))
```

Rubi [A]
time = 0.22, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$-\frac{(3bcd-a(2c^2+d^2))\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{5/2}}-\frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))}+\frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[
c + d]])/((c - d)^(5/2)*(c + d)^(5/2)*f)) + ((b*c - a*d)*Tan[e + f*x])/(2*(
c^2 - d^2)*f*(c + d*Sec[e + f*x])^2) - ((3*a*c*d - b*(c^2 + 2*d^2))*Tan[e +
 f*x])/(2*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-2(ac-bd)-(bc-ad)\sec(e+}{(c+d\sec(e+fx))^2}}{2\left(c^2-d^2\right)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx)}$$

$$= \frac{(2ac^2-3bcd+ad^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ac}{2\left(c^2-d^2\right)f(}$$

Mathematica [A]
time = 0.85, size = 172, normalized size = 1.04

$$\frac{-\frac{2\left(-3bcd+a\left(2c^2+d^2\right)\right)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\left(c^2-d^2\right)^{5/2}} + \frac{d(-bc+ad)\sin(e+fx)}{c(c-d)(c+d)(d+c\cos(e+fx))^2} + \frac{\left(ad\left(-4c^2+d^2\right)+bc\left(2c^2+d^2\right)\right)\sin(e+fx)}{c(c-d)^2(c+d)^2(d+c\cos(e+fx))}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((-2*(-3*b*c*d + a*(2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/(c^2 - d^2)^(5/2) + (d*(-(b*c) + a*d)*Sin[e + f*x])/(c*(c - d)
*(c + d)*(d + c*Cos[e + f*x])^2) + ((a*d*(-4*c^2 + d^2) + b*c*(2*c^2 + d^2)
)*Sin[e + f*x])/(c*(c - d)^2*(c + d)^2*(d + c*Cos[e + f*x])))/(2*f)
```

Maple [A]
time = 0.36, size = 236, normalized size = 1.42

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(4acd+d^2a-2b\,c^2-bcd-2b\,d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{\left(4acd-d^2a-2b\,c^2+bcd-2b\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c^2-2cd+d^2\right)}\right)}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}+\frac{\left(2c^2a+d^2a-3bcd\right)\operatorname{arctanh}\cdots}{\left(c^4-2c^2d^2+d^4\right)\sqrt{\cdots}}}{f}$
default	$\frac{-\frac{2\left(-\frac{\left(4acd+d^2a-2b\,c^2-bcd-2b\,d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c-d)\left(c^2+2cd+d^2\right)}+\frac{\left(4acd-d^2a-2b\,c^2+bcd-2b\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c^2-2cd+d^2\right)}\right)}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^2}+\frac{\left(2c^2a+d^2a-3bcd\right)\operatorname{arctanh}\cdots}{\left(c^4-2c^2d^2+d^4\right)\sqrt{\cdots}}}{f}$
risch	$\frac{i\left(-5a\,c^3d^2\mathrm{e}^{3i(fx+e)}+2ac\,d^4\mathrm{e}^{3i(fx+e)}+3b\,c^4d\,\mathrm{e}^{3i(fx+e)}-4a\,c^4d\,\mathrm{e}^{2i(fx+e)}-7a\,c^2d^3\mathrm{e}^{2i(fx+e)}+2a\,d^5\mathrm{e}^{2i(fx+e)}+2b\,c^5\mathrm{e}^{2i(fx\cdots}\right.}{c^2\left(-c^2+d^2\right)^{2\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(-1/2*(4*a*c*d+a*d^2-2*b*c^2-b*c*d-2*b*d^2)/(c-d)/(c^2+2*c*d+d^2)*t
an(1/2*f*x+1/2*e)^3+1/2*(4*a*c*d-a*d^2-2*b*c^2+b*c*d-2*b*d^2)/(c+d)/(c^2-2*
c*d+d^2)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2
-c-d)^2+(2*a*c^2+a*d^2-3*b*c*d)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arc
tanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see 'assume?' for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(158) = 316$.
time = 3.04, size = 772, normalized size = 4.65

$$\left[\frac{\left(2\,ac^{2}d^{2}-3\,bcd^{3}+ad^{4}+\left(2\,ac^{4}-3\,bc^{3}d+ac^{2}d^{2}\right)\cos\left(fx+e\right)^{2}+2\,\left(2\,ac^{3}d-3\,bc^{2}d^{2}+acd^{3}\right)\cos\left(fx+e\right)\right)\sqrt{c^{2}-d^{2}}\log\left(\frac{2\,cd\cos\left(fx+e\right)-\left(c^{2}-2\,d^{2}\right)\cos\left(fx+e\right)^{2}+2\,\sqrt{c^{2}-d^{2}}\left(d\cos\left(fx+e\right)+c\right)\sin\left(fx+e\right)+2\,c^{2}-d^{2}}{c^{2}\cos\left(fx+e\right)^{2}+2\,cd\cos\left(fx+e\right)+d^{2}}\right)+2\,\left(bc^{4}d-3\,ac^{3}d^{2}+bc^{2}d^{3}+3\,acd^{4}-2\,bd^{5}+\left(2\,bc^{5}-4\,ac^{4}d-bc^{3}d^{2}+5\,ac^{2}d^{3}-bcd^{4}-ad^{5}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{4\,\left(\left(c^{8}-3\,c^{6}d^{2}+3\,c^{4}d^{4}-c^{2}d^{6}\right)f\cos\left(fx+e\right)^{2}+2\,\left(c^{7}d-3\,c^{5}d^{3}+3\,c^{3}d^{5}-cd^{7}\right)f\cos\left(fx+e\right)+\left(c^{6}d^{2}-3\,c^{4}d^{4}+3\,c^{2}d^{6}-d^{8}\right)f\right)}, \frac{\left(2\,ac^{2}d^{2}-3\,bcd^{3}+ad^{4}+\left(2\,ac^{4}-3\,bc^{3}d+ac^{2}d^{2}\right)\cos\left(fx+e\right)^{2}+2\,\left(2\,ac^{3}d-3\,bc^{2}d^{2}+acd^{3}\right)\cos\left(fx+e\right)\right)\sqrt{-c^{2}+d^{2}}\arctan\left(-\frac{\sqrt{-c^{2}+d^{2}}\left(d\cos\left(fx+e\right)+c\right)}{\left(c^{2}-d^{2}\right)\sin\left(fx+e\right)}\right)+\left(bc^{4}d-3\,ac^{3}d^{2}+bc^{2}d^{3}+3\,acd^{4}-2\,bd^{5}+\left(2\,bc^{5}-4\,ac^{4}d-bc^{3}d^{2}+5\,ac^{2}d^{3}-bcd^{4}-ad^{5}\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{2\,\left(\left(c^{8}-3\,c^{6}d^{2}+3\,c^{4}d^{4}-c^{2}d^{6}\right)f\cos\left(fx+e\right)^{2}+2\,\left(c^{7}d-3\,c^{5}d^{3}+3\,c^{3}d^{5}-cd^{7}\right)f\cos\left(fx+e\right)+\left(c^{6}d^{2}-3\,c^{4}d^{4}+3\,c^{2}d^{6}-d^{8}\right)f\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*
cos(f*x + e)^2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(c
^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c
^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e
)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3
*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d
^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 - 3*c^6*d^2 + 3*c^4*d^4 - c^2
*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*cos(f*
x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f), 1/2*((2*a*c^2*d^2 - 3*
b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a
*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt
(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^4*d -
3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^
3*d^2 + 5*a*c^2*d^3 - b*c*d^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 -
3*c^6*d^2 + 3*c^4*d^4 - c^2*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 +
3*c^3*d^5 - c*d^7)*f*cos(f*x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)
*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \sec{\left(e + f x \right)}\right) \sec{\left(e + f x \right)}}{\left(c + d \sec{\left(e + f x \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(153) = 306$.
time = 0.55, size = 399, normalized size = 2.40

$$\frac{\frac{\left(2\,ac^{2}-3\,bcd+ad^{2}\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,c+2\,d\right)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^{2}+d^{2}}}\right)\right)}{\left(c^{4}-2\,c^{2}d^{2}+d^{4}\right)\sqrt{-c^{2}+d^{2}}}-\frac{2\,bc^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-4\,ac^{2}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-bc^{2}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+3\,acd^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+bcd^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+ad^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,bd^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}-2\,bc^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,ac^{2}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-bc^{2}d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,acd^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-bcd^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-ad^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,bd^{3}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(c^{4}-2\,c^{2}d^{2}+d^{4}\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c-d\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] ((2*a*c^2 - 3*b*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2
*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
 d^2)))/((c^4 - 2*c^2*d^2 + d^4)*sqrt(-c^2 + d^2)) - (2*b*c^3*tan(1/2*f*x +
 1/2*e)^3 - 4*a*c^2*d*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d*tan(1/2*f*x + 1/2*e)
^3 + 3*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + b*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a*
d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*c^3*tan(1
/2*f*x + 1/2*e) + 4*a*c^2*d*tan(1/2*f*x + 1/2*e) - b*c^2*d*tan(1/2*f*x + 1/
2*e) + 3*a*c*d^2*tan(1/2*f*x + 1/2*e) - b*c*d^2*tan(1/2*f*x + 1/2*e) - a*d^
3*tan(1/2*f*x + 1/2*e) - 2*b*d^3*tan(1/2*f*x + 1/2*e))/((c^4 - 2*c^2*d^2 +
d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B]
time = 4.99, size = 250, normalized size = 1.51

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,d^2+2\,b\,c^2+2\,b\,d^2-4\,a\,c\,d-b\,c\,d\right)}{(c+d)\,\left(c^2-2\,c\,d+d^2\right)}-\frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(2\,b\,c^2-a\,d^2+2\,b\,d^2-4\,a\,c\,d+b\,c\,d\right)}{{(c+d)}^2\,(c-d)}}{f\,\left(2\,c\,d-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(2\,c^2-2\,d^2\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4\,\left(c^2-2\,c\,d+d^2\right)+c^2+d^2\right)}+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c-2\,d\right)\,\left(c^2-2\,c\,d+d^2\right)}{2\,\sqrt{c+d}\,{(c-d)}^{5/2}}\right)\,\left(2\,a\,c^2-3\,b\,c\,d+a\,d^2\right)}{f\,{\left(c+d\right)}^{5/2}\,{\left(c-d\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)*(a*d^2 + 2*b*c^2 + 2*b*d^2 - 4*a*c*d - b*c*d))/((c + d
)*(c^2 - 2*c*d + d^2)) - (tan(e/2 + (f*x)/2)^3*(2*b*c^2 - a*d^2 + 2*b*d^2 -
 4*a*c*d + b*c*d))/((c + d)^2*(c - d)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2
*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) + (a
tanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*
(c - d)^(5/2)))*(2*a*c^2 + a*d^2 - 3*b*c*d))/(f*(c + d)^(5/2)*(c - d)^(5/2)
)
```

3.251 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal. Leaf size=237

$$\frac{(2ac^3-4bc^2d+3acd^2-bd^3)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{7/2}(c+d)^{7/2}f}+\frac{(bc-ad)\tan(e+fx)}{3\,(c^2-d^2)\,f(c+d\sec(e+fx))^3}+\frac{(2bc^2-5a}{6\,(c^2-d^2)}$$

```
[Out] (2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/
(c+d)^(1/2))/(c-d)^(7/2)/(c+d)^(7/2)/f+1/3*(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/
f/(c+d*sec(f*x+e))^3+1/6*(-5*a*c*d+2*b*c^2+3*b*d^2)*tan(f*x+e)/(c^2-d^2)^2/
f/(c+d*sec(f*x+e))^2+1/6*(-11*a*c^2*d-4*a*d^3+2*b*c^3+13*b*c*d^2)*tan(f*x+e
)/(c^2-d^2)^3/f/(c+d*sec(f*x+e))
```

Rubi [A]
time = 0.38, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$\frac{(-5acd+2bc^2+3bd^2)\tan(e+fx)}{6f\,(c^2-d^2)^2\,(c+d\sec(e+fx))^2}+\frac{(bc-ad)\tan(e+fx)}{3f\,(c^2-d^2)\,(c+d\sec(e+fx))^3}+\frac{(2ac^3+3acd^2-4bc^2d-bd^3)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{7/2}(c+d)^{7/2}}+\frac{(-11ac^2d-4ad^3+2bc^3+13bcd^2)\tan(e+fx)}{6f\,(c^2-d^2)^3\,(c+d\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/((c - d)^(7/2)*(c + d)^(7/2)*f) + ((b*c - a*d)*Tan[e +
 f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((2*b*c^2 - 5*a*c*d + 3*b
*d^2)*Tan[e + f*x])/(6*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x])^2) + ((2*b*c^3
- 11*a*c^2*d + 13*b*c*d^2 - 4*a*d^3)*Tan[e + f*x])/(6*(c^2 - d^2)^3*f*(c +
d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-3(ac-bd)-2(bc-ad)\sec(e+f}{(c+d\sec(e+fx))^3}}{3(c^2-d^2)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(2bc^2-5acd+3bd^2)\tan(e+fx}{6(c^2-d^2)^2 f(c+d\sec(e+fx))^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(2bc^2-5acd+3bd^2)\tan(e+fx}{6(c^2-d^2)^2 f(c+d\sec(e+fx))^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(2bc^2-5acd+3bd^2)\tan(e+fx}{6(c^2-d^2)^2 f(c+d\sec(e+fx))^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(2bc^2-5acd+3bd^2)\tan(e+fx}{6(c^2-d^2)^2 f(c+d\sec(e+fx))^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(2bc^2-5acd+3bd^2)\tan(e+fx}{6(c^2-d^2)^2 f(c+d\sec(e+fx))^2}$$

$$= \frac{(2ac^3-4bc^2d+3acd^2-bd^3)\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{7/2}(c+d)^{7/2}f} + \frac{}{3(c^2}$$

Mathematica [A]

time = 1.04, size = 405, normalized size = 1.71

$$\frac{(d+c\cos(e+fx))\sec^3(e+fx)(a+b\sec(e+fx))\left(\frac{24(-bd(4c^2+d^2)+a(2c^3+3cd^2))\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^3}{\sqrt{c^2-d^2}}-6bc^5\sin(e+fx)+18ac^4d\sin(e+fx)-18bc^3d^2\sin(e+fx)+39ac^2d^3\sin(e+fx)-51bcd^4\sin(e+fx)+18ad^5\sin(e+fx)-12bc^4d\sin(2(e+fx))+54ac^3d^2\sin(2(e+fx))-54bc^2d^3\sin(2(e+fx))+6acd^4\sin(2(e+fx))+6bd^5\sin(2(e+fx))-6bc^5\sin(3(e+fx))+18ac^4d\sin(3(e+fx))-10bc^3d^2\sin(3(e+fx))-5ac^2d^3\sin(3(e+fx))+bcd^4\sin(3(e+fx))+2ad^5\sin(3(e+fx))\right)}{24(-c^2+d^2)^3f(b+a\cos(e+fx))(c+d\sec(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^3*(a + b*Sec[e + f*x])*((24*(-(b*d*(4*c^
2 + d^2)) + a*(2*c^3 + 3*c*d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c
^2 - d^2]]*(d + c*Cos[e + f*x])^3)/Sqrt[c^2 - d^2] - 6*b*c^5*Sin[e + f*x] +
 18*a*c^4*d*Sin[e + f*x] - 18*b*c^3*d^2*Sin[e + f*x] + 39*a*c^2*d^3*Sin[e +
 f*x] - 51*b*c*d^4*Sin[e + f*x] + 18*a*d^5*Sin[e + f*x] - 12*b*c^4*d*Sin[2*
(e + f*x)] + 54*a*c^3*d^2*Sin[2*(e + f*x)] - 54*b*c^2*d^3*Sin[2*(e + f*x)]
+ 6*a*c*d^4*Sin[2*(e + f*x)] + 6*b*d^5*Sin[2*(e + f*x)] - 6*b*c^5*Sin[3*(e
+ f*x)] + 18*a*c^4*d*Sin[3*(e + f*x)] - 10*b*c^3*d^2*Sin[3*(e + f*x)] - 5*a
*c^2*d^3*Sin[3*(e + f*x)] + b*c*d^4*Sin[3*(e + f*x)] + 2*a*d^5*Sin[3*(e + f
*x)]))/(24*(-c^2 + d^2)^3*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4)
```

Maple [A]

time = 0.49, size = 376, normalized size = 1.59

method	result
derivativedivides	$-\frac{2\left(-\frac{\left(6a\,c^2d+3ac\,d^2+2a\,d^3-2b\,c^3-2b\,c^2d-6bc\,d^2-b\,d^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{2\left(9a\,c^2d+a\,d^3-3b\,c^3-7bc\,d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3\left(c^2+2cd+d^2\right)\left(c^2-2cd+d^2\right)}-\frac{(6a\,c \ldots}{}\right.}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^3}$
default	$-\frac{2\left(-\frac{\left(6a\,c^2d+3ac\,d^2+2a\,d^3-2b\,c^3-2b\,c^2d-6bc\,d^2-b\,d^3\right)\left(\tan^5\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{2\left(9a\,c^2d+a\,d^3-3b\,c^3-7bc\,d^2\right)\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3\left(c^2+2cd+d^2\right)\left(c^2-2cd+d^2\right)}-\frac{(6a\,c \ldots}{}\right.}{\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(-1/2*(6*a*c^2*d+3*a*c*d^2+2*a*d^3-2*b*c^3-2*b*c^2*d-6*b*c*d^2-b*d^
3)/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(9*a*c^2*d+a*d^
3-3*b*c^3-7*b*c*d^2)/(c^2+2*c*d+d^2)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1
/2*(6*a*c^2*d-3*a*c*d^2+2*a*d^3-2*b*c^3+2*b*c^2*d-6*b*c*d^2+b*d^3)/(c+d)/(c
^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1
/2*f*x+1/2*e)^2-c-d)^3+(2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)/(c^6-3*c^4*d^2+3
*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(
c-d))^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(229) = 458.
time = 2.97, size = 1264, normalized size = 5.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c^
5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^2
+ 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3 +
3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e
) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*s
in(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2))
 + 2*(2*b*c^5*d^2 - 11*a*c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6
+ 4*a*d^7 + (6*b*c^7 - 18*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11*b*c^3*d
^4 - 7*a*c^2*d^5 + b*c*d^6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*d - 9*a*c
^5*d^2 + 7*b*c^4*d^3 + 8*a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d^7)*cos(f*
x + e))*sin(f*x + e))/((c^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6 + c^3*d^8)
*f*cos(f*x + e)^3 + 3*(c^10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d^7 + c^2*d^9
)*f*cos(f*x + e)^2 + 3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^1
0)*f*cos(f*x + e) + (c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f)
, 1/6*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c^
5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^2
+ 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3 +
3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d
^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2*b*c^5*d^2 - 11*a*
c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6 + 4*a*d^7 + (6*b*c^7 - 18
*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11*b*c^3*d^4 - 7*a*c^2*d^5 + b*c*d^
```

```
6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*d - 9*a*c^5*d^2 + 7*b*c^4*d^3 + 8*
a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d^7)*cos(f*x + e))*sin(f*x + e))/((c
^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6 + c^3*d^8)*f*cos(f*x + e)^3 + 3*(c^
10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d^7 + c^2*d^9)*f*cos(f*x + e)^2 + 3*(c
^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e) + (c^8*
d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(e+fx))\sec(e+fx)}{(c+d\sec(e+fx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(222) = 444.
time = 0.54, size = 693, normalized size = 2.92

$$-\frac{\frac{3\left(2ac^3-4bc^2d+3acd^2-bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{\left(c^6-3c^4d^2+3c^2d^4-d^6\right)\sqrt{-c^2+d^2}}+\frac{6bc^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18ac^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-6bc^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+27ac^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+12bc^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-6ac^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-27bc^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+3acd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+12bcd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-6ad^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+3bd^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12bc^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+36ac^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-16bc^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-32ac^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+28bcd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4ad^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6bc^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-18ac^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+6bc^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-27ac^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+12bc^3d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6ac^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+27bc^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3acd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+12bcd^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-6ad^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-3bd^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c^6-3c^4d^2+3c^2d^4-d^6\right)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
1/2*e))/sqrt(-c^2 + d^2)))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(-c^2 +
 d^2)) + (6*b*c^5*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^4*d*tan(1/2*f*x + 1/2*e)^
5 - 6*b*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 27*a*c^3*d^2*tan(1/2*f*x + 1/2*e)^5
+ 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 6*a*c^2*d^3*tan(1/2*f*x + 1/2*e)^5
- 27*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a*c*d^4*tan(1/2*f*x + 1/2*e)^5 +
12*b*c*d^4*tan(1/2*f*x + 1/2*e)^5 - 6*a*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b*d^
5*tan(1/2*f*x + 1/2*e)^5 - 12*b*c^5*tan(1/2*f*x + 1/2*e)^3 + 36*a*c^4*d*tan
(1/2*f*x + 1/2*e)^3 - 16*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 32*a*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 28*b*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*a*d^5*tan(1/2*
f*x + 1/2*e)^3 + 6*b*c^5*tan(1/2*f*x + 1/2*e) - 18*a*c^4*d*tan(1/2*f*x + 1/
2*e) + 6*b*c^4*d*tan(1/2*f*x + 1/2*e) - 27*a*c^3*d^2*tan(1/2*f*x + 1/2*e) +
 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 6*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + 27*
b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 3*a*c*d^4*tan(1/2*f*x + 1/2*e) + 12*b*c*d^
4*tan(1/2*f*x + 1/2*e) - 6*a*d^5*tan(1/2*f*x + 1/2*e) - 3*b*d^5*tan(1/2*f*x
 + 1/2*e))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*(c*tan(1/2*f*x + 1/2*e)^2 -
 d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

Mupad [B]
time = 6.39, size = 439, normalized size = 1.85

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\,(2\,b\,c^3-2\,a\,d^3+b\,d^3-3\,a\,c\,d^2-6\,a\,c^2\,d+6\,b\,c\,d^2+2\,b\,c^2\,d)}{(c+d)^3\,(c-d)}+\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\,(-3\,b\,c^3+9\,a\,c^2\,d-7\,b\,c\,d^2+a\,d^3)}{3\,(c+d)^2\,(c^2-2\,c\,d+d^2)}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,a\,d^3-2\,b\,c^3+b\,d^3-3\,a\,c\,d^2+6\,a\,c^2\,d-6\,b\,c\,d^2+2\,b\,c^2\,d)}{(c+d)\,(c^3-3\,c^2\,d+3\,c\,d^2-d^3)}}{f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\,(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3)+3\,c\,d^2+3\,c^2\,d+c^3+d^3-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\,(c^3-3\,c^2\,d+3\,c\,d^2-d^3)\right)}+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,c-2\,d)\,(c^3-3\,c^2\,d+3\,c\,d^2-d^3)}{2\,\sqrt{c+d}\,(c-d)^{7/2}}\right)\,(2\,a\,c^3-4\,b\,c^2\,d+3\,a\,c\,d^2-b\,d^3)}{f\,(c+d)^{7/2}\,(c-d)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(2*b*c^3 - 2*a*d^3 + b*d^3 - 3*a*c*d^2 - 6*a*c^2*d +
 6*b*c*d^2 + 2*b*c^2*d))/((c + d)^3*(c - d)) + (4*tan(e/2 + (f*x)/2)^3*(a*d
^3 - 3*b*c^3 + 9*a*c^2*d - 7*b*c*d^2))/(3*(c + d)^2*(c^2 - 2*c*d + d^2)) -
(tan(e/2 + (f*x)/2)*(2*a*d^3 - 2*b*c^3 + b*d^3 - 3*a*c*d^2 + 6*a*c^2*d - 6*
b*c*d^2 + 2*b*c^2*d))/((c + d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(tan(e/
2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(
3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/
2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (atanh((tan(e/2 + (f*x)/
2)*(2*c - 2*d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3))/(2*(c + d)^(1/2)*(c - d)^(7
/2)))*(2*a*c^3 - b*d^3 + 3*a*c*d^2 - 4*b*c^2*d))/(f*(c + d)^(7/2)*(c - d)^(
7/2))
```

3.252 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=247

$$\frac{d^3(4bc-ad)\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}+\frac{2(bc-ad)^4\,\mathrm{ta}\cdots}{\sqrt{\cdots}}$$

```
[Out] 1/2*d^3*(-a*d+4*b*c)*arctanh(sin(f*x+e))/b^2/f+d*(-a*d+2*b*c)*(a^2*d^2-2*a*
b*c*d+2*b^2*c^2)*arctanh(sin(f*x+e))/b^4/f+2*(-a*d+b*c)^4*arctanh((a-b)^(1/
2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/b^4/f/(a-b)^(1/2)/(a+b)^(1/2)+d^4*tan(f*
x+e)/b/f+d^2*(a^2*d^2-4*a*b*c*d+6*b^2*c^2)*tan(f*x+e)/b^3/f+1/2*d^3*(-a*d+4
*b*c)*sec(f*x+e)*tan(f*x+e)/b^2/f+1/3*d^4*tan(f*x+e)^3/b/f
```

Rubi [A]
time = 0.30, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$,
Rules used = {4073, 3031, 2738, 214, 3855, 3852, 8, 3853}

$$\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f}+\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\tan(e+fx)}{b^3f}+\frac{2(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^4f\sqrt{a-b}\sqrt{a+b}}+\frac{d^3(4bc-ad)\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{d^3(4bc-ad)\tan(e+fx)\sec(e+fx)}{2b^2f}+\frac{d^4\tan^3(e+fx)}{3bf}+\frac{d^4\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d^3*(4*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d*(2*b*c - a*d)*(2*b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*
d)^4*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*
Sqrt[a + b]*f) + (d^4*Tan[e + f*x])/(b*f) + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a
^2*d^2)*Tan[e + f*x])/(b^3*f) + (d^3*(4*b*c - a*d)*Sec[e + f*x]*Tan[e + f*x
])/(2*b^2*f) + (d^4*Tan[e + f*x]^3)/(3*b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \int \frac{(d+c\cos(e+fx))^4\sec^4(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= \int \left(\frac{(bc-ad)^4}{b^4(b+a\cos(e+fx))} + \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\text{se}}{b^4}\right.$$

$$= \frac{d^4\int\sec^4(e+fx)\,dx}{b} + \frac{(bc-ad)^4\int\frac{1}{b+a\cos(e+fx)}\,dx}{b^4} + \frac{(d^3(4bc-a\text{}}{}$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\tanh^{-1}(\sin(e+fx))}{b^4f} + \frac{d^3(4bc}{}$$

$$= \frac{d^3(4bc-ad)\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d(2bc-ad)\left(2b^2c^2-2abcd+\right.}{b^4f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 580 vs. 2(247) = 494.
time = 4.46, size = 580, normalized size = 2.35

$$\frac{\cos^3(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^4\left(-\frac{24(bc-ad)^4\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-6d(8a^2bcd^2-2a^3d^3+4b^3c(2c^2+d^2)-ab^2d(12c^2+d^2))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-6d(-8a^2bcd^2+2a^3d^3-4b^3c(2c^2+d^2)+ab^2d(12c^2+d^2))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{b^2d^3(-3ad+b(12c+d))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{2b^3d^4\sin\left(\frac{1}{2}(e+fx)\right)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{4bd^2(-12abcd+3a^2d^2+2b^2(9c^2+d^2))\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)}+\frac{2b^3d^4\sin\left(\frac{1}{2}(e+fx)\right)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{b^2d^3(-3ad+b(12c+d))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{4bd^2(-12abcd+3a^2d^2+2b^2(9c^2+d^2))\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}\right)}{12b^4f(d+c\cos(e+fx))^4(a+b\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^3*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((-24*(b*c - a*
d)^4*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
- 6*d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c^2
+ d^2))*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 6*d*(-8*a^2*b*c*d^2 + 2*
a^3*d^3 - 4*b^3*c*(2*c^2 + d^2) + a*b^2*d*(12*c^2 + d^2))*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]] + (b^2*d^3*(-3*a*d + b*(12*c + d)))/(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 + (2*b^3*d^4*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3 + (4*b*d^2*(-12*a*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d
^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (2*b^3*d^4*S
in[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (b^2*d^3*(-3*a*d
 + b*(12*c + d)))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*b*d^2*(-12*a
*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])))/(12*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e + f*
x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(232) = 464.
time = 0.63, size = 480, normalized size = 1.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2/b^4*(-a^4*d^4+4*a^3*b*c*d^3-6*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d-b^4*c^4
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))
-1/3*d^4/b/(tan(1/2*f*x+1/2*e)+1)^3-1/2*d*(2*a^3*d^3-8*a^2*b*c*d^2+12*a*b^2
*c^2*d+a*b^2*d^3-8*b^3*c^3-4*b^3*c*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)+1)-1/2*d^
2*(2*a^2*d^2-8*a*b*c*d+a*b*d^2+12*b^2*c^2-4*b^2*c*d+2*b^2*d^2)/b^3/(tan(1/2
*f*x+1/2*e)+1)+1/2*d^3*(a*d-4*b*c+b*d)/b^2/(tan(1/2*f*x+1/2*e)+1)^2-1/3*d^4
/b/(tan(1/2*f*x+1/2*e)-1)^3+1/2*d*(2*a^3*d^3-8*a^2*b*c*d^2+12*a*b^2*c^2*d+a
*b^2*d^3-8*b^3*c^3-4*b^3*c*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)-1)-1/2*d^2*(2*a^2
*d^2-8*a*b*c*d+a*b*d^2+12*b^2*c^2-4*b^2*c*d+2*b^2*d^2)/b^3/(tan(1/2*f*x+1/2
*e)-1)-1/2*d^3*(a*d-4*b*c+b*d)/b^2/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(240) = 480$.
time = 183.45, size = 1119, normalized size = 4.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*sqrt(a^2 - b^2)*cos(f*x + e)^3*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2
)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*
a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + 3*(8*(a^2*b^3
 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*
c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)^3*log(sin(f*x + e) + 1)
 - 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b -
 a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)^3*log(-
sin(f*x + e) + 1) + 2*(2*(a^2*b^3 - b^5)*d^4 + 2*(18*(a^2*b^3 - b^5)*c^2*d^
```

```
2 - 12*(a^3*b^2 - a*b^4)*c*d^3 + (3*a^4*b - a^2*b^3 - 2*b^5)*d^4)*cos(f*x +
 e)^2 + 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*d^4)*cos(f*x + e))*s
in(f*x + e))/((a^2*b^4 - b^6)*f*cos(f*x + e)^3), 1/12*(12*(b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(
f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*
(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x +
e)^3*log(sin(f*x + e) + 1) - 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b
^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)
*d^4)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*(a^2*b^3 - b^5)*d^4 + 2*
(18*(a^2*b^3 - b^5)*c^2*d^2 - 12*(a^3*b^2 - a*b^4)*c*d^3 + (3*a^4*b - a^2*b
^3 - 2*b^5)*d^4)*cos(f*x + e)^2 + 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a
*b^4)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^2*b^4 - b^6)*f*cos(f*x + e)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sec(e + fx))^4 \sec(e + fx)}{a + b\sec(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 606 vs. 2(232) = 464.
time = 0.56, size = 606, normalized size = 2.45

$$\frac{\frac{3(8b^3c^3d - 12ab^2c^2d^2 + 8a^2bcd^3 + 4b^3cd^3 - 2a^3d^4 - ab^2d^4)\log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right|\right)}{b^4} - \frac{3(8b^3c^3d - 12ab^2c^2d^2 + 8a^2bcd^3 + 4b^3cd^3 - 2a^3d^4 - ab^2d^4)\log\left(\left|\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right|\right)}{b^4} - \frac{12(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4)\left(\pi\left\lfloor\frac{fx+e}{2\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(2a - 2b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - b\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{\sqrt{-a^2 + b^2}\, b^4} - [illegible]}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(8*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^
3*d^4 - a*b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^4 - 3*(8*b^3*c^3*d
- 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^3*d^4 - a*b^2*d^4)*l
og(abs(tan(1/2*f*x + 1/2*e) - 1))/b^4 - 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*s
gn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sq
rt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^4) - 2*(36*b^2*c^2*d^2*tan(1/2*f*x + 1
/2*e)^5 - 24*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^5 - 12*b^2*c*d^3*tan(1/2*f*x +
1/2*e)^5 + 6*a^2*d^4*tan(1/2*f*x + 1/2*e)^5 + 3*a*b*d^4*tan(1/2*f*x + 1/2*e
)^5 + 6*b^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 72*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e
)^3 + 48*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*d^4*tan(1/2*f*x + 1/2*e)
```

```
^3 - 4*b^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 36*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)
 - 24*a*b*c*d^3*tan(1/2*f*x + 1/2*e) + 12*b^2*c*d^3*tan(1/2*f*x + 1/2*e) +
6*a^2*d^4*tan(1/2*f*x + 1/2*e) - 3*a*b*d^4*tan(1/2*f*x + 1/2*e) + 6*b^2*d^4
*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*b^3))/f
```

Mupad [B]
time = 11.32, size = 2500, normalized size = 10.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] (atan((((((((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16
*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*
d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^
3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b
^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2
)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*
(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a
*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 + (8*ta
n(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*
b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 +
 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b
^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 19
2*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 -
104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d
^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*
a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c
^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 -
376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*
b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d
^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*
b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^
2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3)*1i)/b^4 - (((((
8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d
+ 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*
b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3
*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2
- 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 + (8*tan(e/2 + (f*x)/2)*(8*a*b^10
- 16*a^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4
*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) -
b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x
)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*
a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*
```

d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3)*1i)/b^4)/((16*(4*a^11*d^12 - 6*a^10*b*d^12 + 16*b^11*c^11*d - a^6*b^5*d^12 + 2*a^7*b^4*d^12 - 5*a^8*b^3*d^12 + 6*a^9*b^2*d^12 - 16*b^11*c^6*d^6 - 64*b^11*c^8*d^4 + 8*b^11*c^9*d^3 - 64*b^11*c^10*d^2 + 72*a*b^10*c^5*d^7 + 32*a*b^10*c^6*d^6 + 368*a*b^10*c^7*d^5 + 62*a*b^10*c^8*d^4 + 440*a*b^10*c^9*d^3 - 24*a*b^10*c^10*d^2 + 12*a^5*b^6*c*d^11 - 24*a^6*b^5*c*d^11 + 60*a^7*b^4*c*d^11 - 72*a^8*b^3*c*d^11 + 72*a^9*b^2*c*d^11 - 129*a^2*b^9*c^4*d^8 - 144*a^2*b^9*c^5*d^7 - 936*a^2*b^9*c^6*d^6 - 496*a^2*b^9*c^7*d^5 - 1422*a^2*b^9*c^8*d^4 - 240*a^2*b^9*c^9*d^3 + 88*a^2*b^9*c^10*d^2 + 116*a^3*b^8*c^3*d^9 + 258*a^3*b^8*c^4*d^8 + 1384*a^3*b^8*c^5*d^7 + 1336*a^3*b^8*c^6*d^6 + 2848*a^3*b^8*c^7*d^5 + 1148*a^3*b^8*c^8*d^4 - 208*a^3*b^8*c^9*d^3 - 54*a^4*b^7*c^2*d^10 - 232*a^4*b^7*c^3*d^9 - 1301*a^4*b^7*c^4*d^8 - 1952*a^4*b^7*c^5*d^7 - 3888*a^4*b^7*c^6*d^6 - 2496*a^4*b^7*c^7*d^5 + 276*a^4*b^7*c^8*d^4 + 108*a^5*b^6*c^2*d^10 + 788*a^5*b^6*c^3*d^9 + 1756*a^5*b^6*c^4*d^8 + 3744*a^5*b^6*c^5*d^7 + 3360*a^5*b^6*c^6*d^6 - 224*a^5*b^6*c^7*d^5 - 294*a^6*b^5*c^2*d^10 - 1008*a^6*b^5*c^3*d^9 - 2556*a^6*b^5*c^4*d^8 - 3072*a^6*b^5*c^5*d^7 + 112*a^6*b^5*c^6*d^6 + 360*a^7*b^4*c^2*d^10 + 1216*a^7*b^4*c^3*d^9 + 1968*a^7*b^4*c^4*d^8 - 32*a^7*b^4*c^5*d^7 - 384*a^8*b^3*c^2*d^10 - 880*a^8*b^3*c^3*d^9 + 4*a^8*b^3*c^4*d^8 + 264*a^9*b^2*c^2*d^10 - 16*a*b^10*c^11*d - 48*a^10*b*c*d^11))/b^9 + (((((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2...

3.253 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{d^3\tanh^{-1}(\sin(e+fx))}{2bf}+\frac{d(3b^2c^2-3abcd+a^2d^2)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,f}$$

```
[Out] 1/2*d^3*arctanh(sin(f*x+e))/b/f+d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*arctanh(sin
(f*x+e))/b^3/f+2*(-a*d+b*c)^3*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^
(1/2))/b^3/f/(a-b)^(1/2)/(a+b)^(1/2)+d^2*(-a*d+3*b*c)*tan(f*x+e)/b^2/f+1/2*
d^3*sec(f*x+e)*tan(f*x+e)/b/f
```

Rubi [A]
time = 0.24, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4073, 3031, 2738, 214, 3855, 3852, 8, 3853}

$$\frac{d(a^2d^2-3abcd+3b^2c^2)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^3f\sqrt{a-b}\sqrt{a+b}}+\frac{d^2(3bc-ad)\tan(e+fx)}{b^2f}+\frac{d^3\tanh^{-1}(\sin(e+fx))}{2bf}+\frac{d^3\tan(e+fx)\sec(e+fx)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d^3*ArcTanh[Sin[e + f*x]])/(2*b*f) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcTanh[(Sqrt[a - b]*Tan[
(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*f) + (d^2*(3*b*c -
 a*d)*Tan[e + f*x])/(b^2*f) + (d^3*Sec[e + f*x]*Tan[e + f*x])/(2*b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \int \frac{(d+c\cos(e+fx))^3\sec^3(e+fx)}{b+a\cos(e+fx)}\,dx$$

$$= \int \left(\frac{(bc-ad)^3}{b^3(b+a\cos(e+fx))} + \frac{d(3b^2c^2-3abcd+a^2d^2)\sec(e+fx)}{b^3} + \right.$$

$$= \frac{d^3\int \sec^3(e+fx)\,dx}{b} + \frac{(bc-ad)^3\int \frac{1}{b+a\cos(e+fx)}\,dx}{b^3} + \frac{(d^2(3bc-ad)}{}$$

$$= \frac{d(3b^2c^2-3abcd+a^2d^2)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{d^3\sec(e+fx)\tan(e}{2bf}$$

$$= \frac{d^3\tanh^{-1}(\sin(e+fx))}{2bf} + \frac{d(3b^2c^2-3abcd+a^2d^2)\tanh^{-1}(\sin(e+f}{b^3f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 389 vs. $2(170) = 340$.
time = 1.46, size = 389, normalized size = 2.29

$$\frac{\cos^2(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^3\left(\frac{8(-bc+ad)^3\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 2d(-6abcd+2a^2d^2+b^2(6c^2+d^2))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right) + 2d(-6abcd+2a^2d^2+b^2(6c^2+d^2))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right) + \frac{b^2d^3}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} + \frac{4bd^2(3bc-ad)\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} - \frac{b^2d^3}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} + \frac{4bd^2(3bc-ad)\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}\right)}{4b^3f(d+c\cos(e+fx))^3(a+b\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((8*(-(b*c) + a
*d)^3*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
 - 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] + 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^3)/(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^2 + (4*b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - (b^2*d^3)/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*
b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
))/(4*b^3*f*(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x]))
```

Maple [A]
time = 0.48, size = 289, normalized size = 1.70

method	result
derivativedivides	$-\frac{2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-c^3b^3\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}} - \frac{d^3}{2b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2} + \frac{d\left(2a^2d^2-6abdc+6b^2c^2+b^2d^2\right.}{2b^3}$

default	$\frac{-\frac{2\left(a^{3} d^{3}-3 a^{2} b c\, d^{2}+3 a\, b^{2} c^{2} d-c^{3} b^{3}\right) \operatorname{arctanh}\left(\frac{(a-b) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{3} \sqrt{(a+b)(a-b)}}-\frac{d^{3}}{2 b\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}+\frac{d\left(2 a^{2} d^{2}-6 a b d c+6 b^{2} c^{2}+b^{2}\right.}{2 b^{3}}}{}$
risch	$-\frac{i d^{2}\left(b d\, \mathrm{e}^{3 i(f x+e)}+2 a d\, \mathrm{e}^{2 i(f x+e)}-6 b c\, \mathrm{e}^{2 i(f x+e)}-b d\, \mathrm{e}^{i(f x+e)}+2 a d-6 b c\right)}{f\, b^{2}\left(\mathrm{e}^{2 i(f x+e)}+1\right)^{2}}-\frac{d^{3} \ln\left(\mathrm{e}^{i(f x+e)}-i\right) a^{2}}{b^{3} f}+\frac{3 d^{2} \ln\left(\mathrm{e}^{i(f x+e)}\right.}{b^{2} f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^3/((a+b)*(a-b))^(1/
2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))-1/2*d^3/b/(tan(1/2
*f*x+1/2*e)+1)^2+1/2*d*(2*a^2*d^2-6*a*b*c*d+6*b^2*c^2+b^2*d^2)/b^3*ln(tan(1
/2*f*x+1/2*e)+1)+1/2*d^2*(2*a*d-6*b*c+b*d)/b^2/(tan(1/2*f*x+1/2*e)+1)+1/2*d
^3/b/(tan(1/2*f*x+1/2*e)-1)^2-1/2*d*(2*a^2*d^2-6*a*b*c*d+6*b^2*c^2+b^2*d^2)
/b^3*ln(tan(1/2*f*x+1/2*e)-1)+1/2*d^2*(2*a*d-6*b*c+b*d)/b^2/(tan(1/2*f*x+1/
2*e)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(163) = 326$.
time = 38.07, size = 803, normalized size = 4.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/4*(2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a^2 - b^2
)*cos(f*x + e)^2*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2
```

```
*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(
f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3
*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(sin(f*x
 + e) + 1) + (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 -
a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*((a^2*b^2 - b
^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)*d^3)*cos(f*x + e))*s
in(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2), 1/4*(4*(b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*
(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e)^2 + (6*(a^2*b
^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*co
s(f*x + e)^2*log(sin(f*x + e) + 1) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b -
a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e
) + 1) + 2*((a^2*b^2 - b^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b
^3)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sec(e + fx))^3 \sec(e + fx)}{a + b\sec(e + fx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(157) = 314.
time = 0.56, size = 339, normalized size = 1.99

$$\frac{\frac{(6\,b^2c^2d - 6\,abcd^2 + 2\,a^2d^3 + b^2d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right)}{b^3} - \frac{(6\,b^2c^2d - 6\,abcd^2 + 2\,a^2d^3 + b^2d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right)}{b^3} - \frac{4\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a - 2\,b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - b\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{\sqrt{-a^2 + b^2}\,b^3} - \frac{2\left(6\,bcd^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^3 - 2\,ad^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^3 - bd^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^3 - 6\,bcd^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 2\,ad^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - bd^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 - 1\right)^2 b^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*((6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*log(abs(tan(1/2*f*x
+ 1/2*e) + 1))/b^3 - (6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*log(
abs(tan(1/2*f*x + 1/2*e) - 1))/b^3 - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((
a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/(sqrt(-
a^2 + b^2)*b^3) - 2*(6*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*d^3*tan(1/2*f*x
 + 1/2*e)^3 - b*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*b*c*d^2*tan(1/2*f*x + 1/2*e)
 + 2*a*d^3*tan(1/2*f*x + 1/2*e) - b*d^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x
 + 1/2*e)^2 - 1)^2*b^2))/f
```

Mupad [B]
time = 9.66, size = 2500, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)*(b*d^3 - 2*a*d^3 + 6*b*c*d^2))/b^2 + (tan(e/2 + (f*x)/
2)^3*(2*a*d^3 + b*d^3 - 6*b*c*d^2))/b^2)/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e
/2 + (f*x)/2)^2 + 1)) - (atan(((((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c
^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 1
3*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c
^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b
^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*
b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2
+ 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*
b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 +
24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 + (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a
*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*
a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*
b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*(
8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*
c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3)*(b^2*(3*
c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2)*1i)/b^3 + (((8*tan(e/2 + (f*x)/2)*(
8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6
- 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7
*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6
*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*
b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 1
68*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4
*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4
+ 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((8*(4*b^10*c^3
 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 +
 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*
b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 + (8*t
an(e/2 + (f*x)/2)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2)
 + a^2*d^3 - 3*a*b*c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*
d^2))/b^3)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2)*1i)/b^3)/((16*(4
*a^8*d^9 - 6*a^7*b*d^9 - 12*b^8*c^8*d - a^3*b^5*d^9 + 2*a^4*b^4*d^9 - 5*a^5
*b^3*d^9 + 6*a^6*b^2*d^9 + b^8*c^3*d^6 + 12*b^8*c^5*d^4 - 2*b^8*c^6*d^3 + 3
6*b^8*c^7*d^2 - 3*a*b^7*c^2*d^7 - 2*a*b^7*c^3*d^6 - 48*a*b^7*c^4*d^5 - 12*a
*b^7*c^5*d^4 - 178*a*b^7*c^6*d^3 + 12*a*b^7*c^7*d^2 + 3*a^2*b^6*c*d^8 - 6*a
^3*b^5*c*d^8 + 27*a^4*b^4*c*d^8 - 36*a^5*b^3*c*d^8 + 48*a^6*b^2*c*d^8 + 6*a
^2*b^6*c^2*d^7 + 77*a^2*b^6*c^3*d^6 + 66*a^2*b^6*c^4*d^5 + 384*a^2*b^6*c^5*
```

d^4 + 104*a^2*b^6*c^6*d^3 - 48*a^2*b^6*c^7*d^2 - 63*a^3*b^5*c^2*d^7 - 112*a
^3*b^5*c^3*d^6 - 474*a^3*b^5*c^4*d^5 - 324*a^3*b^5*c^5*d^4 + 76*a^3*b^5*c^6
*d^3 + 90*a^4*b^4*c^2*d^7 + 364*a^4*b^4*c^3*d^6 + 432*a^4*b^4*c^4*d^5 - 60*
a^4*b^4*c^5*d^4 - 174*a^5*b^3*c^2*d^7 - 324*a^5*b^3*c^3*d^6 + 24*a^5*b^3*c^
4*d^5 + 144*a^6*b^2*c^2*d^7 - 4*a^6*b^2*c^3*d^6 + 12*a*b^7*c^8*d - 36*a^7*b
*c*d^8))/b^6 - (((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4
*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 -
16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^
6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^
2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a
^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^
2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 15
2*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d -
48*a^6*b*c*d^5))/b^4 + (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b
^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*
a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a
*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*(8*a*b^8 - 16*a^2
*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^7)*(b^
2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3)*(b^2*(3*c^2*d + d^3/2) +
a^2*d^3 - 3*a*b*c*d^2))/b^3 + (((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c
^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 1
3*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c
^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b
^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*
b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2
+ 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*
b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 +
24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a
*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*
a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 ...

3.254 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=103

$$\frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,f}+\frac{d^2\tan(e+fx)}{bf}$$

```
[Out] d*(-a*d+2*b*c)*arctanh(sin(f*x+e))/b^2/f+2*(-a*d+b*c)^2*arctanh((a-b)^(1/2)
*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/b^2/f/(a-b)^(1/2)/(a+b)^(1/2)+d^2*tan(f*x+
e)/b/f
```

Rubi [A]

time = 0.20, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4073, 3031, 2738, 214, 3855, 3852, 8}

$$\frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a-b}\,\sqrt{a+b}}+\frac{d^2\tan(e+fx)}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[
(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*f
) + (d^2*Tan[e + f*x])/(b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx &= \int \frac{(d+c\cos(e+fx))^2\sec^2(e+fx)}{b+a\cos(e+fx)}\,dx\\ &= \int \left(\frac{(bc-ad)^2}{b^2(b+a\cos(e+fx))}+\frac{d(2bc-ad)\sec(e+fx)}{b^2}+\frac{d^2\sec^2(e+\\ &= \frac{d^2\int\sec^2(e+fx)\,dx}{b}+\frac{(bc-ad)^2\int\frac{1}{b+a\cos(e+fx)}\,dx}{b^2}+\frac{(d(2bc-ad))\\ &= \frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f}-\frac{d^2\text{Subst}(\int 1\,dx,x,-\tan(e+fx\\ &= \frac{d(2bc-ad)\tanh^{-1}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan}{\sqrt{a+}}\right.}{\sqrt{a-b}\,b^2\sqrt{a+b}\,f}\end{aligned}$$

Mathematica [A]

time = 0.83, size = 135, normalized size = 1.31

$$\frac{-\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+d\left(-\left((2bc-ad)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)+bd\tan(e+fx)\right)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

```
[Out] ((-2*(b*c - a*d)^2*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + d*(-((2*b*c - a*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + b*d*Tan[e + f*x]))/(b^2*f)
```

Maple [A]
time = 0.37, size = 165, normalized size = 1.60

method	result
derivativedivides	$\frac{-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{d(ad-2bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^2}-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d(ad-2bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2d^2+2abdc-\right.}{b^2}}{f}$
default	$\frac{-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{d(ad-2bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^2}-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d(ad-2bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2d^2+2abdc-\right.}{b^2}}{f}$
risch	$\frac{2id^2}{fb\left(e^{2i(fx+e)}+1\right)}+\frac{d^2\ln\left(e^{i(fx+e)}-i\right)a}{b^2f}-\frac{2d\ln\left(e^{i(fx+e)}-i\right)c}{bf}-\frac{d^2\ln\left(e^{i(fx+e)}+i\right)a}{b^2f}+\frac{2d\ln\left(e^{i(fx+e)}+i\right)c}{bf}+\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-d^2/b/(tan(1/2*f*x+1/2*e)-1)+d*(a*d-2*b*c)/b^2*ln(tan(1/2*f*x+1/2*e)-
1)-d^2/b/(tan(1/2*f*x+1/2*e)+1)-d*(a*d-2*b*c)/b^2*ln(tan(1/2*f*x+1/2*e)+1)-
2/b^2*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/
2*f*x+1/2*e)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(97) = 194$.
time = 8.62, size = 540, normalized size = 5.24

$$\left[\frac{2\,(a^2b-b^3)d^2\sin\left(fx+e\right)+(b^2c^2-2\,abcd+a^2d^2)\sqrt{a^2-b^2}\cos\left(fx+e\right)\log\left(\frac{2\,ab\cos(fx+e)-(a^2-2\,b^2)\cos(fx+e)^2+2\,\sqrt{a^2-b^2}\,(b\cos(fx+e)+a)\sin(fx+e)+2\,a^2-b^2}{a^2\cos(fx+e)^2+2\,ab\cos(fx+e)+b^2}\right)+(2\,(a^2b-b^3)cd-(a^3-ab^2)d^2)\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)-(2\,(a^2b-b^3)cd-(a^3-ab^2)d^2)\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)}{2\,(a^2b^2-b^4)f\cos\left(fx+e\right)},\ \frac{2\,(a^2b-b^3)d^2\sin\left(fx+e\right)+2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(fx+e)+a)}{(a^2-b^2)\sin(fx+e)}\right)\cos\left(fx+e\right)+(2\,(a^2b-b^3)cd-(a^3-ab^2)d^2)\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)-(2\,(a^2b-b^3)cd-(a^3-ab^2)d^2)\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)}{2\,(a^2b^2-b^4)f\cos\left(fx+e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sq
rt(a^2 - b^2)*cos(f*x + e)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x
+ e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)
/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*b - b^3)*c*d -
(a^3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^3)*c*
d - (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2 - b^4
)*f*cos(f*x + e)), 1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + 2*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
 + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e) + (2*(a^2*b - b^3)*c*d - (a^
3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^3)*c*d -
 (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2 - b^4)*f
*cos(f*x + e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+d\sec\left(e+fx\right)\right)^2\sec\left(e+fx\right)}{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(94) = 188$.
time = 0.51, size = 195, normalized size = 1.89

$$-\frac{\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)b}-\frac{(2\,bcd-ad^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^2}+\frac{(2\,bcd-ad^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^2}+\frac{2\,(b^2c^2-2\,abcd+a^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a-2\,b\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,b^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(2*d^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*b) - (2*b*c*d -
a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^2 + (2*b*c*d - a*d^2)*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/b^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) -
b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^2))/f
```

Mupad [B]
time = 7.32, size = 2500, normalized size = 24.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] - (2*d^2*tan(e/2 + (f*x)/2))/(b*f*(tan(e/2 + (f*x)/2)^2 - 1)) - (atan(((((a
 + b)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c
^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c
^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3
*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d
^3))/b^2 + (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2
 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*
a^2*b^5*c*d))/b^3 - (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b
*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)
^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2*1i)/(b^4 - a^2*b^2) + (((a + b)*(a - b))
^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d
^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^
2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*
a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 - (((
a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2
 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/
b^3 + (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^2*(2*a*b^6
 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)^2)/(b^4 - a^2
*b^2))*(a*d - b*c)^2*1i)/(b^4 - a^2*b^2))/((64*(a^4*b*d^6 - a^5*d^6 - 2*b^5
*c^5*d + 4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + a*b^4*c^4*d^2 - 6*a^3*b^2*c*d^5
 - a^4*b*c^2*d^4 + 13*a^2*b^3*c^2*d^4 + 8*a^2*b^3*c^3*d^3 - 5*a^2*b^3*c^4*d
^2 - 12*a^3*b^2*c^2*d^4 + 4*a^3*b^2*c^3*d^3 + 2*a*b^4*c^5*d + 6*a^4*b*c*d^5
))/b^3 - (((a + b)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*
c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2
 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3
 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d
- 8*a^4*b*c*d^3))/b^2 + (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^
2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a
*b^6*c*d + 2*a^2*b^5*c*d))/b^3 - (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(
```

1/2)*(a*d - b*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2))
)*(a*d - b*c)^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2)/(b^4 - a^2*b^2) + (((a + b
)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 -
4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d
^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2
*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))
/b^2 - (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a
^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*
b^5*c*d))/b^3 + (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^
2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)^2)/
(b^4 - a^2*b^2))*(a*d - b*c)^2)/(b^4 - a^2*b^2)))*((a + b)*(a - b))^(1/2)*(
a*d - b*c)^2*2i)/(f*(b^4 - a^2*b^2)) - (d*atan(((d*(a*d - 2*b*c)*((32*tan(e
/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4
+ 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a
^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 +
4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 + (d*(a*d - 2*b*c)*((3
2*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^
4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3 - (32*d*tan(e/2 + (f*
x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/b^4))/b^2)*1i)/b^2 +
(d*(a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4
- 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*
d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^
2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3)
)/b^2 - (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*
c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d
))/b^3 + (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^
3*b^4))/b^4))/b^2)*1i)/b^2)/((64*(a^4*b*d^6 - a^5*d^6 - 2*b^5*c^5*d + 4*b^5
*c^4*d^2 - 12*a*b^4*c^3*d^3 + a*b^4*c^4*d^2 - 6*a^3*b^2*c*d^5 - a^4*b*c^2*d
^4 + 13*a^2*b^3*c^2*d^4 + 8*a^2*b^3*c^3*d^3 - 5*a^2*b^3*c^4*d^2 - 12*a^3*b^
2*c^2*d^4 + 4*a^3*b^2*c^3*d^3 + 2*a*b^4*c^5*d + 6*a^4*b*c*d^5))/b^3 - (d*(a
*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^
4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 -
12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2
+ 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2
+ (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2 +
2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3
- (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4)
)/b^4))/b^2))/b^2 + (d*(a*d - 2*b*c)*((32*tan(e...

3.255 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=76

$$\frac{d\tanh^{-1}(\sin(e+fx))}{bf}+\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,f}$$

```
[Out] d*arctanh(sin(f*x+e))/b/f+2*(-a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*
e)/(a+b)^(1/2))/b/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4083, 3855, 3916, 2738, 214}

$$\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{bf\sqrt{a-b}\sqrt{a+b}}+\frac{d\tanh^{-1}(\sin(e+fx))}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(b*f) + (2*(b*c - a*d)*ArcTanh[(Sqrt[a - b]*Tan[(
e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx &= \frac{d\int \sec(e+fx)\,dx}{b}+\frac{(bc-ad)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{b}\\
&= \frac{d\tanh^{-1}(\sin(e+fx))}{bf}+\frac{(bc-ad)\int \frac{1}{1+\frac{a\cos(e+fx)}{b}}\,dx}{b^2}\\
&= \frac{d\tanh^{-1}(\sin(e+fx))}{bf}+\frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\right.}{b^2f}\\
&= \frac{d\tanh^{-1}(\sin(e+fx))}{bf}+\frac{2(bc-ad)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,f}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 112, normalized size = 1.47

$$\frac{\frac{2(-bc+ad)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+d\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

```
[Out] ((2*(-(b*c) + a*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + d*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]))/(b*f)
```

Maple [A]
time = 0.29, size = 92, normalized size = 1.21

method	result

derivativedivides	$\frac{\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b}-\frac{2(ad-bc)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b}}{f}$
default	$\frac{\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b}-\frac{2(ad-bc)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b}}{f}$
risch	$\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)ad}{\sqrt{a^2-b^2}\,fb}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)c}{\sqrt{a^2-b^2}\,f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ia^2-ib^2\cdots}{a\sqrt{\cdots}}\right.}{\sqrt{a^2-\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(d/b*ln(tan(1/2*f*x+1/2*e)+1)-2*(a*d-b*c)/b/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2))-d/b*ln(tan(1/2*f*x+1/2*e)-1)
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.91, size = 321, normalized size = 4.22

$$\left[\frac{(a^2-b^2)d\log\left(\sin\left(fx+e\right)+1\right)-(a^2-b^2)d\log\left(-\sin\left(fx+e\right)+1\right)-\sqrt{a^2-b^2}\,(bc-ad)\log\left(\frac{2\,ab\cos(fx+e)-(a^2-2\,b^2)\cos(fx+e)^2-2\sqrt{a^2-b^2}\,(b\cos(fx+e)+a)\sin(fx+e)+2\,a^2-b^2}{a^2\cos(fx+e)^2+2\,ab\cos(fx+e)+b^2}\right)}{2\,(a^2b-b^3)f},\ \frac{(a^2-b^2)d\log\left(\sin\left(fx+e\right)+1\right)-(a^2-b^2)d\log\left(-\sin\left(fx+e\right)+1\right)+2\sqrt{-a^2+b^2}\,(bc-ad)\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(fx+e)+a)}{(a^2-b^2)\sin(fx+e)}\right)}{2\,(a^2b-b^3)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x + e)
 + 1) - sqrt(a^2 - b^2)*(b*c - a*d)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)
```

```
*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a
^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/((a^2*b - b^3)*
f), 1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x +
 e) + 1) + 2*sqrt(-a^2 + b^2)*(b*c - a*d)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f
*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/((a^2*b - b^3)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sec(e + fx))\sec(e + fx)}{a + b\sec(e + fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Giac [A]
time = 0.52, size = 127, normalized size = 1.67

$$\frac{\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{b} - \frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{b} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)(bc-ad)}{\sqrt{-a^2+b^2}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b - d*log(abs(tan(1/2*f*x + 1/2*e) -
1))/b - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(b*c - a*d)/(
sqrt(-a^2 + b^2)*b))/f
```

Mupad [B]
time = 2.81, size = 571, normalized size = 7.51

$$\frac{b^2\,c\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(a^2-b^2\right)}^{3/2}}-\frac{a^2\,c\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(a^2-b^2\right)}^{3/2}}-\frac{2\,b\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(a^2-b^2\right)}+\frac{c\,\ln\left(\frac{a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{\left(a+b\right)\,\left(a-b\right)}}{f\,\left(a^2-b^2\right)}-\frac{a\,b\,d\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(a^2-b^2\right)}^{3/2}}+\frac{2\,a^2\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{b\,f\,\left(a^2-b^2\right)}+\frac{a^3\,d\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{b\,f\,{\left(a^2-b^2\right)}^{3/2}}-\frac{a\,d\,\ln\left(\frac{a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{\left(a+b\right)\,\left(a-b\right)}}{b\,f\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] (b^2*c*log((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2
)*(a^2 - b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) - (a^2*c*lo
g((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 -
b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) - (2*b*d*atanh(sin(e
/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)) + (c*log((a*cos(e/2 + (f
*x)/2) + b*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e
```

```
/2 + (f*x)/2))*((a + b)*(a - b))^(1/2))/(f*(a^2 - b^2)) - (a*b*d*log((b*sin
(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 - b^2)^(1/
2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) + (2*a^2*d*atanh(sin(e/2 + (
f*x)/2)/cos(e/2 + (f*x)/2)))/(b*f*(a^2 - b^2)) + (a^3*d*log((b*sin(e/2 + (f
*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e
/2 + (f*x)/2)))/(b*f*(a^2 - b^2)^(3/2)) - (a*d*log((a*cos(e/2 + (f*x)/2) +
b*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e/2 + (f*x
)/2))*((a + b)*(a - b))^(1/2))/(b*f*(a^2 - b^2))
```

3.256 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=121

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,(bc-ad)f}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,\sqrt{c+d}\,(bc-ad)f}$$

```
[Out] 2*b*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(-a*d+b*c)/f/(a-b)^
(1/2)/(a+b)^(1/2)-2*d*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(
-a*d+b*c)/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4073, 3080, 2738, 214}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\,\sqrt{a+b}\,(bc-ad)}-\frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\,\sqrt{c+d}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*b*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*(b*c - a*d)*f) - (2*d*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c
 + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \int \frac{\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}\,dx$$

$$= \frac{b\int \frac{1}{b+a\cos(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{1}{d+c\cos(e+fx)}\,dx}{bc-ad}$$

$$= \frac{(2b)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} - \frac{(2d)\text{Su}}{}$$

$$= \frac{2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,(bc-ad)f} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{c-d}\,\tan}{\sqrt{c+}}\right)}{\sqrt{c-d}\,\sqrt{c+d}\,(bc}$$

Mathematica [A]
time = 0.23, size = 119, normalized size = 0.98

$$-\frac{2b\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(bc-ad)f} - \frac{2d\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(-bc+ad)\sqrt{c^2-d^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*b*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2
]*(b*c - a*d)*f) - (2*d*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/((-(b*c) + a*d)*Sqrt[c^2 - d^2]*f)
```

Maple [A]
time = 0.47, size = 108, normalized size = 0.89

method	result

derivativedivides	$\frac{\frac{2d\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(ad-bc)\sqrt{(c+d)(c-d)}}-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(ad-bc)\sqrt{(a+b)(a-b)}}}{f}$
default	$\frac{\frac{2d\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(ad-bc)\sqrt{(c+d)(c-d)}}-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(ad-bc)\sqrt{(a+b)(a-b)}}}{f}$
risch	$\frac{d\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(ad-bc)f}-\frac{d\ln\left(\mathrm{e}^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}\,(ad-bc)f}+\frac{b\ln\left(\mathrm{e}^{i(fx+e)}-\frac{ia^2-ib^2-\ldots}{a\sqrt{\ldots}}\right.}{\sqrt{a^2-b^2}\,(a\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d/(a*d-b*c)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c
+d)*(c-d))^(1/2))-2*b/(a*d-b*c)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f
*x+1/2*e)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 5.02, size = 1072, normalized size = 8.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((a^2 - b^2)*sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2
)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*
```

```
c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (b*c^2 - b*d^
2)*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*co
s(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b
^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -1/2*((a^2 - b^2)*
sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 +
2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos
(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(b*c^2 - b*d^2)*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))
))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 -
 a*b^2)*d^3)*f), -1/2*(2*(a^2 - b^2)*sqrt(-c^2 + d^2)*d*arctan(-sqrt(-c^2 +
 d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^2 - b*d^2)*sq
rt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sq
rt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x
 + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c
^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -((a^2 - b^2)*sqrt(-c^2
 + d^2)*d*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*
x + e))) - (b*c^2 - b*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/(((a^2*b - b^3)*c^3 - (a^3 - a*
b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(103) = 206.
time = 0.58, size = 522, normalized size = 4.31

$$\frac{\frac{\left(\sqrt{-c^2+d^2}\,b(c-2d)|c-d|+\sqrt{-c^2+d^2}\,ad|c-d|+\sqrt{-c^2+d^2}\,|-bc+ad||c-d|\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-\frac{2ac-2bd+\sqrt{-4(ac+bc+ad+bd)(ac-bc-ad+bd)+4(ac-bd)^2}}{ac-bc-ad+bd}}}\right)\right)}{(bc-ad)^2(c^2-2cd+d^2)+(c^3-2c^2d+cd^2)a|-bc+ad|-(c^2d-2cd^2+d^3)b|-bc+ad|}+\frac{\left(\sqrt{-a^2+b^2}\,bc|a-b|+\sqrt{-a^2+b^2}\,(a-2b)d|a-b|-\sqrt{-a^2+b^2}\,|-bc+ad||a-b|\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-\frac{2ac-2bd-\sqrt{-4(ac+bc+ad+bd)(ac-bc-ad+bd)+4(ac-bd)^2}}{ac-bc-ad+bd}}}\right)\right)}{(a^2-2ab+b^2)(bc-ad)^2-(a^3-2a^2b+ab^2)c|-bc+ad|+(a^2b-2ab^2+b^3)d|-bc+ad|}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((sqrt(-c^2 + d^2)*b*(c - 2*d)*abs(c - d) + sqrt(-c^2 + d^2)*a*d*abs(c - d)
 + sqrt(-c^2 + d^2)*abs(-b*c + a*d)*abs(c - d))*(pi*floor(1/2*(f*x + e)/pi
+ 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d + sqr
t(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2))/(a
```

```
*c - b*c - a*d + b*d))))/((b*c - a*d)^2*(c^2 - 2*c*d + d^2) + (c^3 - 2*c^2*
d + c*d^2)*a*abs(-b*c + a*d) - (c^2*d - 2*c*d^2 + d^3)*b*abs(-b*c + a*d)) +
 (sqrt(-a^2 + b^2)*b*c*abs(a - b) + sqrt(-a^2 + b^2)*(a - 2*b)*d*abs(a - b)
 - sqrt(-a^2 + b^2)*abs(-b*c + a*d)*abs(a - b))*(pi*floor(1/2*(f*x + e)/pi
+ 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d - sqr
t(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2))/(a
*c - b*c - a*d + b*d))))/((a^2 - 2*a*b + b^2)*(b*c - a*d)^2 - (a^3 - 2*a^2*
b + a*b^2)*c*abs(-b*c + a*d) + (a^2*b - 2*a*b^2 + b^3)*d*abs(-b*c + a*d)))/
f
```

Mupad [B]
time = 4.36, size = 2665, normalized size = 22.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))*(c + d/cos(e + f*x))),x)
```

```
[Out] (b*c^2*atan((b^5*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^5*d^2*tan(
e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i + b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2
)^(3/2)*2i + b^5*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a^2*b^3*c^2*
tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^3*b^2*c^2*tan(e/2 + (f*x)/2)*(a
^2 - b^2)^(1/2)*1i - a^2*b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*3i +
a^3*b^2*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - b^3*c*d*tan(e/2 + (f*
x)/2)*(a^2 - b^2)^(3/2)*2i - b^5*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2
i + a*b^2*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i + a*b^4*c^2*tan(e/2 +
 (f*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(
1/2)*1i + a^2*b^3*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a^3*b^2*c*d
*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a*b^2*c*d*tan(e/2 + (f*x)/2)*(a^
2 - b^2)^(3/2)*2i - a*b^4*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i)/(a^6
*d^2 - b^6*c^2 + 2*a^2*b^4*c^2 - a^4*b^2*c^2 + a^2*b^4*d^2 - 2*a^4*b^2*d^2)
)*(a^2 - b^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3
*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) - (b*d^2*atan((b^5*c^2*tan
(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^5*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^
2)^(1/2)*1i + b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i + b^5*d^2*tan
(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a^2*b^3*c^2*tan(e/2 + (f*x)/2)*(a^2
 - b^2)^(1/2)*1i - a^3*b^2*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^2
*b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*d^2*tan(e/2 + (f
*x)/2)*(a^2 - b^2)^(1/2)*1i - b^3*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*
2i - b^5*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a*b^2*c^2*tan(e/2 +
(f*x)/2)*(a^2 - b^2)^(3/2)*2i + a*b^4*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1
/2)*1i + a^4*b*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i + a^2*b^3*c*d*ta
n(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a^3*b^2*c*d*tan(e/2 + (f*x)/2)*(a^2
 - b^2)^(1/2)*2i - a*b^2*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i - a*b^
4*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i)/(a^6*d^2 - b^6*c^2 + 2*a^2*b
```

^4*c^2 - a^4*b^2*c^2 + a^2*b^4*d^2 - 2*a^4*b^2*d^2))*(a^2 - b^2)^(1/2)*2i)/
(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b
^2*c^2*d - a^2*b*c*d^2)) + (a^2*d*atan((a^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d
^2)^(1/2)*1i - b^2*c^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*d^3*ta
n(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i + b^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d
^2)^(1/2)*2i - a^2*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a^2*c^
3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - b^2*c^2*d^3*tan(e/2 + (f*x)
/2)*(c^2 - d^2)^(1/2)*3i + b^2*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)
*1i - a*b*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*d^5*tan(e/2 + (
f*x)/2)*(c^2 - d^2)^(1/2)*2i + a^2*c*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/
2)*2i + a^2*c*d^4*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*c^4*d*tan(e
/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + a*b*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 -
d^2)^(1/2)*2i + a*b*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a*b*c
*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*c*d^4*tan(e/2 + (f*x)/2)
*(c^2 - d^2)^(1/2)*2i)/(a^2*d^6 - b^2*c^6 - 2*a^2*c^2*d^4 + a^2*c^4*d^2 - b
^2*c^2*d^4 + 2*b^2*c^4*d^2))*(c^2 - d^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 +
a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2))
 - (b^2*d*atan((a^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - b^2*c^5*t
an(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*d^3*tan(e/2 + (f*x)/2)*(c^2 -
d^2)^(3/2)*2i + b^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a^2*c^2*d
^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a^2*c^3*d^2*tan(e/2 + (f*x)/2)
*(c^2 - d^2)^(1/2)*1i - b^2*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*3i
 + b^2*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a*b*d^3*tan(e/2 +
(f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2
)*2i + a^2*c*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i + a^2*c*d^4*tan(e/
2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*c^4*d*tan(e/2 + (f*x)/2)*(c^2 - d^2
)^(1/2)*1i + a*b*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i + a*b*c^3*
d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a*b*c*d^2*tan(e/2 + (f*x)/2)*
(c^2 - d^2)^(3/2)*2i - a*b*c*d^4*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i)/(
a^2*d^6 - b^2*c^6 - 2*a^2*c^2*d^4 + a^2*c^4*d^2 - b^2*c^2*d^4 + 2*b^2*c^4*d
^2))*(c^2 - d^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 -
a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2))

3.257 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal. Leaf size=187

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)^2f}-\frac{2d(2bc^2-acd-bd^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2f}+\frac{d^2\sin}{(bc-ad)(c^2-d^2)}$$

```
[Out] -2*d*(-a*c*d+2*b*c^2-b*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1
/2))/(c-d)^(3/2)/(c+d)^(3/2)/(-a*d+b*c)^2/f+d^2*sin(f*x+e)/(-a*d+b*c)/(c^2-
d^2)/f/(d+c*cos(f*x+e))+2*b^2*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^
(1/2))/(-a*d+b*c)^2/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4073, 3135, 3080, 2738, 214}

$$\frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)^2}+\frac{d^2\sin(e+fx)}{f(c^2-d^2)(bc-ad)(c\cos(e+fx)+d)}-\frac{2d(-acd+2bc^2-bd^2)\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(b*c - a*d)^2*f) - (2*d*(2*b*c^2 - a*c*d - b*d^2)*ArcTanh[(Sqrt[c
 - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^(3/2)*(c + d)^(3/2)*(b*c - a
*d)^2*f) + (d^2*Sin[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*(d + c*Cos[e + f*x
]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
```

```
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))^2}\,dx$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{\int \frac{-bcd-\left(acd-b\left(c^2-\right.\right.}{(b+a\cos(e+fx))(}}{(bc-ad)}$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{b^2\int \frac{1}{b+a\cos(e+fx)}}{(bc-ad)^2}$$

$$= \frac{d^2\sin(e+fx)}{(bc-ad)\left(c^2-d^2\right)f(d+c\cos(e+fx))} + \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{}{a}\right.}{}$$

$$= \frac{2b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\,(bc-ad)^2f} - \frac{2d\left(2bc^2-acd-bd^2\right)}{(c-d)^{3/2}(}$$

Mathematica [A]
time = 0.71, size = 229, normalized size = 1.22

$$\frac{-2b^2(c^2-d^2)^{3/2}\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(d+c\cos(e+fx))-\sqrt{a^2-b^2}\,d\left(-2(2bc^2-acd-bd^2)\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))+d(-bc+ad)\sqrt{c^2-d^2}\sin(e+fx)\right)}{\sqrt{a^2-b^2}\,(c-d)(c+d)(bc-ad)^2\sqrt{c^2-d^2}\,f(d+c\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (-2*b^2*(c^2 - d^2)^(3/2)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^
2]]*(d + c*Cos[e + f*x]) - Sqrt[a^2 - b^2]*d*(-2*(2*b*c^2 - a*c*d - b*d^2)*
ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x]) +
 d*(-(b*c) + a*d)*Sqrt[c^2 - d^2]*Sin[e + f*x]))/(Sqrt[a^2 - b^2]*(c - d)*(
c + d)*(b*c - a*d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x]))
```

Maple [A]
time = 1.18, size = 208, normalized size = 1.11

method	result
derivativedivides	$\frac{\frac{2b^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(ad-bc)^2\sqrt{(a+b)(a-b)}}-\frac{2d\left(-\frac{d(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c^2-d^2)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{\left(acd-2b\,c^2+b\,d^2\right)\operatorname{arctan}}{(c+d)(c-d)\sqrt{}}\right.}{(ad-bc)^2}}{f}$
default	$\frac{\frac{2b^2\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(ad-bc)^2\sqrt{(a+b)(a-b)}}-\frac{2d\left(-\frac{d(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c^2-d^2)\left(c\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-c-d\right)}-\frac{\left(acd-2b\,c^2+b\,d^2\right)\operatorname{arctan}}{(c+d)(c-d)\sqrt{}}\right.}{(ad-bc)^2}}{f}$
risch	$\frac{2id^2\left(d\,e^{i(fx+e)}+c\right)}{c(c^2-d^2)(-ad+bc)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{b^2\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(ad-bc)^2f}-\frac{b^2\ln\left(e^{i(fx+e)}-\frac{ia^2}{}\right.}{\sqrt{a^2-b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b^2/(a*d-b*c)^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)
/((a+b)*(a-b))^(1/2))-2*d/(a*d-b*c)^2*(-d*(a*d-b*c)/(c^2-d^2)*tan(1/2*f*x+1
/2*e)/(c*tan(1/2*f*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)^2-c-d)-(a*c*d-2*b*c^2+b*
d^2)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d
)*(c-d))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. 2(169) = 338.
time = 98.58, size = 2863, normalized size = 15.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2
*c*d^4)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^
2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2
*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*b -
b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a^2*b - b^3)*c
^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x + e))*sqrt(c^2
- d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2
- d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2
 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^
2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))/(((a^2*b^2 -
 b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4
*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a^3*b - a
*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*b^2 - b^4)*c^
6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^
3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3
)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), 1/2*(2*(b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*
d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4)*cos(f*x + e))*sqrt(-a^2 + b^2)*
arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) +
 (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a
^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x +
e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2
```

```
- 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*c
os(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^
3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))
/(((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4
)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 -
 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*
b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^
4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(
a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), -1/2*(2*(2*(a^2*b - b^3)*c^
2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a^2*b - b^3)*c^3*d -
(a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)
*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e)))
- (b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d
^4)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*c
os(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2
 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - 2*((a^2*b - b^3)
*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)
*sin(f*x + e))/(((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a
^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 +
b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x
 + e) + ((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b
^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)
*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), ((b^2*c^4*d -
 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4)*cos(f*x +
e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 -
b^2)*sin(f*x + e))) - (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2
*b - b^3)*d^4 + (2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b
^3)*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f
*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + ((a^2*b - b^3)*c^3*d^2 - (a^3 -
a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))/(((
a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^
5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(
a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*b^2
- b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^
3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*
b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))\,(c+d\sec(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2), x)
```

Giac [A]
time = 0.54, size = 331, normalized size = 1.77

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{-a^2+b^2}}+\frac{d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(bc^3-ac^2d-bcd^2+ad^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)}-\frac{(2bc^2d-acd^2-bd^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(b^2c^4-2abc^3d+a^2c^2d^2-b^2c^2d^2+2abcd^3-a^2d^4)\sqrt{-c^2+d^2}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*
x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*b^2/((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)) + d^2*tan(1/2*f*x + 1/2*e)/((b*c^3 - a*
c^2*d - b*c*d^2 + a*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)
^2 - c - d)) - (2*b*c^2*d - a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1
/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e
))/sqrt(-c^2 + d^2)))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 - b^2*c^2*d^2 +
 2*a*b*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)))/f
```

Mupad [B]
time = 15.42, size = 2500, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*tan(e/2 + (f*x)/2))/(f*(c + d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)
)*(a*d^2 + b*c^2 - a*c*d - b*c*d)) - (d*atan(((d*((32*tan(e/2 + (f*x)/2)*(b
^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d +
3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 +
 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*
a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^
2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 -
 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2
*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a
^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*
c^2*d^3 + 2*a*b*c^3*d^2) + (d*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7
*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*
d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2
 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 1
1*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a
^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*
b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^
```

2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2
 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^
4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*
a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^
3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^
2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*
c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*
d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b
^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*
c^5*d - 3*a^2*b*c*d^5) + (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/
2)*(b*d^2 - 2*b*c^2 + a*c*d)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2
*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d
^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7
*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 +
2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*
b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3
*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*
b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*
c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*
a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d
^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b
^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 -
 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c
^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a
^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^
3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^
5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c
^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^
8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3
*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 -
6*a*b*c^5*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^
2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6
 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^
5 - 6*a*b*c^5*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*1
i)/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c
^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*
c^3*d^5 - 6*a*b*c^5*d^3) + (d*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6
 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3
*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13
*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a
^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*
a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3
 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2
*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^
2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3

```
*d^2) - (d*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*
d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 +
 a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6
- 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8
*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^...
```

3.258 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=379

$$\frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f}+\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\tanh^{-1}(\sin(e+fx))}{b^5f}+\frac{2(bc-}{}$$

```
[Out] 1/2*d^4*(-2*a*d+5*b*c)*arctanh(sin(f*x+e))/b^3/f+d^2*(-4*a^3*d^3+15*a^2*b*c
*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*arctanh(sin(f*x+e))/b^5/f+2*(-a*d+b*c)^5*ar
ctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/b^3/(a+b)^(
3/2)/f-(-a*d+b*c)^5*sin(f*x+e)/b^4/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c
)^4*(4*a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/b^5/f
/(a-b)^(1/2)/(a+b)^(1/2)+d^5*tan(f*x+e)/b^2/f+d^3*(3*a^2*d^2-10*a*b*c*d+10*
b^2*c^2)*tan(f*x+e)/b^4/f+1/2*d^4*(-2*a*d+5*b*c)*sec(f*x+e)*tan(f*x+e)/b^3/
f+1/3*d^5*tan(f*x+e)^3/b^2/f
```

Rubi [A]
time = 0.48, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {4073, 3031, 2743, 12, 2738, 214, 3855, 3852, 8, 3853}

$$\frac{d^3(3a^2d^2-10abcd+10b^2c^2)\tan(e+fx)}{b^4f}-\frac{(bc-ad)^5\sin(e+fx)}{b^4f(a^2-b^2)(a\cos(e+fx)+b)}+\frac{d^2(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)\tanh^{-1}(\sin(e+fx))}{b^5f}+\frac{2(bc-ad)^4(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^5f\sqrt{a-b}\sqrt{a+b}}+\frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f}+\frac{d^4(5bc-2ad)\tan(e+fx)\sec(e+fx)}{2b^3f}+\frac{2(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^3f(a-b)^{3/2}(a+b)^{3/2}}+\frac{d^5\tan^3(e+fx)}{3b^2f}+\frac{d^5\tan(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^4*(5*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(2*b^3*f) + (d^2*(10*b^3*c^3 -
20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*ArcTanh[Sin[e + f*x]])/(b^5*f)
 + (2*(b*c - a*d)^5*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a
*(a - b)^(3/2)*b^3*(a + b)^(3/2)*f) + (2*(b*c - a*d)^4*(b*c + 4*a*d)*ArcTan
h[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^5*Sqrt[a +
b]*f) - ((b*c - a*d)^5*Sin[e + f*x])/(b^4*(a^2 - b^2)*f*(b + a*Cos[e + f*x]
)) + (d^5*Tan[e + f*x])/(b^2*f) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2
)*Tan[e + f*x])/(b^4*f) + (d^4*(5*b*c - 2*a*d)*Sec[e + f*x]*Tan[e + f*x])/(
2*b^3*f) + (d^5*Tan[e + f*x]^3)/(3*b^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^5\sec^4(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(\frac{(-bc+ad)^5}{ab^4(b+a\cos(e+fx))^2} + \frac{(-bc+ad)^4(bc+4ad)}{ab^5(b+a\cos(e+fx))} + \frac{d^2(10b^3c^3 -$$

$$= \frac{d^5\int\sec^4(e+fx)\,dx}{b^2} + \frac{(d^4(5bc-2ad))\int\sec^3(e+fx)\,dx}{b^3} - \frac{(bc-a$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\tanh^{-1}(\sin(e+fx))}{b^5f} - \frac{}{b^{}}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2b}{}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2b}{}$$

$$= \frac{d^4(5bc-2ad)\tanh^{-1}(\sin(e+fx))}{2b^3f} + \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2b}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1137 vs. $2(379) = 758$.
time = 6.53, size = 1137, normalized size = 3.00

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (-2*(b*c - a*d)^4*(-(a*b*c) - 4*a^2*d + 5*b^2*d)*ArcTanh[((-a + b)*Tan[(e +
 f*x)/2])/Sqrt[a^2 - b^2]]*Cos[e + f*x]^3*(b + a*Cos[e + f*x])^2*(c + d*Sec
[e + f*x])^5)/(b^5*Sqrt[a^2 - b^2]*(-a^2 + b^2)*f*(d + c*Cos[e + f*x])^5*(a
```

```
 + b*Sec[e + f*x])^2) + ((-20*b^3*c^3*d^2 + 40*a*b^2*c^2*d^3 - 30*a^2*b*c*d
^4 - 5*b^3*c*d^4 + 8*a^3*d^5 + 2*a*b^2*d^5)*Cos[e + f*x]^3*(b + a*Cos[e + f
*x])^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(c + d*Sec[e + f*x])^5)/(2*
b^5*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f*x])^2) + ((20*b^3*c^3*d^2 - 4
0*a*b^2*c^2*d^3 + 30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*C
os[e + f*x]^3*(b + a*Cos[e + f*x])^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]]*(c + d*Sec[e + f*x])^5)/(2*b^5*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f
*x])^2) + ((b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^5*(-60*a^2*b^3*c^2*d^3
*Sin[e + f*x] + 60*b^5*c^2*d^3*Sin[e + f*x] + 45*a^3*b^2*c*d^4*Sin[e + f*x]
 - 45*a*b^4*c*d^4*Sin[e + f*x] - 12*a^4*b*d^5*Sin[e + f*x] + 12*b^5*d^5*Sin
[e + f*x] + 6*b^5*c^5*Sin[2*(e + f*x)] - 30*a*b^4*c^4*d*Sin[2*(e + f*x)] +
60*a^2*b^3*c^3*d^2*Sin[2*(e + f*x)] - 120*a^3*b^2*c^2*d^3*Sin[2*(e + f*x)]
+ 60*a*b^4*c^2*d^3*Sin[2*(e + f*x)] + 90*a^4*b*c*d^4*Sin[2*(e + f*x)] - 90*
a^2*b^3*c*d^4*Sin[2*(e + f*x)] + 30*b^5*c*d^4*Sin[2*(e + f*x)] - 24*a^5*d^5
*Sin[2*(e + f*x)] + 22*a^3*b^2*d^5*Sin[2*(e + f*x)] - 4*a*b^4*d^5*Sin[2*(e
+ f*x)] - 60*a^2*b^3*c^2*d^3*Sin[3*(e + f*x)] + 60*b^5*c^2*d^3*Sin[3*(e + f
*x)] + 45*a^3*b^2*c*d^4*Sin[3*(e + f*x)] - 45*a*b^4*c*d^4*Sin[3*(e + f*x)]
- 12*a^4*b*d^5*Sin[3*(e + f*x)] + 8*a^2*b^3*d^5*Sin[3*(e + f*x)] + 4*b^5*d^
5*Sin[3*(e + f*x)] + 3*b^5*c^5*Sin[4*(e + f*x)] - 15*a*b^4*c^4*d*Sin[4*(e +
 f*x)] + 30*a^2*b^3*c^3*d^2*Sin[4*(e + f*x)] - 60*a^3*b^2*c^2*d^3*Sin[4*(e
+ f*x)] + 30*a*b^4*c^2*d^3*Sin[4*(e + f*x)] + 45*a^4*b*c*d^4*Sin[4*(e + f*x
)] - 30*a^2*b^3*c*d^4*Sin[4*(e + f*x)] - 12*a^5*d^5*Sin[4*(e + f*x)] + 7*a^
3*b^2*d^5*Sin[4*(e + f*x)] + 2*a*b^4*d^5*Sin[4*(e + f*x)]))/(24*b^4*(-a^2 +
 b^2)*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f*x])^2)
```

Maple [A]
time = 1.46, size = 689, normalized size = 1.82

method	result
derivativedivides	$-\frac{2\left(\frac{b\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a-b\right)}-\frac{\left(4a^6d^5-15a^5bc\,d^4+20a^4b^2c^2d^3-5a^4b^2d^5-10a\right.}{}\right.}{b^5}$
default	$-\frac{2\left(\frac{b\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a-b\right)}-\frac{\left(4a^6d^5-15a^5bc\,d^4+20a^4b^2c^2d^3-5a^4b^2d^5-10a\right.}{}\right.}{b^5}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-2/b^5*(b*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2
+5*a*b^4*c^4*d-b^5*c^5)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(a*tan(1/2*f*x+1/2*e)^
2-b*tan(1/2*f*x+1/2*e)^2-a-b)-(4*a^6*d^5-15*a^5*b*c*d^4+20*a^4*b^2*c^2*d^3-
5*a^4*b^2*d^5-10*a^3*b^3*c^3*d^2+20*a^3*b^3*c*d^4-30*a^2*b^4*c^2*d^3+a*b^5*
c^5+20*a*b^5*c^3*d^2-5*b^6*c^4*d)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((
a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2)))-1/3*d^5/b^2/(tan(1/2*f*x+1/2*
e)+1)^3-1/2*d^2*(8*a^3*d^3-30*a^2*b*c*d^2+40*a*b^2*c^2*d+2*a*b^2*d^3-20*b^3
*c^3-5*b^3*c*d^2)/b^5*ln(tan(1/2*f*x+1/2*e)+1)-1/2*d^3*(6*a^2*d^2-20*a*b*c*
d+2*a*b*d^2+20*b^2*c^2-5*b^2*c*d+2*b^2*d^2)/b^4/(tan(1/2*f*x+1/2*e)+1)+1/2*
d^4*(2*a*d-5*b*c+b*d)/b^3/(tan(1/2*f*x+1/2*e)+1)^2-1/3*d^5/b^2/(tan(1/2*f*x
+1/2*e)-1)^3+1/2*d^2*(8*a^3*d^3-30*a^2*b*c*d^2+40*a*b^2*c^2*d+2*a*b^2*d^3-2
0*b^3*c^3-5*b^3*c*d^2)/b^5*ln(tan(1/2*f*x+1/2*e)-1)-1/2*d^3*(6*a^2*d^2-20*a
*b*c*d+2*a*b*d^2+20*b^2*c^2-5*b^2*c*d+2*b^2*d^2)/b^4/(tan(1/2*f*x+1/2*e)-1)
-1/2*d^4*(2*a*d-5*b*c+b*d)/b^3/(tan(1/2*f*x+1/2*e)-1)^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d \sec(e + fx))^5 \sec(e + fx)}{(a + b \sec(e + fx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**5*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(355) = 710.
time = 0.62, size = 857, normalized size = 2.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*(a*b^5*c^5 - 5*b^6*c^4*d - 10*a^3*b^3*c^3*d^2 + 20*a*b^5*c^3*d^2 +
 20*a^4*b^2*c^2*d^3 - 30*a^2*b^4*c^2*d^3 - 15*a^5*b*c*d^4 + 20*a^3*b^3*c*d^
4 + 4*a^6*d^5 - 5*a^4*b^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 +
 b^2)))/((a^2*b^5 - b^7)*sqrt(-a^2 + b^2)) - 12*(b^5*c^5*tan(1/2*f*x + 1/2*
e) - 5*a*b^4*c^4*d*tan(1/2*f*x + 1/2*e) + 10*a^2*b^3*c^3*d^2*tan(1/2*f*x +
1/2*e) - 10*a^3*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 5*a^4*b*c*d^4*tan(1/2*f*
x + 1/2*e) - a^5*d^5*tan(1/2*f*x + 1/2*e))/((a^2*b^4 - b^6)*(a*tan(1/2*f*x
+ 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)) - 3*(20*b^3*c^3*d^2 - 40*a*
b^2*c^2*d^3 + 30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*log(a
bs(tan(1/2*f*x + 1/2*e) + 1))/b^5 + 3*(20*b^3*c^3*d^2 - 40*a*b^2*c^2*d^3 +
30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*log(abs(tan(1/2*f*x
 + 1/2*e) - 1))/b^5 + 2*(60*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 60*a*b*c*d
^4*tan(1/2*f*x + 1/2*e)^5 - 15*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 18*a^2*d^
5*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*d^5*tan(1/2*f*x + 1/2*e)^5 + 6*b^2*d^5*tan
(1/2*f*x + 1/2*e)^5 - 120*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 120*a*b*c*d^
4*tan(1/2*f*x + 1/2*e)^3 - 36*a^2*d^5*tan(1/2*f*x + 1/2*e)^3 - 4*b^2*d^5*ta
n(1/2*f*x + 1/2*e)^3 + 60*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 60*a*b*c*d^4*t
an(1/2*f*x + 1/2*e) + 15*b^2*c*d^4*tan(1/2*f*x + 1/2*e) + 18*a^2*d^5*tan(1/
2*f*x + 1/2*e) - 6*a*b*d^5*tan(1/2*f*x + 1/2*e) + 6*b^2*d^5*tan(1/2*f*x + 1
/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*b^4))/f
```

Mupad [B]
time = 16.95, size = 2500, normalized size = 6.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

[Out] (atan(((((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2 + (f*x)/2)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*1i)/b^5 + (((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d

```
^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 +
 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*
a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^
9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d
^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 8
0*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5
*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7
*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 +
 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*
a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^
4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*
d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a
*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (((8
*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*
c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 -
16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b
^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 2
0*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^
4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^
16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*
d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80
*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 +...
```

3.259 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=297

$$\frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\tanh^{-1}(\sin(e+fx))}{b^4f}+\frac{2(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}}{\sqrt{a+b}}\right.}{a(a-b)^{3/2}b^2(a+b)^{3/2}f}$$

```
[Out] 1/2*d^4*arctanh(sin(f*x+e))/b^2/f+d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*arcta
nh(sin(f*x+e))/b^4/f+2*(-a*d+b*c)^4*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/
(a+b)^(1/2))/a/(a-b)^(3/2)/b^2/(a+b)^(3/2)/f-(-a*d+b*c)^4*sin(f*x+e)/b^3/(a
^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c)^3*(3*a*d+b*c)*arctanh((a-b)^(1/2)*t
an(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/b^4/f/(a-b)^(1/2)/(a+b)^(1/2)+2*d^3*(-a*d+
2*b*c)*tan(f*x+e)/b^3/f+1/2*d^4*sec(f*x+e)*tan(f*x+e)/b^2/f
```

Rubi [A]
time = 0.38, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {4073, 3031, 2743, 12, 2738, 214, 3855, 3852, 8, 3853}

$$\frac{d^2(3a^2d^2-8abcd+6b^2c^2)\tanh^{-1}(\sin(e+fx))}{b^4f}-\frac{(bc-ad)^4\sin(e+fx)}{b^3f\,(a^2-b^2)\,(a\cos(e+fx)+b)}+\frac{2(bc-ad)^3(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(e+fx))}{\sqrt{a+b}}\right)}{ab^4f\sqrt{a-b}\,\sqrt{a+b}}+\frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f}+\frac{2(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(e+fx))}{\sqrt{a+b}}\right)}{ab^2f(a-b)^{3/2}(a+b)^{3/2}}+\frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f}+\frac{d^4\tan(e+fx)\sec(e+fx)}{2b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^4*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2
*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*d)^4*ArcTanh[(Sqrt[a - b
]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b^2*(a + b)^(3/2)*f) + (
2*(b*c - a*d)^3*(b*c + 3*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a
 + b]])/(a*Sqrt[a - b]*b^4*Sqrt[a + b]*f) - ((b*c - a*d)^4*Sin[e + f*x])/(b
^3*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (2*d^3*(2*b*c - a*d)*Tan[e + f*x])
/(b^3*f) + (d^4*Sec[e + f*x]*Tan[e + f*x])/(2*b^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^4\sec^3(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$
$$= \int \left(-\frac{(-bc+ad)^4}{ab^3(b+a\cos(e+fx))^2} - \frac{(-bc+ad)^3(bc+3ad)}{ab^4(b+a\cos(e+fx))} + \frac{d^2(6b^2c^2$$
$$= \frac{d^4\int \sec^3(e+fx)\,dx}{b^2} - \frac{(bc-ad)^4\int \frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab^3} + \frac{(2d^3(2bc-$$
$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\tanh^{-1}(\sin(e+fx))}{b^4f} - \frac{(bc-ad)^4\,\mathrm{si}}{b^3(a^2-b^2)f(b+}$$
$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\tanh^{-1}(\sin(e+}{b^4f}$$
$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\tanh^{-1}(\sin(e+}{b^4f}$$
$$= \frac{d^4\tanh^{-1}(\sin(e+fx))}{2b^2f} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\tanh^{-1}(\sin(e+}{b^4f}$$

Mathematica [A]
time = 4.01, size = 511, normalized size = 1.72

$$\frac{\cos^2(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^4\left(\frac{8(-bc+ad)^3(abc+3a^2d-4b^2d)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(e+fx))}{(a^2-b^2)^{3/2}} - 2d^2(-16abcd+6a^2d^2+b^2(12c^2+d^2))(b+a\cos(e+fx))\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)-\sin\left(\tfrac{1}{2}(e+fx)\right)\right) + 2d^2(-16abcd+6a^2d^2+b^2(12c^2+d^2))(b+a\cos(e+fx))\log\left(\cos\left(\tfrac{1}{2}(e+fx)\right)+\sin\left(\tfrac{1}{2}(e+fx)\right)\right) + \frac{b^2d^4(b+a\cos(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} + \frac{8bd^3(2bc-ad)(b+a\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} - \frac{b^2d^4(b+a\cos(e+fx))}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} + \frac{8bd^3(2bc-ad)(b+a\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)} + \frac{4b(bc-ad)^4\sin(e+fx)}{(-a+b)(a+b)}\right)}{4b^4f(d+c\cos(e+fx))^4(a+b\sec(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((8*(-(b*c) + a
*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[
a^2 - b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) - 2*d^2*(-16*a*b*c*d +
6*a^2*d^2 + b^2*(12*c^2 + d^2))*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] + 2*d^2*(-16*a*b*c*d + 6*a^2*d^2 + b^2*(12*c^2 + d^2))*(
b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^4*(b
+ a*Cos[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (8*b*d^3*(2*b*
c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - (b^2*d^4*(b + a*Cos[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*
```

```
x)/2])^2 + (8*b*d^3*(2*b*c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(C
os[(e + f*x)/2] + Sin[(e + f*x)/2]) + (4*b*(b*c - a*d)^4*Sin[e + f*x])/((-a
 + b)*(a + b))))/(4*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e + f*x])^2)
```

Maple [A]
time = 1.09, size = 465, normalized size = 1.57

method	result
derivativedivides	$\frac{\frac{d^4}{2b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{d^2\left(6a^2d^2-16abdc+12b^2c^2+b^2d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2b^4}+\frac{d^3(4ad-8bc+db)}{2b^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{d^4}{2b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{}$
default	$\frac{\frac{d^4}{2b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{d^2\left(6a^2d^2-16abdc+12b^2c^2+b^2d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2b^4}+\frac{d^3(4ad-8bc+db)}{2b^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{d^4}{2b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(1/2*d^4/b^2/(tan(1/2*f*x+1/2*e)-1)^2-1/2*d^2*(6*a^2*d^2-16*a*b*c*d+12*
b^2*c^2+b^2*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)-1)+1/2*d^3*(4*a*d-8*b*c+b*d)/b^3
/(tan(1/2*f*x+1/2*e)-1)-1/2*d^4/b^2/(tan(1/2*f*x+1/2*e)+1)^2+1/2*d^2*(6*a^2
*d^2-16*a*b*c*d+12*b^2*c^2+b^2*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)+1)+1/2*d^3*(4
*a*d-8*b*c+b*d)/b^3/(tan(1/2*f*x+1/2*e)+1)+2/b^4*(b*(a^4*d^4-4*a^3*b*c*d^3+
6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(a*ta
n(1/2*f*x+1/2*e)^2-b*tan(1/2*f*x+1/2*e)^2-a-b)-(3*a^5*d^4-8*a^4*b*c*d^3+6*a
^3*b^2*c^2*d^2-4*a^3*b^2*d^4+12*a^2*b^3*c*d^3-a*b^4*c^4-12*a*b^4*c^2*d^2+4*
b^5*c^3*d)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)
/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sec{(e + fx)})^4 \sec{(e + fx)}}{(a + b\sec{(e + fx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(275) = 550.
time = 0.55, size = 551, normalized size = 1.86

$$-\frac{\frac{4\,(ab^4c^4-4\,b^5c^3d-6\,a^3b^2c^2d^2+12\,ab^4c^2d^2+8\,a^4bcd^3-12\,a^2b^3cd^3-3\,a^5d^4+4\,a^3b^2d^4)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^4-b^6)\sqrt{-a^2+b^2}}-\frac{4\,(b^4c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-4\,ab^3c^3d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+6\,a^2b^2c^2d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-4\,a^3bcd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^4d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right))}{(a^2b^3-b^5)\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-a-b\right)}-\frac{(12\,b^2c^2d^2-16\,abcd^3+6\,a^2d^4+b^2d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^4}+\frac{(12\,b^2c^2d^2-16\,abcd^3+6\,a^2d^4+b^2d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^4}+\frac{2\left(8\,bcd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,ad^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-bd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-8\,bcd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+4\,ad^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-bd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2b^3}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(a*b^4*c^4 - 4*b^5*c^3*d - 6*a^3*b^2*c^2*d^2 + 12*a*b^4*c^2*d^2 + 8
*a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 - 3*a^5*d^4 + 4*a^3*b^2*d^4)*(pi*floor(1/2*
(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan
(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) -
4*(b^4*c^4*tan(1/2*f*x + 1/2*e) - 4*a*b^3*c^3*d*tan(1/2*f*x + 1/2*e) + 6*a^
2*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) - 4*a^3*b*c*d^3*tan(1/2*f*x + 1/2*e) + a
^4*d^4*tan(1/2*f*x + 1/2*e))/((a^2*b^3 - b^5)*(a*tan(1/2*f*x + 1/2*e)^2 - b
*tan(1/2*f*x + 1/2*e)^2 - a - b)) - (12*b^2*c^2*d^2 - 16*a*b*c*d^3 + 6*a^2*
d^4 + b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^4 + (12*b^2*c^2*d^2 - 1
6*a*b*c*d^3 + 6*a^2*d^4 + b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/b^4 +
 2*(8*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 4*a*d^4*tan(1/2*f*x + 1/2*e)^3 - b*d
```

```
^4*tan(1/2*f*x + 1/2*e)^3 - 8*b*c*d^3*tan(1/2*f*x + 1/2*e) + 4*a*d^4*tan(1/
2*f*x + 1/2*e) - b*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^
2*b^3))/f
```

Mupad [B]
time = 14.37, size = 2500, normalized size = 8.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (atan((((((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 +
 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^
4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2
*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b
^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*
a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2
*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*
b^9) - (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c
*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^
6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*c^2*d^2) +
 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*
d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*
b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^
8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2
- 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*
d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^
5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a
^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6
*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96
*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6
*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5
 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^
6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3
*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*
d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d
^4 - 8*a*b*c*d^3)*1i)/b^4 - (((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3
*d + 4*a^2*b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16
*a^3*b^12*d^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*
b^8*d^4 + 24*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*
b^13*c^3*d + 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16
*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*
d^2 - 24*a^5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b
^12 - a^2*b^10 - a^3*b^9) + (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2)
```

+ 3*a^2*d^4 - 8*a*b*c*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 + (8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3)*1i)/b^4)/((16*(108*a^11*d^12 - 54*a^10*b*d^12 + 4*a^3*b^8*d^12 - 4*a^4*b^7*d^12 + 41*a^5*b^6*d^12 - 9*a^6*b^5*d^12 + 63*a^7*b^4*d^12 + 81*a^8*b^3*d^12 - 216*a^9*b^2*d^12 - 4*b^11*c^3*d^9 - 96*b^11*c^5*d^7 + 32*b^11*c^6*d^6 - 576*b^11*c^7*d^5 + 384*b^11*c^8*d^4 + 12*a*b^10*c^2*d^10 + 4*a*b^10*c^3*d^9 + 417*a*b^10*c^4*d^8 - 96*a*b^10*c^5*d^7 + 3288*a*b^10*c^6*d^6 - 2256*a*b^10*c^7*d^5 + 144*a*b^10*c^8*d^4 - 192*a*b^10*c^9*d^3 - 12*a^2*b^9*c*d^11 + 12*a^3*b^8*c*d^11 - 252*a^4*b^7*c*d^11 + 60*a^5*b^6*c*d^11 - 744*a^6*b^5*c*d^11 - 648*a^7*b^4*c*d^11 + 1872*a^8*b^3*c*d^11 + 432*a^9*b^2*c*d^11 - 12*a^2*b^9*c^2*d^10 - 716*a^2*b^9*c^3*d^9 + 63*a^2*b^9*c^4*d^8 - 7872*a^2*b^9*c^5*d^7 + 5784*a^2*b^9*c^6*d^6 + 192*a^2*b^9*c^7*d^5 + 690*a^2*b^9*c^8*d^4 + 24*a^2*b^9*c^10*d^2 + 606*a^3*b^8*c^2*d^10 + 76*a^3*b^8*c^3*d^9 + 10203*a^3*b^8*c^4*d^8 - 8592*a^3*b^8*c^5*d^7 - 3752*a^3*b^8*c^6*d^6 - 480*a^3*b^8*c^7*d^5 - 144*a^3*b^8*c^8*d^4 - 32*a^3*b^8*c^9*d^3 - 126*a^4*b^7*c^2*d^10 - 7680*a^4*b^7*c^3*d^9 + 8277*a^4*b^7*c^4*d^8 + 11232*a^4*b^7*c^5*d^7 - 1552*a^4*b^7*c^6*d^6 + 384*a^4*b^7*c^7*d^5 - 132*a^4*b^7*c^8*d^4 + 3318*a^5*b^6*c^2*d^10 - 5424*a^5*b^6*c^3*d^9 - 16488*a^5*b^6*c^4*d^8 + 4128*a^5*b^6*c^5*d^7 + 46...

3.260 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=228

$$\frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b(a+b)^{3/2}f}+\frac{2(bc-ad)^2(bc+2ad)\tan\cdots}{a\sqrt{a-b}\,b\cdots}$$

```
[Out] d^2*(-2*a*d+3*b*c)*arctanh(sin(f*x+e))/b^3/f+2*(-a*d+b*c)^3*arctanh((a-b)^(
1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/b/(a+b)^(3/2)/f-(-a*d+b*
c)^3*sin(f*x+e)/b^2/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c)^2*(2*a*d+b*c)
*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/b^3/f/(a-b)^(1/2)/(a
+b)^(1/2)+d^3*tan(f*x+e)/b^2/f
```

Rubi [A]
time = 0.34, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4073, 3031, 2743, 12, 2738, 214, 3855, 3852, 8}

$$-\frac{(bc-ad)^3\sin(e+fx)}{b^2f\left(a^2-b^2\right)(a\cos(e+fx)+b)}+\frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f}+\frac{2(bc-ad)^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^3f\sqrt{a-b}\,\sqrt{a+b}}+\frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{abf(a-b)^{3/2}(a+b)^{3/2}}+\frac{d^3\tan(e+fx)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcT
anh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b*(a + b)
^(3/2)*f) + (2*(b*c - a*d)^2*(b*c + 2*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*
x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^3*Sqrt[a + b]*f) - ((b*c - a*d)^3*Sin
[e + f*x])/(b^2*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (d^3*Tan[e + f*x])/(b
^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx &= \int \frac{(d+c\cos(e+fx))^3\sec^2(e+fx)}{(b+a\cos(e+fx))^2}\,dx \\
&= \int \left(\frac{(-bc+ad)^3}{ab^2(b+a\cos(e+fx))^2} + \frac{(-bc+ad)^2(bc+2ad)}{ab^3(b+a\cos(e+fx))} + \frac{d^2(3bc-}{} \right. \\
&= \frac{d^3\int \sec^2(e+fx)\,dx}{b^2} + \frac{(d^2(3bc-2ad))\int \sec(e+fx)\,dx}{b^3} - \frac{(bc-a}{} \\
&= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f} - \frac{(bc-ad)^3\sin(e+fx)}{b^2\left(a^2-b^2\right)f(b+a\cos(e+} \\
&= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^2(bc+2ad)\tanh^{-}}{a\sqrt{a-b}\,b^3\sqrt{}} \\
&= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^2(bc+2ad)\tanh^{-}}{a\sqrt{a-b}\,b^3\sqrt{}} \\
&= \frac{d^2(3bc-2ad)\tanh^{-1}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{}}\right.}{a(a-b)^{3/2}b(a+b)}
\end{aligned}
$$

Mathematica [A]
time = 1.76, size = 362, normalized size = 1.59

$$
\frac{\cos(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^3\left(-\frac{2(bc-ad)^2(abc+2a^2d-3b^2d)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(e+fx))}{(a^2-b^2)^{3/2}}+d^2(-3bc+2ad)(b+a\cos(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+d^2(3bc-2ad)(b+a\cos(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\frac{bd^3(b+a\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)}+\frac{bd^3(b+a\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}+\frac{b(bc-ad)^3\sin(e+fx)}{(-a+b)(a+b)}\right)}{b^3f(d+c\cos(e+fx))^3(a+b\sec(e+fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((-2*(b*c - a*d)^
2*(a*b*c + 2*a^2*d - 3*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) + d^2*(-3*b*c + 2*a*d)*(b +
 a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + d^2*(3*b*c - 2*
a*d)*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*d^3
*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]) + (b*d^3*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2]) + (b*(b*c - a*d)^3*Sin[e + f*x])/((-a + b)*(a + b))))/(b^3*f*
(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^2)
```

Maple [A]
time = 0.78, size = 314, normalized size = 1.38

method	result
derivativedivides	$\frac{-\frac{d^3}{b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{d^2(2ad-3bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^3}-\frac{d^3}{b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d^2(2ad-3bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^3}-\frac{2\left(\frac{b\left(a^3d^3\cdots\right.}{\left(a^2-b^2\right)\left(\cdots\right.}\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{d^3}{b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{d^2(2ad-3bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^3}-\frac{d^3}{b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d^2(2ad-3bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^3}-\frac{2\left(\frac{b\left(a^3d^3\cdots\right.}{\left(a^2-b^2\right)\left(\cdots\right.}\cdots\right.}{\cdots}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-d^3/b^2/(tan(1/2*f*x+1/2*e)-1)+d^2*(2*a*d-3*b*c)/b^3*ln(tan(1/2*f*x+1
/2*e)-1)-d^3/b^2/(tan(1/2*f*x+1/2*e)+1)-d^2*(2*a*d-3*b*c)/b^3*ln(tan(1/2*f*
x+1/2*e)+1)-2/b^3*(b*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a^2-b^2
)*tan(1/2*f*x+1/2*e)/(a*tan(1/2*f*x+1/2*e)^2-b*tan(1/2*f*x+1/2*e)^2-a-b)-(2
*a^4*d^3-3*a^3*b*c*d^2-3*a^2*b^2*d^3+a*b^3*c^3+6*a*b^3*c*d^2-3*b^4*c^2*d)/(
a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b
))^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(216) = 432.
time = 106.48, size = 1358, normalized size = 5.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5
- 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 -
2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(
b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b
*cos(f*x + e) + b^2)) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*
a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d
^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1)
 - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^
3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3
*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a^4*b^2 - 2*a
^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*c^2*d + 3*(a
^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*
sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2 + (a^4*b^4 -
2*a^2*b^6 + b^8)*f*cos(f*x + e)), 1/2*(2*((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*
(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5 - 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^
4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d
^3)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
 + a)/((a^2 - b^2)*sin(f*x + e))) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 -
 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^
4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f
*x + e) + 1) - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 +
 a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a
^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a
^4*b^2 - 2*a^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*
c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*co
s(f*x + e))*sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2 +
 (a^4*b^4 - 2*a^2*b^6 + b^8)*f*cos(f*x + e))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sec(e+fx))^3\sec(e+fx)}{(a+b\sec(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(210) = 420.

time = 0.58, size = 539, normalized size = 2.36

$$-\frac{\frac{2\,(ab^3c^3-3\,b^4c^2d-3\,a^3bcd^2+6\,ab^3cd^2+2\,a^4d^3-3\,a^2b^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{-a^2+b^2}}-\frac{2\left(b^3c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,ab^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+3\,a^2bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,a^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^2bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+ab^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-b^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-b^3c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,ab^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,a^2bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,a^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+a^2bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-ab^2d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-b^3d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a+b\right)(a^2b^2-b^4)}-\frac{(3\,bcd^2-2\,ad^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^3}+\frac{(3\,bcd^2-2\,ad^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(a*b^3*c^3 - 3*b^4*c^2*d - 3*a^3*b*c*d^2 + 6*a*b^3*c*d^2 + 2*a^4*d^3 -
3*a^2*b^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a
*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^
3 - b^5)*sqrt(-a^2 + b^2)) - 2*(b^3*c^3*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^
2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*d
^3*tan(1/2*f*x + 1/2*e)^3 + a^2*b*d^3*tan(1/2*f*x + 1/2*e)^3 + a*b^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 - b^3*d^3*tan(1/2*f*x + 1/2*e)^3 - b^3*c^3*tan(1/2*f*x
 + 1/2*e) + 3*a*b^2*c^2*d*tan(1/2*f*x + 1/2*e) - 3*a^2*b*c*d^2*tan(1/2*f*x
+ 1/2*e) + 2*a^3*d^3*tan(1/2*f*x + 1/2*e) + a^2*b*d^3*tan(1/2*f*x + 1/2*e)
- a*b^2*d^3*tan(1/2*f*x + 1/2*e) - b^3*d^3*tan(1/2*f*x + 1/2*e))/((a*tan(1/
2*f*x + 1/2*e)^4 - b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
a + b)*(a^2*b^2 - b^4)) - (3*b*c*d^2 - 2*a*d^3)*log(abs(tan(1/2*f*x + 1/2*e
) + 1))/b^3 + (3*b*c*d^2 - 2*a*d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/b^3)
/f
```

Mupad [B]

time = 11.30, size = 2500, normalized size = 10.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (d^2*atan(((d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*
c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a
^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^
3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3
*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^
3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2
 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c
*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) +
 (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b
^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^
4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2
*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 +
 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*d^2*tan(e/2 + (f*
x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^
```

5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c)*1i)/b^3 + (d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*d^2*tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c)*1i)/b^3)/((64*(8*a^8*d^9 - 4*a^7*b*d^9 + 12*a^4*b^4*d^9 + 6*a^5*b^3*d^9 - 20*a^6*b^2*d^9 + 27*b^8*c^4*d^5 - 27*b^8*c^5*d^4 - 90*a*b^7*c^3*d^6 + 99*a*b^7*c^4*d^5 - 9*a*b^7*c^5*d^4 + 18*a*b^7*c^6*d^3 - 60*a^3*b^5*c*d^8 - 39*a^4*b^4*c*d^8 + 96*a^5*b^3*c*d^8 + 24*a^6*b^2*c*d^8 + 111*a^2*b^6*c^2*d^7 - 144*a^2*b^6*c^3*d^6 - 15*a^2*b^6*c^4*d^5 - 39*a^2*b^6*c^5*d^4 - 3*a^2*b^6*c^7*d^2 + 105*a^3*b^5*c^2*d^7 + 113*a^3*b^5*c^3*d^6 + 3*a^3*b^5*c^4*d^5 + 9*a^3*b^5*c^5*d^4 + 2*a^3*b^5*c^6*d^3 - 165*a^4*b^4*c^2*d^7 + 55*a^4*b^4*c^3*d^6 - 12*a^4*b^4*c^4*d^5 + 9*a^4*b^4*c^5*d^4 - 57*a^5*b^3*c^2*d^7 - 23*a^5*b^3*c^3*d^6 - 12*a^5*b^3*c^4*d^5 + 54*a^6*b^2*c^2*d^7 + 4*a^6*b^2*c^3*d^6 - 36*a^7*b*c*d^8))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*d^2*tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c))/b^3 - (d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^

```
5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2
*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^
4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3
+ 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a
*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*
b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^
2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 +
3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9...
```

3.261 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=198

$$\frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}f}+\frac{2(b^2c^2-a^2d^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(}{\sqrt{a+b}}\right.}{a\sqrt{a-b}\,b^2\sqrt{a+b}\,f}$$

```
[Out] d^2*arctanh(sin(f*x+e))/b^2/f+2*(-a*d+b*c)^2*arctanh((a-b)^(1/2)*tan(1/2*f*
x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/(a+b)^(3/2)/f-(-a*d+b*c)^2*sin(f*x+e)/b
/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a^2*d^2+b^2*c^2)*arctanh((a-b)^(1/2)*tan(
1/2*f*x+1/2*e)/(a+b)^(1/2))/a/b^2/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4073, 3031, 2743, 12, 2738, 214, 3855}

$$\frac{2(b^2c^2-a^2d^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f\sqrt{a-b}\,\sqrt{a+b}}-\frac{(bc-ad)^2\sin(e+fx)}{bf\,(a^2-b^2)\,(a\cos(e+fx)+b)}+\frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{af(a-b)^{3/2}(a+b)^{3/2}}+\frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[(Sqrt[a - b]
*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*f) + (2*(b^
2*c^2 - a^2*d^2)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sq
rt[a - b]*b^2*Sqrt[a + b]*f) - ((b*c - a*d)^2*Sin[e + f*x])/(b*(a^2 - b^2)*
f*(b + a*Cos[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(d+c\cos(e+fx))^2\sec(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(-\frac{(-bc+ad)^2}{ab(b+a\cos(e+fx))^2} + \frac{b^2c^2-a^2d^2}{ab^2(b+a\cos(e+fx))} + \frac{d^2\sec(e}{b^2}\right.$$

$$= \frac{d^2\int\sec(e+fx)\,dx}{b^2} - \frac{(bc-ad)^2\int\frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab} + \frac{(b^2c^2-a^2d}{}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} - \frac{(bc-ad)^2\sin(e+fx)}{b\,(a^2-b^2)\,f(b+a\cos(e+fx))} + \frac{(bc-}{}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2(b^2c^2-a^2d^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(e+}{\sqrt{a+b}}\right.}{a\sqrt{a-b}\,b^2\sqrt{a+b}\,f}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2(b^2c^2-a^2d^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(e+}{\sqrt{a+b}}\right.}{a\sqrt{a-b}\,b^2\sqrt{a+b}\,f}$$

$$= \frac{d^2\tanh^{-1}(\sin(e+fx))}{b^2f} + \frac{2(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(e+fx))}{\sqrt{a+b}}\right.}{a(a-b)^{3/2}(a+b)^{3/2}f}$$

<u>Mathematica</u> [A]
time = 0.71, size = 180, normalized size = 0.91

$$\frac{\frac{2(2b^3cd+a^3d^2-ab^2(c^2+2d^2))\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - d^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right) + d^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right) + \frac{b(bc-ad)^2\sin(e+fx)}{(-a+b)(a+b)(b+a\cos(e+fx))}}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((2*(2*b^3*c*d + a^3*d^2 - a*b^2*(c^2 + 2*d^2))*ArcTanh[((-a + b)*Tan[(e + 
f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - d^2*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]] + d^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*(b*c -
 a*d)^2*Sin[e + f*x])/((-a + b)*(a + b)*(b + a*Cos[e + f*x])))/(b^2*f)
```

<u>Maple</u> [A]
time = 0.56, size = 215, normalized size = 1.09

method	result

derivativedivides	$\frac{\frac{d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}-\frac{d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^2}+\frac{\frac{2b\left(a^2d^2-2abdc+b^2c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a-b\right)}-\frac{2\left(a^3d^2-b^2c^2a-2b^2d^2a+\cdots\right.}{(a+b)(a\cdots}}{b^2}}{f}$
default	$\frac{\frac{d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}-\frac{d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^2}+\frac{\frac{2b\left(a^2d^2-2abdc+b^2c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a-b\right)}-\frac{2\left(a^3d^2-b^2c^2a-2b^2d^2a+\cdots\right.}{(a+b)(a\cdots}}{b^2}}{f}$
risch	$-\frac{2i(a^2d^2-2abdc+b^2c^2)\left(b\,e^{i(fx+e)}+a\right)}{(a^2-b^2)fba\left(a\,e^{2i(fx+e)}+2b\,e^{i(fx+e)}+a\right)}+\frac{\ln\left(e^{i(fx+e)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)a^3d^2}{\sqrt{a^2-b^2}\,(a+b)(a-b)f\,b^2}-\frac{\ln\left(e^{i(fx+e)}-\frac{ia^2-ib^2-\cdots}{a\sqrt{\cdots}}\right.}{\sqrt{a^2-b^2}\,(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(d^2/b^2*ln(tan(1/2*f*x+1/2*e)+1)-d^2/b^2*ln(tan(1/2*f*x+1/2*e)-1)+2/b^
2*(b*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(a*tan(1/2*f*
x+1/2*e)^2-b*tan(1/2*f*x+1/2*e)^2-a-b)-(a^3*d^2-a*b^2*c^2-2*a*b^2*d^2+2*b^3
*c*d)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+
b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(185) = 370.
time = 14.50, size = 820, normalized size = 4.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*((a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c^2 - 2*a*
b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*c
os(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x +
 e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x +
e) + b^2)) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b
^3 + b^5)*d^2)*log(sin(f*x + e) + 1) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f
*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) + 2*((a^2*b
^3 - b^5)*c^2 - 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x +
e))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5 +
b^7)*f), 1/2*(2*(a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c
^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) + (
(a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)
*log(sin(f*x + e) + 1) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4
*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) - 2*((a^2*b^3 - b^5)*c^2
- 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x + e))/((a^5*b^2
- 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5 + b^7)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d\sec(e + fx))^2 \sec(e + fx)}{(a + b\sec(e + fx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [A]
time = 0.51, size = 267, normalized size = 1.35

$$\frac{\frac{d^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{b^2} - \frac{d^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{b^2} - \frac{2\,(ab^2c^2-2\,b^3cd-a^3d^2+2\,ab^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^2-b^4)\sqrt{-a^2+b^2}} + \frac{2\,(b^2c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2\,abcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+a^2d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right))}{(a^2b-b^3)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a-b\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (d^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^2 - d^2*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/b^2 - 2*(a*b^2*c^2 - 2*b^3*c*d - a^3*d^2 + 2*a*b^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) -
b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2
```

```
)) + 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1/2*e) + a^2
*d^2*tan(1/2*f*x + 1/2*e))/((a^2*b - b^3)*(a*tan(1/2*f*x + 1/2*e)^2 - b*tan
(1/2*f*x + 1/2*e)^2 - a - b)))/f
```

Mupad [B]
time = 9.75, size = 2500, normalized size = 12.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] - (d^2*atan(((d^2*((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^
4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d
^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^
2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (d^
2*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4
*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*
c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (
32*d^2*tan(e/2 + (f*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*
a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2)*1i)/b^
2 + (d^2*((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5
*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^
6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b
^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (d^2*((32*(a
*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2
+ a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a
^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*d^2*ta
n(e/2 + (f*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 -
 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2)*1i)/b^2)/((64*(
a^5*d^6 + 2*a*b^4*d^6 - a^4*b*d^6 - 2*b^5*c*d^5 + 2*a^2*b^3*d^6 - 3*a^3*b^2
*d^6 + 4*b^5*c^2*d^4 + a*b^4*c^2*d^4 - 4*a*b^4*c^3*d^3 + 2*a^2*b^3*c*d^5 +
2*a^3*b^2*c*d^5 - a^4*b*c^2*d^4 + 3*a^2*b^3*c^2*d^4 + a^2*b^3*c^4*d^2 - a^3
*b^2*c^2*d^4 - 6*a*b^4*c*d^5))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (d^2*((3
2*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2
*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 +
4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4
*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (d^2*((32*(a*b^8*c^2 - b
^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^
2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d -
 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*d^2*tan(e/2 + (f*x
)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))
/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2))/b^2 + (d^2*((32*tan(e/2 +
(f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3
*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*
```

```
d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d
))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (d^2*((32*(a*b^8*c^2 - b^9*d^2 + 2*a
*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^
6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c
*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*d^2*tan(e/2 + (f*x)/2)*(2*a*b^
9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4
 + b^5 - a^2*b^3 - a^3*b^2))))/b^2))/b^2))*2i)/(b^2*f) - (2*tan(e/2 + (f*x)
/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(f*(a + b)*(a*b - b^2)*(a + b - tan(e/
2 + (f*x)/2)^2*(a - b))) - (atan(((((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6
*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*
d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 -
 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3
 - a^3*b^2) + ((a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*c^2 - b^
9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2
 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d -
2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*tan(e/2 + (f*x)/2)*
(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*(2*a*b^9
- 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5
 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2*d - 2*
b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a*d - b*c)*((a +
b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*1i)/(b^8 - 3*a^2*b^6 + 3*a^
4*b^4 - a^6*b^2) + (((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*
d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2
*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*
d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (
(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8
*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^
2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))
/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((a
 + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*(2*a*b^9 - 2*a^2*b^8 - 4
*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^
3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2*d - 2*b^2*d + a*b*c))
/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a*d - b*c)*((a + b)^3*(a - b)^3)
^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*1i)/(b^8 - 3*a...
```

3.262 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}f}-\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(a-b)^(3/2)
/(a+b)^(3/2)/f-(-a*d+b*c)*tan(f*x+e)/(a^2-b^2)/f/(a+b*sec(f*x+e))
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$\frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}}-\frac{(bc-ad)\tan(e+fx)}{f\,(a^2-b^2)\,(a+b\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*f) - ((b*c - a*d)*Tan[e + f*x])/((a^2 - b^2)*f*(a + b
*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx &= -\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}+\frac{\int \frac{(-ac+bd)\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{-a^2+b^2}\\ &= -\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}+\frac{(ac-bd)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{a^2-b^2}\\ &= -\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}+\frac{(ac-bd)\int \frac{1}{1+\frac{a\cos(e+fx)}{b}}\,dx}{b\,(a^2-b^2)}\\ &= -\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}+\frac{(2(ac-bd))\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x}\right.}{b\,(a^2-b^2}\\ &= \frac{2(ac-bd)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}f}-\frac{(bc-ad)\tan(e+}{(a^2-b^2)\,f(a+b\sec(e}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 97, normalized size = 0.97

$$\frac{-\frac{2(ac-bd)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{(-bc+ad)\sin(e+fx)}{(a-b)(a+b)(b+a\cos(e+fx))}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(a*c - b*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2) + ((-(b*c) + a*d)*Sin[e + f*x])/((a - b)*(a + b)*(b + a*Cos[e
 + f*x])))/f
```

Maple [A]
time = 0.24, size = 132, normalized size = 1.32

method	result
derivativedivides	$\frac{-\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a-b\right)}+\frac{2(ac-db)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{f}$
default	$\frac{-\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^{2}-b^{2}\right)\left(a\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a-b\right)}+\frac{2(ac-db)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{f}$
risch	$\frac{2i(ad-bc)\left(b\,\mathrm{e}^{i(fx+e)}+a\right)}{a(a^{2}-b^{2})f\left(a\,\mathrm{e}^{2i(fx+e)}+2b\,\mathrm{e}^{i(fx+e)}+a\right)}+\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ia^{2}-ib^{2}+b\sqrt{a^{2}-b^{2}}}{a\sqrt{a^{2}-b^{2}}}\right)ac}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)f}-\frac{\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ia^{2}-ib^{2}+b\sqrt{a}}{a\sqrt{a^{2}-}}\right.}{\sqrt{a^{2}-b^{2}}\,(a+b)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(a*tan(1/2*f*x+1/2*e)^2-b*ta
n(1/2*f*x+1/2*e)^2-a-b)+2*(a*c-b*d)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*f*x+1/2*e)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.77, size = 408, normalized size = 4.08

$$\left[\frac{(abc-b^{2}d+(a^{2}c-abd)\cos(fx+e))\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(fx+e)-(a^{2}-2\,b^{2})\cos(fx+e)^{2}+2\sqrt{a^{2}-b^{2}}(b\cos(fx+e)+a)\sin(fx+e)+2\,a^{2}-b^{2}}{a^{2}\cos(fx+e)^{2}+2\,ab\cos(fx+e)+b^{2}}\right)-2\,((a^{2}b-b^{3})c-(a^{3}-ab^{2})d)\sin(fx+e)}{2\,((a^{5}-2\,a^{3}b^{2}+ab^{4})f\cos(fx+e)+(a^{4}b-2\,a^{2}b^{3}+b^{5})f)},\ \frac{(abc-b^{2}d+(a^{2}c-abd)\cos(fx+e))\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}(b\cos(fx+e)+a)}{(a^{2}-b^{2})\sin(fx+e)}\right)-((a^{2}b-b^{3})c-(a^{3}-ab^{2})d)\sin(fx+e)}{(a^{5}-2\,a^{3}b^{2}+ab^{4})f\cos(fx+e)+(a^{4}b-2\,a^{2}b^{3}+b^{5})f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*b*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2
*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos
(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(
f*x + e) + b^2)) - 2*((a^2*b - b^3)*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^
5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f), ((a*b
*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) - ((a^2*b - b^3)
*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x +
e) + (a^4*b - 2*a^2*b^3 + b^5)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d\sec(e+fx))\sec(e+fx)}{(a+b\sec(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Giac [A]
time = 0.50, size = 173, normalized size = 1.73

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)(ac-bd)}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a-b\right)(a^2-b^2)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*
x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(a*c - b*d)/((a^2 -
 b^2)*sqrt(-a^2 + b^2)) - (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x + 1/2
*e))/((a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)*(a^2 -
b^2)))/f
```

Mupad [B]

time = 2.20, size = 106, normalized size = 1.06

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)\,\left(a\,c-b\,d\right)}{f\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,d-b\,c\right)}{f\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(b-a\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (2*atanh((tan(e/2 + (f*x)/2)*(a - b)^(1/2))/(a + b)^(1/2))*(a*c - b*d))/(f*
(a + b)^(3/2)*(a - b)^(3/2)) + (2*tan(e/2 + (f*x)/2)*(a*d - b*c))/(f*(a + b
)*(a - b)*(a + b - tan(e/2 + (f*x)/2)^2*(a - b)))
```

3.263 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=186

$$\frac{2b(abc-2a^2d+b^2d)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2f}+\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}\,(bc-ad)^2f}-\frac{b^2\,\mathrm{si}}{(a^2-b^2)\,(bc-a}$$

```
[Out] 2*b*(-2*a^2*d+a*b*c+b^2*d)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/
2))/(a-b)^(3/2)/(a+b)^(3/2)/(-a*d+b*c)^2/f-b^2*sin(f*x+e)/(a^2-b^2)/(-a*d+b
*c)/f/(b+a*cos(f*x+e))+2*d^2*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(
1/2))/(-a*d+b*c)^2/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4073, 3135, 3080, 2738, 214}

$$-\frac{b^2\sin(e+fx)}{f\,(a^2-b^2)\,(bc-ad)(a\cos(e+fx)+b)}+\frac{2b(-2a^2d+abc+b^2d)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2}+\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[
a + b]])/((a - b)^(3/2)*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTanh[(Sq
rt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a
*d)^2*f) - (b^2*Sin[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*(b + a*Cos[e + f*x
]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
```

```
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \int \frac{\cos^2(e+fx)}{(b+a\cos(e+fx))^2(d+c\cos(e+fx))}\,dx$$

$$= -\frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))} - \frac{\int \frac{-abd-(abc-a^2d+}{(b+a\cos(e+fx))(}}{(a^2-b^2)(}$$

$$= -\frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))} + \frac{d^2\int \frac{1}{d+c\cos(e+fx)}}{(bc-ad)^2}$$

$$= -\frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))} + \frac{(2d^2)\,\mathrm{Subst}\Big(\int \frac{}{c}}{}$$

$$= \frac{2b(abc-2a^2d+b^2d)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2f} + \frac{2d^2\,\mathrm{ta}}{\sqrt{}}$$

Mathematica [A]
time = 1.03, size = 176, normalized size = 0.95

$$\frac{\frac{2b(abc-2a^2d+b^2d)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{2(a^2-b^2)d^2\tanh^{-1}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+\frac{b^2(bc-ad)\sin(e+fx)}{b+a\cos(e+fx)}}{(-a+b)(a+b)(bc-ad)^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] ((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] + (2*(a^2 - b^2)*d^2*ArcTanh[((-c + d)*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + (b^2*(b*c - a*d)*Sin[e + f*x])
/(b + a*Cos[e + f*x]))/((-a + b)*(a + b)*(b*c - a*d)^2*f)
```

Maple [A]
time = 1.20, size = 210, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{2d^2\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(ad-bc)^2\sqrt{(c+d)(c-d)}}+\frac{2b\left(-\frac{b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a-b\right)}-\frac{(2a^2d-abc-db^2)\operatorname{arct}\cdots}{(a+b)(a-b)\sqrt{\cdots}}\right.}{(ad-bc)^2}\cdots}{f}$
default	$\frac{\frac{2d^2\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(ad-bc)^2\sqrt{(c+d)(c-d)}}+\frac{2b\left(-\frac{b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-b\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-a-b\right)}-\frac{(2a^2d-abc-db^2)\operatorname{arct}\cdots}{(a+b)(a-b)\sqrt{\cdots}}\right.}{(ad-bc)^2}\cdots}{f}$
risch	$\frac{2ib^2\left(b\,e^{i(fx+e)}+a\right)}{a(a^2-b^2)(ad-bc)f\left(a\,e^{2i(fx+e)}+2b\,e^{i(fx+e)}+a\right)}+\frac{2b\ln\left(e^{i(fx+e)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)a^2d}{\sqrt{a^2-b^2}\,(ad-bc)^2(a+b)(a-b)f}-\frac{b^2\ln\left(e^{i(fx+e)}\cdots\right.}{\sqrt{a^2-b^2}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*d^2/(a*d-b*c)^2/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)
/((c+d)*(c-d))^(1/2))+2*b/(a*d-b*c)^2*(-b*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1
/2*e)/(a*tan(1/2*f*x+1/2*e)^2-b*tan(1/2*f*x+1/2*e)^2-a-b)-(2*a^2*d-a*b*c-b^
2*d)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a+b
)*(a-b))^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(172) = 344$.
time = 166.85, size = 2900, normalized size = 15.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/2*((a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^
4)*d^3 + (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b
- a*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2
- 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x +
e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - ((a^5
- 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt
(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt
(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
 e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*
b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*sin(f*x + e))/(
((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d
+ (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^
2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f*cos(f*x + e) + ((a^4*b^3
- 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3
 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f), 1/2*(2*((a^5 - 2*a^3*b^2 + a*b^4)
*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(-c^2 + d^2)*arctan(
-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (a*b^3
*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3 + (a^2
*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3)
*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(
```

```
f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 -
b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - 2*((a^2*b^3 - b^5)*
c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d
^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^
3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6
*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f*cos(f*
x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)
*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3
*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f), 1/2*(2*(a*b^3*
c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3 + (a^2*
b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3)*
cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a
)/((a^2 - b^2)*sin(f*x + e))) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e)
 + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) -
 (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(
f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) -
2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 +
(a^3*b^2 - a*b^4)*d^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 -
2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3
*b^4)*d^4)*f*cos(f*x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 -
 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 +
 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)
*f), ((a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4
)*d^3 + (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b -
 a*b^3)*d^3)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2
*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(-c^2 + d^2)*arctan(-sqr
t(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - ((a^2*b^3
- b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a
*b^4)*d^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2
*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 +
 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f
*cos(f*x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 +
 a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2
- 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec{(e+fx)}}{(a+b\sec{(e+fx)})^2\,(c+d\sec{(e+fx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))**2*(c + d*sec(e + f*x))), x)
```

Giac [A]
time = 0.52, size = 330, normalized size = 1.77

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{-c^2+d^2}}+\frac{b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(a^2bc-b^3c-a^3d+ab^2d)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a-b\right)}-\frac{(ab^2c-2a^2bd+b^3d)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^2c^2-b^4c^2-2a^3bcd+2ab^3cd+a^4d^2-a^2b^2d^2)\sqrt{-a^2+b^2}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f
*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + b^2*tan(1/2*f*x + 1/2*e)/((a^2*b*c -
 b^3*c - a^3*d + a*b^2*d)*(a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e
)^2 - a - b)) - (a*b^2*c - 2*a^2*b*d + b^3*d)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*
e))/sqrt(-a^2 + b^2)))/((a^2*b^2*c^2 - b^4*c^2 - 2*a^3*b*c*d + 2*a*b^3*c*d
+ a^4*d^2 - a^2*b^2*d^2)*sqrt(-a^2 + b^2)))/f
```

Mupad [B]
time = 15.56, size = 2500, normalized size = 13.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^2*(c + d/cos(e + f*x))),x)
```

```
[Out] (d^2*atan(((d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*
d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5
*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2
- 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d -
8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 +
 a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2
*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/
(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^
2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3
*b^2*c*d) + (d^2*(c^2 - d^2)^(1/2)*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7
+ 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b
^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^
5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6
*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d
+ 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8
*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 -
 a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*
```

d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*
b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 -
21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c
^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^
7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a
^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3
*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d +
3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a
*b^5*c^2*d - 3*a^5*b*c*d^2) + (32*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*
(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 +
4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d
^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10
*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d
^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*
d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d
- 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d -
6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5
- 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c
^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*
a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*
d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b
^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 +
16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c
^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*
b^9*c^6*d + 2*a^9*b*c*d^6))/((a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2
- 2*a*b*c*d^3 + 2*a*b*c^3*d)*(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 +
a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4
*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d))))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2
+ b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c
^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d) + (d^2*(c^2 - d^2)^(1/2)*
((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 -
a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a
^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^
2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d
- 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*
d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5
*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a
^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*
c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) - (d^2*(c^2 - d^2)^(1/2)
((32(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^
7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d
^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*
c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c
^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6
*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 +

```
 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*
d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b
^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 -
21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c
^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5...
```

3.264 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=213

$$\frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx}{a-b}}}{df}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/d/f-2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/
f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4054, 3917, 4058}

$$\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\text{ArcSin}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{df(c+d)\sqrt{-\tan^2(e+fx)}\,\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4054

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d, Int[Csc[e + f*
```

```
x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Csc[e + f*x]/
(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d}}{d}$$

$$= \frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{df}$$

Mathematica [A]
time = 3.78, size = 183, normalized size = 0.86

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)(c+d)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)+2(bc-ad)\Pi\left(\frac{c-d}{c+d};\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sqrt{a+b\sec(e+fx)}}{(c-d)(c+d)f(b+a\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/
((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A]
time = 4.94, size = 355, normalized size = 1.67

method	result

default	$\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\,(\cos(fx+e)+1)^2\left(\mathrm{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)ac+\mathrm{Ellip}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(
(a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(EllipticF((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c+EllipticF((-1+cos(f*x+e))
/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d-EllipticF((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*b*c-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^
(1/2))*b*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b)
)^(1/2))*a*d+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+
b))^(1/2))*b*c)*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+e)^2/(c+d)/(c-d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec{(e+fx)}}\,\sec{(e+fx)}}{c+d\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

3.265 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=196

$$\frac{2\cot(e+fx)\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}}{d\sqrt{\frac{a+b}{c+d}}f}$$

```
[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec
(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*
x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)
*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/d/f/((a+b)/(c+d))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {4067}

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{df\sqrt{\frac{a+b}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(d*Sqrt[(a + b)/(c + d)]*f)
```

Rule 4067

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-}{(a-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.52, size = 44216, normalized size = 225.59

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]], x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 4.16, size = 351, normalized size = 1.79

method	result
default	$\frac{2\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)a-\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)b+2\,\mathrm{EllipticPi}\left(\right.}{f(-1+\cos(fx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/
(a-b)/(c+d))^(1/2))*a-EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x
+e),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b+2*EllipticPi((-1+cos(f*x+e))*((a-b)/
(a+b))^(1/2)/sin(f*x+e),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)
)*b)*cos(f*x+e)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*((a*cos(f*x+e
```

```
)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)^2*((a*cos(f*x+e)+b)/cos(f*x+e))
^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))
/(a*cos(f*x+e)+b)/((a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b \sec{\left(e + fx \right)}} \sec{\left(e + fx \right)}}{\sqrt{c + d \sec{\left(e + fx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos(e+f\,x)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)), x
)
```

3.266 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=192

$$\frac{2\sqrt{a+b}\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}}{\sqrt{c+d}(bc-ad)f}$$

```
[Out] 2*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(
1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*
(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {4069}

$$\frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{f\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e
+ f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b
)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x]
)))]*(c + d*Sec[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{c+d}\,\sqrt{a+b\sec(e+f}}{\sqrt{a+b}\,\sqrt{c+d\sec(e+f}}\right.\right.}{}$$

Mathematica [A]
time = 3.52, size = 233, normalized size = 1.21

$$\frac{4\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}\csc(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}}{\sqrt{2}}\right)|\frac{2(bc-ad)}{(a+b)(c-d)}\right)\sqrt{a+b\sec(e+fx)}\sin^2\left(\frac{1}{2}(e+fx)\right)}{(a+b)f\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqr
t[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]
], (2*(b*c - a*d))/((a + b)*(c - d))]*Sqrt[a + b*Sec[e + f*x]]*Sin[(e + f*x
)/2]^2)/((a + b)*f*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(
b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [A]
time = 3.93, size = 219, normalized size = 1.14

method	result
default	$\frac{2\,\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)\cos(fx+e)\left(\sin^2(fx+e)\right)\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{d+c\cos(fx+e)}{\cos(fx+e)}}\sqrt{\frac{d+c}{(\cos(f}}}{f(-1+\cos(fx+e))(d+c\cos(fx+e))(a\cos(fx+e)+b)\sqrt{\frac{a-b}{a+b}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/f*EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*cos(f*x+e)*sin(f*x+e)^2*((a*cos(f*x+e)+b)/cos(f*x+e))^(1
/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c
+d))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)/(-1+cos(f*x+e))/(d
+c*cos(f*x+e))/(a*cos(f*x+e)+b)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/(b*
d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
 x)
```

3.267 $\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx$

Optimal. Leaf size=110

$$\frac{2\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{2+3\sec(e+fx)}}{\sqrt{5}\sqrt{-4+5\sec(e+fx)}}\right)\middle|45\right)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{1+\ldots}{4-\ldots}}}{f}$$

```
[Out] 2*cot(f*x+e)*EllipticF(1/5*(2+3*sec(f*x+e))^(1/2)*5^(1/2)/(-4+5*sec(f*x+e))
^(1/2),3*5^(1/2))*(4-5*sec(f*x+e))*((1-sec(f*x+e))/(4-5*sec(f*x+e)))^(1/2)*
((1+sec(f*x+e))/(4-5*sec(f*x+e)))^(1/2)/f
```

Rubi [A]

time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {4069}

$$\frac{2\cot(e+fx)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{\sec(e+fx)+1}{4-5\sec(e+fx)}}F\left(\text{ArcSin}\left(\frac{\sqrt{3\sec(e+fx)+2}}{\sqrt{5}\sqrt{5\sec(e+fx)-4}}\right)\middle|45\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]]),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[2 + 3*Sec[e + f*x]]/(Sqrt[5]*Sqrt[-4
+ 5*Sec[e + f*x]])], 45]*(4 - 5*Sec[e + f*x])*Sqrt[(1 - Sec[e + f*x])/(4 -
5*Sec[e + f*x])]*Sqrt[(1 + Sec[e + f*x])/(4 - 5*Sec[e + f*x])])/f
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{2+3\sec(e+fx)}}{\sqrt{5}\sqrt{-4+5\sec(e+fx)}}\right)\middle|\ldots\right.}{\ldots}$$

Mathematica [A]
time = 1.80, size = 176, normalized size = 1.60

$$-\frac{4\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{-\left((3+2\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\sqrt{-\left((-5+4\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\csc(e+fx)F\left(\text{ArcSin}\left(\sqrt{\frac{5}{22}}\sqrt{\frac{-5+4\cos(e+fx)}{-1+\cos(e+fx)}}\right)|\frac{44}{45}\right)\sec(e+fx)\sin^4\left(\frac{1}{2}(e+fx)\right)}{3\sqrt{5}f\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]])
,x]
```

```
[Out] (-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44/45
]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[2 + 3*Sec[e + f*x]]*Sq
rt[-4 + 5*Sec[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 9.73, size = 177, normalized size = 1.61

method	result
default	$\frac{i\cos(fx+e)\,\text{EllipticF}\left(\frac{i(-1+\cos(fx+e))\sqrt{5}}{5\sin(fx+e)},3\sqrt{5}\right)\sqrt{\frac{2\cos(fx+e)+3}{\cos(fx+e)}}\sqrt{-\frac{4\cos(fx+e)-5}{\cos(fx+e)}}\left(\sin^2(fx+e)\right)\sqrt{10}\sqrt{\frac{2\cos(fx+}{\cos(fx+e}}}{5f(8(\cos^3(fx+e))-6(\cos^2(fx+e))-17\cos(fx+e)+15)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/5*I/f*cos(f*x+e)*EllipticF(1/5*I*(-1+cos(f*x+e))*5^(1/2)/sin(f*x+e),3*5^(
1/2))*((2*cos(f*x+e)+3)/cos(f*x+e))^(1/2)*(-(4*cos(f*x+e)-5)/cos(f*x+e))^(1
/2)*sin(f*x+e)^2*10^(1/2)*((2*cos(f*x+e)+3)/(cos(f*x+e)+1))^(1/2)*(-2*(4*co
s(f*x+e)-5)/(cos(f*x+e)+1))^(1/2)/(8*cos(f*x+e)^3-6*cos(f*x+e)^2-17*cos(f*x
+e)+15)*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)*sec(f*x + e)/(15
*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec{\left(e + fx \right)}}{\sqrt{3\sec{\left(e + fx \right)} + 2}\,\sqrt{5\sec{\left(e + fx \right)} - 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))**(1/2)/(-4+5*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(3*sec(e + f*x) + 2)*sqrt(5*sec(e + f*x) - 4)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{\frac{3}{\cos\left(e+f\,x\right)}+2}\,\sqrt{\frac{5}{\cos\left(e+f\,x\right)}-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(5/cos(e + f*x) - 4)^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(5/cos(e + f*x) - 4)^(1/2)),
 x)
```

3.268 $\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx$

Optimal. Leaf size=125

$$\frac{2i\cot(e+fx)F\left(i\sinh^{-1}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{2+3\sec(e+fx)}}\right)|\frac{1}{45}\right)\sqrt{\frac{1-\sec(e+fx)}{2+3\sec(e+fx)}}\sqrt{\frac{1+\sec(e+fx)}{2+3\sec(e+fx)}}(2+3}{3\sqrt{5}f}$$

```
[Out] 2/15*I*cot(f*x+e)*EllipticF(I*5^(1/2)*(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e
))^(1/2),1/15*5^(1/2))*(2+3*sec(f*x+e))*((1-sec(f*x+e))/(2+3*sec(f*x+e)))^(
1/2)*((1+sec(f*x+e))/(2+3*sec(f*x+e)))^(1/2)/f*5^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {4069}

$$\frac{2i\cot(e+fx)\sqrt{\frac{1-\sec(e+fx)}{3\sec(e+fx)+2}}\sqrt{\frac{\sec(e+fx)+1}{3\sec(e+fx)+2}}(3\sec(e+fx)+2)F\left(i\sinh^{-1}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{3\sec(e+fx)+2}}\right)|\frac{1}{45}\right)}{3\sqrt{5}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),x]
```

```
[Out] (((2*I)/3)*Cot[e + f*x]*EllipticF[I*ArcSinh[(Sqrt[5]*Sqrt[4 - 5*Sec[e + f*x
]])/Sqrt[2 + 3*Sec[e + f*x]]], 1/45]*Sqrt[(1 - Sec[e + f*x])/(2 + 3*Sec[e +
 f*x])]*Sqrt[(1 + Sec[e + f*x])/(2 + 3*Sec[e + f*x])]*(2 + 3*Sec[e + f*x]))
/(Sqrt[5]*f)
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\,\sqrt{2+3\sec(e+fx)}}\,dx = \frac{2i\cot(e+fx)F\left(i\sinh^{-1}\left(\frac{\sqrt{5}\,\sqrt{4-5\sec(e+fx)}}{\sqrt{2+3\sec(e+fx)}}\right)|\right.}{}$$

Mathematica [A]
time = 0.45, size = 176, normalized size = 1.41

$$-\frac{4\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\,\sqrt{-\left((3+2\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\,\sqrt{-\left((-5+4\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\csc(e+fx)F\left(\text{ArcSin}\left(\sqrt{\frac{5}{22}}\sqrt{\frac{-5+4\cos(e+fx)}{-1+\cos(e+fx)}}\right)|\frac{44}{45}\right)\sec(e+fx)\sin^4\left(\frac{1}{2}(e+fx)\right)}{3\sqrt{5}\,f\sqrt{4-5\sec(e+fx)}\,\sqrt{2+3\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),
x]
```

```
[Out] (-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44/45
]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[4 - 5*Sec[e + f*x]]*Sq
rt[2 + 3*Sec[e + f*x]])
```

Maple [A]
time = 11.07, size = 170, normalized size = 1.36

method	result
default	$-\frac{i\sqrt{-\frac{2(4\cos(fx+e)-5)}{\cos(fx+e)+1}}\,\sqrt{10}\,\sqrt{\frac{2\cos(fx+e)+3}{\cos(fx+e)+1}}\,(\sin^2(fx+e))\cos(fx+e)\sqrt{\frac{4\cos(fx+e)-5}{\cos(fx+e)}}\,\sqrt{\frac{2\cos(fx+e)+3}{\cos(fx+e)}}\,\text{Elliptic}}{15f(8(\cos^3(fx+e))-6(\cos^2(fx+e))-17\cos(fx+e)+15)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/15*I/f*(-2*(4*cos(f*x+e)-5)/(cos(f*x+e)+1))^(1/2)*10^(1/2)*((2*cos(f*x+e
)+3)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)^2*cos(f*x+e)*((4*cos(f*x+e)-5)/cos(f*
x+e))^(1/2)*((2*cos(f*x+e)+3)/cos(f*x+e))^(1/2)*EllipticF(3*I*(-1+cos(f*x+e
))/sin(f*x+e),1/15*5^(1/2))/(8*cos(f*x+e)^3-6*cos(f*x+e)^2-17*cos(f*x+e)+15
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4)*sec(f*x + e)/(
15*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\,\sqrt{3\sec(e+fx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))**(1/2)/(2+3*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(4 - 5*sec(e + f*x))*sqrt(3*sec(e + f*x) + 2)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4))
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\sqrt{\frac{3}{\cos(e+f\,x)}+2}\,\sqrt{4-\frac{5}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(4 - 5/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(4 - 5/cos(e + f*x))^(1/2)),
 x)
```

3.269 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=396

$$\frac{2\cot(e+fx)\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}}{bd\sqrt{\frac{a+b}{c+d}}\,f}$$

```
[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec
(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*
x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)
*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/b/d/f/((a+b)/(c+d))^(1/2)-2*a
*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*s
ec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(1
/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(
1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/b/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {4070, 4069, 4067}

$$\frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\Pi\left(\frac{b(c+d)}{(a+b)d};\text{ArcSin}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{bdf\sqrt{\frac{a+b}{c+d}}}-\frac{2a\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}F\left(\text{ArcSin}\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)}{bf\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(b*d*Sqrt[(a + b)/(c + d)]*f) - (2*a*S
qrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]
*(c + d*Sec[e + f*x]))/(b*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 4067

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
```

```
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x]))))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x]))))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4070

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-a/b, Int[Csc
[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dis
t[1/b, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{b} - \frac{a\int \frac{}{\sqrt{a+b\sec(e+}}}{}$$

$$= \frac{2\cot(e+fx)\Pi\left(\frac{b(c+d)}{(a+b)d};\sin^{-1}\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+}}{\sqrt{a+b\sec(e+fx)}}\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.33, size = 39039, normalized size = 98.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]
),x]
```

```
[Out] Result too large to show
```

Maple [A]
time = 4.19, size = 291, normalized size = 0.73

method	result
default	$-\frac{2\left(\mathrm{EllipticF}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)-2\,\mathrm{EllipticPi}\left(\frac{(-1+\cos(fx+e))\sqrt{\frac{a-b}{a+b}}}{\sin(fx+e)},\frac{a+b}{a-b},\frac{\sqrt{\frac{c-d}{c+d}}}{\sqrt{\frac{a-b}{a+b}}}\right)\right)\cos(fx+e)\ \sqrt{}}{f(-1+\cos(fx+e))(d+c\cos(fx+e))(a\cos(f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -2/f*(EllipticF((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f*x+e),((a+b)*(c-d)
/(a-b)/(c+d))^(1/2))-2*EllipticPi((-1+cos(f*x+e))*((a-b)/(a+b))^(1/2)/sin(f
*x+e),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)))*cos(f*x+e)*((a*
cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/
(c+d))^(1/2)*sin(f*x+e)^2*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*((d+c*cos(f*x
+e))/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/(d+c*cos(f*x+e))/(a*cos(f*x+e)+b)/((
a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
), x)
```

3.270 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{2dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{bf\sqrt{c+d\sec(e+fx)}}+\frac{2(bc-ad)g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e$$

```
[Out] 2*d*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2,2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f
*x+e))^(1/2)/b/f/(c+d*sec(f*x+e))^(1/2)+2*(-a*d+b*c)*g*(cos(1/2*f*x+1/2*e)^
2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*a/(a+b),2^(1/2)
*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x+e))^(1/2)/b/(a+
b)/f/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {4056, 3944, 2886, 2884, 4060}

$$\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{bf(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e + f*x]),
x]
```

```
[Out] (2*d*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/
(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x
)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*(a + b)*f*Sqrt[c + d*Sec[e + f
*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4056

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx}{b} - \frac{(-bc+ad)\int \frac{(g\sec(e-}{(a+b\sec(e+fx))\sqrt{c}}}{b}$$

$$= \frac{\left(dg\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+}}}{b\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{\left(dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+}{\sqrt{\frac{d}{c+d}+\frac{c\cos}{}}}}{b\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{bf\sqrt{c+d\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.60, size = 223, normalized size = 1.31

$$-\frac{2ig\sqrt{-\frac{c(-1+\cos(e+fx))}{c+d}}\sqrt{\frac{c(1+\cos(e+fx))}{c-d}}\cot(e+fx)\left(\Pi\left(1-\frac{c}{d};i\sinh^{-1}\left(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e+fx)}\right)|\frac{-c+d}{c+d}\right)-\Pi\left(\frac{a(-c+d)}{-bc+ad};i\sinh^{-1}\left(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e+fx)}\right)|\frac{-c+d}{c+d}\right)\right)\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{b\sqrt{\frac{1}{c-d}}f\sqrt{d+c\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x
]))/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]
*Sqrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(a*(-c + d))/(-(
b*c) + a*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c +
d)/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(b*Sqrt[(c - d)
^(-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 6.34, size = 481, normalized size = 2.83

method	result
default	$\frac{2i\sqrt{\frac{d+c\cos(fx+e)}{(\cos(fx+e)+1)(c+d)}}\left(\text{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{c-d}{c+d}}\right)abc-\text{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{c-d}{c+d}}\right)abd+\text{EllipticF}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] 2*I/f*((d+c*cos(f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*(EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*c-EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*d+EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,(-(c-d)/(c+d))^(1/2))*b^2*c-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)
/(c+d))^(1/2))*b^2*d-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/
(c+d))^(1/2))*a^2*d+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/(
c+d))^(1/2))*b^2*d+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(a-b)/(a+b),I
*((c-d)/(c+d))^(1/2))*a^2*d-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(a-b
)/(a+b),I*((c-d)/(c+d))^(1/2))*a*b*c)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(
-1+cos(f*x+e))*cos(f*x+e)^2*(g/cos(f*x+e))^(3/2)/(d+c*cos(f*x+e))/(1/(cos(f
*x+e)+1))^(3/2)/sin(f*x+e)^2/b/(a+b)/(a-b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e) +
 a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}\sqrt{c+d\sec{(e+fx)}}}{a+b\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(c + d*sec(e + f*x))/(a + b*sec(e + f*
x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e) +
 a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{a+\frac{b}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(a + b/cos(e + f*x)
),x)
```

```
[Out] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(a + b/cos(e + f*x)
), x)
```

3.271 $\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+
1/2*e),2*a/(a+b),2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g
*sec(f*x+e))^(1/2)/(a+b)/f/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4060, 2886, 2884}

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\left(g\sqrt{d+c\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{}{(b+a\cos(e+fx))\sqrt{c}}}{\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{\left(g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\sqrt{g\sec(e+fx)}\right)\int \frac{}{(b+a\cos(e+fx))\sqrt{}}}{\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+f}}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

Mathematica [A]
time = 0.25, size = 83, normalized size = 1.00

$$\frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f
*x]]),x]
```

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

Maple [C] Result contains complex when optimal does not.
time = 6.54, size = 254, normalized size = 3.06

method	result

default	$-\frac{2i\left(2a\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},-\frac{a-b}{a+b},i\sqrt{\frac{c-d}{c+d}}\right)-a\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{c-d}{c+d}}\right)-b\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)}\right.}{f(d+c\cos(fx+e))\left(\frac{1}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*I/f*(2*a*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(a-b)/(a+b),I*((c-d)/(
c+d))^(1/2))-a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))
-b*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2)))*((d+c*cos(
f*x+e))/(cos(f*x+e)+1)/(c+d))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*(-1
+cos(f*x+e))*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(d+c*cos(f*x+e))/(1/(cos(f*x
+e)+1))^(3/2)/sin(f*x+e)^2/(a-b)/(a+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
+ c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/((a + b*sec(e + f*x))*sqrt(c + d*sec(e + f
*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
+ c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{g}{\cos(e+f\,x)}\right)}^{3/2}}{\left(a+\frac{b}{\cos(e+f\,x)}\right)\,\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)
), x)
```

3.272 $\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx$

Optimal. Leaf size=168

$$\frac{2d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{af\sqrt{c+d\sec(e+fx)}}+\frac{2(ac-bd)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(e+\ldots\right.}{a(a+b)f\sqrt{c+d\sec(e\ldots}}$$

```
[Out] 2*d*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+
1/2*e),2,2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x
+e))^(1/2)/a/f/(c+d*sec(f*x+e))^(1/2)+2*(a*c-b*d)*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(a+b),2^(1/2)*(c/(
c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x+e))^(1/2)/a/(a+b)/f/
(c+d*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3041, 4056, 3944, 2886, 2884, 4060}

$$\frac{2(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{af(a+b)\sqrt{c+d\sec(e+fx)}}+\frac{2d\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)}{af\sqrt{c+d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*x]),x]
```

```
[Out] (2*d*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/(c
 + d)]*Sqrt[g*Sec[e + f*x]])/(a*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(a*c - b*d
)*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*b)/(a + b), (e + f*x)/2,
 (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(a*(a + b)*f*Sqrt[c + d*Sec[e + f*x]]
)
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
```

```
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
(c_))^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist
[g^m, Int[(g*Csc[e + f*x])^(p - m)*(b + a*Csc[e + f*x])^m*(c + d*Csc[e + f*
x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[m]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4056

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\sec(e+fx)}\,\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \frac{\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{b+a\sec(e+fx)}\,dx}{g}$$

$$= \frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx}{ag} + \frac{(ac-bd)\int \frac{(g\sec(e+f}{(b+a\sec(e+fx))\sqrt{c+}}}{ag}$$

$$= \frac{\left(d\sqrt{d+c\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+}}}{a\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{\left(d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+f}{\sqrt{\frac{d}{c+d}+\frac{c\cos}{}}}}{a\sqrt{c+d\sec(e+fx)}}$$

$$= \frac{2d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{af\sqrt{c+d\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.51, size = 222, normalized size = 1.32

$$-\frac{2i\sqrt{-\frac{c(-1+\cos(e+fx))}{c+d}}\sqrt{\frac{c(1+\cos(e+fx))}{c-d}}\cot(e+fx)\left(\Pi\left(1-\frac{c}{d};i\sinh^{-1}\left(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e+fx)}\right)|\frac{-c+d}{c+d}\right)-\Pi\left(\frac{b(-c+d)}{-ac+bd};i\sinh^{-1}\left(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e+fx)}\right)|\frac{-c+d}{c+d}\right)\right)\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a\sqrt{\frac{1}{c-d}}f\sqrt{d+c\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*
x]),x]
```

```
[Out] ((-2*I)*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x])
)/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]*S
qrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(b*(-c + d))/(-(a*
c) + b*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c + d)
/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a*Sqrt[(c - d)^(
-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 6.23, size = 461, normalized size = 2.74

method	result

default	$-\frac{2i\left(2\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\frac{a-b}{a+b},i\sqrt{\frac{c-d}{c+d}}\right)abc-2\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\frac{a-b}{a+b},i\sqrt{\frac{c-d}{c+d}}\right)b^2d-\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx}{\sin(fx+e}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+cos(f*x+e)*b),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*I/f*(2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),(a-b)/(a+b),I*((c-d)/(c+d
))^(1/2))*a*b*c-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),(a-b)/(a+b),I*((c
-d)/(c+d))^(1/2))*b^2*d-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d
))^(1/2))*a^2*c+EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2)
)*a^2*d-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*c+
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(c-d)/(c+d))^(1/2))*a*b*d-2*Ellipt
icPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/(c+d))^(1/2))*a^2*d+2*Ellipti
cPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((c-d)/(c+d))^(1/2))*b^2*d)*cos(f*x+e
)*(g/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e))/cos(f*x+e))^(1/2)*((d+c*cos(f*x+e)
)/(cos(f*x+e)+1)/(c+d))^(1/2)/(d+c*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)/a/(
a-b)/(a+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) + a
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g \sec{(e + fx)}}\,\sqrt{c + d \sec{(e + fx)}}}{a + b \cos{(e + fx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*cos(f*x+e)),x)
[Out] Integral(sqrt(g*sec(e + f*x))*sqrt(c + d*sec(e + f*x))/(a + b*cos(e + f*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) + a
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{\cos\left(e+f\,x\right)}}\,\sqrt{\frac{g}{\cos\left(e+f\,x\right)}}}{a+b\,\cos\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(1/2))/(a + b*cos(e + f*x)
),x)
[Out] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(1/2))/(a + b*cos(e + f*x)
), x)
```

3.273 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal. Leaf size=95

$$\frac{E\left(\text{ArcSin}\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

```
[Out] EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*(1/(1+sec(f*x+e)))
^(1/2)*(a+b*sec(f*x+e))^(1/2)/c/f/((a+b*sec(f*x+e))/(a+b)/(1+sec(f*x+e)))^(
1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {4053}

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}\,E\left(\text{ArcSin}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[e + f*x])^(-1)]*Sqrt[a + b*Sec[e + f*x]])/(c*f*Sqrt[(a + b*Sec[e +
f*x])/((a + b)*(1 + Sec[e + f*x]))])
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{E\left(\sin^{-1}\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\,\sqrt{a+b\sec(e+f}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 264 vs. 2(95) = 190.
time = 5.36, size = 264, normalized size = 2.78

$$\frac{\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\,\sqrt{a+b\sec(e+fx)}\left(\frac{2\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\,E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{1+\sec(e+fx)}}{\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}}+\frac{\sec^5\left(\frac{1}{2}(e+fx)\right)\sqrt{1+\sec(e+fx)}\left(-\sin\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{3}{2}(e+fx)\right)\right)}{\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}}-8\sqrt{\sec(e+fx)}\left(\sin(e+fx)-\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{4cf(1+\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

```
[Out] (Cos[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/
(a + b)]*Sec[(e + f*x)/2]^4*Sqrt[1 + Sec[e + f*x]])/(((1 + Cos[e + f*x])^(-
1))^(3/2)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]) + (Sec[(
e + f*x)/2]^5*Sqrt[1 + Sec[e + f*x]]*(-Sin[(e + f*x)/2] + Sin[(3*(e + f*x))
/2]))/((1 + Cos[e + f*x])^(-1))^(3/2) - 8*Sqrt[Sec[e + f*x]]*(Sin[e + f*x]
- Tan[(e + f*x)/2])))/(4*c*f*(1 + Sec[e + f*x]))
```

Maple [A]
time = 5.01, size = 153, normalized size = 1.61

method	result
default	$-\frac{\text{EllipticE}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(-1+\cos(fx+e))\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\,(\cos(fx-}{cf(a\cos(fx+e)+b)\sin(fx+e)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/c/f*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*((a*cos(f*
x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-1+c
os(f*x+e))*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2/(a*cos(f*x+
e)+b)/sin(f*x+e)^2*(-a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a+b\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + c/cos(e + f*x))),x)
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + c/cos(e + f*x))), x)
```

3.274 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal. Leaf size=295

$$\frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}+\frac{(a-b)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

```
[Out] -g*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/f/(c+c*cos(f*x+e))/(a+b
*sec(f*x+e))^(1/2)+g*(b+a*cos(f*x+e))*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*
f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x
+e))^(1/2)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e))^(1/2)+(a-b)*
g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2
*e),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^
(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)+2*b*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2
*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {4056, 3944, 2886, 2884, 4060, 2847, 2831, 2742, 2740, 2734, 2732}

$$-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\sqrt{a+b\sec(e+fx)}}+\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),
x]
```

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
 + f*x]])/(c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e + f*x]])
 + ((a - b)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2*b*g*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a + b)]*
Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a*Cos[e + f*
x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/(f*(c + c*Cos[e + f*x])*Sqrt[a + b*S
ec[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4056

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = -\left((-a+b)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx\right.$$

$$= -\frac{\left((-a+b)g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{}{\sqrt{b\cdots}}}{\sqrt{a+b\sec(e+fx)}}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(a(-\cdots\right.}{}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+f\cdots}}{cf\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+f\cdots}}{cf\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} + \cdots$$

Mathematica [F]
time = 19.73, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
f*x]),x]
```

```
[Out] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
f*x]), x]
```

Maple [C] Result contains complex when optimal does not.
time = 6.76, size = 308, normalized size = 1.04

method	result
default	$-\frac{i\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\left(2a\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)-2b\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)-a\,\mathrm{Elliptic}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] -I/c/f*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(2*a*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-2*b*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-a*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b
))^(1/2))+4*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2
))*b)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*cos(f*x+e)^2*(g/c
os(f*x+e))^(3/2)/(a*cos(f*x+e)+b)/(1/(cos(f*x+e)+1))^(3/2)/sin(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e) +
 c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}\sqrt{a+b\sec(e+fx)}}{\sec(e+fx)+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(a + b*sec(e + f*x))/(sec(e + f*x) + 1
), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e) +
 c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c+\frac{c}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + c/cos(e + f*x)
),x)
```

```
[Out] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + c/cos(e + f*x)
), x)
```

3.275 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=209

$$-\frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx}{a-b}}}{(a-b)cf}$$

```
[Out] -2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(
1/2)/(a-b)/c/f+EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*(1/
(1+sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sec(f*x+e))/(a
+b)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4057, 3917, 4053}

$$\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}\,E\left(\text{ArcSin}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}-\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{cf(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[e + f*x]))/(a - b))])/((a - b)*c*f) + (EllipticE[ArcSin[Tan[e + f*x
]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[
a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1 + S
ec[e + f*x]))])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))])]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 4057

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[
Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[Csc
[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^
2 - d^2, 0])
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = -\frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{(a-b)c} - \frac{c\int \frac{\sec(e+fx)\sqrt{a+b\sec(e-\ldots}}{c+c\sec(e+fx)}}{-ac+bc}$$

$$= -\frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\middle|\ldots\right.}{(a-b)\ldots}$$

Mathematica [A]
time = 13.98, size = 375, normalized size = 1.79

$$\frac{\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(b+a\cos(e+fx))\sec^2(e+fx)\left(\frac{2\sin(e+fx)}{-a+b}-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{-a+b}\right)}{f\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))} - \frac{2\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^{\frac{3}{2}}(e+fx)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}\left((a-b)E\left(\mathrm{ArcSin}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{(b+a\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)}{a+b}}+\sqrt{2}\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}(b+a\cos(e+fx))\tan\left(\frac{1}{2}(e+fx)\right)\right)\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{\left(\frac{a-b}{a+b}\right)^{3/2}(a+b)f\sqrt{\cos(e+fx)\sec^4\left(\frac{1}{2}(e+fx)\right)}\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*Sec[e + f*x]^2*((2*Sin[e + f*x])
/(-a + b) - (2*Tan[(e + f*x)/2])/(-a + b)))/(f*Sqrt[a + b*Sec[e + f*x]]*(c
+ c*Sec[e + f*x])) - (2*Cos[e/2 + (f*x)/2]^2*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e
 + f*x)/2]^2*Sec[e + f*x]]*((a - b)*EllipticE[ArcSin[Sqrt[(a - b)/(a + b)]*
Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[e + f*x])*Sec[(e + f*x
)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos
```

```
[e + f*x])]*(b + a*Cos[e + f*x])*Tan[(e + f*x)/2])*(-1 + Tan[(e + f*x)/2]^2
))/(((a - b)/(a + b))^(3/2)*(a + b)*f*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]
*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
```

Maple [A]
time = 4.81, size = 225, normalized size = 1.08

method	result
default	$-\frac{\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\,(\cos(fx+e)+1)^2(-1+\cos(fx+e))\left(2\,\mathrm{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\right.\right.}{cf(a\cos(fx+e)+b)\sin(fx+e)^2(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/c/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*
x+e))*(2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b-a*Elli
pticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-b*EllipticE((-1+cos(f
*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)))/(a*cos(f*x+e)+b)/sin(f*x+e)^2/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(b*c*sec(f*x + e)^2 + (a + b
)*c*sec(f*x + e) + a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec (e+fx)}{\sqrt{a+b\sec (e+fx)}\sec (e+fx)+\sqrt{a+b\sec (e+fx)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a + b*s
ec(e + f*x))), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos (e+f\,x)\,\sqrt{a+\frac{b}{\cos (e+f\,x)}}\,\left(c+\frac{c}{\cos (e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))), x)
```

3.276 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=214

$$\frac{2a\sqrt{a+b}\,\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx}{a-b}}}{(a-b)bcf}$$

[Out] 2*a*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/b/c/f-EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*(1/(1+sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sec(f*x+e))/(a+b)/(1+sec(f*x+e)))^(1/2)

Rubi [A]
time = 0.25, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4061, 3917, 4053}

$$\frac{2a\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bcf(a-b)}-\frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}\,E\left(\text{ArcSin}\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/((a - b)*b*c*f) - (EllipticE[ArcSin[Tan[e +
f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sq
rt[a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1
+ Sec[e + f*x]))])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 4061

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[-a/(b*c - a*d), I
nt[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[c/(b*c - a*d), Int[
Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ
[c^2 - d^2, 0])
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{(a-b)c} + \frac{c\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+}}{c+c\sec(e+fx)}}{-ac+bc}$$

$$= \frac{2a\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\right.}{(a-b)bc}$$

Mathematica [A]
time = 4.84, size = 156, normalized size = 0.73

$$\frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a+b)E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-2aF\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)}{(-a+b)cf\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\,(1+\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^4*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x])
)]*((a + b)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*a*Elli
pticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/((-a + b)*c*f*Sqrt[Cos[e
 + f*x]/(1 + Cos[e + f*x])]*(1 + Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A]
time = 5.12, size = 224, normalized size = 1.05

method	result
default	$\frac{\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\,(\cos(fx+e)+1)^2\left(2\,\mathrm{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)a-a\,\mathrm{Ellipt}\right.}{cf(a\cos(fx+e)+b)\sin(fx+e)^2(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/c/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(2*Elliptic
F((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a-a*EllipticE((-1+cos(f*x
+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-b*EllipticE((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+e)^2/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)^2/(b*c*sec(f*x + e)^2 + (a +
 b)*c*sec(f*x + e) + a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sec(e+fx)+\sqrt{a+b\sec(e+fx)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a +
b*sec(e + f*x))), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{c}{\cos\left(e+f\,x\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))), x)
```

3.277 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=229

$$\frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e}}{cf\sqrt{a+b\sec(e+fx)}}$$

```
[Out] -g*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/(a-b)/f/(c+c*cos(f*x+e)
)/(a+b*sec(f*x+e))^(1/2)+g*(b+a*cos(f*x+e))*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*(g*s
ec(f*x+e))^(1/2)/(a-b)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e))^
(1/2)+g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f
*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*
x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {4060, 2847, 2831, 2742, 2740, 2734, 2732}

$$-\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),
x]
```

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e +
 f*x]]) + (g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a
*Cos[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e +
 f*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = \frac{\left(g\sqrt{b+a\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{}{\sqrt{b+a\cos(e+}}}{\sqrt{a+b\sec(e+fx)}}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} - \frac{\left(ag\sqrt{\right.}{}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(g\sqrt{\right.}{}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(g(b\right.}{}$$

$$= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\sqrt{a+b\sec(e+fx)}} + \frac{g}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 8.31, size = 1019, normalized size = 4.45

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*(g*Sec[e + f*x])^(3/2)*((2*Csc[e
])/((-a + b)*f) + (2*Sec[e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/((-a + b)*f)
))/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])) + (AppellF1[1/2, 1/2, 1/
2, 3/2, (Csc[e]*(b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]])
)/(a*Sqrt[1 + Cot[e]^2]*(1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2]))), (Csc[e]*(
b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]))/(a*Sqrt[1 + Cot
[e]^2]*(-1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2])))]*Cos[e/2 + (f*x)/2]^2*Sqrt
[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(g*Sec[e + f*x])^(3/2)*Sec[f*x - Arc
Tan[Cot[e]]]*Sqrt[(a*Sqrt[1 + Cot[e]^2] - a*Sqrt[1 + Cot[e]^2]*Sin[f*x - Ar
cTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] - b*Csc[e])]*Sqrt[(a*Sqrt[1 + Cot[e]^2
```

```
] + a*Sqrt[1 + Cot[e]^2]*Sin[f*x - ArcTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] +
 b*Csc[e])]*Sqrt[b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]]
)/((-a + b)*f*Sqrt[1 + Cot[e]^2]*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*
x])) + (a*Cos[e/2 + (f*x)/2]^2*Sqrt[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(
g*Sec[e + f*x])^(3/2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[e]*(b + a*Co
s[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^2]*(
1 - (b*Sec[e])/(a*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(b + a*Cos[e]*Cos[f*x +
 ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^2]*(-1 - (b*Sec[e]
)/(a*Sqrt[1 + Tan[e]^2]))))]*Sin[f*x + ArcTan[Tan[e]]]*Tan[e])/(Sqrt[1 + Ta
n[e]^2]*Sqrt[(a*Sqrt[1 + Tan[e]^2] - a*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + T
an[e]^2])/(b*Sec[e] + a*Sqrt[1 + Tan[e]^2])]*Sqrt[(a*Sqrt[1 + Tan[e]^2] + a
*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(b*Sec[e]) + a*Sqrt[1 + Ta
n[e]^2])]*Sqrt[b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])
- ((Sin[f*x + ArcTan[Tan[e]]]*Tan[e])/Sqrt[1 + Tan[e]^2] + (2*a*Cos[e]*(b +
 a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a^2*Cos[e]^2 + a^
2*Sin[e]^2))/Sqrt[b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]
]))/(2*(-a + b)*f*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
```

Maple [C] Result contains complex when optimal does not.
time = 6.82, size = 238, normalized size = 1.04

method	result
default	$-\frac{i\left(2a\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)-a\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)-b\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\right.\right.}{cf(a\cos(fx+e)+b)\left(\frac{1}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -I/c/f*(2*a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-a*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2)))*((a*cos(f*x+e)+b)/(cos(f*
x+e)+1)/(a+b))^(1/2)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(g
/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(a*cos(f*x+e)+b)/(1/(cos(f*x+e)+1))^(3/2)/s
in(f*x+e)^2/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 535, normalized size = 2.34

$$-\frac{6ag\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+\sqrt{2}\left(i(3a-2b)g\cos(fx+e)+i(3a-2b)g\right)\sqrt{ag}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(fx+e)+3ia\sin(fx+e)+2b}{3a}\right)+\sqrt{2}\left(-i(3a-2b)g\cos(fx+e)-i(3a-2b)g\right)\sqrt{ag}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(fx+e)-3ia\sin(fx+e)+2b}{3a}\right)-3\sqrt{2}\left(iag\cos(fx+e)+iag\right)\sqrt{ag}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(fx+e)+3ia\sin(fx+e)+2b}{3a}\right)\right)-3\sqrt{2}\left(-iag\cos(fx+e)-iag\right)\sqrt{ag}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(fx+e)-3ia\sin(fx+e)+2b}{3a}\right)\right)}{6\left((a^2-ab)cf\cos(fx+e)+(a^2-ab)cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] -1/6*(6*a*g*sqrt((a*cos(f*x + e) + b)/cos(f*x + e))*sqrt(g/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + sqrt(2)*(I*(3*a - 2*b)*g*cos(f*x + e) + I*(3*a -
2*b)*g)*sqrt(a*g)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) + 3*I*a*sin(f*x + e) + 2*b)/a) + sqr
t(2)*(-I*(3*a - 2*b)*g*cos(f*x + e) - I*(3*a - 2*b)*g)*sqrt(a*g)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(f*x + e) - 3*I*a*sin(f*x + e) + 2*b)/a) - 3*sqrt(2)*(I*a*g*cos(f*x + e) +
 I*a*g)*sqrt(a*g)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) + 3*I*a*sin(f*x + e) + 2*b)/a)) - 3*sqrt(
2)*(-I*a*g*cos(f*x + e) - I*a*g)*sqrt(a*g)*weierstrassZeta(-4/3*(3*a^2 - 4*
b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) - 3*I*a*sin(f*x
+ e) + 2*b)/a)))/((a^2 - a*b)*c*f*cos(f*x + e) + (a^2 - a*b)*c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a+b\sec(e+fx)}\sec(e+fx)+\sqrt{a+b\sec(e+fx)}}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + s
qrt(a + b*sec(e + f*x))), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
), x)
```

3.278 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$

Optimal. Leaf size=312

$$-\frac{g^2(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\,\mathrm{se}}}{cf\sqrt{a+b\sec(e+fx)}}$$

```
[Out] g^2*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/(a-b)/f/(c+c*cos(f*x+e
))/(a+b*sec(f*x+e))^(1/2)-g^2*(b+a*cos(f*x+e))*(cos(1/2*f*x+1/2*e)^2)^(1/2)
/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*(
g*sec(f*x+e))^(1/2)/(a-b)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e
))^(1/2)-g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(
1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*s
ec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)+2*g^2*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/2)*(a/(a+b))^(1
/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e
))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {4064, 3944, 2886, 2884, 4060, 2847, 2831, 2742, 2740, 2734, 2732}

$$\frac{g^2\sin(e+fx)\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,F\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{g\sec(e+fx)}\,(a\cos(e+fx)+b)E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\sqrt{a+b\sec(e+fx)}}+\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),
x]
```

```
[Out] -((g^2*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Se
c[e + f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec
[e + f*x]])) - (g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/
2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2
*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a
 + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (g^2*(b + a*C
os[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e + f
*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4064

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c*(g/d),
 Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx = -\left(g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx\right) + \frac{g\int}{}$$

$$= -\frac{\left(g^2\sqrt{b+a\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{}{\sqrt{b+a\cos}}}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\,\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(ag^2\right.}{}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+f}}{cf\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+f}}{cf\sqrt{a+b\sec(e+fx)}}$$

$$= -\frac{g^2(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\sqrt{a+b\sec(e+fx)}}$$

Mathematica [F]
time = 14.73, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+c\sec(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])),x]
```

```
[Out] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])), x]
```

Maple [C] Result contains complex when optimal does not.
time = 6.55, size = 357, normalized size = 1.14

method	result
default	$-\frac{i\left(4a\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)-2b\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)-a\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -I/c/f*(4*a*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-2*
b*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-a*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))-4*EllipticPi(I*(-1+cos(f*x+e))/sin(f
*x+e),-1,I*((a-b)/(a+b))^(1/2))*a+4*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e)
,-1,I*((a-b)/(a+b))^(1/2))*b)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)
*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))^2*(g/cos(f*x+e))^(5/2)
*cos(f*x+e)^3/(a*cos(f*x+e)+b)/(1/(cos(f*x+e)+1))^(5/2)/sin(f*x+e)^4/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(\frac{g}{\cos(e+f\,x)}\right)}^{5/2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
), x)
```

3.279 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=213

$$\frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/d/f-2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/
f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4054, 3917, 4058}

$$\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{df}-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\text{ArcSin}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{df(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4054

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d, Int[Csc[e + f*
```

```
x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Csc[e + f*x]/
(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+\cdots}}{d}$$

$$= \frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b\cdots}{\cdots}}}{df}$$

Mathematica [A]
time = 0.41, size = 183, normalized size = 0.86

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)(c+d)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)+2(bc-ad)\Pi\left(\frac{c-d}{c+d};\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sqrt{a+b\sec(e+fx)}}{(c-d)(c+d)f(b+a\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/
((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A]
time = 5.24, size = 355, normalized size = 1.67

method	result

default	$\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\,(\cos(fx+e)+1)^2\left(\mathrm{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)ac+\mathrm{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(
(a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(EllipticF((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*c+EllipticF((-1+cos(f*x+e))
/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*d-EllipticF((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*b*c-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^
(1/2))*b*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+b)
)^(1/2))*a*d+2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c-d)/(c+d),((a-b)/(a+
b))^(1/2))*b*c)*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+e)^2/(c+d)/(c-d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sec(e+fx)}\,\sec(e+fx)}{c+d\sec(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

3.280 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal. Leaf size=170

$$\frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}}-\frac{2(bc-ad)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e \text{[cut off]}}{d(c+d)f\sqrt{a+b\sec(e \text{[cut off]}}}$$

```
[Out] 2*b*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f
*x+e))^(1/2)/d/f/(a+b*sec(f*x+e))^(1/2)-2*(-a*d+b*c)*g*(cos(1/2*f*x+1/2*e)^
2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*c/(c+d),2^(1/2)
*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/d/(c+
d)/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {4056, 3944, 2886, 2884, 4060}

$$\frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),
x]
```

```
[Out] (2*b*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*(b*c - a
*d)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x
)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f
*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4056

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{b\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{(g\sec(e+f}{\sqrt{a+b\sec(e+f}}}{d}$$

$$= \frac{\left(bg\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+}}}{d\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{\left(bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+}{\sqrt{\frac{b}{a+b}+\frac{a\,\mathrm{c}}{}}}}{d\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.48, size = 223, normalized size = 1.31

$$-\frac{2ig\sqrt{-\frac{a(-1+\cos(e+fx))}{a+b}}\sqrt{\frac{a(1+\cos(e+fx))}{a-b}}\cot(e+fx)\left(\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(e+fx)}\right)|\frac{-a+b}{a+b}\right)-\Pi\left(\frac{(a-b)c}{-bc+ad};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(e+fx)}\right)|\frac{-a+b}{a+b}\right)\right)\sqrt{g\sec(e+fx)}\sqrt{a+b\sec(e+fx)}}{\sqrt{\frac{1}{a-b}}\,df\sqrt{b+a\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*x
]))/(a - b)]*Cot[e + f*x]*(EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)] - EllipticPi[((a - b)*c)/(-(b
*c) + a*d), I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b
)/(a + b)])*Sqrt[g*Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[(a - b)^(-
1)]*d*f*Sqrt[b + a*Cos[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 6.86, size = 481, normalized size = 2.83

method	result
default	$\frac{2i\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\left(\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)acd+\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)a\,d^2-\mathrm{EllipticF}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] 2*I/f*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*a*c*d+EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),(-(a-b)/(a+b))^(1/2))*a*d^2-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,(-(a-b)/(a+b))^(1/2))*b*c*d-EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)
/(a+b))^(1/2))*b*d^2-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/
(a+b))^(1/2))*b*c^2+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(
a+b))^(1/2))*b*d^2-2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(c-d)/(c+d),I
*((a-b)/(a+b))^(1/2))*a*c*d+2*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(c-d
)/(c+d),I*((a-b)/(a+b))^(1/2))*b*c^2)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(
-1+cos(f*x+e))*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(a*cos(f*x+e)+b)/(1/(cos(f
*x+e)+1))^(3/2)/sin(f*x+e)^2/d/(c+d)/(c-d)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) +
 c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}\sqrt{a+b\sec{(e+fx)}}}{c+d\sec{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*
x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) +
 c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,{\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{c+\frac{d}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
),x)
```

```
[Out] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
), x)
```

3.281 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=102

$$\frac{2\Pi\left(\frac{2d}{c+d};\text{ArcSin}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[Out] 2*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)*(b/(a+b))^(
1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/(c+d)/f/(a+b*sec(f*x+e))^(1
/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {4058}

$$\frac{2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\text{ArcSin}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{f(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/
(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/((c + d)*f*Sqrt[a
 + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{2\Pi\left(\frac{2d}{c+d};\sin^{-1}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e}{a+b}}}{(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+}}$$

Mathematica [A]
time = 6.74, size = 187, normalized size = 1.83

$$\frac{2\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((c+d)F\left(\operatorname{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|\frac{a-b}{a+b}\right)-2d\Pi\left(\frac{c-d}{c+d};\operatorname{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|\frac{a-b}{a+b}\right)\right)\sqrt{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}}{(c-d)(c+d)f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*Ellipti
cF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*d*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[Cos[e + f*x]*Sec[(e +
 f*x)/2]^2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/((c - d)*(c + d)*f*S
qrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(97) = 194$.
time = 5.59, size = 236, normalized size = 2.31

method	result
default	$\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}(\cos(fx+e)+1)^2\left(\operatorname{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)c+\operatorname{Elliptic}\right.}{f(a\cos(fx+e)+b)\sin(fx+e)^2(c-d)(c+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(
(a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(EllipticF((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c+EllipticF((-1+cos(f*x+e))/s
in(f*x+e),((a-b)/(a+b))^(1/2))*d-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),(c
-d)/(c+d),((a-b)/(a+b))^(1/2))*d)*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+
e)^2/(c-d)/(c+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.282 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=209

$$\frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bdf}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/b/d/f-2*c*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)
*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/f/(a+b*
sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4062, 3917, 4058}

$$\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bdf}-\frac{2c\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\Pi\left(\frac{2d}{c+d};\text{ArcSin}\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right)|\frac{2b}{a+b}\right)}{df(c+d)\sqrt{-\tan^2(e+fx)}\,\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(b*d*f) - (2*c*EllipticPi[(2*d)/(c + d), ArcSi
n[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e +
 f*x]^2])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
```

```
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4062

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[1/d, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c/d, Int[Csc[e + f*x]/(Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{c\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec}}{d}$$

$$= \frac{2\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+}{a-}}{bdf}$$

Mathematica [A]
time = 3.08, size = 165, normalized size = 0.79

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((c+d)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-2c\Pi\left(\frac{c-d}{c+d};\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sec(e+fx)}{(c-d)(c+d)f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] - 2*c*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(
e + f*x)/2]], (a - b)/(a + b)])*Sec[e + f*x])/((c - d)*(c + d)*f*Sqrt[a + b
*Sec[e + f*x]])
```

Maple [A]
time = 5.82, size = 236, normalized size = 1.13

method	result

default	$-\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\,(\cos(fx+e)+1)^2\left(\operatorname{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)c+\operatorname{Ellipt}\right.}{f(a\cos(fx+e)+b)\sin(fx+e)^2(c-d)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(EllipticF((
-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c+EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*d-2*c*EllipticPi((-1+cos(f*x+e))/sin(f*x+e)
,(c-d)/(c+d),((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x
+e)^2/(c-d)/(c+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\cos\left(e+f\,x\right)}^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos\left(e+f\,x\right)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.283 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=83

$$\frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+
1/2*e),2*c/(c+d),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g
*sec(f*x+e))^(1/2)/(c+d)/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4060, 2886, 2884}

$$\frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{\left(g\sqrt{b+a\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{}{\sqrt{b+a\cos(e}}}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{\left(g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\sqrt{g\sec(e+fx)}\right)\int \frac{}{\sqrt{\frac{b}{a+b}+\frac{a}{}}}}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)\middle|\frac{2a}{a+b}\right)\sqrt{g\sec(e+}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [A]
time = 0.24, size = 83, normalized size = 1.00

$$\frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)\middle|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Maple [C] Result contains complex when optimal does not.
time = 6.74, size = 254, normalized size = 3.06

method	result

default	$-\frac{2i\left(2c\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},-\frac{c-d}{c+d},i\sqrt{\frac{a-b}{a+b}}\right)-c\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)-d\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{\right.}\right.}{f(a\cos(fx+e)+b)\left(\frac{1}{\cos(fx+e)+1}\right)^{\frac{3}{2}}\sin(f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*I/f*(2*c*EllipticPi(I*(-1+cos(f*x+e))/sin(f*x+e),-(c-d)/(c+d),I*((a-b)/(
a+b))^(1/2))-c*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))
-d*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2)))*((a*cos(f*
x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(-1
+cos(f*x+e))*(g/cos(f*x+e))^(3/2)*cos(f*x+e)^2/(a*cos(f*x+e)+b)/(1/(cos(f*x
+e)+1))^(3/2)/sin(f*x+e)^2/(c-d)/(c+d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{a+b\sec{(e+fx)}}\,(c+d\sec{(e+fx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f
*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{g}{\cos(e+f\,x)}\right)}^{3/2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

3.284 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$

Optimal. Leaf size=166

$$\frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}}-\frac{2cg^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\right.}{d(c+d)f\sqrt{a+b\sec(e+f}}$$

```
[Out] 2*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f
*x+e))^(1/2)/d/f/(a+b*sec(f*x+e))^(1/2)-2*c*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*c/(c+d),2^(1/2)*(a/(a+
b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/d/(c+d)/f/(a
+b*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {4064, 3944, 2886, 2884, 4060}

$$\frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}}-\frac{2cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\Pi\left(\frac{2c}{c+d};\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*c*g^2*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)/2, (2*
a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4064

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c*(g/d),
 Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx = \frac{g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(cg)\int \frac{(g\sec(e+fx))^{3/}}{\sqrt{a+b\sec(e+fx)}\,(c}}{d}$$

$$= \frac{\left(g^2\sqrt{b+a\cos(e+fx)}\,\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e-}}}{d\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{\left(g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e-}{\sqrt{\frac{b}{a+b}+\frac{a}{}}}}{d\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx}}{df\sqrt{a+b\sec(e+fx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.68, size = 246, normalized size = 1.48

$$-\frac{2ig\sqrt{-\frac{a(-1+\cos(e+fx))}{a+b}}\sqrt{\frac{a(1+\cos(e+fx))}{a-b}}\sqrt{b+a\cos(e+fx)}\cot(e+fx)\left((-bc+ad)\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(e+fx)}\right)|\frac{-a+b}{a+b}\right)+bc\Pi\left(\frac{(a-b)c}{-bc+ad};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(e+fx)}\right)|\frac{-a+b}{a+b}\right)\right)(g\sec(e+fx))^{3/2}}{\sqrt{\frac{1}{a-b}}\,bd(-bc+ad)f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*x
]))/(a - b)]*Sqrt[b + a*Cos[e + f*x]]*Cot[e + f*x]*((-(b*c) + a*d)*Elliptic
Pi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b
)/(a + b)] + b*c*EllipticPi[((a - b)*c)/(-(b*c) + a*d), I*ArcSinh[Sqrt[(a -
 b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)])*(g*Sec[e + f*x])^(3
/2))/(Sqrt[(a - b)^(-1)]*b*d*(-(b*c) + a*d)*f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 6.28, size = 346, normalized size = 2.08

method	result
default	$\frac{2i\left(\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)dc+\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},\sqrt{-\frac{a-b}{a+b}}\right)d^2+2c^2\,\mathrm{EllipticPi}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},-1,i\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 2*I/f*(EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*d*c+Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),(-(a-b)/(a+b))^(1/2))*d^2+2*c^2*Ellipti
cPi(I*(-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2))-2*EllipticPi(I*(
-1+cos(f*x+e))/sin(f*x+e),-1,I*((a-b)/(a+b))^(1/2))*d^2-2*c^2*EllipticPi(I*
(-1+cos(f*x+e))/sin(f*x+e),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2)))*((a*cos(f*x
+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(-1+
cos(f*x+e))^2*(g/cos(f*x+e))^(5/2)*cos(f*x+e)^3/(a*cos(f*x+e)+b)/(1/(cos(f*
x+e)+1))^(5/2)/sin(f*x+e)^4/d/(c+d)/(c-d)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

3.285 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx$

Optimal. Leaf size=67

$$\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}$$

```
[Out] 1/20*cot(1/2*f*x+1/2*e)^5/c^7/f-1/14*cot(1/2*f*x+1/2*e)^7/c^7/f+1/36*cot(1/
2*f*x+1/2*e)^9/c^7/f
```

Rubi [A]
time = 0.22, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {12, 276}

$$\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] Cot[(e + f*x)/2]^5/(20*c^7*f) - Cot[(e + f*x)/2]^7/(14*c^7*f) + Cot[(e + f*
x)/2]^9/(36*c^7*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{2\text{Subst}\left(\int -\frac{(1-x^2)^2}{8c^7x^{10}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^{10}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{4c^7f}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{10}}-\frac{2}{x^8}+\frac{1}{x^6}\right)\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{4c^7f}$$
$$= \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f} - \frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f} + \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(67) = 134.
time = 0.97, size = 151, normalized size = 2.25

$$\frac{\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(-971082\sin\left(\frac{fx}{2}\right)-718830\sin\left(e+\frac{fx}{2}\right)+467208\sin\left(e+\frac{3fx}{2}\right)+659400\sin\left(2e+\frac{3fx}{2}\right)-303192\sin\left(2e+\frac{5fx}{2}\right)-179640\sin\left(3e+\frac{5fx}{2}\right)+30753\sin\left(3e+\frac{7fx}{2}\right)+89955\sin\left(4e+\frac{7fx}{2}\right)-13427\sin\left(4e+\frac{9fx}{2}\right)+15\sin\left(5e+\frac{9fx}{2}\right)\right)}{23063040c^7f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^9*(-971082*Sin[(f*x)/2] - 718830*Sin[e + (f*x)/2
] + 467208*Sin[e + (3*f*x)/2] + 659400*Sin[2*e + (3*f*x)/2] - 303192*Sin[2*
e + (5*f*x)/2] - 179640*Sin[3*e + (5*f*x)/2] + 30753*Sin[3*e + (7*f*x)/2] +
 89955*Sin[4*e + (7*f*x)/2] - 13427*Sin[4*e + (9*f*x)/2] + 15*Sin[5*e + (9*
f*x)/2]))/(23063040*c^7*f)
```

Maple [A]
time = 0.25, size = 49, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}}{4fc^7}$
default	$\frac{\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}}{4fc^7}$
risch	$\frac{2i\left(315\,e^{8i(fx+e)}-630\,e^{7i(fx+e)}+2310\,e^{6i(fx+e)}-2520\,e^{5i(fx+e)}+3402\,e^{4i(fx+e)}-1638\,e^{3i(fx+e)}+1062\,e^{2i(fx+e)}-108\,e^{i}\right.}{315fc^7\left(e^{i(fx+e)}-1\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/c^7*(1/9/tan(1/2*f*x+1/2*e)^9-2/7/tan(1/2*f*x+1/2*e)^7+1/5/tan(1/2*f*
x+1/2*e)^5)
```

Maxima [A]
time = 0.28, size = 74, normalized size = 1.10

$$-\frac{\left(\frac{90\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{63\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-35\right)\left(\cos\left(fx+e\right)+1\right)^9}{1260\,c^7f\sin\left(fx+e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="maxima")
```

```
[Out] -1/1260*(90*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 63*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 35)*(cos(f*x + e) + 1)^9/(c^7*f*sin(f*x + e)^9)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(61) = 122.
time = 2.45, size = 131, normalized size = 1.96

$$\frac{47\,\cos\left(fx+e\right)^5+127\,\cos\left(fx+e\right)^4+101\,\cos\left(fx+e\right)^3+11\,\cos\left(fx+e\right)^2-8\,\cos\left(fx+e\right)+2}{315\left(c^7f\cos\left(fx+e\right)^4-4\,c^7f\cos\left(fx+e\right)^3+6\,c^7f\cos\left(fx+e\right)^2-4\,c^7f\cos\left(fx+e\right)+c^7f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="fricas")
```

```
[Out] 1/315*(47*cos(f*x + e)^5 + 127*cos(f*x + e)^4 + 101*cos(f*x + e)^3 + 11*cos
(f*x + e)^2 - 8*cos(f*x + e) + 2)/((c^7*f*cos(f*x + e)^4 - 4*c^7*f*cos(f*x
+ e)^3 + 6*c^7*f*cos(f*x + e)^2 - 4*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\tan^4\left(e+fx\right)\sec\left(e+fx\right)}{\sec^7\left(e+fx\right)-7\sec^6\left(e+fx\right)+21\sec^5\left(e+fx\right)-35\sec^4\left(e+fx\right)+35\sec^3\left(e+fx\right)-21\sec^2\left(e+fx\right)+7\sec\left(e+fx\right)-1}\,dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**7,x)
```

```
[Out] -Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**6
 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e
+ f*x)**2 + 7*sec(e + f*x) - 1), x)/c**7
```

Giac [A]
time = 1.26, size = 47, normalized size = 0.70

$$\frac{63\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-90\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35}{1260\,c^7f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="giac")
```

```
[Out] 1/1260*(63*tan(1/2*f*x + 1/2*e)^4 - 90*tan(1/2*f*x + 1/2*e)^2 + 35)/(c^7*f*
tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B]
time = 1.93, size = 47, normalized size = 0.70

$$\frac{63\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-90\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+35}{1260\,c^7\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(cos(e + f*x)*(c - c/cos(e + f*x))^7),x)
```

```
[Out] (63*tan(e/2 + (f*x)/2)^4 - 90*tan(e/2 + (f*x)/2)^2 + 35)/(1260*c^7*f*tan(e/
2 + (f*x)/2)^9)
```

3.286 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx$

Optimal. Leaf size=89

$$\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}-\frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f}-\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}$$

```
[Out] 1/40*cot(1/2*f*x+1/2*e)^5/c^8/f-3/56*cot(1/2*f*x+1/2*e)^7/c^8/f+1/24*cot(1/
2*f*x+1/2*e)^9/c^8/f-1/88*cot(1/2*f*x+1/2*e)^11/c^8/f
```

Rubi [A]
time = 0.23, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {12, 276}

$$-\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f}-\frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f}+\frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

```
[Out] Cot[(e + f*x)/2]^5/(40*c^8*f) - (3*Cot[(e + f*x)/2]^7)/(56*c^8*f) + Cot[(e
+ f*x)/2]^9/(24*c^8*f) - Cot[(e + f*x)/2]^11/(88*c^8*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{2\text{Subst}\left(\int \frac{(1-x^2)^3}{16c^8x^{12}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^{12}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{8c^8f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^{12}}-\frac{3}{x^{10}}+\frac{3}{x^8}-\frac{1}{x^6}\right)\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{8c^8f}$$
$$= \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f} - \frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f} + \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f} - \frac{\cot^{11}\left(\frac{1}{2}(e+\right.}{88c^8f}$$

Mathematica [A]

time = 1.14, size = 175, normalized size = 1.97

$$-\frac{\csc\left(\frac{e}{2}\right)\csc^{11}\left(\frac{1}{2}(e+fx)\right)\left(425964\sin\left(\frac{fx}{2}\right)+486024\sin\left(e+\frac{fx}{2}\right)-351450\sin\left(e+\frac{3fx}{2}\right)-299970\sin\left(2e+\frac{3fx}{2}\right)+145695\sin\left(2e+\frac{5fx}{2}\right)+180015\sin\left(3e+\frac{5fx}{2}\right)-63580\sin\left(3e+\frac{7fx}{2}\right)-44990\sin\left(4e+\frac{7fx}{2}\right)+6710\sin\left(4e+\frac{9fx}{2}\right)+15004\sin\left(5e+\frac{9fx}{2}\right)-1975\sin\left(5e+\frac{11fx}{2}\right)+\sin\left(6e+\frac{11fx}{2}\right)\right)}{15375360c^8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

```
[Out] -1/15375360*(Csc[e/2]*Csc[(e + f*x)/2]^11*(425964*Sin[(f*x)/2] + 486024*Sin
[e + (f*x)/2] - 351450*Sin[e + (3*f*x)/2] - 299970*Sin[2*e + (3*f*x)/2] + 1
45695*Sin[2*e + (5*f*x)/2] + 180015*Sin[3*e + (5*f*x)/2] - 63580*Sin[3*e +
(7*f*x)/2] - 44990*Sin[4*e + (7*f*x)/2] + 6710*Sin[4*e + (9*f*x)/2] + 15004
*Sin[5*e + (9*f*x)/2] - 1975*Sin[5*e + (11*f*x)/2] + Sin[6*e + (11*f*x)/2])
)/(c^8*f)
```

Maple [A]

time = 0.27, size = 62, normalized size = 0.70

method	result
derivativedivides	$\frac{\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}}{8f\,c^8}$
default	$\frac{\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}}{8f\,c^8}$
risch	$\frac{2i\left(1155\,e^{10i(fx+e)}-3465\,e^{9i(fx+e)}+13860\,e^{8i(fx+e)}-23100\,e^{7i(fx+e)}+37422\,e^{6i(fx+e)}-32802\,e^{5i(fx+e)}+27060\,e^{4i(fx+e}\right.}{1155f\,c^8\left(e^{i(fx+e)}-1\right)^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/c^8*(1/5/tan(1/2*f*x+1/2*e)^5-3/7/tan(1/2*f*x+1/2*e)^7+1/3/tan(1/2*f*
x+1/2*e)^9-1/11/tan(1/2*f*x+1/2*e)^11)
```

Maxima [A]
time = 0.29, size = 96, normalized size = 1.08

$$\frac{\left(\frac{385 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{495 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{231 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 105\right)(\cos(fx+e)+1)^{11}}{9240\, c^8 f \sin(fx+e)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="maxima")
```

```
[Out] 1/9240*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 495*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 231*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 105)*(cos(f*x +
e) + 1)^11/(c^8*f*sin(f*x + e)^11)
```

Fricas [A]
time = 2.64, size = 158, normalized size = 1.78

$$\frac{152 \cos(fx+e)^6 + 395 \cos(fx+e)^5 + 289 \cos(fx+e)^4 + 15 \cos(fx+e)^3 - 19 \cos(fx+e)^2 + 10 \cos(fx+e) - 2}{1155 \left(c^8 f \cos(fx+e)^5 - 5 c^8 f \cos(fx+e)^4 + 10 c^8 f \cos(fx+e)^3 - 10 c^8 f \cos(fx+e)^2 + 5 c^8 f \cos(fx+e) - c^8 f\right) \sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="fricas")
```

```
[Out] 1/1155*(152*cos(f*x + e)^6 + 395*cos(f*x + e)^5 + 289*cos(f*x + e)^4 + 15*c
os(f*x + e)^3 - 19*cos(f*x + e)^2 + 10*cos(f*x + e) - 2)/((c^8*f*cos(f*x +
e)^5 - 5*c^8*f*cos(f*x + e)^4 + 10*c^8*f*cos(f*x + e)^3 - 10*c^8*f*cos(f*x
+ e)^2 + 5*c^8*f*cos(f*x + e) - c^8*f)*sin(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\tan^4(e+fx)\sec(e+fx)}{\sec^8(e+fx) - 8\sec^7(e+fx) + 28\sec^6(e+fx) - 56\sec^5(e+fx) + 70\sec^4(e+fx) - 56\sec^3(e+fx) + 28\sec^2(e+fx) - 8\sec(e+fx) + 1}\, dx}{c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**8,x)
```

```
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**8 - 8*sec(e + f*x)**7
+ 28*sec(e + f*x)**6 - 56*sec(e + f*x)**5 + 70*sec(e + f*x)**4 - 56*sec(e +
 f*x)**3 + 28*sec(e + f*x)**2 - 8*sec(e + f*x) + 1), x)/c**8
```

Giac [A]
time = 1.34, size = 60, normalized size = 0.67

$$\frac{231 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^6 - 495 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 385 \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 105}{9240\, c^8 f \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="giac")
```

```
[Out] 1/9240*(231*tan(1/2*f*x + 1/2*e)^6 - 495*tan(1/2*f*x + 1/2*e)^4 + 385*tan(1
/2*f*x + 1/2*e)^2 - 105)/(c^8*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B]
time = 2.43, size = 60, normalized size = 0.67

$$\frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6}{5}-\frac{3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{7}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}-\frac{1}{11}}{8\,c^8\,f\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(cos(e + f*x)*(c - c/cos(e + f*x))^8),x)
```

```
[Out] (tan(e/2 + (f*x)/2)^2/3 - (3*tan(e/2 + (f*x)/2)^4)/7 + tan(e/2 + (f*x)/2)^6
/5 - 1/11)/(8*c^8*f*tan(e/2 + (f*x)/2)^11)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```